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Climate Policy Design with Correlated Uncertainties in Offset Supply and Abatement Cost

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RESOURCES FOR THE FUTURE

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Abstract

Current and proposed greenhouse gas cap-and-trade systems allow regulated entities to offset abatement requirements by paying unregulated entities to abate. These offsets from unregulated entities are believed to contain system costs and stabilize allowance prices. However, the supply of offsets is highly uncertain. Furthermore, the offset supply uncertainty may be correlated with other sources of uncertainty in emissions trading systems. This paper presents a model that incorporates both uncertainties in the supply of offsets *and* in abatement costs. Using numerical methods we solve the model under a variety of parameter settings, including a system that includes allowance price controls. We find that as uncertainty in offsets and uncertainty in abatement costs become more negatively correlated, expected abatement plus offset purchase costs increase, as does the variability in allowance prices and emissions from the regulated sector. Imposing an allowance price collar that limits the upper and lower cost substantially mitigates cost increases as well as the variability in prices and emissions, while roughly maintaining expected environmental outcomes.

Key Words: climate change, offsets, cap-and-trade, price collars, stochastic dynamic programming

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Contents

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Introduction

Recently proposed U.S. cap and trade legislation aimed at reducing CO_2 emissions allows regulated entities to cover as much as one-third of their emissions with offsets from exempted sources, both domestic and international. Intended as a low-cost means of meeting domestic emissions goals, offsets for exempted sources represent a large area of uncertainty. While the legislation is clear in setting aggregate and, in some cases, individual *ceilings* for the offsets, their *actual* use remains an open issue. Arguably, actual use is related to the technical, economic and, particularly, the market supply of offsets. In fact, especially in a globally interrelated economy, it is likely that at a time of high U.S. GDP growth, exempted sources, both domestic and international, would be less interested in selling offsets because of the higher opportunity cost of the resources. That is, there could be a negative correlation between the demand for and supply of offsets. Clearly, a negative correlation of this type could add to other sources of abatement cost uncertainty and increase price volatility as well as total costs.

This paper examines the implications of heavy reliance on offsets as a policy tool in the context of a cap-and-trade system where abatement costs are inherently uncertain. Specifically, we build on a previously developed abatement cost uncertainty model by including uncertainty in offset supply. We parameterize the new model with values relevant to the pending climate legislation to explore the effect of this added offset uncertainty on abatement costs, cumulative emissions, offset purchases, and emission allowance price paths. Of particular interest is the possible role of a price collar – a mechanism that constrains both upside and downside allowance price swings – in limiting adverse consequences in the face of added uncertainty due to offsets. Overall, we find that such a price collar can play a quite constructive economic role while achieving approximately the same expected emissions outcome as a system without a collar. Interestingly, the case for a price collar is even greater with a heavy reliance on offsets than without it.

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The organization of this paper is as follows: In Section II we outline the basic model, including a discussion of uncertainty, offset purchasing and the price collar. Section III describes the numerical solution algorithm, including the model parameterization. Section IV presents the results for the net present value (NPV) of costs and offset purchases, cumulative emissions and offsets, and the emission price paths. The final section offers concluding observations.

Model Setup

This section presents the basic reduced form, dynamic-stochastic abatement cost model. Because of the model's simplicity, this exercise is not intended to provide comprehensive estimates of abatement costs and emissions outcomes. Rather, it offers a tractable way to compare outcomes of multiple policies and parameter settings with important parameters keyed toward values relevant to the U.S. climate policy debate. Additionally, the analysis does not quantify emission abatement benefits. Benefits are ignored here because, even though the variance of cumulative emission outcomes may vary across different policies and parameter settings reviewed, expected values of cumulative emissions are quite similar across model runs. The near-linear nature of typical abatement benefit functions used in climate change models (see Pizer 2002) and the slow atmospheric decay of CO_2 imply that different policies and parameter settings with similar expected cumulative emissions will have similar expected abatement benefits.

With respect to emission regulation policies, we focus our attention on quantity-based policies (i.e. cap-and-trade) and hybrid policies, which combine elements of quantity policies and price policies (i.e., emission taxes). Additionally, we employ a representative firm framework in modeling emissions decisions. Though in reality proposed quantity-based instruments are operationally carried out under cap-and-trade systems in a multifirm setting, we justify the use of the representative firm based on the result that the emission outcome of a competitive cap-and-trade market are equivalent to that of a centralized planner (i.e., a representative firm) under the goal of minimizing abatement costs (see Rubin 1996 and Cronshaw and Kruse 1996). From here forward, unless otherwise noted, we discuss the model in terms of a representative firm's behavior, but this reference is meant to be synonymous with a competitive market outcome.

As in Fell, et al. (2008) and Fell and Morgenstern (2009), we begin by assuming a simple convex abatement cost function at any given period t for the representative firm:

$$\frac{c_t}{2}\left(\overline{q}_t + \theta_t - q_t\right)^2 \tag{1}$$

where c_t represents the slope of the marginal abatement cost function, $\bar{q}_t$ is the expected profit-maximizing level of emissions in the absence of a regulation (i.e. expected baseline emissions), θ_t is a shock to the baseline emissions, and q_t is the chosen level of emissions. In a regulatory system with no permit banking and borrowing, the level of emissions in each period t, q_t, would be equal to the total number of emission allowances allocated, y_t, plus the number of offsets entering the system, z_t. In reality, however, proposed cap-and-trade legislation allow for permit banking and generally some form of limited borrowing. Such provisions transform the static cost minimization in to a dynamic problem.

The dynamic cost minimization problem can be written as:

$$\min_{q,z} \sum_{t=1}^{T} \beta^{t-1} \left[\frac{c_t}{2} \left(\bar{q}_t + \theta_t - q_t \right)^2 + P_t^z z_t \right] \tag{2}$$

subject to

$$\begin{aligned} B_{t+1} &= R_t B_t + y_t + z_t - q_t \\ c_{t+1} &= c_0 (1 + g_c)^{t-1} \end{aligned} \tag{3}$$

$$\begin{aligned} & B_t \geq B_{\min,t}, 0 \leq q_t \leq \bar{q}_t + \theta_t, 0 \leq z_t \leq z_{\max} \\ & R_t = \begin{cases} 1 + r_{bank} & \text{if } B_t \geq 0 \\ 1 + r_{borr.} & \text{otherwise} \end{cases} \end{aligned} \tag{4}$$

where P_t^z is the price paid for offsets, B_t is the bank level at t, and R_t is one plus the interest rate paid (charged) on banked (borrowed) allowances.[1] Equations in (3) gives the transition equations for the bank level and slope of the marginal abatement cost curve. The bank transition equation states that the bank next period will be equal to the current bank with interest applied plus the number of allowances and offsets purchased less the emission level chosen at t. The c_t transition equation states that the slope of the marginal abatement cost curve evolves at a constant rate over time.[2] Condition (4) sets boundary constraints on state variable B_t and the control variables q_t and z_t. It should also be noted that the formulation of (2) implies the representative firm cannot price discriminate in the purchase of offsets. Obviously, this assertion causes the results to differ from

[1] Most current policies do not allow for an interest rate to be paid on banked allowances, so below we assume that the interest rate for the allowance bank is zero.

[2] This specification implicitly assumes that innovation aimed at reducing abatement costs evolves exogenously overtime. That is, we do not consider endogenous technological change in this model.

those in which it is assumed there is a centralized buyer of offsets who *can* price discriminate among offset suppliers.

Uncertainty

Starting with Weitzman (1974), abatement cost uncertainty has been included in many emission regulation models (e.g. Stavins 1996; Schennach 2000; Hoel and Karp 2002; Newell and Pizer 2003; Newell, et al. 2005). In this model, we introduce abatement cost uncertainty through two sources of uncertainty: uncertainty in baseline emissions and uncertainty in offset supply. The uncertainty to baseline emissions, as noted above, is represented by θ_t. Since emission levels are positively correlated with economic activity and since output measures, such as GDP, have been found to be highly persistent in numerous empirical studies, we assume that the shocks to baseline emissions are temporally correlated. As in Newell and Pizer (2003), this correlation is invoked by modeling the evolution of θ_t as an AR(1) process:

$$\begin{aligned} &\theta_t = \phi_1\theta_{t-1} + \varepsilon_{1t} \\ &\varepsilon_{1t} \sim \text{iid}\, N(0,\sigma_1^2) \\ &0 < \phi_1 < 1 \end{aligned} \qquad (5).$$

With respect to uncertainty in offset supply, we begin by assuming that offsets are a linear function of the offset price:

$$z_t = \gamma_t P_t \qquad (6).$$

We impose uncertainty in offset supply through the supply slope parameter, γ_t, in the form:

$$\gamma_t = \overline{\gamma}_t + \mu_t \qquad (7)$$

where $\overline{\gamma}_t$ is the unconditional mean of γ_t and μ_t is a random variable.

There are many potential reasons why the offset supply curve may deviate from expectations and, thus, many sensible ways one could model μ_t. In our representation of this deviation, we model μ_t in such a way to account for two particularly salient features likely to be associated with offset markets. First, as with the shocks to baseline emissions, we allow μ_t to be positively correlated over time. This positive correlation over time implies a "stickiness" to the offset supply state. Such persistence in the offset supply states would seem likely if, for instance, there is a time lag between application and approval for new offset projects and/or if there are contractual obligations that inhibit offset suppliers' abilities to remove their projects from the offset system.

The second feature we account for in the uncertainty of offset supply is the potential correlation between the uncertainty in baseline emissions (θ_t) and the uncertainty in the slope of the offset supply curve (γ_t). Correlation between offset uncertainty and shocks to domestic economic activity, which are associated with θ_t realizations, seems particularly relevant in a global economy context, yet to our knowledge this correlation has not been considered in other models. For example, correlation between γ_t and θ_t could occur if an increase in economic activity in the United States increases the opportunity cost of providing offsets by increasing the demand for the offset projects' alternative uses. An increase in the demand for alternative uses of offset-providing infrastructure would in turn decrease the quantity of offsets available at each offset price. This implies a negative correlation between γ_t and θ_t. To capture these two offset market features, we use the following specification for μ_t:

$$\begin{aligned} &\mu_t = \phi_2 \mu_{t-1} + \phi_3 \theta_t + \varepsilon_{2t} \\ &\left(\varepsilon_{2t}\right) \sim \text{iid}\, N\left(0, \sigma_2^2\right) \end{aligned} \qquad (8).$$

Note that in specification (8), μ_t is modeled as a function θ_t. However, it may be the case that offset supplies are not able to adjust to contemporaneous economic activity shocks, but rather adjust to lagged θ_t values. This would seem particularly true for agriculture-based offsets given the short-term irreversibilities of agricultural land use. We did consider specifications of (8) where θ_t was replaced with θ_{t-1}, but the results were not materially different from those presented here, so they are omitted.

Offset Purchasing

Assuming the representative agent is not a price taking entity with respect to offsets, we can plug (6) into (2) to yield the following Bellman equation for this cost minimization

$$V_t\left(B_t, \theta_t, \gamma_t\right) = \max_{q,z} \left\{ -\frac{c_t}{2}\left(\bar{q}_t + \theta_t - q_t\right)^2 - \frac{z_t^2}{\gamma_t} + \beta \mathrm{E}\left[V_t\left(B_{t+1}, \theta_{t+1}, \gamma_{t+1}\right) \mid B_t, \theta_t, \gamma_t\right] \right\} \qquad (9)$$

subject to (3)-(8). The first order conditions for minimization, assuming non-binding state variables, are:

$$\frac{\partial V_t}{\partial q_t} = c_t\left(\bar{q}_t + \theta_t - q_t\right) - \beta \mathrm{E}\left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \mid B_t, \theta_t, \gamma_t\right] = 0 \qquad (10)$$

$$\frac{\partial V_t}{\partial z_t} = -\frac{2z_t}{\gamma} + \beta \mathrm{E}\left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \mid B_t, \theta_t, \gamma_t\right] = -2P_t^z + \beta \mathrm{E}\left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \mid B_t, \theta_t, \gamma_t\right] = 0 \qquad (11)$$

and the envelope condition is:

$$\frac{\partial V_t}{\partial B_t} = R_t \beta \mathrm{E}\left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \mid B_t, \theta_t, \gamma_t\right] - \eta_t \tag{12}$$

where η is the shadow value of the banking constraint such that $\eta_t = 0$ if $B_t > B_{min,t}$ and $\eta_t < 0$ otherwise.

From (10) and (11) we get the condition:

$$c_t\left(\overline{q}_t + \theta_t - q_t\right) = 2P_t^z \quad (13).$$

Equation (13) states that the representative agent should purchase offsets until the marginal cost of abatement is equal to twice the price of offsets. This result is a familiar prescription for monopsonistic behavior when the firm cannot price discriminate and supply curves are linear. Note that a multifirm competitive market model where individual agents are purchasing offsets at a single market clearing price would not obtain such a result because the agents would not account for the pecuniary externalities of offset purchases. Individual agents would recognize the offset price as their individual marginal cost. However, we can preserve the equivalence of our representative agent framework as written above and a multifirm competitive market outcome if we assume that for the multifirm model offsets enter the market via an intermediary monopsonistic offset purchaser, such as an independent government corporation, as suggested by Purvis, Kopp and Stevenson (2009), that is purchasing offsets to minimize system-wide costs (abatement costs plus offset purchase costs) and provides the offsets to the market at cost.[3] A single aggregating offset-purchasing agency may provide a variety of functions in addition to purchasing offsets for the covered sectors. It may serve as the evaluator of the credibility of emissions reductions, use its purchasing power to direct investments toward nations that are taking steps toward making and fulfilling commitments within an international climate policy regime, and, more generally, direct revenue in a way that complements international aid policy. Although considering these additional roles of an offset purchasing agency is important, their consideration is beyond the scope of this paper.[4]

From (12) and (11) we can get the following Euler equation:

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[3] An additional consideration with a single offset purchaser is how offsets are distributed to the covered firms. There are multiple ways in which this could be accomplished, including options from auctioning to free allocation. Given the representative firm framework here we do not consider the offset rationing problem in this paper.

[4] It is also possible that the aggregators could be private firms. Though we do not consider such a situation see Fulton and Vercammen (2009) for more on this form of offset provision.

$$P_t^z = \beta \mathrm{E}\left[P_{t+1} \mid B_t, \theta_t, \gamma_t\right] - \eta_t \quad (14)$$

Equation (14) states that expected offset prices should rise at the rate of interest except for cases where the bank constraint is binding. Similarly, the Euler equation can be solved in terms of marginal abatement costs, stating that the marginal cost of abatement should rise at the rate of interest in expectation, except for periods of binding bank constraints. Thus, in a competitive emissions trading market where marginal abatement cost equals the price of an allowance, allowance prices will follow a modified Hotelling path as in Rubin (1996).

Price Collars

The problem described above is for a standard quantity-based emissions regulation, such as a cap-and-trade program with banking and limited borrowing. Recently, hybrid mechanisms, which combine elements of a price policy and a quantity policy, have garnered considerable attention. In this analysis, we consider a particular hybrid policy commonly referred to as a price collar. Collar mechanisms work by imposing a price floor, P_t^f, and a price ceiling, P_t^c, on emission allowance prices (i.e. marginal abatement costs) in a cap-and-trade system. The price ceiling is enforced by selling an unlimited number of additional allowances into the market at P_t^c. The price floor could be implemented in one of two ways. First, if emission allowances are auctioned, the regulator could set a minimum price for the auction, often referred to as a reserve price, which would serve as the floor. Alternatively, the government could promise to always buy and retire allowances from covered entities at P_t^f. For modeling simplicity, we consider the latter price floor design.[5]

Given this price collar setup, the next step is to determine emissions and offset purchasing decisions when the price ceiling or price floor is binding. We begin with the price ceiling case. If the firm has marginal abatement cost such that the price ceiling is binding, the banking dynamics equation becomes:

$$B_{t+1} = R_t B_{t+1} + y_t + z_t + y_t^c - q_t \quad (15)$$

[5] The price floor implemented via an auction floor is more complicated to model because with an auction and banking orborrowing the representative firm would have to choose allowances purchased as well as emission levels. This adds an additional control variable to the optimal control problem and thus complicates matters. However, since we are considering only abatement costs without endogenous technological investment, results under the two possible price floor designs will be the same.

where y_t^c are the additional allowances the firm purchases at the ceiling price and $y_t^c \geq 0$. With this adjusted banking dynamics equation and assuming that the regulator has provided some level of offsets, z_t, at a price P_t^z, the Bellman equation with a binding price ceiling can be written as:

$$V_t(B_t, \theta_t, \mu_t) = \max_{q, y^c} \left\{ -\frac{c_t}{2}\left(\overline{q}_t + \theta_t - q_t\right) - P_t^c y_t^c - P_t^z z_t + \beta \mathrm{E}\left[V_{t+1}(B_{t+1}, \theta_{t+1}, \mu_{t+1})\right] \right\} \quad (16)$$

subject to (15) and (4). Differentiating (16) with respect to q_t and y_t^c yields the optimal emission decision and allowance purchasing rule under a price ceiling:

$$c_t\left(\overline{q}_t + \theta_t - q_t\right) = P_t^c \quad (17)$$

$$P_t^c = \beta \mathrm{E}\left[\partial V_{t+1} / \partial B_{t+1} \right] \quad (18).$$

Equation (17) states that with a price ceiling in effect, the representative firm will emit up to the point where the marginal cost of abatement equals the price ceiling. Equation (18) states that additional allowances will be purchased up to the point where the discounted expected marginal value of the bank in $t + 1$ is equal to the price ceiling.

As stated above, the model presented in this analysis is consistent with a situation in which a regulator with monopsonistic power in the offset market is supplying offsets to the market in a way that minimizes total system costs (abatement costs plus offset purchasing costs) for a given emissions trajectory and policy design. Therefore, from the regulator's perspective, the government's gain from sales of allowances at the price ceiling is cancelled out by the firm's losses from the purchase of these allowances. This implies that the optimal purchase decision of offsets given in (13) still holds under a price ceiling. With the optimal emissions decision from (17), it is straight forward to see that when the price ceiling is in effect, offsets will be supplied such that the price ceiling equals two times the price of offsets, $2P_t^z = P_t^c$, barring any offset supply constraint conditions.

With the price floor in effect, the banking dynamic equation becomes:

$$B_{t+1} = R_t B_t + y_t + z_t - y_t^f - q_t \quad (19)$$

where y_t^f are the allowances *sold* back to the government at the price floor and $y_t^f \geq 0$. Given some level of offset provision, the Bellman equation with a price floor in effect becomes:

$$V_t(B_t, \theta_t, \mu_t) = \max_{q, y^f} \left\{ -\frac{c_t}{2}\left(\overline{q}_t + \theta_t - q_t\right) + P_t^f y_t^f - P_t^z z_t + \beta \mathrm{E}\left[V_{t+1}(B_{t+1}, \theta_{t+1}, \mu_{t+1})\right] \right\} \quad (20)$$

subject to (19) and (4). Differentiating (20) with respect to q_t and y_t^f yields the following:

$$c_t\left(\overline{q}_t + \theta_t - q_t\right) = P_t^f \qquad (21)$$

$$P_t^f = \beta \mathrm{E}\left[\partial V_{t+1} \big/ \partial B_{t+1}\right] \qquad (22)$$

As with the price ceiling, (21) states that the representative firm will emit up to the point where the marginal abatement cost equals the price floor and will sell allowances up to the point where the discounted expected marginal value of the bank in $t + 1$ equals the price floor. The offset purchase decision at the floor, however, is not as simple.

Assume that for (22) to hold, $B_{t+1} = B^*$. By rearranging (19) under this condition we have:

$$y_t^f = y_t - \left(B^* - R_t B_t\right) - q_t + z_t$$

where q_t solves (21). If $y_t - (B^* - R_t B_t) - q_t > 0$, then any offsets purchased by the regulator and sold to the firm at price P^z will just be sold back to the government at P^f. The payment to the firms for the offsets purchased at the price floor does not affect total system costs since it simply represents a transfer of funds, but the purchase of the offsets themselves does increase the system-wide costs. We assume the government retires the offsets, as well as allowances, that it buys at the price floor. That is, we assume the government does not use the allowances or offsets bought at the price floor to reduce future abatement requirements.[6] Therefore, offsets purchased when the price floor is in effect *and* $y_t - (B^* - R_t B_t) - q_t > 0$ only increase system-wide costs. If the offset aggregator is purchasing offsets with the goal of minimizing system-wide costs, no offsets should be purchased if the price floor is binding and $y_t - (B^* - R_t B_t) - q_t > 0$. By similar reasoning, if $y_t - (B^* - R_t B_t) - q_t < 0$, it is clear that system-wide costs are minimized if offsets are purchased up to the point where no allowances are bought back by the government at the price floor ($y_t^f = 0$) while the price floor is in effect.

The solution algorithm and the results presented below proceed based on these price collar implementation rules, with accompanying offset provision rules.

[6] Similarly, we assume that additional allowances sold at the price ceiling will not result in a reduction in future allocation of allowances. For systems with this structure see Murray, et al. (2008).

Numerical Solution Algorithm

Ideally, one would like to solve emissions and offset provision rules as a function of the continuous state variables and other known model parameters. Given the correlated uncertainty of θ_t and μ_t and boundary constraints on bank levels and per period offset purchases, an analytic optimal control solution for the emissions and offset purchases is not forthcoming. We therefore solve the optimal emissions and offset provision rules numerically through a backward recursion process over a discretized state space. The steps to this numerical solution technique are given below.

First, we descritize the state space. It should be noted that the time paths of c_t, y_t, $\overline{q}_t$, and $\overline{\gamma}_t$ are assumed to be known; thus the discretization is needed only for the variables B_t, θ_t, and μ_t. For B_t, we use 201 possible discrete bank levels such that bank levels are evenly distributed between $B_{min,t}$ and zero and between zero and $B_{max,t}$. This guarantees the existence of a zero bank state, which we employ in the terminal condition.

To discretize θ_t and μ_t, we first calculate the long-run covariance matrix of (θ_t, μ_t) based on the specification of (5) and (8). Given these calculated long-run variances for θ_t and μ_t and that the unconditional mean of both variables is zero, we set the discretized space for both θ_t and μ_t as evenly spaced values between ± 2 standard deviations, based on their respective variances, away from zero. For θ_t, we use 101 discrete values and for μ_t we use 25 discrete values. [7]

Next, we solve for the probability transition matrix P. The probability transition matrix gives us the probabilities of moving from each possible state in period t to each possible state in t + 1, where a state is defined by the B_t, θ_t, and μ_t realization. This in turn allows us to form expectations about the future value function, V_{t+1}, at time period t. In forming P, note that the evolution of the bank state is deterministic given emissions and offset choice. Therefore, the probability matrix can be written in terms of moving from states (θ_t, μ_t) to states (θ_{t+1}, μ_{t+1}). The transition matrix will be determined by the specifications of (5) and (8).

After discretizing the state space and calculating P, we begin the backward recursion step. To start the backward recursion, we impose a terminal condition on the bank state that $B_{T+1} \geq 0$.[8] Cost minimization will occur when the terminal condition is binding for states where the

[7] Details of the discretization procedure are given in the appendix.

[8] The bank state B_t can be thought of as the bank level at the start of the period, before an emissions level and offset provision are chosen. Therefore, B_{T+1} is the bank level at the end of the final period of the regulation.

constraints on emissions are not binding. Therefore, the optimal emissions quantities in the last period for all bank and shock states are known. Additionally, since the offsets provision decision, z_t, is a function of optimal q_t, final period optimal z_T's are also known. Given, z_T and q_T for each bank-shock state combination, the values of $V_T\ (B_T, \theta_T, \mu_T)$ are known.[9] Knowing the values of the final period value function, the probability transition matrix, and the state dynamics, we can step back one period to T-1 and determine the optimal q_{T-1}, and thus z_{T-1}, for each $(B_{T-1}, \theta_{T-1}, \mu_{T-1})$ state. These optimal decisions, along with the discounted $V_T\ (B_T, \theta_T, \mu_T)$, again give us values for $V_{T-1}\ (B_{T-1}, \theta_{T-1}, \mu_{T-1})$. We continue this backward stepping procedure to period $t = 1$, where we get $V_1\ (B_1, \theta_1, \mu_1)$.

The matrix of values for $V_1\ (B_1, \theta_1, \mu_1)$ gives us the expected discounted value of the expected net present value of system-wide costs at time $t = 1$ for every given initial (B_1, θ_1, μ_1) combination. In addition to getting a matrix of values for the value function at each time step for each possible state, the backward recursion process also collects the optimal emissions and offset provision choices for each state in each time period. These optimal decision matrices can then be used in simulation analyses. The simulation analyses are conducted by first simulating multiple draws of the shock paths for θ_t and μ_t. Then, given an initial bank value $B_1 = 0$ and the optimal emissions decision matrices, emission and offset paths are determined, which form the basis for our expected cumulative emissions and cumulative offsets estimates. In addition, the emission paths coupled with the shock paths allow us to calculate multiple emission price paths (where emission price equals marginal abatement cost) and total abatement cost estimates. The results from these simulation exercises are presented below.

Model Parameterization

To conduct the backward recursion solution algorithm and the subsequent simulations, we must first parameterize the model. Many parameters used here are keyed directly to the recent climate change legislation introduced by Reps. Henry Waxman (D-CA) and Edward Markey (D-MA), H.R. 2454 (U.S. Congress 2009a) and to the analysis of H.R. 2454 by the Energy Information Administration (EIA 2009). However, as stated above, this research should be seen not as a substitute for the EIA analysis but rather as a tractable way to compare policies and

[9] The value function $V_t(B_t, \theta_t, \mu_t)$ is equivalent to the value function $V_t(B_t, \theta_t, \gamma_t)$ given that $\overline{\gamma}_t$ is deterministic. We therefore use the value functions interchangeably.

parameter settings not possible in EIA's more comprehensive analysis of the bill. The parameterization is explained below and summarized Table 1.

To begin, we set the terminal period of the model at $T = 39$ to simulate H.R. 2454's 2012 – 2050 timeframe. The emission allowances, y_t, are taken directly from H.R. 2454 as laid out in Section 702 of the bill. With respect to banking and borrowing restrictions, in accordance with H.R. 2454, we set the minimum bank level at $-0.15y_t$, the interest rate paid on banked permits at $r_{bank} = 0$, and the interest charged on borrowed permits at $r_{borr.} = 0.08$. No limit is given for the maximum bank level, but for discretization we set it at 45 giga-metric tons of CO_2 equivalent ($GmtCO_2e$), which was sufficiently high to not be binding for all simulations conducted. With respect to offsets, we use the 2 $GmtCO_2e$ annual maximum limit given in H.R. 2454.[10] For the expected baseline emissions path, $\overline{q}_t$, we use the baseline emissions path given in EIA (2009) for 2012 – 2030, EIA's analysis period, and extend the trend of EIA's baseline emissions from 2025 – 2030 for the remaining years of our analysis. As stated above, we assume an exogenous decline in the slope of the marginal abatement cost curve. Unfortunately, to our knowledge, there is no direct corollary to this decline rate that has been empirically estimated or estimated via simulation. We set the decline rate, g_c, at what we feel is a modest -1.25% annual rate. Given this value g_c, we set the initial value of c_0 = \$64/$mtCO_2e$ per $GmtCO_2e$ to approximate the price path in EIA (2009) for EIA's low discount rate case, which sets the discount rate to 0.05, with this model under no uncertainty ($\theta_t = \mu_t = 0, \ \forall \ t$). For the price collar case, we set the initial values of the price floor and price ceiling, P_1^f and P_1^c, at \$10/tCO_2 and \$28/tCO_2, respectively, and increased both at a rate of 5 percent annually. This setting is similar to that described in the Kerry-Boxer bill, S. 1733 (U.S. Congress 2009b).

For the unconditional expected offset supply curve slope, $\overline{\gamma}_t$, we use the offset supply curves provided in EIA (2009). EIA provides offset supply schedules for every fifth year from 2010 to 2050, broken down by sector and location (i.e. industry-based, domestic vs. international).[11] We aggregate the predictions that are categorized as "offset applicable"; these include the "Non-Covered US Sector," "US Sequestration," and "International" offsets. After making several unit conversions, we derive aggregate offset supply schedules for every fifth year through 2050 (i.e. for years 2010, 2015, 2020, ...). These offset supply schedules imply non-

[10] H.R. 2454 puts specific limits on the maximum number of international offsets and domestic offsets, but we make no such distinction here.

[11] The offset supply schedules provide a quantity of offsets at a given price for a range of prices. The schedules are quoted for every fifth year starting in 2010.

linear supply curves. To get an estimate of the slope for each of these years, we find the slope at the offset price equal to one-half of the EIA's expected emissions price for the corresponding years.[12] This gives us offset supply slopes for every fifth year starting at 2015. To get an expected offset supply slope for every year, we fit a smoothed curve to the slope estimates derived in the previous step. A plot of the initial offset supply slopes and the smoothed series is given in Figure 1. The smoothed estimates form our $\bar{\gamma}_t$ values.

To parameterize the evolution equation of θ_t, equation (5), we use historic emissions data to guide our estimates. The expected baseline emissions represent an approximately linear upward trend. Therefore, we fit a linear trend to U.S. CO_2 emissions for the period 1949 – 2007, and fit an AR(1) process to the resulting residuals to get estimates for ϕ_1 and σ_1^2.[13] The results of this AR(1) estimation imply $\phi_1 = 0.9$ and $\sigma_1^2 = 0.01$. For sensitivity analysis we also consider a higher variance case, $\sigma_1^2 = 0.10$.

For the parameters of the offset supply shock dynamics given in (8), we use a range of values to explore how various levels of shock persistence and levels of correlation with baseline emissions shocks affect model outcomes. For ϕ_2 we use values of 0.0, 0.4, and 0.8. To keep θ_t realizations from completely dominating the realization of μ_t the magnitude of the ϕ_3 values is a factor of 10 smaller than ϕ_2 values at ϕ_3 = 0.0, -0.04, and -0.08. Also, given the relatively low values of the expected slope of the offset supply curve, $\bar{\gamma}_t$, we set σ_2^2 to a small value, 1/3500, to reduce the spread of μ_t. Given this variance value, along with our other parameters, the percentage of the time that γ_t is negative in the first five periods, implying no offsets available, is between approximately 5 and 25 percent, depending on the combinations of ϕ_2 and ϕ_3. This means that our parameterization of offset uncertainty highlights a wide range of offset availability, particularly in the early years of the program.

Results

This section discusses the results for expected net present value of abatement costs plus offset purchase costs (NPV of costs), emission outcomes, offset quantities, and emission price paths under a variety of parameter settings and for emission regulation policies with and without

[12] In EIA (2009), emission allowance price forecasts are only given out to 2030. For years beyond 2030, we simply extend the five percent annual price growth trend of the EIA's price forecast.

[13] Historic CO_2 emissions for the U.S. used for this procedure are available through the EIA at http://www.eia.doe.gov/environment.html.

price collars. Results presented for expected NPV of costs are based directly on the backward recursion determination of V_1, with the initial bank equal to zero and $\theta_0 = \mu_0 = 0$, as explained above. It should be noted that for the collar mechanisms, these costs do not include costs of purchasing additional allowances at the price ceilings or reductions in costs due to allowance sales at the price floor.

Emission outcomes, offset quantities, price paths, and confidence intervals for the NPV of costs are derived from simulation analysis. For the simulation analysis, we generate 10,000 time paths for θ and μ based on the discretized values of these variables and the derived probability transition matrix. Given these shock draws and the optimal emission and offset decisions solved in the backward recursion algorithm, we generate 10,000 emission and offset paths. These emission paths form the basis of our emission price paths, since emission price is assumed to be equal to the marginal cost of abatement. We also get NPV of cost estimates from these simulated and derived paths that form the basis of the cost confidence intervals presented below.

NPV of Abatement Costs plus Offset Purchase Costs

At the top of Table 2, the NPV of abatement costs plus offset purchase costs for the case with no uncertainty in baseline emissions or offset supply (the No Uncertainty case). The top two panels of Table 2 present the NPV of abatement costs plus offset purchase costs, with 95 percent confidence intervals given in parentheses, over a range of ϕ_2 and ϕ_3 combinations for the cases where σ_1^2 = 0.01, 0.10 and no price collars are imposed. The inclusion of uncertainty, both in the form of offset supply uncertainty and baseline emissions uncertainty, increases expected costs compared to the situation with no uncertainty. The results of both panels show that as the persistence of μ_t increases (ϕ_2 increases) or the negative correlation between μ_t and θ_t increases in magnitude (ϕ_3 decreases), expected NPV of costs increase. The cost increases from the no offset uncertainty persistence and no uncertainty correlation setting ($\phi_2 = \phi_3 = 0$) to the high persistence and high correlation setting ($\phi_2 = 0.8$, $\phi_3 = -0.08$) are quite substantial in the σ_1^2 = 0.10 case, increasing expected NPV of costs by 35 percent.

There are primarily two reasons for this pattern. The first is essentially a Jensen's inequality application resulting from the convex cost function. Regardless of the values of ϕ_2 and ϕ_3, the unconditional mean of μ_t, as well as θ_t, is still zero. However, as ϕ_2 and ϕ_3 increase in magnitude the tails of the distribution of μ_t get more probability mass. To put it another way, with larger absolute values ϕ_2 and ϕ_3 both states where offsets are relatively cheap (states where γ_t is relatively large compared with $\bar{\gamma}_t$) and states where offsets are relatively expensive (states where γ_t is relatively small compared with $\bar{\gamma}_t$) are more likely. Because the availability of offsets affect abatement costs and we have convex abatement costs, increasing the probability of offset availability states further from its expected mean increases expected costs via a Jensen's inequality result.

The second reason is associated with the offset constraints. Note that in Table 3, discussed in more detail below, even with low persistence in offset supply shocks and low correlation between offset supply and baseline emissions shocks (low ϕ_2 and ϕ_3 combinations), cumulative offsets purchased are around 63GmtCO_2e, which is about 81 percent of the maximum allowable cumulative offsets of 78 GmtCO_2e (2 GmtCO_2e/year x 39 years). Since, in expectation, the cumulative offset limit is met even with low absolute values of ϕ_2 and ϕ_3, increasing the magnitude of these parameters, which increases the probability of being in low offset cost states *and* high offset cost states, effectively only lowers expected cumulative offset purchases through being in the high offset cost states more frequently. The reduction in offset purchases for cases without price collars results in increased abatement and consequently increases NPV of costs.[14] This effect can also be noticed in the increasing spread of the 95 percent confidence intervals. All parameter settings have similar lower bounds of cost, but the upper bound of costs increase dramatically as the absolute values ϕ_2 and ϕ_3 increases. The observed pattern is exacerbated for the case where σ_1^2 = 0.10 because the range of possible μ_t and θ_t values increases, due to the larger σ_1^2's effect on the long-run variances of μ_t and θ_t.

The bottom two panels of Table 2 show the NPV of costs results for σ_1^2 = 0.01 and σ_1^2 = 0.10 when price collars are implemented. For the cases with the price collar, increasing absolute values of ϕ_2 or ϕ_3 does not necessarily lead to increasing NPV of costs as observed for the cases without price collars. In fact, for σ_1^2 = 0.01 settings the NPV of costs is falling with increasing absolute values of ϕ_2 or ϕ_3, except for the case where ϕ_2 = 0.8 and ϕ_3 = -0.08. The NPV of costs across ϕ_2 and ϕ_3 settings when σ_1^2 = 0.10 and the price collar is applied appears more random. However, upon further inspection these results are quite intuitive.

Consider the cases for a given ϕ_3 where the NPV of costs falls as the ϕ_2 value increases. From Table 3**,** we see that for the corresponding ϕ_2 and ϕ_3 parameter settings, the price floor

[14] Being in a low offset cost state does lower total system costs (abatement cost plus offset purchase costs) even when at z_{max} because the offset limit is reached at a lower cost. However, these savings do not compensate for the additional costs incurred from being in the high offset cost states more frequently.

rarely binds and the percentage of time the price ceiling binds is similar across the three ϕ_2 and ϕ_3 combinations. In these instances, the upper bound of costs for each of the ϕ_2 and ϕ_3 combinations will be similar, as can be seen in Table 2. Since a larger ϕ_2 for a given ϕ_3 increases the probability of reaching low offset cost states and the floor is rarely binding, the larger ϕ_2 values for these cases decrease the expected NPV of costs. To put it more succinctly, for the ϕ_3 settings where we observe expected NPV of costs decline as ϕ_2 increases, the price ceiling effectively caps the upper bound of costs, but the floor does not effectively restrict lower bound costs.

For the last column of Table 2's lower left panel and the last two columns of Table 2's lower right panel, NPV of costs initially decreases as ϕ_2 increases (ϕ_2 = 0.0 to ϕ_2 = 0.4), but as ϕ_2 continues to increase (ϕ_2 = 0.4 to ϕ_2 = 0.8), NPV of costs increases. As before, note that for the initial increase in ϕ_2 for these columns Table 3 shows that the corresponding ϕ_2 and ϕ_3 values have rarely binding price floors and the price ceiling is binding only slightly more often for the larger ϕ_2 settings. As explained above, this will lead to a similar upper bound of costs for these ϕ_2 and ϕ_3 combinations, but the larger ϕ_2 values (ϕ_2 = 0.4) will have smaller lower bound values of costs and, consequently, smaller expected NPV of costs. For the increase from ϕ_2 = 0.4 to ϕ_2 = 0.8, Table 3 shows that the price ceiling is binding significantly more frequently for the settings where ϕ_2 = 0.8, leading to significantly higher upper bound of costs for these ϕ_2 = 0.8 cases. The higher upper-bound costs for these ϕ_2 = 0.8 cases more than compensate for the cost savings offered by being in low offset cost states more frequently with the larger ϕ_2 values, resulting in higher expected NPV of costs for the ϕ_2 = 0.8 cases compared with the ϕ_2 = 0.4 cases.

It should be noted that the price collar results described in this section cannot be generalized for all collar specifications. The collar used here, which is similar to that proposed in S. 1733 (U.S. Congress 2009b), does not start at values centered on the initial expected emissions price of approximately \$23/mtCO_2e. Given that the price ceiling is much closer to the expected emissions price path than the floor and the wide spread between the ceiling and floor, this particular collar acts more like a one-sided safety valve than a symmetric price collar (a price collar with expected price centered between the price floor and ceiling). Results, not shown here, from a tighter collar centered on the initial expected emissions price lead to a NPV of costs pattern over the ϕ_2 and ϕ_3 values similar to that observed for the no collar scenarios, but with tighter confidence interval bands about the expected NPVs of costs and less disparity between the NPV of costs for the $\phi_2 = \phi_3 = 0$ case and the $\phi_2 = 0.8, \phi_3 = -0.08$ case.

For comparison purposes, we also include the expected NPV of costs with no offset supply uncertainty ($\mu_t = 0, \forall\, t$) and with no offsets ($z_t = 0, \forall\, t$) below each panel in Table 2. With no offsets NPV of costs are significantly higher than when offsets are included, even if the supply is uncertain, highlighting the cost saving potential of offsets. However, with respect to the no offset supply uncertainty cases, note that the NPV of abatement costs are lower for some low persistence and low correlation cases than for the cases without offset supply uncertainty. This result may seem counterintuitive given the convex cost function. The result is in fact an artifact of the parameterization used here. Offset supplies, z_t, must satisfy $z_t \geq 0$. Given this condition and the low initial values of the unconditional expectations of the offset supply slopes, $\bar{\gamma}_t$, the majority of the negative effects of the offset uncertainty, μ_t, are truncated away in the early periods. This means μ_t provides a benefit in early periods of the program in terms of creating an opportunity for more low offset cost opportunities while doing little to increase the cost of offsets beyond the expected value. For low persistence and low correlation cases this benefit dominates the Jensen inequality effect that generally causes costs to increase with greater uncertainty and convex cost curves.

Cumulative Emissions and Offsets

Unlike a standard quantities based regulation, a quantities based regulation with offsets will not guarantee a specific quantity of emissions from the covered industries. Tables 4 and 5 report expected cumulative emissions (the expected sum of emissions over the 39 periods) and expected cumulative offsets, respectively, over a range of ϕ_2 and ϕ_3 combinations.[15] The 95 percent confidence interval for each combination is given in brackets below the expected value. The top two panels of each table give the results for runs with no price collars. As expected, the expected cumulative emission and cumulative offsets, along with their respective 95 percent confidence intervals, are quite similar across most ϕ_2 and ϕ_3 combinations, for a given σ_1^2 without price collars. The exceptions are the noticeably lower cumulative emissions and cumulative offset values that occur in cases where both ϕ_2 and ϕ_3 are relatively large in magnitude and $\sigma_1^2 = 0.10$. The reasoning for this pattern follows from the explanation given above of the NPV of costs pattern over the ϕ_2 and ϕ_3 values for the no collar cases. Again, since, in expectation, per period offsets limits are met in most periods even for low absolute values of ϕ_2 and ϕ_3, increasing the magnitude of these parameters, which increases the probability of being in both high and low offset cost states, effectively only lowers offset purchases through being in the high

[15] We report only cumulative emissions results because CO_2 is a global stock pollutant so the time path of emissions is relatively unimportant from an environmental perspective, assuming that any non-linearities in the emission damages function are sufficiently out of the range of possible emission outcomes over the time span analyzed.

offset cost states more frequently. If offset purchases are low, cumulative emissions must also decrease to meet emission goals outlined in the y_t schedule, which in turn lowers the expected value of cumulative emissions.[16] As with the NPV of costs, this pattern is magnified for the larger σ_1^2 case as it increases the range θ_t and μ_t values.

For the parameter settings with the price collar implemented, the expected cumulative emissions are the same as or slightly higher than the ϕ_2 and ϕ_3 counterparts without the price collar. This is as expected since the price ceiling is binding much more frequently than the price floor, and therefore, on net, more allowances are added to the system at the price ceiling than removed with a binding price floor. Comparing results across ϕ_2 and ϕ_3 values for the collar scenarios, we again see similar cumulative emission values for a given σ_1^2. In fact, for the case where σ_1^2 = 0.10, the range of expected cumulative emissions across ϕ_2 and ϕ_3 values is tighter with the price collar than without the collar. Additionally, for combinations of ϕ_2 and ϕ_3 where both values are relatively large in absolute terms, the confidence intervals for cumulative emissions are narrower with price collars than without price collars, albeit with significantly larger lower bounds. The finding that price collars can actually reduce the variance in emission outcomes compared with a case without collars is a direct contradiction of previous price collar analyses that have not considered offsets (e.g. Philibert 2008; Burtraw, et al. 2009; Fell and Morgenstern 2009). The explanation for the potential reduction in cumulative emissions volatility is quite straightforward. In instances where offset supply is low, emission targets must be met with additional abatement. With price collars, the reduction in emissions will trigger the price ceiling and add allowances in the system, thereby increasing the lower bound of emission outcomes and narrowing the spread of possible emission outcomes.

With respect to the comparison of offset purchases between the no-collar and collar scenarios, the expected cumulative purchases of offsets are either the same across the two scenarios or slightly larger for the no-collar case. That offset purchasing can be less with price collars is as expected, since, when binding, both the floor and the ceiling reduce the purchases of offsets relative to the case of no collar.

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[16] Without per period offset limits, expected cumulative emission values across all parameter settings for the case with no price collars will be essentially the same. This is because for large absolute values of ϕ_2 and ϕ_3 offsets above the per period cap will enter during the more likely low offset cost states, counter-acting the lower emissions required during the high offset cost states.

Emission Price Paths

Figures 2 and 3 plot emission price paths and the corresponding 95 percent confidence intervals over four separate ϕ_2 and ϕ_3 combinations, without price collars, for $\sigma_1^2 = 0.01$ and $\sigma_1^2 = 0.10$, respectively. For the cases where $\sigma_1^2 = 0.01$, relative to the $\phi_2 = \phi_3 = 0$ case, other ϕ_2 and ϕ_3 combinations have upper confidence interval bounds that are distinctly greater in the early periods. This is again because as the persistence and negative correlation increase, the probability of reaching high offset cost states, and remaining in these high cost states, increases. Being in high offset cost states, in turn, increases abatement and thus emission prices. In the beginning periods, the firm has not built up a bank of allowances and therefore cannot smooth out these high cost states, leading to the noticeable bump in early period upper bound prices. The upper bound confidence interval of the price recedes, rather quickly in some settings, around year 2022 because at that point, under the parameter settings $\sigma_1^2 = 0.01$, $\gamma_t > 0$ for all possible discretized values of μ_t. With $\gamma_t > 0$ at least some supply of offsets enters the market, which provides some immediate allowance price relief.

For the plots with $\sigma_1^2 = 0.10$, we see, as expected, much larger spreads in the confidence intervals, though roughly similar expected price paths compared with the $\sigma_1^2 = 0.01$ cases. The initial bump in the upper bound of the confidence interval is even more pronounced with the larger σ_1^2. The reasoning is the same as above, but the with the larger possible θ_t values possible with the larger σ_1^2, the $\phi_3\theta_t$ term of (8) can dominate the determination of μ_t, making a high offset cost state even more likely. Note also for the case where μ_t is both highly persistent and negatively correlated with θ_t, there is great uncertainty in the possible price. This figure again highlights a particularly relevant finding of this work: although offsets are largely considered a way to add certainty to the market and reduce costs, if the supply of offsets is negatively correlated with economic activity shocks and if shocks to the offset supply are persistent, uncertainty in prices can still exist and be quite large.

Figures 4 and 5 plot the price paths with 95 percent confidence intervals for ϕ_2 and ϕ_3 cases with the price collar implemented. The price collar is also given on these plots. For $\sigma_1^2 = 0.01$, the plots with the price collar are similar to those without the price collar, though with slightly less exaggerated bumps in early upper-bound price confidence intervals due to binding price constraints. The similar shapes in the price paths with and without the price for this σ_1^2 value is as expected given how infrequently the upper or lower price control is binding.

For the cases where $\sigma_1^2 = 0.10$, the upper bound of the 95 percent confidence interval is the same as the price ceiling in almost all periods for all ϕ_2 and ϕ_3 combinations. Conversely, the lower bound of the 95 percent confidence interval overlaps with the price floor in only the last few periods. This, again, is due to the particular setting of the collar used here, and the result cannot be generalized for all collars. Nevertheless, it is clear that any reasonably assigned price collar for the $\sigma_1^2 = 0.10$ cases has the potential to significantly reduce the possible range of price paths compared with the no-collar cases.

To compare price path variation across parameter settings more succinctly we also provide root mean square error (RMSE) calculations in Table 6. We calculate RMSE for each setting as:

$$RMSE = \frac{1}{sims}\sum_{i=1}^{sims}\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(P_{it}-\overline{P}_t\right)^2}$$

where *sims* is the number of simulations (10,000), P_{it} is the marginal abatement cost of simulation *i* in time period *t*, and $\overline{P}_t$ is the expected price path at time *t*.

Results in Table 6 show, as expected, that increasing persistence and correlation in the sources uncertainty increase variability in prices. Importantly, we find that it is possible that variability in prices can be greater with offsets than without offsets. This shows that while it is generally thought that offsets reduce volatility, if offset supply uncertainty is highly persistent and/or there is a high degree of negative correlation between offset supply uncertainty and general macroeconomic uncertainty the inclusion of offsets can actually *increase* price variability compared to a case without offsets.[17] The variability in price for the offset uncertainty cases is considerably mitigated with the inclusion of the price collar as seen in the bottom two panels of Table 6. The cases with price collars also remove nearly all of the price volatility in the cases without offset, but this is due to a near-always binding price ceiling (see Table 4).

Conclusion

Recently proposed U.S. federal legislation aimed at reducing greenhouse gas emissions (e.g., U.S. Congress 2009a, 2009b), allow regulated entities to cover a significant portion of their emissions through the use of carbon offsets. Although offsets ostensibly provide a low-cost means to meet emission reduction goals, uncertainty remains about the quantity of offsets that

[17] Though not shown, it should be noted that the expected price path of the “No Offsets” case is always greater than any of the uncertain offset supply cases examined.

will be available at any given time period. Furthermore, little is known about the relationship of this offset supply uncertainty to other sources of abatement cost uncertainty. In this paper, we present a model with uncertainty in abatement costs and uncertainty in offsets. We explore a range of possible parameters to determine how persistence in the uncertainties and correlation between the uncertainties affects total costs, emission outcomes, offset purchases, and emission price paths. This parameter exploration is conducted under two emission regulation policies: a quantity-based regulation with banking and limited borrowing and a quantity-based regulation with banking, limited borrowing, and an emissions' price floor and ceiling (i.e., a price collar).

Using a parameterization of the model keyed toward recently proposed federal cap and trade legislation H.R. 2454 (U.S. Congress 2009a), we find that for a quantity-based regulation without emission price limitations (i.e., no price collar), as the persistence to the shock to offset supply increases and as the negative correlation between offset uncertainty and baseline emissions uncertainty increases, the expected NPV of abatement costs plus offset purchase costs increases. This result is in part due to the specification of a convex cost curve and the limitations on offset supply. For the quantity-based regulation without a price collar, we find that the range of possible cost, cumulative emissions, and price paths can be quite large in situations where both the offset supply shock is highly persistent and the negative correlation between offset uncertainty and baseline emissions uncertainty is large in magnitude. In fact, we find that under certain persistence and uncertainty source correlation settings the variability in prices can actually be *greater* with offsets than without offsets. Additionally, for the cases without price limitations we find the potential for substantial price variability in the early periods because of the relatively high probability of limited offset supply availability. A wide range of possible prices in the early periods may be particularly worrisome for policy makers and market participants as great price variability will surely open the system to considerable criticism.

With the price collar, which is patterned after the pending Senate bill, S.1733 (U.S. Congress 2009b), we find that for the same persistence and correlation parameters the expected NPV of costs is lower and the range of cost outcomes is narrower than without price collars, while expected emissions increase only slightly with the collar compared with the no-collar cases. Importantly, we also find that the range of possible cumulative emission outcomes can actually be smaller with a price collar compared with a no-collar policy if both the offset supply shock is highly persistent and the negative correlation between offset uncertainty and baseline emissions uncertainty is large in magnitude. For cases with low offset uncertainty persistence or low magnitudes of negative correlation between the sources of uncertainty, we obtain the standard result that price collars increase the range of possible cumulative emission outcomes. In addition, our results show that cumulative offset purchases decrease slightly with the price collar implemented compared with the no-collar cases. As expected, the range of possible price paths is smaller than with no-collar policies.

This research also opens the door to several additional considerations. In particular, this model assumes that offsets are purchased by a regulator with monopsonistic offset purchasing power and with the objective of minimizing system costs. Alternatively, it is quite possible that the regulated entities, which presumably do not have monopsonistic offset purchasing power, will be allowed to purchase offsets directly from suppliers. These two offset purchasing regimes will obviously lead to different outcomes. The picture could be further complicated if we consider informational asymmetries regarding model uncertainties and international competition for offsets. Comparing the cost, price, and emission outcomes under these two scenarios, with different information assumptions and foreign competition assumptions, is an important policy relevant exercise.

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Tables

Table 1. Parameter Values

Parameter	Value	Definition	Justification
T	39	Terminal period	Consistent with current climate legislation for 2012 – 2050
β	0.952	Discount factor	Assumes commonly used 5% discount rate
r_{bank}	0.0	Interest on banked permits	From H.R. 2454
$r_{borr.}$	0.08	Interest charged on borrowed permits	From H.R. 2454
ϕ_1	0.9	AR(1) parameter for baseline emissions shock	Based on regression results of historic U.S. CO_2 emissions
σ_1^2	(0.01, 0.10)	Variance of error term in baseline emissions shock	0.01 value is based on regression results; 0.10 value is for sensitivity analysis
σ_2^2	1/3500	Variance of random error term in offset supply shock	Set at low value because of relatively low values of $\overline{\gamma}_t$
g_c	-0.0125	Rate of decline for slope of marginal abatement cost curve	Modest rate set to reflect technological innovation in abatement costs
c_0	\$63/mtCO_2e per GmtCO_2e	Initial slope of marginal abatement cost curve	Set to approximate the emissions price path of EIA's H.R. 2454 analysis for low discount case
z_{max}	2 GmtCO_2e	Maximum offset provision	From H.R. 2454
$B_{min,t}$	$-0.15y_t$	Minimum bank level	From H.R. 2454
y_t	-	Allowance allocation	From H.R. 2454
$\overline{q}_t$	-	Expected baseline emissions	From EIA's H.R. 2454 analysis
$\overline{\gamma}_t$	-	Expected slope of offset supply curve	Based on offset supply schedules and emission prices in EIA's H.R. 2454 analysis
P_1^c	\$28/mtCO_2e	Initial period maximum allowance price with price collar implemented	From S. 1733
P_1^f	\$10/mtCO_2e	Initial period minimum allowance price with price collar implemented	From S. 1733

Table 2. Expected NPV of Abatement Costs plus Offset Purchase Costs

No Uncertainty: 817.3

Without Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	821.0 [658, 1003]	821.6 [652, 1011]	822.3 [644, 1024]
0.4	821.1 [658, 1005]	822.2 [643, 1023]	824.1 [626, 1047]
0.8	821.3 [652, 1011]	826.1 [614, 1077]	835.0 [577, 1160]

No Offset Supply Uncertainty: 821.9

No Offsets: 2830.4

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	868.5 [410, 1509]	879.9 [406, 1575]	895.5 [386, 1663]
0.4	868.8 [409, 1509]	892.5 [384, 1621]	915.8 [359, 1797]
0.8	870.3 [406, 1516]	937.7 [339, 1933]	1173.7 [304, 3269]

No Offset Uncertainty: 869.4

No Offsets: 2904.6

Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	820.3 [658, 1001]	819.7 [651, 1005]	818.7 [643, 1013]
0.4	820.0 [658, 1001]	819.1 [643, 1014]	817.7 [626, 1025]
0.8	819.0 [652, 1004]	818.1 [614, 1045]	819.2 [576, 1095]

No Offset Supply Uncertainty: 821.3

No Offsets: 977.9

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	808.4 [418, 1149]	799.8 [411, 1143]	792.2 [396, 1149]
0.4	807.9 [418, 1147]	794.6 [395, 1150]	790.2 [379, 1177]
0.8	806.5 [416, 1147]	800.4 [361, 1252]	803.4 [338, 1253]

No Offset Supply Uncertainty: 808.6

No Offsets: 978.1

Notes: Values given in billions of dollars. The expected costs are based on ϕ_2 and ϕ_3 combinations, with ϕ_2 values along the rows and ϕ_3 along the columns. For example, the case without a price collar, ϕ_2 = 0.4, ϕ_3 = -0.08, and σ_1^2 = 0.1 has an expected NPV of costs at $915.8B. 95% confidence intervals of costs are given in parentheses below the expected values. "No Uncertainty" refers $\theta_t = \mu_t = 0\ \forall\ t$, "No Offset Supply Uncertainty" refers to $\mu_t = 0\ \forall\ t$, and "No Offsets" refers to $z_t = 0\ \forall\ t$.

Table 3. Expected Cumulative Offset Purchases

No Uncertainty: 62.8

Without Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	62.9 [62, 64]	62.9 [62, 64]	63.0 [61, 65]
0.4	62.9 [61, 65]	62.9 [61, 65]	62.9 [60, 66]
0.8	63.0 [60, 67]	62.8 [58, 68]	62.8 [56, 70]

No Offset Supply Uncertainty: 62.8

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	63.1 [61, 65]	63.0 [61, 65]	63.1 [58, 68]
0.4	63.1 [61, 66]	63.0 [59, 67]	63.0 [56, 70]
0.8	63.1 [59, 67]	62.6 [54, 70]	57.9 [26, 73]

No Offset Supply Uncertainty: 63.0

Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	62.9 [62, 64]	62.9 [62, 64]	62.9 [61, 65]
0.4	62.9 [61, 65]	62.9 [61, 65]	62.9 [59, 66]
0.8	62.9 [60, 67]	62.9 [58, 68]	62.8 [56, 70]

No Offset Supply Uncertainty: 62.8

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	62.8 [61, 65]	62.7 [60, 65]	62.8 [57, 68]
0.4	62.8 [61, 65]	62.7 [58, 67]	62.7 [55, 70]
0.8	62.8 [59, 67]	62.3 [54, 70]	57.0 [24, 73]

No Offset Supply Uncertainty: 62.7

Notes: Values given in GmtCO_2e. The reported numbers are cumulative quantity of offsets purchased over the entire T periods. For example, the case without a price collar, $\phi_2 = 0.4$, $\phi_3 = -0.08$, and $\sigma_1^2 = 0.1$ has an expected cumulative offsets quantity of 63.0 GmtCO_2e. 95% confidence intervals of cumulative offsets are given in parentheses below the expected values. "No Uncertainty" refers $\theta_t = \mu_t = 0 \;\forall\, t$ and "No Offset Supply Uncertainty" refers to $\mu_t = 0 \;\forall\, t$.

Table 4. Frequency of Binding Price Collar

Binding Price Ceiling

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	1.0%	1.6%	2.3%
0.4	1.2%	2.1%	3.1%
0.8	1.9%	3.5%	5.5%

No Offset Supply Uncertainty: 0.9%
No Offsets: 99.3%

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	11.3%	12.7%	14.2%
0.4	11.4%	13.9%	15.8%
0.8	11.7%	16.2%	20.8%

No Offset Supply Uncertainty: 11.8%
No Offsets: 99.7%

Binding Price Floor

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	0.0%	0.0%	0.0%
0.4	0.0%	0.0%	0.0%
0.8	0.0%	0.0%	0.0%

No Offset Supply Uncertainty: 0.0%
No Offsets: 0.0%

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	0.4%	0.4%	0.5%
0.4	0.4%	0.5%	0.6%
0.8	0.4%	0.7%	0.8%

No Offset Supply Uncertainty: 0.4%
No Offsets: 0.0%

Notes: Values represent the percentage of all simulated price values (T x 3500 = 136,500 simulated prices) where either the price ceiling or price floor was binding for the particular parameter setting. For example, the case with $\phi_2 = 0.4$, $\phi_3 = -0.08$, and $\sigma_1^2 = 0.10$ had a binding price ceiling in 16% of the simulated prices. "No Uncertainty" refers $\theta_t = \mu_t = 0 \ \forall \ t$, "No Offset Supply Uncertainty" refers to $\mu_t = 0 \ \forall \ t$, and "No Offsets" refers to $z_t = 0 \ \forall \ t$.

Table 5. Expected Cumulative Emissions

No Uncertainty: 195.0

Without Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	195.1 [194, 196]	195.1 [194, 196]	195.2 [193, 197]
0.4	195.1 [193, 197]	195.2 [193, 197]	195.2 [192, 199]
0.8	195.2 [192, 199]	195.1 [190, 200]	195.1 [188, 202]

No Offset Supply Uncertainty: 195.0

No Offsets: 132.2

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	195.3 [193, 197]	195.2 [193, 197]	195.3 [190, 200]
0.4	195.3 [193, 198]	195.2 [191, 199]	195.2 [188, 203]
0.8	195.3 [192, 199]	194.8 [186, 203]	190.1 [158, 206]

No Offset Supply Uncertainty: 195.2

No Offsets: 132.2

Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	195.2 [194, 196]	195.2 [194, 196]	195.3 [193, 197]
0.4	195.2 [194, 197]	195.2 [193, 197]	195.3 [192, 199]
0.8	195.3 [192, 199]	195.4 [192, 200]	195.5 [191, 202]

No Offset Supply Uncertainty: 195.1

No Offsets: 180.5

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	196.8 [193, 207]	197.2 [194, 206]	197.6 [193, 206]
0.4	196.8 [193, 207]	197.4 [193, 206]	198.0 [193, 207]
0.8	196.9 [192, 207]	197.9 [192, 207]	197.9 [191, 207]

No Offset Supply Uncertainty: 196.7

No Offsets: 180.6

Notes: Values given in giga-metric tons of CO_2 equivalent ($GmtCO_2e$). The reported numbers are cumulative emissions over the entire T periods. For example, the case without a price collar, $\phi_2 = 0.4$, $\phi_3 = -0.08$, and $\sigma_1^2 = 0.1$ has an expected cumulative emissions of 195.2 $GmtCO_2e$. 95% confidence intervals of cumulative emissions are given in parentheses below the expected values. "No Uncertainty" refers $\theta_t = \mu_t = 0 \ \forall\ t$, "No Offset Supply Uncertainty" refers to $\mu_t = 0 \ \forall\ t$, and "No Offsets" refers to $z_t = 0 \ \forall\ t$.

Table 6. Mean Squared Error of Price Paths

Without Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	49.4	52.2	57.3
0.4	50.3	56.1	68.1
0.8	55.7	75.1	105.3

No Offset Uncertainty: 48.2
No Offsets: 66.0

$\sigma_1^2 = 0.10$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	444.7	472.6	553.9
0.4	449.1	536.6	646.1
0.8	458.6	668.8	1610.7

No Offset Uncertainty: 438.2
No Offsets: 564.0

Price Collar

$\sigma_1^2 = 0.01$

ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	48.9	51.1	54.8
0.4	49.7	54.0	62.7
0.8	54.0	67.9	87.8

No Offset Uncertainty: 47.9
No Offsets: 0.06

$\sigma_1^2 = 0.10$

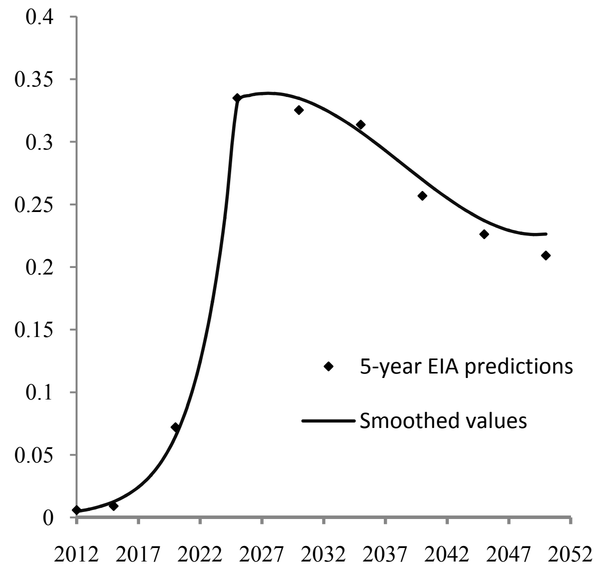
ϕ_2 \ ϕ_3	0	-0.04	-0.08
0	293.7	294.2	316.6
0.4	293.9	313.2	342.7
0.8	297.4	340.5	381.8

No Offset Uncertainty: 289.6
No Offsets: 0.03

Notes: Measures the deviation of permit price from mean permit price at each period, over all simulations. Mean Squared Error takes the average across all 39 periods. Low MSEs for "Price Collar – No Offsets" result from a binding price ceiling in over 99% of simulations, which removes nearly all price variability. "No Offset Supply Uncertainty" refers to $\mu_t = 0 \;\forall\; t$ and "No Offsets" refers to $z_t = 0 \;\forall\; t$.

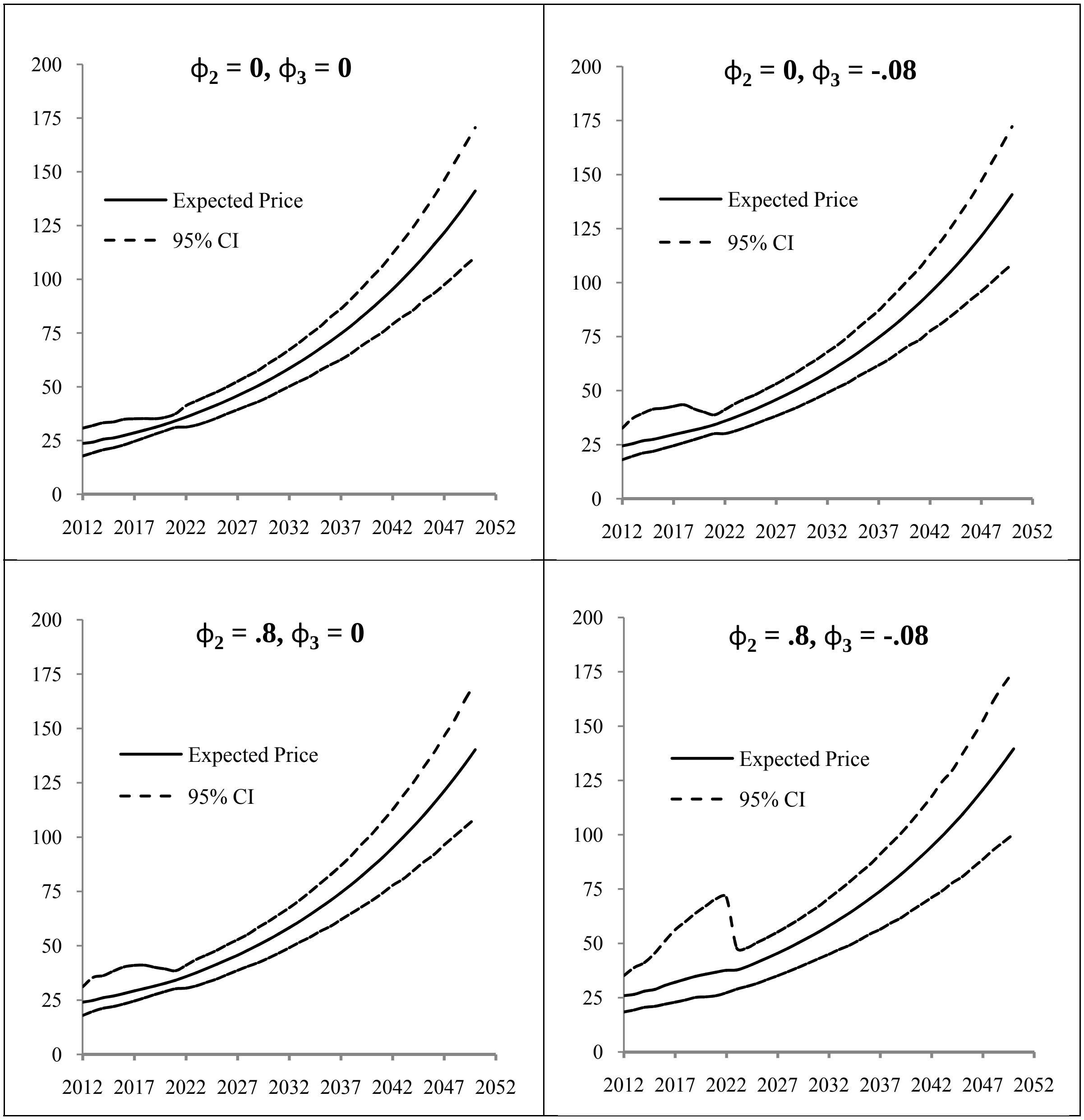
Figures

Figure 1. Expected Slope of Offset Supply Curve ($\bar{\gamma}_t$) Values



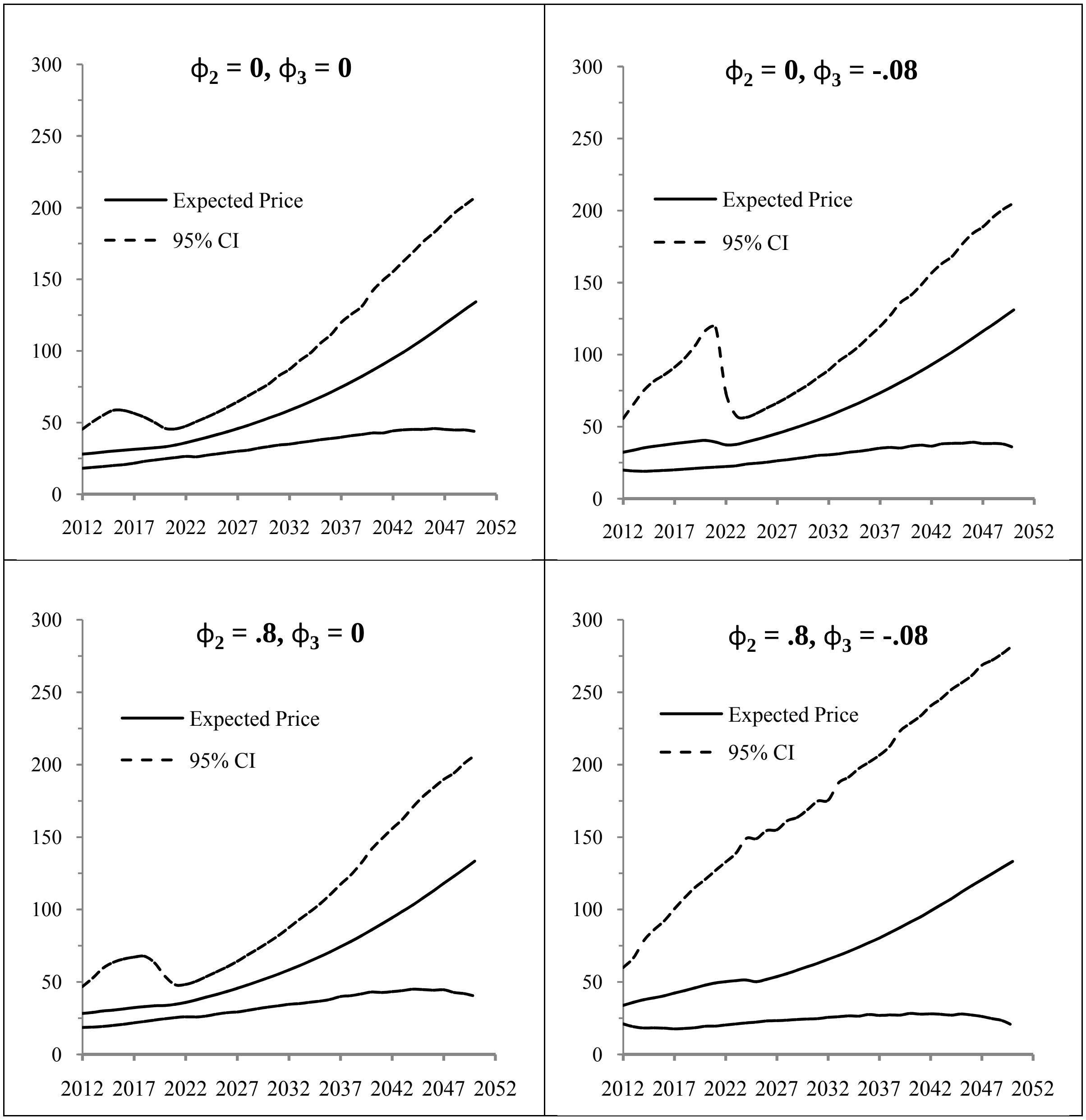
Notes: Five-year EIA predictions show the slope derived using EIA offset supply projections and EIA predicted offset price, for every 5th year of the program. We used both exponential and cubic fits to calculate the smoothed curve, which we took as our parameter values in the model.

Figure 2. Expected Price without Price Collar, $\sigma_1^2 = 0.01$



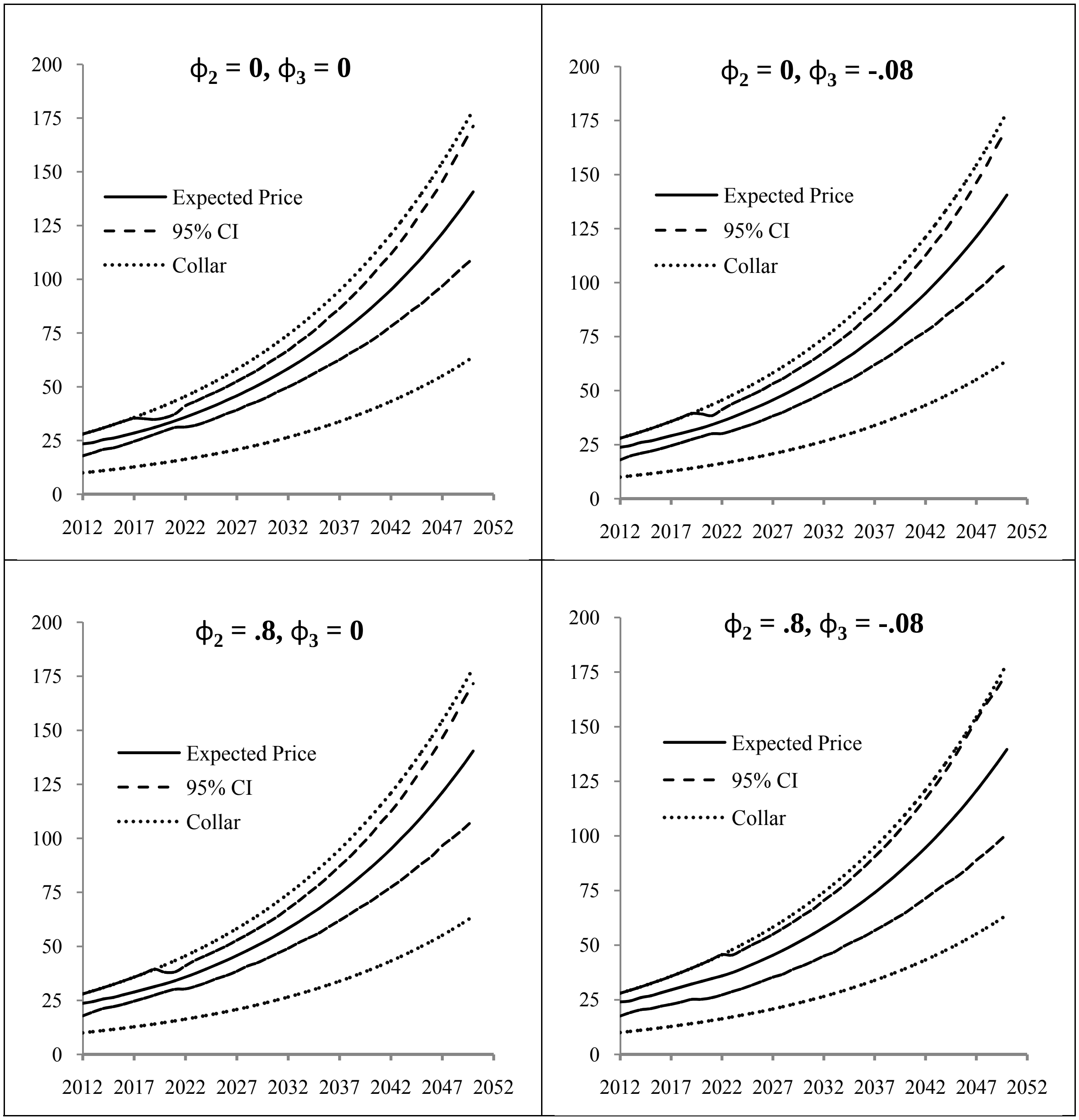
Notes: Prices are on the vertical axis and are given in 2007$ per metric ton of CO_2 equivalent.

Figure 3. Expected Price without Price Collar, $\sigma_1^2 = 0.10$



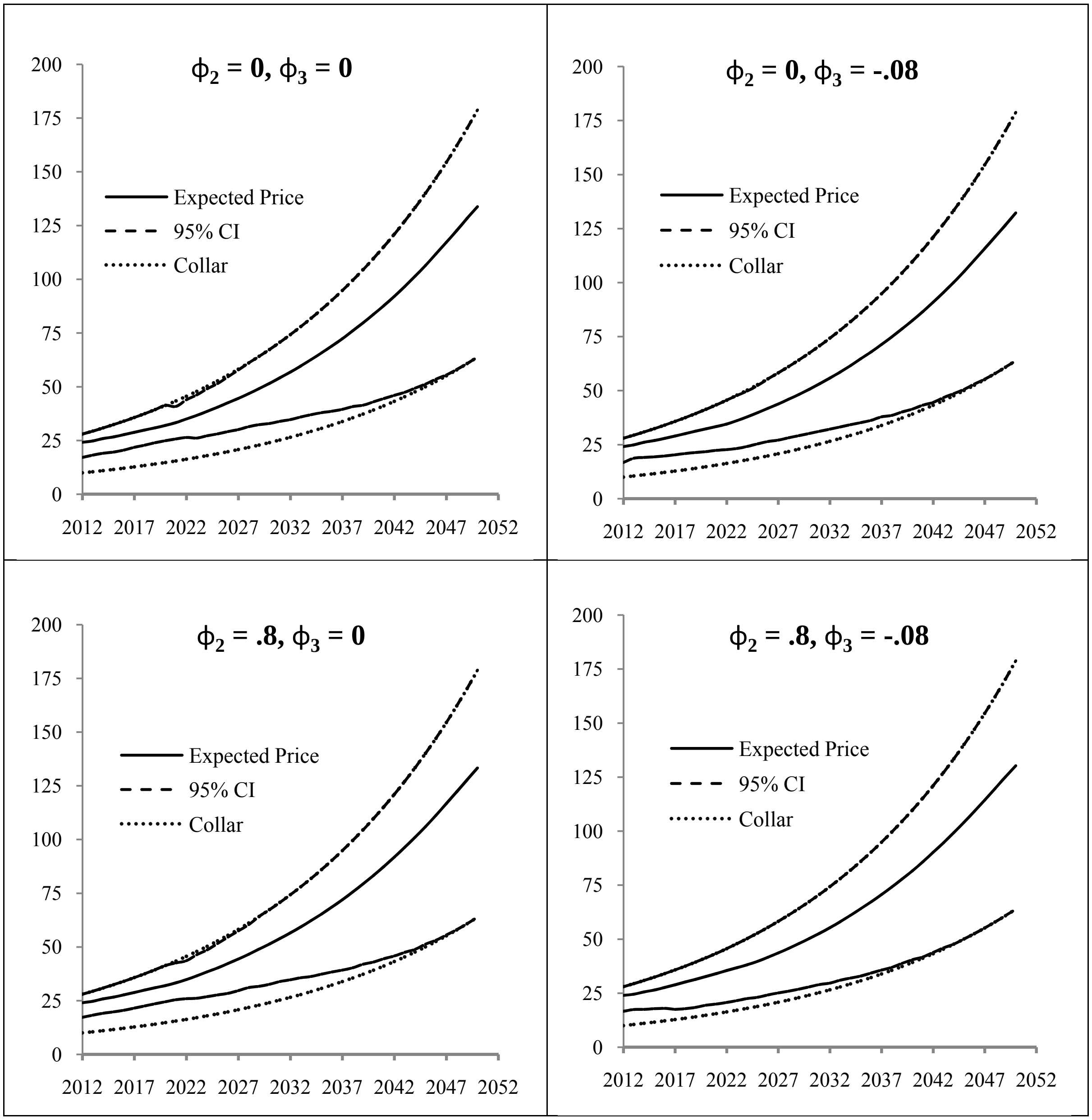
Notes: Prices are on the vertical axis and are given in 2007$ per metric ton of CO_2 equivalent.

Figure 4. Expected Price with Price Collar, $\sigma_1^2 = 0.01$



Notes: Prices are on the vertical axis and are given in 2007$ per metric ton of CO_2 equivalent.

Figure 5. Expected Price with Price Collar, $\sigma_1^2 = 0.10$



Notes: Prices are on the vertical axis and are given in 2007$ per metric ton of CO_2 equivalent.

Appendix

Discretization of θ_t and μ_t

Following Tauchen (1986), we set the discretized space for θ_t and μ_t at n_1 and n_2, respectively, equally spaced values between $\pm m$ standard deviations, based on θ_t and μ_t respective unconditional variances, from their common unconditional mean of zero. To do this we must first find the unconditional covariance matrix for θ_t and μ_t . Define the θ_t and μ_t data generating processes in vector autoregressive (VAR) form:

$$A_0 X_t = A_1 X_{t-1} + \varepsilon_t \tag{23}$$

where

$$X_t = \begin{bmatrix} \theta_t \\ \mu_t \end{bmatrix}, \qquad A_0 = \begin{bmatrix} 0 & 0 \\ \phi_3 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix},\ \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},\ \varepsilon_t \sim N\left(0, \Sigma\right)$$

Equation (23) can be rewritten as:

$$X_t = \overline{A}_1 X_{t-1} + \overline{\varepsilon}_t \tag{24}$$

where $\overline{A}_1 = A_0^{-1} A_1$ and $\overline{\varepsilon}_t \sim N\left(0, \overline{\Sigma}\right)$, with $\overline{\varepsilon}_t = A_0^{-1}\varepsilon_t$ and $\overline{\Sigma} = A_0^{-1}\Sigma A_0^{-1'}$. Given that θ_t and μ_t are both stationary processes, a stationary unconditional covariance matrix Σ^* exists. Σ^* can be found by iterating on the equation:

$$\Sigma_{t+1}^* = \overline{A}_1 \Sigma_t^* \overline{A}_1' + \overline{\Sigma}$$

until convergence, with initial Σ^* guess of $\overline{\Sigma}$. Taking the square root of the diagonal elements Σ^* we get the standard deviations of θ_t and μ_t. We then assign values to θ_t and μ_t, such that the n_1 and n_2 respective values are equi-spaced between $\pm m$ times their respective standard deviations. As state above, we form the discretization based on $n_1 = 101$, $n_2 = 25$, and $m = 2$.