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# Soft and Hard Price Collars in a Cap-and- Trade System

*A Comparative Analysis*

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# **Soft and Hard Price Collars in a Cap-and-Trade System: A Comparative Analysis**

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## **Abstract**

We use a stochastic dynamic framework to compare price collars (price ceilings and floors) in a cap-and-trade system. Sources of uncertainty include shocks to baseline emissions, affecting corresponding abatement costs, and shocks to the supply of offsets. We consider a continuum between soft collars, which have a limited volume of additional emission allowances (a reserve) available at the price ceiling, and hard collars, which provide an unlimited supply of additional allowances, thereby preventing allowance prices from exceeding the price ceiling. For all cases considered, we set the price floors and ceiling such that the expected cumulative emissions net of offsets are equal to the cumulative allowances. Consequently, increasing the size of the allowance reserve requires higher price ceilings and floors, and a lower probability of reaching the ceiling. Across most parameter values examined, we find that increasing the size of the allowance reserve leads to lower expected net present values of compliance costs, although the differences are not large. However, when offset supply shocks are highly persistent and exhibit strong (negative) correlation with baseline emission shocks, hard collars deliver noticeably lower expected costs, though with a wider range of emission outcomes than the soft collars.

**Key Words:** climate change, offsets, cap-and-trade, price collars, stochastic dynamic programming

**JEL Classification Numbers:** Q54, Q58, C61

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## Soft and Hard Price Collars in a Cap-and-Trade System: A Comparative Analysis

Harrison Fell, Dallas Burtraw, Richard Morgenstern, Karen Palmer, and Louis Preonas\*

### Introduction

Concerns about allowance price volatility have hampered efforts to adopt a U.S. cap-and-trade policy to regulate emissions of carbon dioxide (CO<sub>2</sub>) and other greenhouse gases (GHG). Hybrid policies, in particular price collars, which add allowance price floors and ceilings, have recently garnered much attention as a way to alleviate price volatility in a cap-and-trade system.<sup>1</sup> Price collars also appear to have gained political traction, having been included in some form in the most recently proposed GHG mitigation bills (e.g., U.S. Congress 2009a and 2009b).

Price collars may contain costs, but they also make emission outcomes more variable (see Fell and Morgenstern 2010), which calls into question the environmental integrity of such systems. Specifically, because hitting the price ceiling introduces additional emissions allowances, there is a concern that if prices hit the ceiling frequently, it will not be possible to meet the emission reduction goals of the system. As a compromise between the concerns of environmental integrity and cost volatility, Murray et al. (2009) propose a reserve allowance, or *soft*, price collar system in which there is a pre-set maximum of allowances that can be added to the system in any given period when the price ceiling is hit. This is the *allowance reserve*. If the additional allowances in the reserve are not sufficient to satisfy demand, the allowance price can rise above the price ceiling. The soft collar plan differs from the *hard* price collar, in which the price ceiling is strictly enforced in each period no matter how many allowances must be added to the system.<sup>2</sup>

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<sup>1</sup> Hybrid policies, combining price policies with quantity policies, are nothing new in the academic literature. Original contributions in this area can be found in Roberts and Spence (1976) and Weitzman (1978), and more recently in Pizer (2002), Philibert (2008), Burtraw et al. (2010), and Fell and Morgenstern (2010).

<sup>2</sup> In principle, one could also consider different limits on the quantity of allowances that would be withdrawn to support the price floor, although this has not been a major issue in public debates.

Although the environmental and cost-containing tradeoffs between the hard and soft collars seem quite obvious, little has been done in the way of formal comparisons of the two systems. In this paper, we use simulation analysis to address this gap. Using parameters relevant to U.S. climate policy, we compare the two systems over a variety of policy design specifications. We compare net present value (NPV) of regulatory costs, emission outcomes, and allowance price variability for hard and soft collar systems. We consider a range of collar widths (price ceiling minus price floor) and soft collar designs under various allowance reserve sizes. Importantly, for each system specification we set the collars such that all policy designs lead to the same level of *expected* cumulative emission outcomes net of offset purchases. By doing so, we are able to get a more apples-to-apples comparison of the different approaches, at least in terms of environmental integrity.

Following Fell et al. (2010), we adopt a dynamic stochastic framework that includes uncertainty in future baseline emission levels (average cost minimizing level of emissions without regulation), and thus future abatement costs, as well as uncertainty in the availability of emission offsets. Additionally, the model allows for these sources of uncertainty to be contemporaneously correlated. In this paper, we extend the previously developed hard collar mechanism to include soft collar provisions.

Our results indicate that for most of the parameter settings examined, use of a hard collar leads to lower expected NPV of compliance costs, although soft collar performance, in terms of costs and emission outcomes, is not materially different from that of the hard collar. However, in cases where offset supply shocks are both highly persistent and exhibit strong (negative) correlation with baseline emission shocks, we find a noticeable difference in the performance of hard and soft collars, with hard collars delivering lower expected costs and a wider range of emission outcomes than the soft collars.

The remainder of the paper is organized as follows. In Section II, we briefly describe the model, including the implementation of the hard and soft collars. In Section III, we describe the simulation analysis and model parameterization. Section IV describes the results of the analysis and we discuss our conclusions in Section V.

## II. Model Setup

Rubin (1996) showed in a dynamic cap-and-trade model with banking and inter-firm allowance trading, that the market equilibrium of a multi-firm analysis results in the minimization of total costs. Thus, we can represent the market outcome of a cap-and-trade

regulation using a cost minimization model of a single representative firm, whose costs proxy for the market's total cost. Additionally, if we assume offsets enter the market via an intermediary monopsonistic offset purchaser seeking to minimize system-wide costs (abatement costs plus offset purchase costs), we can also represent the market outcome by including an offset purchasing decision in the representative firm emissions decision model. Below we briefly describe such a representative firm model. A more detailed description of this model can be found in Fell et al. (2010).

We begin by setting up the stochastic dynamic modeling framework without price collars. As noted, we consider uncertainty in both baseline emissions and offset supply. The baseline emissions uncertainty can be thought of as uncertainty about general future macroeconomic conditions; when economic growth is greater than expected, baseline emissions and thus the abatement costs of achieving a particular emissions target are greater than expected. The future offset supply uncertainty reflects the lack of certainty about the availability, and thus costs, of future levels of offsets. It is also reasonable to think that these two sources of uncertainty may be correlated. For instance, higher than expected economic growth would most likely lead to an increase in commodity prices. Higher commodity prices would increase the opportunity costs for many types of offsets, particularly agricultural and forest offsets, and thus decrease offset supply.

Given these sources of uncertainty, and assuming a commonly used convex abatement cost function, we can write the dynamic cost minimization problem over a finite regulation period from  $t = 1$  to  $t = T$  as

$$\min_{q,z} \sum_{t=1}^T \beta^{t-1} E_t \left[ \frac{c_t}{2} (\bar{q}_t + \theta_t - q_t)^2 + P_t^z z_t \right] \quad (1)$$

where  $\beta$  is the discount factor,  $E_t$  is the expectations operator at time  $t$ ,  $c_t$  is the slope of the marginal abatement cost curve,  $\bar{q}_t$  is the expected baseline emissions,  $\theta_t$  is the shock to baseline emissions,  $q_t$  is the quantity the firm chooses to emit,  $P_t^z$  is the price of offsets, and  $z_t$  is the

quantity of offsets purchased.<sup>3</sup> We also allow banking and borrowing, so (1) is subject to the following constraints

$$B_{t+1} = R_t B_t + y_t + z_t - q_t \quad (2)$$

$$c_t = c_1 (1 + g_c)^{t-1} \quad (3)$$

and

$$B_t \geq B_{\min,t}, \quad B_{T+1} \geq 0, \quad 0 \leq z_t \leq z_{\max,t} \quad (4)$$

$$R_t = \begin{cases} 1 + r_{bank} & \text{if } B_t \geq 0 \\ 1 + r_{borr.} & \text{otherwise} \end{cases}$$

where  $B_t$  is the bank level at  $t$ ,  $y_t$  are the allocated allowances at time  $t$ , and  $R_t$  is the interest rate paid (charged) on banked (borrowed) allowances.<sup>4</sup> Given the banking dynamics in (2), offsets are essentially treated as equivalent to allowances. Additionally, from (3), we assume that the slope of marginal abatement costs fall at a constant rate. This is tantamount to assuming exogenous technological change.

From the inequality constraints in (4), note that the problem contains a minimum limit for the level of the bank (i.e., a borrowing limit), a maximum limit on the number of offsets that can be purchased, and a terminal condition on the bank level. These inequality constraints are important for several reasons. First, the terminal condition requires that the firm repay all borrowed allowances by the end of the final period. This condition allows us to solve the model recursively as described below. Second, the limits on borrowing, as well as the limits on offset purchases, can create a situation where there is pre-cautionary banking if the probability of these limits binding is positive.<sup>5</sup> This means that the emissions behavior generated by this modeling

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<sup>3</sup> With respect to the expectations operator, we assume that all random variables in the current period are known at time  $t$ , but future realizations of the random variables are unknown.

<sup>4</sup> Allowance interest paid or charged due to banking and borrowing is in terms of emission allowances and not financial payments. For instance if  $r_{borr.} = 0.1$  and a firm borrows 1 allowance in year  $t$ , the firm will owe 1.1 allowances in period  $t + 1$ .

<sup>5</sup> There is an extensive literature on precautionary banking in macroeconomics. For prominent examples of the effect of borrowing constraints on precautionary banking incentives, see among others Deaton (1991) and Aiyagari (1994).

framework is fundamentally different from that under a deterministic setting. The  $R_t$  specification allows for the interest paid on banking to be different from the interest charged on allowance borrowing, a feature common in recently proposed U.S. climate legislation, such as U.S. Congress (2007) and (2009a).

Uncertainty in offsets enters via the slope of the offset supply equation. We assume the offset supply curve is a linear function of the offset price, such that

$$z_t = \gamma_t P_t^z \quad (5)$$

$$\gamma_t = \bar{\gamma}_t + \mu_t \quad (6)$$

where  $\bar{\gamma}_t$  is the unconditional mean of the offset supply slope and  $\mu_t$  is a random variable. We model the evolution of both random variables,  $\theta_t$  and  $\mu_t$ , as AR(1) processes:

$$\begin{aligned} \theta_t &= \phi_1 \theta_{t-1} + \varepsilon_{1t} \\ \varepsilon_{1t} &\sim \text{iid } N(0, \sigma_1^2) \\ 0 &< \phi_1 < 1 \end{aligned} \quad (7)$$

and

$$\begin{aligned} \mu_t &= \phi_2 \mu_{t-1} + \phi_3 \theta_t + \varepsilon_{2t} \\ \varepsilon_{2t} &\sim \text{iid } N(0, \sigma_2^2) \end{aligned} \quad (8).$$

The parameters  $\phi_1$  and  $\phi_2$  determine the persistence of the process, such that as these parameters become closer to one the respective random variable becomes a more persistent process (i.e., the shock state is more permanent). Correlation among the sources of uncertainty exists if  $\phi_3 \neq 0$ . As discussed above, it is reasonable to assume that the offset uncertainty shock and the baseline emissions shock are negatively correlated. Thus, we only consider values of  $\phi_3$  such that  $\phi_3 \leq 0$ .

### Price Collars

The goal of the price collar is to keep the price of emission allowances, modeled as the marginal abatement cost  $c_t(\bar{q}_t + \theta_t - q_t)$ , between a pre-determined price ceiling,  $P_t^c$ , and price floor,  $P_t^f$ . If the price ceiling is triggered in period  $t$ , the cost function given in (1) is altered as

$$\frac{c_t}{2}(\bar{q}_t + \theta_t - q_t)^2 + \max(P_t^c, c_t(\bar{q}_t + \theta_t - q_t))y_t^c + P_t^z z_t \quad (9)$$

where  $y_t^c$  is the quantity of additional allowances the firm buys from the regulator such that  $y_t^c \leq y_{\max,t}^c$ . In the case of a hard collar,  $y_{\max,t}^c = \infty$  and the price of emissions never exceeds  $P_t^c$ . It can also be shown that, with a hard collar, when the price ceiling is binding emissions are such that  $c_t(\bar{q}_t + \theta_t - q_t) = P_t^c$  (see Fell and Morgenstern 2010). Under a soft collar,  $y_{\max,t}^c < \infty$ . In this case, it is possible that the demand for additional allowances will exceed the limit  $y_{\max,t}^c$  and, thus, the marginal cost of abatement may exceed  $P_t^c$ .<sup>6</sup> Note that (9) is written such that we assume the firm will pay the market equilibrium allowance price for additional allowances purchased when the price ceiling is triggered.<sup>7</sup>

The price floor can be implemented in several ways. The two most obvious approaches are to auction off emission allowances in a minimum price auction, as is done in the Regional Greenhouse Gas Initiative, or to force the regulator to have a standing offer to buy allowances at price  $P_t^f$ . We choose the latter implementation strategy here due to the relative ease of representing it in the model.<sup>8</sup> Given this implementation form, the cost at time  $t$  when the floor is binding is

$$\frac{c_t}{2}(\bar{q}_t + \theta_t - q_t)^2 - P_t^f y_t^f + P_t^z z_t \quad (10)$$

where  $y_t^f$  is the quantity of allowances the firm sells to the regulator at the price floor. With this set-up, it can be shown that when the price floor is binding, emissions will be such that  $c_t(\bar{q}_t + \theta_t - q_t) = P_t^f$  (see Fell and Morgenstern 2010). As in Fell et al. (2010), we also assume that offsets are only made available up to the point where the price floor is binding. This precludes the possibility that the firm will resell offset credits back to the government.

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<sup>6</sup> This would be consistent with a system where additional allowances enter the market through an auction with a reserve price at  $P_t^c$ . Under a soft collar, the demand might exceed the supply of reserve prices at this price, which leads to a price above this level.

<sup>7</sup> Given the limited-quantity nature of the price ceiling under the soft collar, such a system could be vulnerable to the types of speculative attacks discussed in Salant (1983). However, we do not consider speculative attacks in this paper.

<sup>8</sup> To assume allowances are auctioned would require us to incorporate another control variable, the number of allowances purchased, into the model. This would make the problem considerably more difficult from a computational perspective.

### III. Simulation Analysis

Given equations (1) – (10), the cost minimization problem for the firm can be written in Bellman equation form. Ideally, one would then solve for the emissions choice  $q_t$  and offset choice  $z_t$  as functions of the state variables,  $B_t$ ,  $\theta_t$ , and  $\mu_t$ . However, given the form of the uncertainty, the limitations on borrowing and offset purchases, and the price collar constraints, a closed form solution to this problem does not exist. Instead, we parameterize the model and discretize the state space so that we can solve it numerically.

To solve this problem numerically, we begin by discretizing the state variables  $B_t$ ,  $\theta_t$ , and  $\mu_t$  such that the states can take on  $N_B$ ,  $N_\theta$ , and  $N_\mu$  possible values, respectively.<sup>9</sup> With the discretized spaces for  $\theta_t$  and  $\mu_t$  and based on equations (7) and (8), we can form a probability transition matrix to describe the probability of moving from any discrete  $(\theta_t, \mu_t)$  pair to any other  $(\theta_{t+1}, \mu_{t+1})$  pair. Imposing the terminal condition for  $B_{T+1}$  in (4), we can solve for the cost in the final period for all given state outcomes  $(B_T, \theta_T, \mu_T)$ . Knowing the final period's cost for all possible states and the probability transition matrix, we recursively solve for the optimal  $q_t$  and  $z_t$  for every  $(B_t, \theta_t, \mu_t)$ . This recursive solution process results in three-dimensional matrices  $(N_B \times (N_\theta \cdot N_\mu) \times T)$  for  $q_t$  and  $z_t$  that give each control variable's optimal level for every possible realized state in each time period.<sup>10</sup>

Knowing the optimal control sequence for the control variables and the bank state dynamics given in (2), we can then conduct simulation analyses. To perform these analyses, we first generate  $N_{sim}$  different paths for  $\theta$  and  $\mu$ , where a path is a  $1 \times T$  vector of outcomes. Given the simulated shock paths and an initial bank condition,  $B_1 = 0$ , we can use the optimal control matrices of  $q_t$  and  $z_t$  to map out  $N_{sim}$  emission, offset, and bank path realizations. The path realizations become the basis for the cost, cumulative emission, cumulative offsets, and price variability metrics we use to compare various policy designs under a range of parameter specifications.

#### **Model Parameterization**

The model is keyed primarily to H.R. 2454, the recent bill put forth by Representatives Henry Waxman and Edward Markey (U.S. Congress 2009a) and to the analysis of this bill

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<sup>9</sup> All other time-dependent state variables are assumed to evolve in a deterministic manner exogenous to the control variables  $q_t$  and  $z_t$ .

<sup>10</sup> See Fell et al. (2010) for a more detailed description of the recursive solution algorithm.

performed by the U.S. Department of Energy's Energy Information Administration (EIA 2009). With the exception of the collar specifications, the parameters used here are the same as those given in Fell et al. (2010). A detailed description of the parameterization, including the methodology to construct  $\bar{\gamma}_t$ , can be found in Fell et al. (2010). Rather than reconstruct that description here, we summarize the key parameters in Table 1.

We design the collars such that both the floor and the ceiling rise at the discount rate,  $P_t^i = P_1^i(1+r)^{t-1}$  for  $i = f, c$ . We take the width of the initial collar,  $X_1 = P_1^c - P_1^f$ , as a pre-determined variable and consider  $X_1 = (10, 15, 20)$ . As mentioned above, initial values of the collars,  $P_1^f$  and  $P_1^c$ , are set such that the *expected* cumulative emissions net of offsets,

$\sum_{t=1}^T (q_t - z_t)$ , are equal to the cumulative allowances,  $\sum_{t=1}^T y_t$ . Expected cumulative emissions net

of offsets are derived as the mean cumulative emissions minus offsets from the simulation analysis. Clearly, these initial values will depend on  $y_{\max,t}^c$ . For the soft collar cases, we set  $y_{\max,t}^c = \alpha y_t$ , where  $\alpha = (10\%, 20\%, 30\%)$ . Finally, we design all of the different collar specifications under various assumptions about the persistence of the offset supply shock and about the correlation among the shocks. More specifically, we consider cases where  $\phi_2 = (0.0, 0.8)$  and  $\phi_3 = (0.0, -0.08)$ .

#### IV. Results

The results of the simulation analysis, with  $N_{sim} = 10,000$ , are given in Tables 3 – 6, one table for each of the  $(\phi_2, \phi_3)$  combinations. Each table initially includes outcomes from the case with no collars included as a baseline for the effectiveness of the collars. Then, the tables present collar design specifications ( $X_1$ ,  $\alpha$ , and expected  $P_1^c$ ) needed to equate expected cumulative domestic emissions less offsets to cumulative allowances.<sup>11</sup> With respect to environmental outcomes, the tables also include results on expected domestic cumulative emissions and expected cumulative emissions net of offset purchases, along with the 95 percent confidence intervals for each metric. We also report the NPV of system costs and the 95 percent confidence intervals of these costs.<sup>12</sup> The system costs include abatement costs plus offset purchase costs.

<sup>11</sup> The initial price floor is not given, but can be calculated by subtracting the collar width,  $X_1$ , from  $P_1^c$ .

<sup>12</sup> In Tables 3 – 6, the 95 percent confidence intervals for cumulative emissions, cumulative emissions net of offsets, and NPV of costs are given in the brackets below the respective expected values.

Importantly, the calculated costs exclude costs the firm incurs from buying additional allowances when the price ceiling is triggered, as well as the regulator's cost from buying back allowances at the price floor. These costs are excluded because they represent transfers within the national system.<sup>13</sup> Price variability is captured in several metrics: the percentage of periods the floor and ceiling are triggered and, for the soft collars, the percentage of periods the allowance reserve is exceeded leading to an equilibrium price that is greater than the price ceiling.<sup>14</sup> Additionally, allowance price variability is captured by the root mean squared error of the price growth rate (RMSE) where

$$RMSE = \frac{1}{N_{sims}} \sum_{i=1}^{N_{sims}} \sqrt{\frac{1}{T-1} \sum_{t=2}^T (P_{it}^g - \bar{P}_i^g)^2}$$

$$P_{it}^g = \frac{(P_{it} - P_{it-1})}{P_{it-1}}, \bar{P}_i^g = \frac{1}{N_{sims}} \sum_{i=1}^{N_{sims}} \frac{(P_{it} - P_{it-1})}{P_{it-1}}$$

Several generalizable and intuitive results hold across all  $\phi_2$  and  $\phi_3$  settings examined. First, we find that the inclusion of price collars, even at the smallest allowance reserve considered ( $\alpha = 10\%$ ), noticeably lowers expected NPV of system costs. This is essentially a Jensen's inequality result – because price collars reduce the variance in cost outcomes compared to the case without collars and the cost function is convex, price collars will lead to lower expected NPV of system costs. It is, however, somewhat surprising that for all cases considered, the majority of the cost savings achieved with a price collar can be realized with an  $\alpha = 10\%$ .

With respect to collar design, we find that as  $\alpha$  increases, which increases the supply of reserve allowances available (i.e., the soft collar becomes more like a hard collar), the initial values of the price collar,  $P_1^f$  and  $P_1^c$ , increase or remain the same.<sup>15</sup> This result is as expected, because increasing  $\alpha$  increases the number of additional allowances that can enter the system when the ceiling is triggered. If the expected cumulative emissions are to remain constant, a higher ceiling is required to decrease the frequency of triggering the ceiling and a higher floor is

<sup>13</sup> If offsets are produced domestically, then offset purchase costs would also represent a transfer among domestic firms. However, many of the offsets are expected to be from international sources (see EIA 2009), which would represent a true cost to the domestic system.

<sup>14</sup> The percentages are based on the simulation analysis and calculated as (number of periods event occurred)/(T x  $N_{sim}$ ).

<sup>15</sup> Recall the initial difference between the predetermined price ceiling and floor remains unchanged.

required to increase the probability that allowances will be removed from the system. We see that the probability of triggering and exceeding the ceiling increases as  $\alpha$  decreases.

With respect to price variability, as measured by RMSE of price growth rates, we find that as  $\alpha$  increases, price variability decreases or remains constant. This result is expected because if  $\alpha$  is sufficiently low the price ceiling is not strictly maintained, thus creating the potential for more price variability.

Regarding emission outcomes, we see that expected cumulative domestic emissions,  $\sum_t q_t$ , are almost constant across  $\alpha$  for a given  $X_1$ , but the spread of emission outcomes, as measured by the 95 percent confidence intervals, increases or remains constant as  $\alpha$  increases. The same holds for cumulative emissions net of offsets. This result is expected because as  $\alpha$  increases, more allowances are allowed to enter the system when the price ceiling is triggered, permitting higher emission outcomes; while increasing price floors as  $\alpha$  increases yields fewer emissions allowances and lower emissions and creates incentives for over compliance (i.e., lower emissions) when abatement and offset costs are low.

Finally, with respect to costs, we find that as  $\alpha$  increases the range of system costs narrows, as measured by the spread of the 95 percent confidence intervals for the NPV of system costs. This is an intuitive result. As the soft collar becomes more like a hard collar, the price ceiling is more effective at constraining high cost outcomes, while the higher price floors as  $\alpha$  increases force an increase in the lower bound of costs. Via the same reasoning, we also find that the range of costs increases as the initial spread of the collar,  $X_1$ , increases.

Comparing the results across tables we can see how changing the persistence of the offset supply shock (i.e., changing  $\phi_2$ ) and/or changing the level of negative correlation between the offset supply shock and the baseline emissions shock (i.e., changing  $\phi_3$ ) affects the results. The most interesting of these comparisons is with respect to costs. For most  $\phi_2$  and  $\phi_3$  combinations, we see that expected NPV of costs falls as the size of the allowance reserve ( $\alpha$ ) increases for a given width of the price collar ( $X_1$ ), with  $\alpha = 30\%$ , yielding cost results quite similar to the hard collar. The exception to this observation is when the initial spread between the price ceiling and price floor is relatively close ( $X_1 = \$10$ ) and the offset supply shock persistence parameter is relatively small ( $\phi_2 = 0.0$ ), while the parameter determining the negative correlation between the sources of uncertainty is relatively large in magnitude, ( $\phi_3 = -0.08$ ). Because we consider a maximum  $\alpha$  of 30% in Tables 3 – 6, it may be possible that other  $\phi_2$  and  $\phi_3$  combinations have soft collar specifications that are both materially different from the hard collar and have lower expected NPV of costs than the hard collar. To explore this possibility further, we searched for

the soft collar specification, in terms of  $\alpha$ ,  $P_1^c$ , and  $P_1^f$ , for each  $(X_1, \phi_2, \phi_3)$  that minimizes expected NPV of costs. We show the results of this search procedure in Table 7. For most cases provided in Table 7, we see that the cost minimizing soft collar design leads to an expected NPV of costs equivalent or nearly equivalent to that of the hard collar and the price ceiling is rarely if ever exceeded. In addition, many of the soft collar designs lead to price floors and ceilings that are identical to that of the hard collar, implying that these soft collars are essentially mimicking a hard collar by providing sufficiently large allowance reserves.<sup>16</sup> As one would expect, we find that in order for the soft collar to mimic the hard collar,  $\alpha$  must increase as  $X_1$  decreases.

In a somewhat surprising result, we still find that the only cases in which the cost-minimizing soft collar design leads to a materially different expected NPV of costs than the corresponding hard collar is the case where  $X_1 = \$10$ ,  $\phi_2 = 0.0$ , and  $\phi_3 = -0.08$ . Why do we get this result? As discussed above, when the collar design goal is such that the expected cumulative emissions net of offsets are the same regardless of the level of  $\alpha$ , lower  $\alpha$ 's allow for lower collars. If the cost-reducing benefits of lower collar levels outweigh the additional cost of having greater exposure to extreme high cost outcomes, it is possible that the expected NPV of costs can be lower with a true soft collar (i.e., a soft collar where it is possible and more likely that the reserve will be exhausted) than with a hard collar. This scenario would appear likely if shock processes were such that the probability of exceeding the price ceiling is relatively low compared to the probability of triggering the price ceiling. In reviewing the columns “Ceiling Binds” and “Above Ceiling” in Tables 3 – 6, we do indeed see that the ratio of “Above Ceiling” to “Ceiling Binds” is relatively low for the  $X_1 = \$10$ ,  $\phi_2 = 0.0$ , and  $\phi_3 = -0.08$  case. Thus, it would appear that this parameter setting is the best candidate for a soft collar having a lower expected NPV of cost than the corresponding hard collar.

How likely is it that this scenario would occur in a real-world situation? We justify how one might think there could be a negative correlation between offset supply shocks and baseline emission shocks. However, it also seems likely that persistence in the offset supply state would be high, given the lengthy offset contracts and what will likely be a lengthy approval time for new offset projects, and the general persistence of business cycles. It would thus seem unlikely that such a parameterization would occur in reality, though finding the answer to this question is

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<sup>16</sup> Because these soft collar designs are mimicking a hard collar, increasing  $\alpha$  beyond what is presented would lead to the same results. That is, for the cases in Table 7 that lead to essentially the same results as their corresponding hard collars, the  $\alpha$ 's we present are the minimum reserve size needed to mimic a hard collar.

ultimately an empirical exercise and we cannot provide any probabilities to the likelihood of any parameterization considered. Additionally, to keep the analysis tractable, we do not present an exhaustive sensitivity analysis. Therefore, it is possible that under different parameter settings than those considered here, for a given  $X_1$ , expected NPV of costs could be minimized at some soft collar that is materially different from a hard collar.

Several other cross-table comparisons are also worth noting. First, emission outcomes and cost outcomes are similar across Tables 3 – 5, but are markedly different in Table 6, the scenarios in which offset supply shocks are both highly persistent and exhibit relatively large negative correlation with baseline emission shocks. More specifically, for the cases where  $\phi_2 = 0.8$  and  $\phi_3 = -0.08$ , the expected cumulative domestic emissions are noticeably lower and the range of emissions net of offsets are wider than those of the corresponding collar specifications under other  $\phi_2$  and  $\phi_3$  parameterizations considered. This result is in part due to the offset purchasing limitation. As  $\phi_2$  and  $\phi_3$  increase in magnitude, the variance of the offset supply shock,  $\mu_t$ , increases, simultaneously creating a higher probability of high offset supply outcomes and low offset supply outcomes. Increasing the variance of  $\mu_t$  can lower expected offset purchases because increasing the probability of realizations where offset cost are low has limited benefit, in terms of being able to purchase more offsets, if the cumulative offset cap is close to being met in expectation. Conversely, increasing the probability of realizations where offset cost are high will pull down the expected offset purchases.<sup>17</sup> With lower expected offset purchases, domestic emissions must fall to meet the emissions cap. In addition, the greater variability in  $\mu_t$  leads to a greater variability in the quantity of offsets purchased and hence a greater range in the outcomes of domestic emissions net of offsets.

The lower quantity of expected offset purchases and the greater variation in offset purchasing outcomes observed when  $\phi_2$  and  $\phi_3$  increase in magnitude lead to both higher expected NPV of costs and a greater range in outcomes for NPV of costs. This is most notably true for the collars where  $\alpha$  is relatively small. In these soft collar cases, we see the  $\phi_2 = 0.8$ ,  $\phi_3 = -0.08$  parameterization leads to much higher upper bound costs compared to the other  $\phi_2$  and  $\phi_3$  parameterizations. This result is as expected because the  $\phi_2 = 0.8$ ,  $\phi_3 = -0.08$  parameterization increases the frequency and duration of states with high offset costs, putting more pressure on the

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<sup>17</sup> This is the case in this model as we see the expected cumulative offset purchase for cases with relatively low variance in  $\mu_t$  is roughly 60 gmtCO<sub>2</sub>, compared to a cumulative cap of 78 gmtCO<sub>2</sub>e (max of 2gmtCO<sub>2</sub>e/year X 39 years). See Fell et al. (2010) for a more indepth discussion of this point.

price ceiling to contain costs. Because collars with smaller  $\alpha$  values have more limited price ceiling support, these collars are unable to effectively rein in costs, hence the greater number of periods with prices above the ceiling price and the larger upper bound of cost outcomes. Despite the rising expected NPV of costs and expanding range of cost outcomes as  $\phi_2$  and  $\phi_3$  increase in magnitude, we also find that increasing benefits of collars relative to no collar cases, in terms of reducing expected NPV of costs. Again, this is as expected because collars are most valuable when shocks to the system have large variances, as they do when  $\phi_2$  and  $\phi_3$  are large in magnitude.

## V. Conclusion

This paper uses a simulation analysis keyed to parameters relevant in the U.S. climate policy debate to compare the performance of various soft and hard price collars in emission allowance trading systems. The soft and hard collar characterizations are distinguished by the volume of allowances in an allowance reserve that would become available when the emissions allowance price reaches the price ceiling. Under a hard collar, an unlimited amount would be available from the reserve; here, the outcomes we explore vary according to the volume of reserve allowances that would be available. In this paper, unlike much of the prior research on price collars, the future states of baseline emissions and offset supply conditions are unknown to the representative firm and price collars are designed to achieve expected cumulative emissions net of offsets and reserve allowances equal to the cumulative allowances.

Several of the more intuitive results derived from this analysis are as follows.

- To meet cumulative emissions goals, the price collar values must be shifted up as the collar moves from a soft collar to a hard collar (that is, as more allowances are available from the allowance reserve). Correspondingly, price ceilings are binding or exceeded less frequently and price floors are binding more frequently as the collar moves from a soft collar to a hard collar.
- Allowance price variability decreases as the collar moves from a soft collar to a hard collar.
- Although the expected cumulative domestic emissions (not accounting for offsets) are almost constant with respect to the size of the allowance reserve for a given width of the price collar, the range of emissions outcomes tends to increase as the reserve size grows.
- The range of NPV of system cost outcomes narrows as the collar moves from a soft collar to a hard collar.
- The range of NPV of system cost increases as the initial spread of the collar increases.

In addition to these relatively straightforward results, we show that, depending upon the specification of the shock processes, it is possible for expected NPV of system costs to be minimized using a soft collar rather than a hard collar. This result is possible when the gains from shifting the level of the price collars down by using “softer” price collars (i.e., few reserve allowances available at the price ceiling) outweigh the reduced protection against high abatement cost and high offset cost outcomes associated with the softer price collar. Although we are able to show such a result is possible from our Monte Carlo analysis, it remains to be seen if the parameterizations needed to produce this result are truly realistic.

In contrast, for a majority of the parameter values that we explore, a “harder” price collar (that is, a greater volume reserve allowances along with higher price collar prices such that expected emissions net of offsets are constant) leads to lower NPV of system costs. In most cases, a hard price collar yields the lowest system costs. We emphasize that the set of parameters we explore are plausible, but it is not known how they compare to the actual distribution of possible empirical outcomes.

Despite the differences we are able to show between the soft and hard collars and across different uncertainty parameterizations, many of the collar designs, for a given parameterization of the shocks, lead to roughly similar results in terms of expected emissions, NPV of costs, and price variability. The exceptions are for the parameterizations where we allow offset supply shocks to be both highly persistent and negatively correlated with baseline emission shocks. Under this parameterization, in which a *perfect storm* outcome is more likely to occur (meaning outcomes that simultaneously have high domestic demand for allowances and reduced supplies of offsets), we find much larger cost disparity between the soft and hard collars, and more significant differences in the spread of outcomes in cumulative emissions net of offset purchases for a soft collar versus a hard collar.

Finally, although this analysis informs the debate on soft versus hard collars, a number of caveats are particularly relevant. First, we assumed in our modeling that the slope of marginal abatement costs declines at a constant and exogenous rate over time. Thus, we do not capture the interaction of collar design and endogenous technological growth. Second, we consider a particular form of uncertainty that allows baseline emissions and offset supply curve slopes to drift from their expected values, but are mean reverting in the long run. Other shock specifications, for example shocks that allow for drastic and permanent changes to key parameters, may be appropriate for certain types of technology adoption processes. Further, we based our analysis on a single, globally convex abatement cost for the entire economy. Clearly, this is a major simplifying assumption and although we can generate cost results in-line with

more comprehensive modeling approaches such as those undertaken by the EIA and the U.S. Environmental Protection Agency, this model should not be considered a substitute for those more robust analyses of specific detailed policy scenarios. Rather, the modeling approach here should be seen as a way to tractably analyze multiple scenarios in a context that allows for true parameter uncertainty.

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## Tables

Table 1. Parameter Values and Description

Parameter	Value	Definition	Justification
$T$	39	Terminal period	Consistent with current climate legislation for 2012 – 2050
$\beta$	0.952	Discount factor	Assumes commonly used 5% discount rate
$r_{bank}$	0.0	Interest on banked permits	From H.R. 2454
$r_{borr.}$	0.08	Interest charged on borrowed permits	From H.R. 2454
$\phi_1$	0.9	AR(1) parameter for persistence of baseline emissions shock	Based on regression results of historic U.S. CO <sub>2</sub> emissions
$\sigma_1^2$	0.10	Variance of error term in baseline emissions shock	Based on regression results of historic U.S. CO <sub>2</sub> emissions
$\sigma_2^2$	1/3500	Variance of random error term in offset supply shock	Set at low value because of relatively low values of $\bar{\gamma}_t$
$g_c$	-0.0125	Rate of decline for slope of marginal abatement cost curve	Modest rate set to reflect technological innovation in abatement costs
$c_0$	\$63/mtCO <sub>2</sub> e per GmtCO <sub>2</sub> e	Initial slope of marginal abatement cost curve	Set to approximate the emissions price path of EIA's H.R. 2454 analysis for low discount case
$z_{max}$	2 GmtCO <sub>2</sub> e	Maximum offset provision	From H.R. 2454
$B_{min,t}$	-0.15 $y_t$	Minimum bank level	From H.R. 2454
$y_t$	-	Allowance allocation	From H.R. 2454
$\bar{q}_t$	-	Expected baseline emissions	From EIA's H.R. 2454 analysis
$\bar{\gamma}_t$	-	Expected slope of offset supply curve	Based on offset supply schedules and emission prices in EIA's H.R. 2454 analysis. Explanation in Fell et al. (2010)

Notes: mtCO<sub>2</sub>e stands for metric tons of CO<sub>2</sub> equivalent and GmtCO<sub>2</sub>e stands for giga-metric tons of CO<sub>2</sub> equivalent.

Table 2. Varied Parameters

Parameter	Values	Definition
$X_1$	(10, 15, 20)	Initial width of price collar ( $X_1 = P_1^c - P_1^f$ )
$\alpha$	(10%, 20%, 30%)	Maximum percentage of each period's allowances that can be purchased at the price ceiling
$\phi_2$	(0, 0.8)	Persistence factor for shocks to offset supply
$\phi_3$	(0, -0.08)	Parameter that determines correlation between offset supply and baseline emission shocks

**Table 3. Low Persistence of Offset Supply Shock and No Correlation of Baseline Emission and Offset Supply Shocks ( $\phi_2 = 0.0$ ,  $\phi_3 = 0.0$ )**

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
No Collar			195.3 [193, 197]	132.2 [132.2, 132]	868.6 [410, 1503]	11.33	-	-	-
10	10%	27.3	194.5 [180, 203]	132.2 [120, 139]	853.2 [598, 1304]	8.54	3.0%	16.6%	6.8%
10	20%	27.8	194.5 [179, 205]	132.2 [119, 141]	850.7 [620, 1232]	8.25	3.8%	13.1%	3.3%
10	30%	28.0	194.4 [178, 206]	132.2 [118, 142]	849.9 [629, 1193]	8.15	4.0%	11.6%	2.0%
10	Hard	28.2	194.4 [178, 207]	132.2 [118, 143]	848.8 [635, 1159]	8.02	3.3%	10.9%	0.0%
15	10%	30.4	194.8 [184, 201]	132.2 [124, 136]	857.6 [527, 1370]	9.46	1.7%	8.3%	3.1%
15	20%	30.8	194.7 [183, 202]	132.2 [123, 138]	855.8 [542, 1328]	9.29	2.4%	6.8%	1.4%
15	30%	31.0	194.7 [183, 202]	132.2 [123, 138]	855.1 [546, 1309]	9.23	2.1%	6.3%	0.7%
15	Hard	31.1	194.7 [183, 202]	132.2 [123, 139]	854.7 [549, 1298]	9.19	2.0%	6.3%	0.0%
20	10%	33.8	195.0 [188, 199]	132.2 [127, 135]	861.2 [478, 1429]	10.08	1.1%	4.1%	1.1%
20	20%	34.0	195.0 [187, 200]	132.2 [127, 135]	860.3 [485, 1410]	9.96	1.3%	3.5%	0.4%
20	30%	34.1	195.0 [187, 200]	132.2 [127, 136]	860.0 [487, 1402]	9.94	1.3%	3.4%	0.1%
20	Hard	34.1	194.9 [187, 200]	132.2 [127, 136]	860.0 [487, 1401]	9.94	1.1%	3.4%	0.0%

Notes:  $X_1$  and  $P_1^c$  values given \$/ton CO<sub>2</sub>. Domestic Emissions and Emissions net of Offsets are in GmtCO<sub>2</sub>e. NPV of costs is in billions of real U.S. dollars (base year 2005). 95% confidence intervals for Domestic Emissions, Emissions net Offsets, NPV of Costs are in brackets below their respective expected values.

**Table 4: High Persistence of Offset Supply Shock and No Correlation of Baseline Emission and Offset Supply Shocks ( $\phi_2 = 0.8$ ,  $\phi_3 = 0.0$ )**

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
No Collar			195.3 [192, 199]	132.2 [132, 132]	870.4 [415, 1517]	11.71	-	-	-
10	10%	27.4	194.6 [180, 203]	132.2 [119, 139]	853.6 [600, 1309]	12.36	2.8%	16.6%	7.0%
10	20%	27.9	194.5 [179, 205]	132.2 [118, 141]	851.2 [621, 1237]	11.23	3.9%	13.2%	3.3%
10	30%	28.1	194.5 [179, 206]	132.2 [118, 142]	850.2 [629, 1198]	10.71	3.6%	11.7%	2.0%
10	Hard	28.3	194.4 [178, 207]	132.2 [118, 144]	849.4 [635, 1162]	9.84	3.5%	11.0%	0.0%
15	10%	30.6	194.8 [184, 201]	132.2 [123, 137]	857.9 [533, 1376]	9.54	2.0%	8.4%	3.3%
15	20%	31.0	194.8 [183, 202]	132.2 [122, 138]	856.0 [548, 1333]	9.26	2.0%	7.0%	1.3%
15	30%	31.1	194.7 [183, 202]	132.2 [122, 139]	855.3 [552, 1312]	9.21	2.3%	6.5%	0.7%
15	Hard	31.2	194.7 [183, 203]	132.2 [122, 139]	855.0 [555, 1301]	9.17	2.2%	6.5%	0.0%
20	10%	34.1	195.0 [187, 200]	132.2 [126, 135]	861.4 [488, 1434]	10.14	1.0%	4.3%	1.3%
20	20%	34.3	194.9 [187, 200]	132.2 [126, 136]	860.3 [496, 1415]	9.97	1.7%	3.7%	0.3%
20	30%	34.4	194.9 [186, 200]	132.2 [126, 136]	860.1 [497, 1410]	9.95	1.5%	3.6%	0.1%
20	Hard	34.4	194.9 [186, 200]	132.2 [126, 136]	860.1 [497, 1409]	9.95	1.4%	3.6%	0.0%

Notes:  $X_1$  and  $P_1^c$  values given \$/ton CO<sub>2</sub>. Domestic Emissions and Emissions net of Offsets are in GmtCO<sub>2</sub>e. NPV of costs are in billions of real U.S. dollars (base year 2005). 95% confidence intervals for Domestic Emissions, Emissions net Offsets, NPV of Costs are in brackets below their respective expected values.

**Table 5: Low Persistence of Offset Supply Shock and Negative Correlation of Baseline Emission and Offset Supply Shocks ( $\phi_2 = 0.0$ ,  $\phi_3 = -0.08$ )**

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
No Collar			195.2 [190, 200]	132.2 [132, 132]	895.6 [381, 1672]	13.87	-	-	-
10	10%	27.6	194.5 [180, 201]	132.2 [116, 141]	856.8 [623, 1356]	9.20	3.5%	19.9%	8.7%
10	20%	28.2	194.4 [179, 203]	132.2 [114, 145]	850.8 [646, 1256]	8.10	3.9%	14.8%	3.4%
10	30%	28.4	194.3 [178, 204]	132.2 [114, 145]	851.1 [654, 1216]	7.87	4.4%	13.6%	1.8%
10	Hard	28.7	194.1 [178, 205]	132.2 [115, 146]	855.9 [666, 1188]	7.74	5.0%	12.7%	0.0%
15	10%	31.0	194.7 [184, 200]	132.2 [119, 138]	862.4 [564, 1434]	10.12	2.5%	11.4%	4.7%
15	20%	31.5	194.7 [183, 200]	132.2 [118, 141]	856.5 [582, 1368]	9.24	3.0%	9.0%	1.4%
15	30%	31.6	194.7 [183, 201]	132.2 [118, 141]	855.7 [584, 1345]	9.07	3.2%	8.7%	0.6%
15	Hard	31.6	194.7 [182, 201]	132.2 [118, 142]	855.5 [585, 1335]	9.04	2.7%	8.7%	0.0%
20	10%	34.9	194.8 [186, 200]	132.2 [122, 137]	867.6 [520, 1501]	10.82	1.9%	6.8%	2.6%
20	20%	35.3	194.8 [185, 200]	132.2 [121, 139]	862.8 [536, 1465]	10.12	2.1%	5.9%	0.5%
20	30%	35.3	194.8 [185, 200]	132.2 [121, 139]	862.6 [534, 1459]	10.04	2.3%	5.8%	0.1%
20	Hard	35.3	194.8 [185, 200]	132.2 [121, 139]	862.6 [534, 1458]	10.04	1.8%	5.8%	0.0%

Notes:  $X_1$  and  $P_1^c$  values given \$/ton CO<sub>2</sub>. Domestic Emissions and Emissions net of Offsets are in GmtCO<sub>2</sub>e. NPV of costs are in billions of real U.S. dollars (base year 2005). 95% confidence intervals for Domestic Emissions, Emissions net Offsets, NPV of Costs are in brackets below their respective expected values.

**Table 6: High Persistence of Offset Supply Shock and Negative Correlation of Baseline Emission and Offset Supply Shocks ( $\phi_2 = 0.8$ ,  $\phi_3 = -0.08$ )**

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
No Collar			190.0 [158, 206]	132.2 [132, 132]	1173.4 [307, 3280]	16.70	-	-	-
10	10%	27.9	189.1 [168, 201]	132.2 [109, 144]	1080.0 [610, 2689]	10.92	2.9%	30.9%	18.0%
10	20%	29.9	188.2 [171, 199]	132.2 [106, 153]	1063.2 [696, 2342]	8.96	4.7%	22.3%	9.3%
10	30%	30.5	188.1 [171, 199]	132.2 [105, 159]	1049.5 [727, 2128]	8.44	5.4%	19.2%	6.9%
10	Hard	31.7	188.0 [170, 199]	132.2 [102, 174]	1018.7 [788, 1458]	7.23	5.1%	15.7%	0.0%
15	10%	31.6	189.3 [167, 202]	132.2 [112, 143]	1088.0 [564, 2738]	11.64	2.7%	22.9%	13.7%
15	20%	33.1	189.1 [171, 200]	132.2 [109, 150]	1058.3 [623, 2424]	10.29	3.0%	17.4%	8.0%
15	30%	34.2	188.5 [173, 200]	132.2 [108, 155]	1056.3 [672, 2233]	9.62	4.0%	15.7%	5.8%
15	Hard	35.4	188.3 [172, 198]	132.2 [105, 167]	1030.0 [730, 1659]	8.54	4.5%	13.4%	0.0%
20	10%	35.6	189.4 [166, 203]	132.2 [115, 142]	1096.8 [526, 2789]	12.36	2.2%	17.5%	10.8%
20	20%	36.9	189.3 [171, 201]	132.2 [111, 148]	1068.7 [578, 2499]	11.23	2.7%	14.2%	6.6%
20	30%	37.6	189.2 [173, 201]	132.2 [110, 153]	1054.7 [605, 2314]	10.71	2.8%	13.4%	4.8%
20	Hard	38.8	188.8 [176, 200]	132.2 [108, 162]	1038.3 [659, 1881]	9.84	3.4%	11.9%	0.0%

Notes:  $X_1$  and  $P_1^c$  values given \$/ton CO<sub>2</sub>. Domestic Emissions and Emissions net of Offsets are in GmtCO<sub>2</sub>e. NPV of costs are in billions of real U.S. dollars (base year 2005). 95% confidence intervals for Domestic Emissions, Emissions net Offsets, NPV of Costs are in brackets below their respective expected values.

**Table 7: Cost-Minimizing Soft Collar Designs**

$\phi_2 = 0, \phi_3 = 0$

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
10	104%	28.2	194.4 [178, 207]	132.2 [118, 143]	848.8 [635, 1159]	8.02	3.7%	10.9%	0.0%
15	45%	31.0	194.7 [183, 202]	132.2 [123, 139]	854.7 [548, 1298]	9.19	2.2%	6.2%	0.2%
20	36%	34.1	195.0 [187, 200]	132.2 [127, 136]	859.9 [487, 1402]	9.94	1.4%	3.4%	0.0%

$\phi_2 = 0.8, \phi_3 = 0$

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
10	80%	28.3	194.4 [178, 207]	132.2 [118, 144]	849.4 [635, 1162]	7.99	4.4%	10.9%	0.1%
15	45%	31.2	194.7 [183, 203]	132.2 [122, 139]	854.9 [555, 1302]	9.17	1.9%	6.4%	0.2%
20	36%	34.4	194.9 [186, 200]	132.2 [126, 136]	860.0 [497, 1409]	9.95	1.6%	3.6%	0.0%

$\phi_2 = 0, \phi_3 = -0.08$

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
10	27%	28.3	194.4 [179, 204]	132.2 [114, 145]	850.3 [651, 1224]	7.89	4.7%	13.9%	2.1%
15	50%	31.6	194.7 [182, 201]	132.2 [118, 141]	855.4 [585, 1335]	9.04	3.3%	8.7%	0.1%
20	28%	35.3	194.8 [185, 200]	132.2 [121, 139]	862.5 [535, 1459]	10.05	1.7%	5.8%	0.1%

$\phi_2 = 0.8, \phi_3 = -0.08$

$X_1$	$\alpha$	$P_1^c$	Domestic Emissions	Emissions net of Offsets	NPV of Cost	RMSE ( $\times 10^{-2}$ )	Floor Binds	Ceiling Binds	Above Ceiling
10	208%	31.7	188.0 [169, 199]	132.2 [102, 174]	1018.7 [788, 1457]	7.23	5.0%	15.7%	0.0%
15	159%	35.4	188.3 [172, 198]	132.2 [105, 167]	1030.0 [730, 1658]	8.55	4.5%	13.4%	0.0%
20	84%	38.6	189.0 [176, 200]	132.2 [108, 161]	1036.5 [652, 1905]	9.94	3.2%	12.0%	0.9%

Notes:  $X_1$  and  $P_1^c$  values given \$/ton CO<sub>2</sub>. Domestic Emissions and Emissions net of Offsets are in GmtCO<sub>2</sub>e.

NPV of costs are in billions of real U.S. dollars (base year 2005). 95% confidence intervals for Domestic Emissions, Emissions net Offsets, NPV of Costs are in brackets below their respective expected values.