

## **Simplified Marginal Effects in Discrete Choice Models**

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### **Abstract**

We show that after a simple normalization of explanatory variables so that they equal zero at some desired reference point, marginal effects for continuous variables in probit and logit models simplify dramatically, becoming a function of only the estimated constant term. We present similar simplifications for computation of the asymptotic variance of marginal effects, as well as for the effects of dummy variables on predicted probabilities. We provide a simple table, which in combination with raw probit or logit estimates, is all one needs to compute the desired effects.

Key Words: logit, probit, discrete choice, binary choice, marginal effect, data normalization

JEL Classification Numbers: C25, C51, C81

## **Contents**

<b>1. Introduction.....</b>	<b>1</b>
<b>2. Normalization of explanatory variables.....</b>	<b>2</b>
<b>3. Marginal and discrete effects of explanatory variables.....</b>	<b>2</b>
<b>4. Variances of marginal and discrete effects.....</b>	<b>5</b>
<b>5. Concluding remarks .....</b>	<b>7</b>

# Simplified Marginal Effects in Discrete Choice Models

Soren Anderson and Richard Newell\*

## 1. Introduction

It is well known that parameter estimates from discrete choice models, such as probit and logit, must be transformed to yield estimates of the marginal effects—that is, the change in predicted probability associated with changes in the explanatory variables (see, for example, Greene 2003, p. 667). The marginal effects are nonlinear functions of the parameter estimates and the levels of the explanatory variables, so they cannot generally be inferred directly from the parameter estimates. Some statistical packages will not directly compute these effects, forcing users to program these procedures themselves. Beyond any computational issues, we believe the approach we suggest builds intuition and clarifies the relationship between discrete choice parameter estimates and their associated marginal and dummy variable effects.

We show that after a simple normalization of the explanatory variables so that they equal zero at the desired reference point, marginal effects for continuous variables simplify dramatically, becoming a function of only the estimated constant term. We present similar simplifications for computation of the asymptotic variance of marginal effects, as well as for the effects of dummy variables on predicted probabilities.

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## 2. Normalization of explanatory variables

The first step in this simplification is to center all continuous variables at the desired reference point. The most commonly chosen reference point for calculating marginal effects in models with non-linear explanatory variables is at the variable means. Taking deviations from means will yield a zero value for the normalized variable at the mean of the original variable. If a log form is being used, the variable can first be normalized to equal one at the mean (i.e., by dividing by the variable's mean), and then logs can be taken so that the variable equals zero at its logged mean.<sup>1</sup> Of course, the reference point is not limited to variables means; variables can be normalized to equal zero at any desired value. Categorical or dummy variables can be treated similarly, and should be coded (i.e., as zero or one) so that the constant term corresponds to the desired reference group for which the marginal effects will be calculated (i.e., the omitted group).<sup>2</sup>

## 3. Marginal and discrete effects of explanatory variables

The predicted probability from a binary choice model is given by

$$E[y | \mathbf{x}] = F(\boldsymbol{\beta}'\mathbf{x}), \quad (1)$$

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<sup>1</sup> Under this model specification, marginal effects are interpreted as the change in predicted probability associated with percent changes in the continuous independent variables.

<sup>2</sup> Note that while model coefficients are invariant to centering of first-order terms, they are not invariant to centering of higher-order terms such as variable interactions or quadratic terms.

where  $y$  is a choice variable,  $\mathbf{x}$  is a vector of explanatory variables,  $\boldsymbol{\beta}$  is a vector of parameter estimates, and  $F$  is an assumed cumulative distribution function. Assuming  $F$  is the standard normal distribution ( $\Phi$ ) produces the probit model, while assuming  $F$  is the logistic distribution ( $\Lambda$ ) produces the logit model, where  $\Lambda(\boldsymbol{\beta}'\mathbf{x}) = \exp(\boldsymbol{\beta}'\mathbf{x}) / (1 + \exp(\boldsymbol{\beta}'\mathbf{x}))$ .

In these models, marginal effects for continuous variables (i.e., the marginal changes in expected probability  $\partial E[y] / \partial \mathbf{x}$ ) are equal to

$$\partial E[y | \mathbf{x}] / \partial \mathbf{x} = f(\boldsymbol{\beta}'\mathbf{x})\boldsymbol{\beta}, \quad (2)$$

where  $f$  is the corresponding probability density function. For the probit model  $f$  is given by  $\phi$ , the standard normal density function, where

$$\phi(\boldsymbol{\beta}'\mathbf{x}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\boldsymbol{\beta}'\mathbf{x})^2\right), \quad (3)$$

while for the logit model the logistic density function is given by

$$\gamma(\boldsymbol{\beta}'\mathbf{x}) = \Lambda(\boldsymbol{\beta}'\mathbf{x})(1 - \Lambda(\boldsymbol{\beta}'\mathbf{x})). \quad (4)$$

The density function  $f(\boldsymbol{\beta}'\mathbf{x})$  can therefore be thought of as a scale factor that translates raw parameter estimates into marginal effects. The point of this note is to find simple forms for this scale factor, as well as analogous factors for the effects of dummy variables and the variances of these effects.

With all continuous variables normalized to equal zero at the desired reference point,  $\boldsymbol{\beta}'\mathbf{x}$  simplifies to  $c$ , and  $f(\boldsymbol{\beta}'\mathbf{x})$  simplifies to  $f(c)$ , where  $c$  is the estimated constant term. As  $c$  gets increasingly positive,  $F(c)$  approaches 1,  $f(c)$  approaches zero, and the marginal effects

therefore approach zero. Similarly, as  $c$  becomes increasingly negative,  $F(c)$  approaches 0, and  $f(c)$  and the marginal effects again approach 0. This pattern is shown in more detail in Table 1, which gives the full range of values of  $f(c)$  for associated values of  $c$ . To convert parameter vector  $\beta$  to its associated marginal effects, one can simply multiply by the value of  $f(c)$  for the estimated value of  $c$ . Note that because both distributions are symmetric,  $f(-c) = f(c)$ , and the predicted probability at  $-c$  is given by  $1 - F(c)$ .

The discrete effect of a dummy variable is found by taking the difference in the predicted probability with and without that dummy variable equal to one. Given the normalizations described above, this results in the following simple relationship for the discrete probability effect of a dummy variable:

$$E[y | d = 1] - E[y | d = 0] = F(c + d) - F(c), \quad (5)$$

where  $d$  is the estimated parameter for the dummy variable. As  $c$  becomes increasingly positive, both the first and second terms of this expression approach 1, so the net effect of the dummy variable approaches zero. As  $c$  becomes increasingly negative, both the first and second terms approach zero and, again, the net effect of the dummy variable approaches zero. The effect of a dummy variable for any value of  $c$  and  $d$  can be readily found using Table 1.<sup>3</sup>

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<sup>3</sup> Simply substitute  $c + d$  for  $c$  to find the value of  $F(c + d)$  from Table 1.

#### 4. Variances of marginal and discrete effects

Variances for marginal effects can be calculated using the linear approximation approach (delta method). The asymptotic covariance matrix for the marginal effects is given by

$$\text{Asy. Var}[\phi(\boldsymbol{\beta}'\mathbf{x})\boldsymbol{\beta}] = \phi(\boldsymbol{\beta}'\mathbf{x})^2 [\mathbf{I} - (\boldsymbol{\beta}'\mathbf{x})\boldsymbol{\beta}\mathbf{x}'] \mathbf{V} [\mathbf{I} - (\boldsymbol{\beta}'\mathbf{x})\boldsymbol{\beta}\mathbf{x}'] \quad (6)$$

for the probit model, and

$$\text{Asy. Var}[\gamma(\boldsymbol{\beta}'\mathbf{x})\boldsymbol{\beta}] = \gamma(\boldsymbol{\beta}'\mathbf{x})^2 [\mathbf{I} + (1 - 2\Lambda(\boldsymbol{\beta}'\mathbf{x}))\boldsymbol{\beta}\mathbf{x}'] \mathbf{V} [\mathbf{I} + (1 - 2\Lambda(\boldsymbol{\beta}'\mathbf{x}))\boldsymbol{\beta}\mathbf{x}'] \quad (7)$$

for the logit model, where  $\mathbf{V}$  is the estimated asymptotic covariance matrix of  $\boldsymbol{\beta}$  (Greene 2003, p. 675). Given the above normalizations, one can show that the asymptotic variance for the marginal effect of a particular continuous coefficient estimate  $\beta$  is given by

$$\text{Asy. Var}[\phi(\boldsymbol{\beta}'\mathbf{x})\beta] = \phi(c)^2 \left( \sigma_\beta^2 + c^2 \beta^2 \sigma_c^2 - 2c\beta\sigma_{\beta c} \right) \quad (8)$$

for the probit model, and

$$\text{Asy. Var}[\gamma(\boldsymbol{\beta}'\mathbf{x})\beta] = \gamma(c)^2 \left( \sigma_\beta^2 + (1 - 2\Lambda(c))^2 \beta^2 \sigma_c^2 + 2(1 - 2\Lambda(c))\beta\sigma_{\beta c} \right) \quad (9)$$



for the logit model, where  $\sigma_c^2$  is the asymptotic variance for the constant,  $\sigma_\beta^2$  is the asymptotic variance for the estimate of parameter  $\beta$ , and  $\sigma_{\beta c}$  is the asymptotic covariance between  $\beta$  and  $c$ .<sup>4</sup>

One can now compute the asymptotic variance for the marginal effect using only Table 1 along with the estimated values of  $\beta$ ,  $c$ ,  $\sigma_\beta^2$ ,  $\sigma_c^2$ , and  $\sigma_{\beta c}$ .

To calculate the asymptotic variance for the effect of a dummy variable, we begin with the asymptotic covariance matrix for the predicted probabilities, which one can also compute using the delta method (Greene, 2003, p. 674):

$$\text{Asy. Var}[F(\beta'x)] = f(\beta'x)^2 x'Vx. \quad (10)$$

Given the above normalizations, one can show that the asymptotic variance of the predicted probability for the reference group is  $f(c)^2 \sigma_c^2$ , and the asymptotic variance for the dummy variable group is  $f(c+d)^2 (\sigma_c^2 + \sigma_d^2 + 2\sigma_{cd})$ , where  $\sigma_d^2$  is the asymptotic variance for dummy variable parameter  $d$ , and  $\sigma_{cd}$  is the asymptotic covariance between  $c$  and  $d$ . One can further show that the asymptotic variance of the effect of the dummy variable (i.e., of the difference in predicted probability) is

$$\text{Asy. Var}[F(c+d) - F(c)] = f(c)^2 \sigma_c^2 + f(c+d)^2 (\sigma_c^2 + \sigma_d^2 + 2\sigma_{cd}), \quad (11)$$

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<sup>4</sup> Although this may not appear to be a dramatic simplification, we point out that without our normalizations, this calculation could potentially involve all  $n \times n$  entries in the estimated covariance matrix, where  $n$  is the number of model parameter estimates. With the simplification it can be done on a hand calculator.

which, as before, can be easily computed using Table 1 along with the estimated values of  $c$ ,  $d$ ,  $\sigma_c^2$ ,  $\sigma_d^2$ , and  $\sigma_{cd}$ .

## 5. Concluding remarks

The need to compute marginal effects from probit and logit models rather than simply looking at raw parameter estimates, is one of the few inconveniences of an otherwise extremely convenient modeling specification. We show that even this inconvenience can be minimized with an appropriate normalization of the explanatory variables. Using only the raw logit or probit estimates and the table given herein, one can compute all the desired marginal and discrete effects, along with their variances. In addition to obviating the need for extra programming or reliance on software capabilities, this approach helps build intuition and clarifies the relationship between discrete choice parameter estimates and their associated effects.

**Reference**

Greene, William H. 2003. *Econometric Analysis*, Fifth edition  
(Prentice Hall, Upper Saddle River, NJ).

**Table 1. Lookup table for computing marginal/discrete effects and their variances**

Constant term $c$	Distribution function $F(c)$ (predicted probability)		Density function $f(c)$ (marginal effect scale factor)	
	Logit $\Lambda(c)$	Probit $\Phi(c)$	Logit $\gamma(c)$	Probit $\phi(c)$
0.000	0.500	0.500	0.250	0.399
0.213	0.553	0.584	0.247	0.390
0.312	0.577	0.622	0.244	0.380
0.388	0.596	0.651	0.241	0.370
0.453	0.611	0.675	0.238	0.360
0.512	0.625	0.696	0.234	0.350
0.565	0.638	0.714	0.231	0.340
0.616	0.649	0.731	0.228	0.330
0.664	0.660	0.747	0.224	0.320
0.710	0.670	0.761	0.221	0.310
0.755	0.680	0.775	0.218	0.300
0.799	0.690	0.788	0.214	0.290
0.841	0.699	0.800	0.210	0.280
0.884	0.708	0.812	0.207	0.270
0.925	0.716	0.823	0.203	0.260
0.967	0.724	0.833	0.200	0.250
1.008	0.733	0.843	0.196	0.240
1.050	0.741	0.853	0.192	0.230
1.091	0.749	0.862	0.188	0.220
1.133	0.756	0.871	0.184	0.210
1.175	0.764	0.880	0.180	0.200
1.218	0.772	0.888	0.176	0.190
1.262	0.779	0.896	0.172	0.180
1.306	0.787	0.904	0.168	0.170
1.352	0.794	0.912	0.163	0.160
1.399	0.802	0.919	0.159	0.150
1.447	0.810	0.926	0.154	0.140
1.498	0.817	0.933	0.149	0.130
1.550	0.825	0.939	0.144	0.120
1.605	0.833	0.946	0.139	0.110
1.664	0.841	0.952	0.134	0.100
1.726	0.849	0.958	0.128	0.090
1.793	0.857	0.963	0.122	0.080
1.866	0.866	0.969	0.116	0.070
1.938	0.874	0.974	0.110	0.061
2.063	0.887	0.980	0.100	0.047
2.197	0.900	0.986	0.090	0.036
2.342	0.912	0.990	0.080	0.026
2.502	0.924	0.994	0.070	0.017
2.681	0.936	0.996	0.060	0.011
2.887	0.947	0.998	0.050	0.006
3.134	0.958	0.999	0.040	0.003
3.444	0.969	1.000	0.030	0.000
3.871	0.980	1.000	0.020	0.000
4.586	0.990	1.000	0.010	0.000
$\geq 7.621$	1.000	1.000	0.000	0.000

Note: For negative values of  $c$ , note that  $f(-c) = f(c)$  and  $F(-c) = 1 - F(c)$ .