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Explaining Sprawl with an Agent-Based Model of Exurban Land and Housing Markets

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RESOURCES FOR THE FUTURE

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Abstract

This paper develops a model of land use in a growing community on the urban fringe and uses it to explore the spatial patterns and time path of development. The model is an agent-based model (ABM) of housing and land markets that includes as agents farmer/landowners, a developer who buys land and builds houses, and consumers who purchase housing. Housing is characterized by lot size and house size. As in all ABMs, macro-scale patterns emerge from many micro-scale interactions between individual agents, which are modeled computationally. In contrast to many other ABMs, however, the fundamentals of microeconomic decisionmaking are built into the model—consumers choose houses to maximize utility; farmers compare returns from agriculture to the expected value of their land in development; and developers purchase land and build houses so as to maximize profits. Model simulations reveal some aspects of sprawl such as "leapfrog" development, yet also confirm some results from traditional urban economic models, such as declining density and rent (land price) gradients. Sensitivity analyses on the utility function parameters, the distribution of agricultural productivity, and the travel costs highlight the importance of the economic features of the model.

Key Words: housing density, farmland, development, urban fringe, computational methods

JEL Classification Numbers: C63, R14, R31, R21, Q24, C63

Contents

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Introduction

Residential development patterns defined by discontinuous or "leapfrog" development and large average lot sizes are characteristic of American exurbia. Urban economics models in the Alonso-Muth-Mills tradition do not obtain this kind of development pattern as an equilibrium outcome (Alonso 1964; Muth 1969; Mills 1967). Rather they show more orderly patterns characterized by continuously declining density gradients from a central business district (CBD). Variations of the model that incorporate traffic congestion, spatially varying amenities, zoning, and uncertainty (in a dynamic version of the model) have generated equilibria with some of the features of sprawl—increasing density gradients in some cases, dispersed development patterns and/or pockets of vacant land in others—but the characterization of space in these models is limited (Wheaton 1982; Fujita 1989; Pasha 1996; Wu 2006). Newer equilibrium sorting models in the Tiebout (1956) tradition allow for more spatial variation but concentrate their attention on the sorting of individuals across communities with different levels of public goods and accompanying taxes (Epple and Sieg 1989; Bayer and Timmins 2007; Kuminoff, Smith, and Timmins 2010). They disregard sorting within communities and at a finer level of geographic detail, leaving most aspects of sprawl unexplored.

In this paper, we develop a model of land use in a growing urban fringe community and show that sprawl can occur as a natural result of heterogeneity in landowner price expectations and consumer preferences and variations in farmland productivity across the landscape. The model is an agent-based model (ABM) of housing and land markets that includes as agents farmer/landowners, a developer who buys land and builds houses, and consumers who purchase housing. As in all ABMs, macro-scale patterns emerge from many micro-scale interactions between individual agents, which are modeled computationally.[1] The model is dynamic; results are generated over a 20-year time horizon. All agents are heterogeneous and the landscape is modeled at the 1-acre "cellular" level.

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[1] For a comprehensive review of ABM techniques for modeling emergent spatial patterns, see Parker et al. (2003) and Heckbert et al. (2010).

Although the model has a “bottom-up” structure, it is nonetheless similar in many ways to a traditional spatial equilibrium model. All agents optimize—consumers maximize utility subject to a budget constraint; the developer and farmers maximize profits. Each consumer locates in the house that gives her the greatest utility within her budget constraint. Farmers compare the returns from agriculture to the expected profits earned from selling their land. The model’s advantage is its ability to incorporate far more heterogeneity in agents and the landscape than traditional economic models and its ability to characterize out-of-equilibrium market dynamics. Local land and housing prices are determined through interactions of the optimizing agents. In addition, modeling at the one-acre level allows for a more nuanced description of spatial outcomes than most economic models, which often divide the landscape into city and suburbs, with an arbitrary boundary between the two, or characterize location only as distance to a CBD.

The agent-based approach has been used to model land use change in several recent studies, but most of these lack the fundamentals of microeconomic decisionmaking. An exception is the work of Tatiana Filatova, Dawn Parker, and colleagues (Filatova et al. 2007; Parker and Filatova 2008; Filatova et al. 2009). In those studies, the authors model the transactions of farmer/landowners and consumers in the land market and trace out the conversion of rural land to developed uses. Their models do not include a housing market, however. Ettema (2010) develops an economic ABM of the housing market but does not look at land use change and thus cannot analyze sprawl. Robinson and Brown (2009) model both housing and land markets but do not incorporate economic behavior to motivate the agents. We advance this literature by modeling not just the conversion of farmland but also housing construction, incorporating both house and lot size, and we explicitly consider the economic aspects of agent decisionmaking. Modeling the choice over lot size allows us to analyze a key aspect of sprawl—the size of individual lots in ex-urban areas and the resulting density of development.

We model a hypothetical exurban area calibrated to data from the Mid-Atlantic region of the U.S. The exurban area has a relatively small developed region akin to a CBD, which we call the Suburban Development District (SDD). The remaining land area begins as agricultural land. We show the model’s predicted results—the amount and type of housing development over the landscape as the region grows over a 20-year simulation period—and we conduct sensitivity analyses of various parameters of the model. Some aspects of sprawl are revealed as an outcome—namely leapfrog development in which some agricultural land remains near the SDD while areas farther away are developed—yet findings of traditional urban economic models are confirmed as well, such as declining density and rent (land price) gradients. Sensitivity analyses reveal the importance of the agricultural productivity and utility function parameters and the travel costs. Agricultural productivity is key in determining which farms are converted to development; parameters of the consumers’ utility functions impact the house types built, including lot sizes; and travel costs have an important effect on the location of development. Moreover, the utility function parameters and travel costs are shown to have a significant effect on the total amount of development, land and housing prices, and the incomes of consumers who locate in the community. In these regards, the sensitivity analyses confirm the importance of the microeconomic aspects of the model.[2]

Communities across the United States have implemented a variety of programs and policies to combat sprawl. These include various kinds of zoning regulations to increase density, promote mixed use development, and preserve open space and farmland; incentives for transit-oriented and infill development; clustering requirements for subdivisions; development impact fees; urban growth boundaries; and more. Evaluation of these policies is difficult. As pointed out by Quigley and Rosenthal (2005), empirical work has been stymied by the difficulty in isolating the effects of the policies from other influences on urban structure over time and from distinguishing one policy's impacts from another's. Whether models that simulate the urban landscape can help depends on the extent to which those models reflect a reasonable representation of reality. Do they generate the sprawl patterns that are typical of American exurbia? Traditional economic models often come up short, but models of land use from other disciplines usually lack the fundamental underpinnings of economics: optimizing behavior by landowners, consumers, and other economic actors, explicit descriptions of private markets and market equilibria, and capitalization of spatial features into land values. The ABM developed here takes some steps toward resolving these shortcomings in the extant literature.

Section II lays out a brief literature review. The model is described in Section III and the baseline modeling results shown in Section IV. Sensitivity analyses for key model parameters are conducted in Section V and the final section of the paper provides some concluding remarks.

Literature Review

Economic models of urban land use are typically built on the assumption of spatial equilibrium. These models assume that over the long run land and housing prices will reach equilibrium and offset differences in spatially heterogeneous attributes such as transportation costs to a central business district (CBD), neighborhood amenities, and access to employment. Early models in the urban economics literature used a monocentric city framework in which location is defined purely by distance to a CBD where all jobs are located (Alonso 1964; Muth 1969; Mills 1972). Decreasing land prices and density gradients are a feature of these monocentric models—i.e., average land prices and number of houses per acre fall as distance to the CBD increases. The basic monocentric framework has been expanded to incorporate growth and uncertainty, environmental and open space amenities, zoning and other regulations, and

[2] In a companion paper, we use the model to analyze the impacts of large lot zoning (Magliocca et al. 2010).

other factors (Capozza and Helsley 1990; Pasha 1996; Wheaton 1982; Wu and Plantinga 2003; Wu 2006).

In recent years, economists have merged this traditional urban economics approach to location choice with the theory of household sorting based on public goods and taxes that was developed by Tiebout (1956). These so-called equilibrium sorting models relax the assumption of homogeneity of households that is typical in the monocentric city models and allow different groups to sort into different communities based on preferences and income (Epple and Sieg 1999; Sieg et al. 2002; Walsh 2007; Smith and Timmins 2010). Natural amenities and publicly-provided goods often vary across communities in these models, and land and housing prices are endogenously determined. However, the focus is across communities that have different levels of public goods and taxes and not within communities, which is where the key aspects of sprawl are often most relevant. [3]

Although spatial equilibrium models have many desirable features—a rigorous representation of agent behavior and capitalization of spatial differences in amenities and other factors into land values (Irwin, 2010)—several strong assumptions are made to ensure analytical tractability. First, spatial equilibrium is a particularly restrictive assumption, because out-of-equilibrium dynamics, such as path dependence of development location, are important drivers of urban systems (Arthur, 2006; Brown et. al., 2005; Irwin, 2010; Tesfatsion, 2006). Second, in order to ensure analytical tractability, agent heterogeneity is typically quite limited. Third, the description of geographic space in the models is typically limited.

Agent-based modeling provides an alternative approach. Parker et al. (2003) provide a detailed review of the different types and applications of ABMs for modeling land use change. Although the models differ widely in their focus, assumptions, and formalizations of agent interactions, they all rely on interactions between many distributed agents to form emergent larger-scale patterns (Manson, 2001). Thus, microeconomic fundamentals can be incorporated into individual agents' decisionmaking rules to simulate emergent trends in a spatially explicit framework.

However, examples of incorporating microeconomic decisionmaking rules into ABMs are few. Filatova et al. (2009) and earlier papers (Filatova et al., 2007; Parker and Filatova, 2008) present the fullest, economically-based implementation of an agent-based land market to date. The authors relax the conventional spatial equilibrium assumption by explicitly modeling decentralized, bilateral transactions between land buyers and sellers. Transaction prices for land

[3] A recent paper by Epple, Gordon, and Sieg (2010) models the location decisions of households both within and across communities, but the authors solve for the specific assumptions that allow them to assume a single house price within a community and focus attention only across communities. Those assumptions center on limiting preference heterogeneity.

are determined by specifying a buyer's and seller's willingness to pay and willingness to accept, respectively, which are then adjusted to form bid and asking prices accounting for different market power scenarios. The authors have provided valuable insights into methods for relaxing spatial equilibrium assumptions and incorporating microeconomic decisionmaking into the ABM framework. However, their model lacks a housing market and cannot capture the feedbacks between land and housing markets that influence spatial rent structures and that can be used to more fully analyze development patterns.

Ettema (2010) presents an economic ABM of a housing market, which explicitly simulates relocation and price setting processes. Housing prices are produced through bilateral transactions between a buyer and seller, and are constrained by the agents' perceptions of market conditions and by the buyer's budget constraint and housing preferences. However, the model's design cannot accommodate spatial characteristics of housing goods or the formation of spatially heterogeneous price expectations.

Robinson and Brown (2009) present a detailed spatial representation of regional development patterns in a GIS-based ABM. Land and housing markets are integrated by the conversion of farm parcels to residential subdivisions of different densities by developers, and the acquisition of deeds to subdivision lots by residential household agents. In addition, township agents are able to specify zoning and land acquisition polices to alter development patterns. However, land conversion is not based on microeconomic fundamentals. Farm and residential parcel sales probabilistically occur on the basis of land or lot characteristics. No markets are represented in which competing land uses can be valued, and the economic constraints or opportunity costs of the acting agents are not considered.

The model we describe in this paper builds upon the above ABMs by integrating many of their innovations into one framework capable of simulating development density patterns through coupled housing and land markets. Similar to Robinson and Brown (2009), housing and land markets are linked through the supply and demand functions of the developer and consumer households, respectively; however, our agents respond directly to and create market prices subject to economic constraints. This allows for more dynamics in land and housing markets based on the economic conditions in these markets. Mechanisms of land and housing transactions in the model are built upon the bilateral transaction framework developed by Parker and Filatova (2008) but are expanded to link the developer's rent expectations in the housing market to his bid prices in the land market. Price expectations play a role similar to that in Ettema's model (2010). Expectations of future prices and market conditions are used to compare present and potential future transactions, directly influencing the timing of transactions. In addition, our agents' price expectation models are designed to capture spatially dependent price trends that directly affect the location of housing and land sales, and they are updated each period to reflect their performance accuracy. These advances allow us to investigate both the supply- and demand-side forces driving spatial patterns of land conversion and development density over time.

The Model

In our model, we consider a single jurisdiction in an ex-urban setting. The jurisdiction contains a Suburban Development District (SDD) that has some initial residential development; remaining land is in agriculture. Our objective is to characterize the additional residential development that takes place over time as the population grows and land is converted from farming. The jurisdiction is not strictly "open" or "closed" in the traditional economic sense. The population size is endogenous consistent with an open city model. On the other hand, the utilities and incomes of residents are also endogenous, which is typical of a closed city.[4] We elaborate further on this point below.

The model incorporates the decisions of three types of optimizing agents: consumers, a single developer, and farmer/landowners.[5] Consumers are motivated to choose housing and other goods to maximize utility subject to a budget constraint, where housing is characterized by house size, lot size, and location relative to the SDD. Consumers are differentiated by income and preferences over different types of housing. Farmers compare the returns from farming each period to the expected profit from selling to developers. Farmers differ in both farm size and productivity and in how they form expectations about future land prices. The developer forecasts future housing sales, and purchases land from farmers to build housing to maximize profits. We abstract from any consideration of externalities, and we do not explicitly model the government sector so the model does not incorporate property or other taxes or provision of public services.

The landscape is modeled at a 1x1-acre level and the full landscape covers 6,400 acres (80 acres square), or 10 square miles. Each period, land use decisions are made for each 1-acre cell. The landscape is highly stylized and does not represent an actual jurisdiction, though it is parameterized using information on agricultural values, incomes, house prices, and so forth from ex-urban areas of the Mid-Atlantic region. Our purpose here is not to replicate the development patterns in one specific location. Rather, we want to illustrate how a zoning policy can be analyzed within the ABM construct and compare the spatial patterns and time paths of development predicted in such a model with and without zoning.

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[4] See Pines and Sadka (1986) for more on the distinction between open and closed city models.

[5] The assumption of a single developer makes the already very complex model easier to solve. Having multiple developers would add a layer of complexity and provide little additional insight for the purpose of modeling spatial patterns of development. We do not assume that the developer has monopoly power.

Consumer Utility Maximization and Willingness to Pay for Housing

A consumer c derives utility from a general consumption good and a housing good. Each housing good can be considered a 'bundle' of one of eighteen different housing types, which are distinguished by different combinations of three different house sizes (h) – 1,500, 2,000, and 2,500 square feet – and six different lot sizes (l) – ¼ acre, ½ acre, 1 acre, 2 acre, 5 acre, and 10 acre; these lot and house sizes are meant to represent those available in a typical ex-urban area. We assume that consumer c's utility function has a Cobb-Douglas form:

$$U(c,n) = \left(I_c - P_{ask|n} - \psi_n\right)^{\alpha_c} h_n^{\beta_c} l_n^{\gamma_c} \quad ; \qquad (1)$$

where I_c is income, ψ_n is the travel cost from the location of house n to the CBD, and β_c and γ_c are the consumer's idiosyncratic preferences for house and lot sizes, respectively. $P_{ask|n}$ is the developer's asking price for house n, which we say more about below.

The willingness to pay (WTP) of consumer c for any given house n is defined as being the portion of the consumer's income he is willing to pay for housing as given by the Cobb-Douglas structure:

$$WTP(c,n) = \left(I_c - \psi_n\right)\left(\beta_c + \gamma_c\right); \qquad (2)$$

Although this functional form for the utility function implies that consumers would pay the same amount for all housing net of transportation costs, in our discrete framework consumers identify the housing option among those available that provides the greatest utility and adjust their bids on other houses relative to this most preferred option. Specifically, the maximum utility across all houses is found, U^*. Then, holding U^*constant for all housing options, the rent that would produce this level of utility—i.e., an optimal rent such that the consumer is indifferent among housing options—is calculated for each house:

$$R^*(c,n) = I_c - \psi_n - \left(\frac{U^*}{h_n^{\beta_c} l_n^{\gamma_c}}\right)^{\frac{1}{\alpha_c}}; \qquad (3)$$

Consumers are therefore willing to bid more or less than the constant share of income for housing depending on their income and idiosyncratic preferences for house and lot size and on the seller's asking prices for the houses that are actually available at a point in time. In section II.D below, we describe how the consumers choose exactly which houses to bid on, the bid price for each house, and the process by which consumers are matched to houses and the housing market clears.

Developer Purchase of Land and House Construction

There is a single developer in the model who buys land from farmers and builds the number and type of houses that he expects will maximize expected profits. To calculate expected profits, the developer needs to form predictions about housing rents for houses of different types. He starts each period with information about the sales of housing of different types and locations from past periods. He obtains information on past rents, lot sizes, house sizes, number of bidders before sale, percent that sale price was above/below the original asking price, and the number of houses of each type in any given neighborhood.

The developer uses this information to form expectations of future rents for each undeveloped cell. The rent projections for each housing type account for the distance of the given cell from the SDD and associated travel costs. Projected rents are a combination of weighting between local (when it is available) and regional (the entire developing area) rent information. [6] More detail on the approach used to form these price expectations can be found in Magliocca at al. (2010a).

Based on projected rents, the developer's potential (annualized) returns can be calculated for every housing type in every undeveloped cell by subtracting the costs of construction and the price of land for the given cell. The developer determines the maximum return for each cell from all of the returns over all possible housing types for the given cell. Those returns will vary across any given farm because of distance from the SDD. The developer's willingness to pay for a farm, on a per acre basis, WTP_F, is then the average of these maximum returns over the extent of the farm:[7]

$$WTP_F = \frac{\sum_{i_F} R_{i_F}^{max}}{A_F} \tag{4}$$

where R_{iF}^{max} is the maximum return for cell i on farm F and A_F is the acreage of farm F. In Section II.D.1 below, we describe how these WTP_F*'s* are used to form the developer's bid prices for farmland and the resulting final land transaction prices.

[6] There are situations when there is no past information about the price of a particular house type in one region because it has never been built there. In these cases, expected rent must be inferred from houses built elsewhere in the region.

[7] This methodology assumes that the developer earns zero economic profit.

Farmer Land Conversion Decisions

Farmers provide a supply of land for future residential development. In each model period, a farmer decides whether to sell his land to a developer or continue farming until the next period when he takes in new information and goes through the same decision process again. Each farmer's decision to sell is based on the expected return from selling his farm relative to the value of the farm's agricultural return per acre in perpetuity. When a farmer decides to sell his land, he sells his entire farm.[8] Farms are heterogeneous in size, productivity, and operating costs.

Heterogeneity also exists in how farmers form expectations of future land prices. Using an approach adapted from price expectation formation in the agent-based financial literature (e.g. Arthur, 1994, 2006; Axtell, 2005), each farmer is randomly assigned a set of prediction models that vary in, for example, the length of time over which past prices matter, the functional form of the effect of past prices on current prices, and the amount of remaining land in the surrounding area (to reflect supply constraints).[9] Farmers adapt their prediction models over time according to the success of past predictions. Based on the accuracy of their predictions of the price of land in period *t-1*, each farmer uses his most successful prediction model to set his willingness to accept (WTA_F) for sale of his land in the current period. As long as this price is above the returns from agriculture, it represents a price floor for the farmer in negotiations with the developer. This procedure enables the farmer to capture speculative gains from sale of his land when development pressure is high, while enforcing a rational threshold below which the farmer would be better-off farming.

Market Interactions

Figure 1 shows a schematic of agent decisionmaking and market interactions in the model, along with the sequence of events. In the following sections, we describe how we use the information on agent decisionmaking that we outlined above to characterize the interactions in the marketplace and how final transaction prices are obtained for both housing and land.

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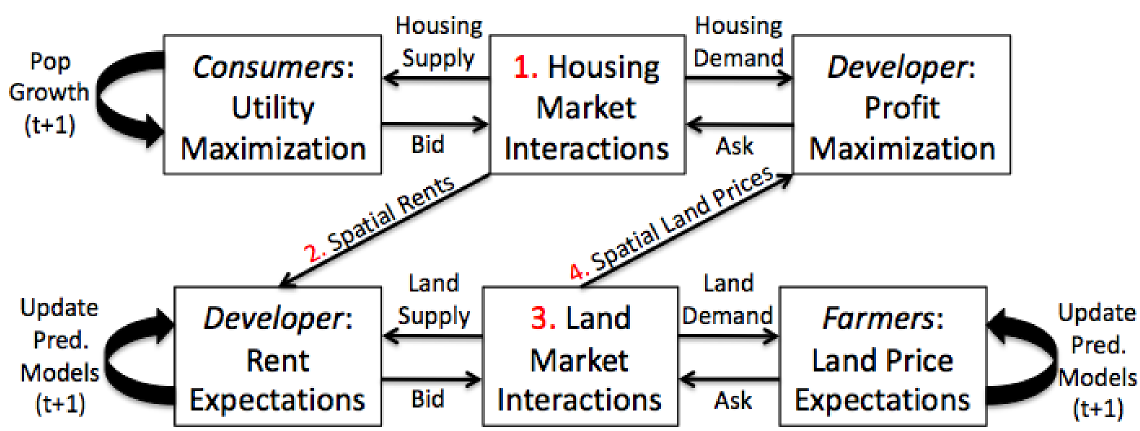
[8] Selling portions of farms would greatly complicate the model. In addition, selling an entire farm is typical practice in many land markets; for example, our review of land sales data and discussions with developers and farmers in exurban counties in Maryland suggests this is the norm there.

[9] Details are provided in the Appendix to Magliocca et al. (2010a).

Figure 1. Interactions among Agents in the Land and Housing Markets



Note: The numbers indicate the (counter-clockwise) sequence of events within one simulated time period (t). Agents (*italics)* are labeled with the underlying conceptual model that governs their behavior. Inter-temporal processes (t+1) shown include updating developer's rent prediction models, updating the farmers' land price prediction models, and exogenous growth of the consumer population.

The Land Market

If the developer's WTP_F for a given farm is greater than the farmer's WTA_F, then the two enter into bilateral negotiation to determine the final transaction price of each parcel. We assume that the transaction price will depend on the degree of bargaining power that each agent has. Our measure of bargaining power in the land market, ε, is adapted from Parker and Filatova (2008) and captures differences in the developer's demand for and the farmers' supply of land at the initial WTP_F of the developer.

$$\varepsilon = \frac{\left(d_{Land} - A_{F^*}\right)}{\left(d_{Land} + A_{F^*}\right)}; \tag{5}$$

where d_{Land} is the acreage demanded by the developer and A_{F*} is the acreage supplied by participating farmers. $F*$ is the subset of all farmers who participate, i.e., farmers for whom the condition $WTP_F > WTA_F$ is true. If the developer demands more land than farmers supply, ε is positive and farmers set their asking prices above their WTA_F*'s*. If farmers supply more land than is demanded by the developer, ε is negative and the developer will bid below his initial WTP_F. Bargaining power is dynamic because the amount of land supplied by farmers depends on the initial WTP_F of the developer. Also, the developer's WTP_F for a given farm depends on the level of rents in the housing market. Thus, the housing and land markets are explicitly linked. After bargaining power is observed, farmers participating in the market form an asking price that is the greater of $WTA_F(1+\varepsilon)$ and the returns from agriculture. The developer forms a bid price for each farm that is the lesser of his initial WTP_F and $WTP_F(1+\varepsilon)$. We assume that the final transaction price for each farm is the average of the bid price and the asking price.

This method for endogenously generating land prices has several important consequences. First, the land market is responding to the behavior of farmers who are responding to uncertain future prices, each with idiosyncratic approaches to predicting those prices. Predictions are made based on past and present trends in land prices and substantial uncertainty about future trends. In general, farmers with the highest agricultural productivity will have the lowest probability of selling and the highest asking prices all else equal, but the process of land sale is less orderly than in traditional economic models. The timing of farm sales is not based purely on relative values in agriculture and development because of the uncertainty in future prices and farmer heterogeneity in predicting those future prices.

The Housing Market

To allocate houses to consumers, the model goes through a careful matching process. The first step in the process is to calculate the price that each consumer is willing to bid on each house. This bid price will depend on the optimal rent for each house, which is determined by preferences and income and is given by equation (3) above, and on the level of competition that the consumer faces for each house from other consumers. We define a competition factor faced by consumer *c*, by comparing the number of houses consumer *c* will bid on, N_c, to the number of other consumers bidding on the same houses, M_{Nc}. This competition factor, HMC_c, is given by:

$$HMC_c = (N_c - M_{Nc})/(N_c + M_{Nc}) \quad (6)$$

HMC_c is positive if there are more consumers bidding than there are houses the consumer is bidding on; it is negative if there are fewer consumers bidding than houses the consumer is bidding on. We use HMC_c to adjust the optimal rent from equation (3) above for each house in the affordable set of houses. The extent of the change in the bid price for a house depends on the difference between the asking price for the house and the maximum the consumer will pay for a house out of income ($WTP(c,n)$) from equation (2)). Consumers with higher income or with a higher preference for housing out of income, for example, would adjust their bid prices more for any house given the level of competition. The determination of the bid price by consumer *c* for house *n* is defined as:

$$P_{bid}(c,n) = R^*(c,n) + HMC_c[WTP(c,n) - P_{ask}(n)] \quad (7)$$

where $P_{ask}(n)$ is the developer's asking price for house *n* and all other variables are as previously defined.

The adjustment of consumers' bid prices in response to market conditions allows consumers to try to maximize utility in the housing choice but also improve the likelihood that they will be the highest bidder.[10]

After the bidding process is completed, the highest bidder on each house is identified. For each consumer who has at least one "winning bid", the house or set of houses for which the consumer owns the highest bid is identified. The consumer's utility is recalculated (using eq. 1) for each of these houses using the winning bid instead of the initial asking price. Given these new levels of utility, the consumer is matched with the house for which that consumer is the highest bidder and derives the highest utility. Once a consumer is matched with a house, both the consumer and house are removed from the market. The matching process continues with the remaining consumers until all consumers are matched with houses or until all houses are occupied or all positive bids are exhausted. This process is carried out at each time step.

Model Parameterization

Figure 2 shows the hypothetical ex-urban area that we are modeling. Total region size is 6,400 acres (80 acres square), or 10 square miles. The established developed area, the SDD, is shown as the dark blue half-moon shaped region at the top of the figure. There are initially 334 households located in the SDD, and housing there includes four lot sizes -- ¼ acre, ½ acre, 1 acre and 2 acre -- and three housing sizes, small (1,500 square feet), medium (2,000 square feet) and large (2,500 square feet). All remaining land is initially in agriculture. The region has 50 farms, delineated in Figure 2 by the different colored areas. Farmers are endowed with heterogeneous plots of land that differ from each other by their size, agricultural productivity, and operating costs, and the farms are randomly assigned to the landscape.[11]

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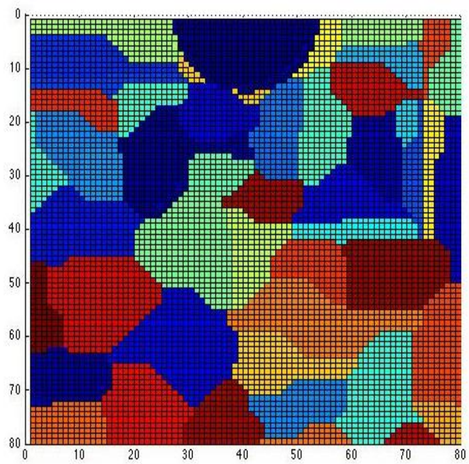
[10] See Magliocca et al (2010a) for more detail on the specifics of the housing competition.

[11] The colors in Figure 2 have no intrinsic meaning and are not related to colors in subsequent figures in the paper; they are simply used as a way to distinguish one farm from another.

Figure 2. Initial Landscape Configuration and Location of 50 Farms



The number of households wanting to move into the region is assumed to grow at 10 percent per year. Developers buy land from farmers and build houses of varying types (lot size and house size) to maximize profits. The model tracks growth over a 20-year period.

Table 1 shows the parameters used for the baseline model. The first several rows show baseline assumptions for the farm sector, including the distribution of farm size and agricultural returns, or productivity. Both the size and productivity distributions are based on data from farms in the Mid-Atlantic region from the 2007 Census of Agriculture. The standard deviation on farm productivity is relatively small, so there is not a great deal of variation in land productivity in this model. In addition, we assume no scale economies in farming, so that farm size does not affect average return per acre.

In addition to the four lot sizes that exist in the SDD, we allow for 5-acre and 10-acre lot sizes as well. This means that there are potentially 18 house and lot size combinations (6 lot sizes and 3 house sizes).

The costs of housing construction include building construction costs and infrastructure costs such as streets and sewers or septic systems. We use an average of construction costs in urban areas of the Mid-Atlantic region, using a range of $85 to $165 per square foot (US Census Bureau)). Infrastructure costs include estimates of road costs and sewer and septic costs, with estimates derived from Frank (1989), Fodor (1997), and more recent evidence from Juntunen and Knaap (forthcoming).[12] Incomes of the households who want to move into the region are

[12] In the case of 5- and 10-acre lots, we assume there are septic systems instead of sewers.

distributed according to a log-normal distribution and range from $40,000 for the lowest quintile to $200,000 for the highest quintile. These data are based on median household incomes for suburban counties in the Mid-Atlantic region (Delaware, Maryland, Pennsylvania, and Virginia) from the 2000 Census.

Table 1. Key Parameter Values

Number of farms	50
Mean (std. dev) farm size, in acres[1]	128 (71)
Mean (std. dev) agricultural return, in $/acre[1,2]	$2,486 ($249)
Housing construction cost per square foot[3]	$85- 165
Infrastructure cost per housing unit[4]	
One acre lots or smaller	$10,000 - $20,000
2 acre lots	$23,000
5+ acre lots	$30,000 - $40,000
Household Income Distribution[5]	
Income range	$40,000 - $200,000
Low income range	$40,000 - $59,999
Middle income range	$60,000 - $99,999
High income range	$100,000 - $200,000
Mean (Std. dev.)[6]	$86,493 ($39,302)
Share of income on housing expenditure , $\beta+\gamma$[7]	
Low income consumers	.35 - .42
Middle income consumers	.27 - .34
High income consumers	.18 - .26
Proportion of housing exp. on land, $\gamma/(\beta+\gamma)$[8]	.10 - .90
Transportation costs ($/mile)	
Time	$1.30
Out of pocket	$0.54

[1]Data from Census of Agriculture (2007).
[2]Agricultural return is the discounted net present value of average farm income divided by total farm acreage for mid-Atlantic states (Delaware, Maryland, Pennsylvania, and Virginia).
[3]U.S. Census Bureau, Manufacturing, Mining and Construction Statistics. http://www.census.gov/const/www/charindex.html#singlecomplete.
[4]From Frank (1989), Fodor (1997), Juntunen and Knaap (forthcoming).
[5]Based on median household incomes for suburban counties in the Mid-Atlantic region (Delaware, Maryland, Pennsylvania, and Virginia) from the 2000 Census.
[6]Household income is log-normally distributed.
[7]Safirova et al. (2006). Calculations in that study based on U.S. Bureau of Labor Statistics' Consumer Expenditure Survey.
[8]Carliner (2002). Range expanded to allow for more heterogeneity.

Parameters of the consumer utility functions were developed based on an examination of available evidence in the literature and from Census data. The share of income spent on housing is assumed to vary within income groups (Safirova et al. 2006). Within each income group, parameter values are randomly drawn from the range of values shown in Table 1. Travel costs for households are assumed to depend both on time and monetary costs, and both are specified based on available evidence. Time costs are assumed to be $1.30/mile, and monetary costs are $0.54/mile (Bureau of Transportation Statistics, 2007). [13]

Baseline Model Results

The model was run 30 times and each run tracks growth over a 20-year simulation period. Farmers' locations and agricultural returns, and the assignment of prediction models for farmers and developers, were held constant across all runs, as were the initial distribution and location of housing types in the SDD. Draws from income and consumer preference distributions were allowed to vary randomly across each of the 30 runs. Holding landscape features constant across runs eliminates sources of geographic variability and allows us to focus on the effects of agent heterogeneity on development patterns. Stochastic elements in the model limit the insight of any single model realization. Instead, we show average outcomes, including maps of the most likely, or 'average', development patterns.[14] Figure 3 shows these average spatial outcomes at four periods, with the colors denoting the housing types as shown in the label at the side of the maps; type 18, for example, shown in dark red, is the largest house and lot type (2,500 square foot house on a 10-acre lot) while type 1, shown in dark blue, is the smallest house and lot type (1,500 square foot house on a ¼ acre lot). The darkest blue area on the maps is the undeveloped farmland remaining at each time step. Figures 4 and 5 show the number of houses of each of the six possible lot sizes and the number of each of the three house sizes, respectively, at each time step.

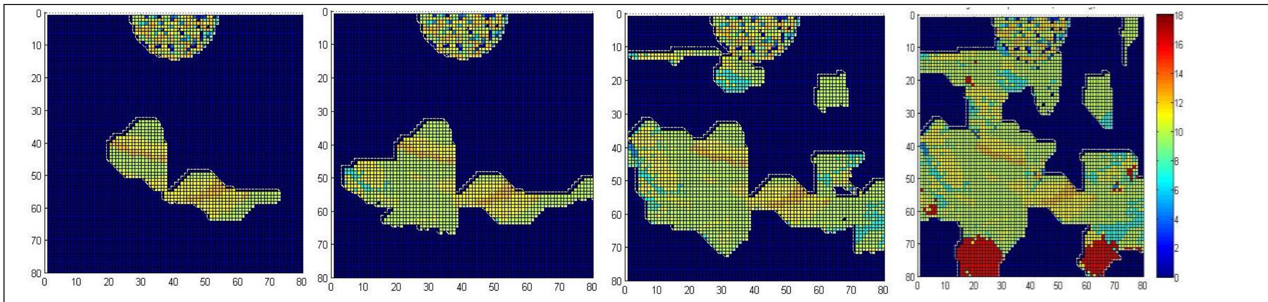
As Figure 3 and 4 make clear, the relatively large 1- and 2-acre lots are the most prevalent. Two-acre lots are particularly noticeable. These are house types 10, 11, and 12, which are the pale green, yellow, and light orange colors in the Figure 3 maps. In the bar graph in Figure 4, the 2-acre lots are shown by the purple bars. Of the 2,488 houses in existence by T=20, 68 percent are on 2-acre lots and 71 percent are on either 1 or 2-acre lots. This outcome results from a combination of relatively low land to housing costs and infrastructure costs that increase with lot size but at a decreasing rate (see Table 1). In ex-urban areas in the mid-Atlantic states,

[13] We assumed time costs to be a function of average road speed (30 mph), average number of workers per house (2), average wage per person ($30/hour), value of time as a percent of wage (50%), and the road network indirectness coefficient (0.3), which is the ratio of network distance to the Euclidian distance.

[14] For each time step displayed, the development pattern consists only of cells that were developed above a threshold frequency, which was calibrated to produce an 'average' development pattern that closely approximated the calculated average percent-developed area and dispersion across 30 runs. Within each of those cells, the housing type with the highest probability of occurrence is mapped. In Magliocca et al. (2010b), we discuss the distributional results in more detail; here we focus on average outcomes.

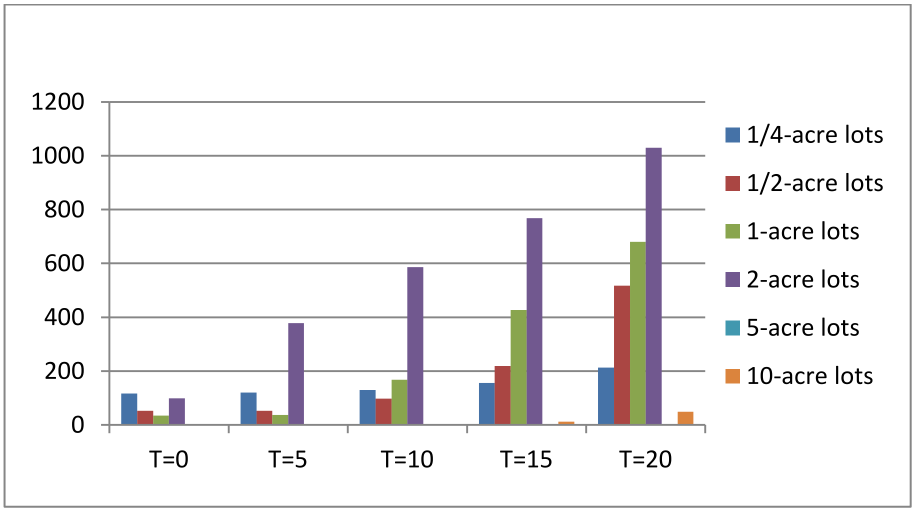
these relatively large 1- and 2-acre lots are quite common.[15] Figure 5 shows that small houses (1,500 square feet) are the most common of the three sizes we include in the model: 46% of houses in existence in the final time period are small, 32% are medium (2,000 square feet), and 22% are large (2,500 square feet). The prevalence of the smaller houses is an economic outcome, as these are the least expensive options.

Figure 3. Spatial Patterns of Development at T=5, 10, 15, and 20



Note: Color scale at right shows lot types, where Type 18 (dark red) is large house on 10-acre lot, Type 17 is medium house on 10-acre lot, and so forth down to Type 1 (dark blue), small house on ¼-acre lot.

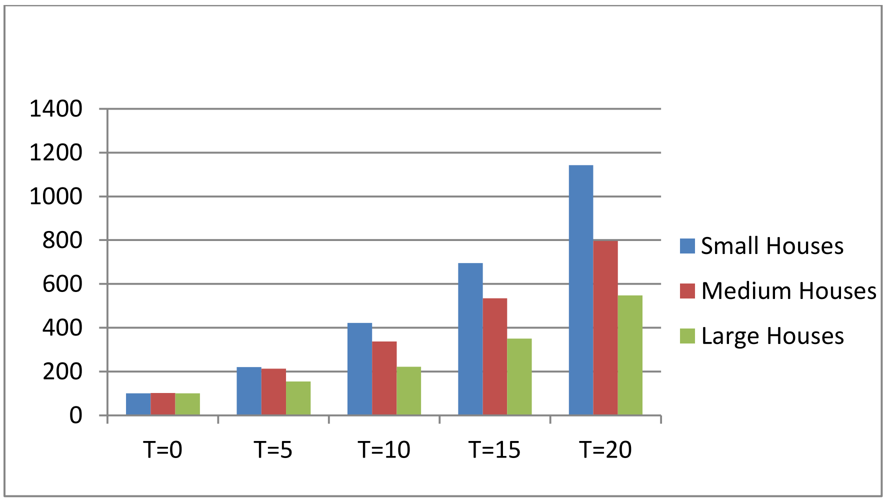
Figure 4. Number of Houses in Each Time Period, by Lot Size



[15] In McConnell, Walls, and Kopits (2007), one county on the fringes of Washington, DC, had an average lot size of 2.6 acres. Lichtenberg, Tra, and Hardie (2007), using data from suburban and ex-urban counties in Maryland, report an average lot size of 0.4 acres in areas with access to public sewers and average lot size of 3.0 acres in areas with septic systems.

The spatial patterns observed in the maps are a result of a combination of factors. Agricultural productivity plays a role: much of the development that takes place in the South, away from the SDD, is a result of comparatively cheaper farmland. Our price prediction models also play a role. Because developers form predictions about future housing prices in part using evidence about past prices in the local region, profitable development in a particular region tends to lead to more development in the future in that same region. Eventually this development increases land prices, which shifts development to other areas, but some path dependence in development patterns is observed.[16]

Figure 5. Number of Houses in Each Time Period, by House Size



The results clearly exhibit some of the sprawl development patterns that we often observe on the urban fringe. In particular: (i) a significant amount of large-lot development and resulting large average lot sizes—the average at T=20 is 1.42 acres; (2) some dispersed development—i.e., not all developed land is contiguous; and (3) leapfrog development—i.e., farmland closer to the SDD remains in agriculture while land farther out is converted to development. On the other hand, some of the general patterns predicted by economic theory are also observed. Figures 6 and 7 show the density and land price gradients, respectively—i.e., the average number of houses per acre and the average land price per acre as functions of distance from the SDD.

[16] In O'Sullivan's (2009) agent-based model of residential sorting, he finds that segregation of neighborhoods can occur because of what he refers to as "self-reinforcing changes" in household locations. This is similar to the path dependence we are referring to.

The density gradient declines as expected; the estimated slope of the function is -0.162—i.e., a 1-mile increase in distance from the SDD leads to an average reduction in houses per acre of 0.16. Since the average density is 0.703 houses per acre, this is about a 23 percent reduction for each mile farther from the SDD. However, Figure 6 makes clear that the model does not predict a constant decline; density declines sharply closer to the SDD then levels out.

Figure 6. Density Gradient

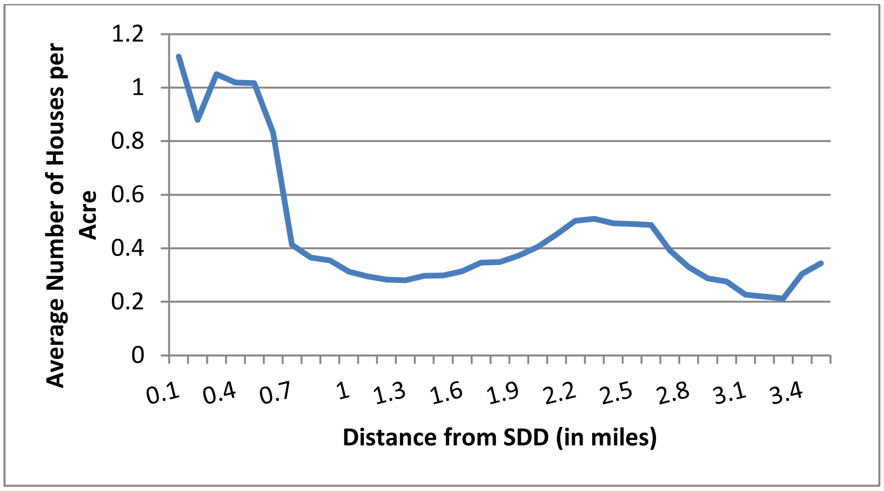


Figure 7. Land Price Gradient

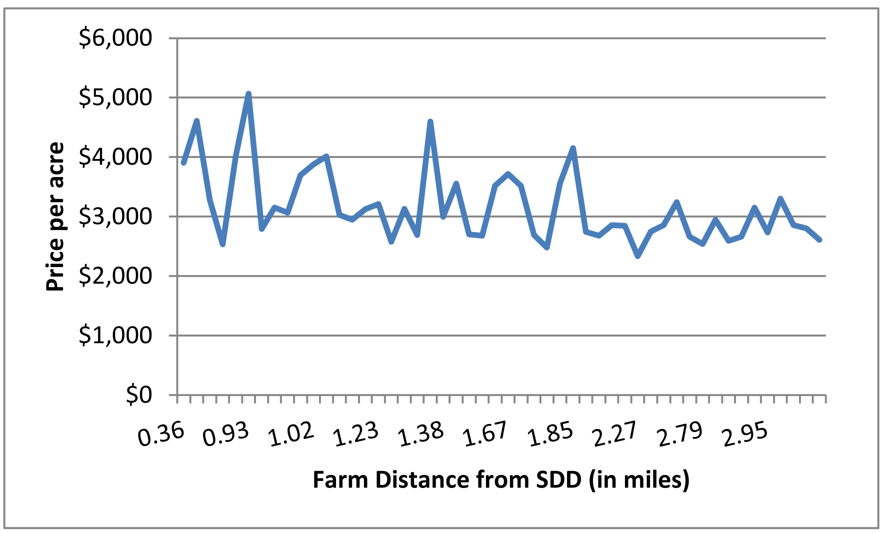


Figure 7 shows that average per-acre land prices tend to decline with distance from the SDD—the farm closest to the SDD that sells during the 20-year period receives a price of $3,902 per acre while the farm farthest from the SDD sells for $2,607 per acre—but spikes upward and downward are observed at various distances. This is a result of variations in agricultural productivity across the landscape and the timing of land sales. Land prices tend to rise over time, as shown below in Figure 8, due to increasing land scarcity and the greater productivity of remaining farms. Some of the farms that lie farther from the SDD may sell later and receive a higher price. We find that land prices increase, on average, about 1.7 percent per year.

Figure 8. Average Land Prices Over Time

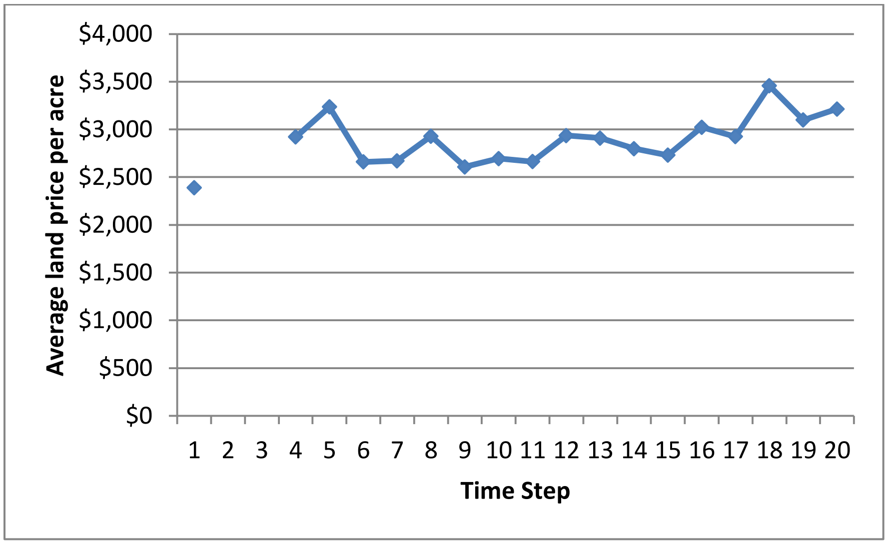
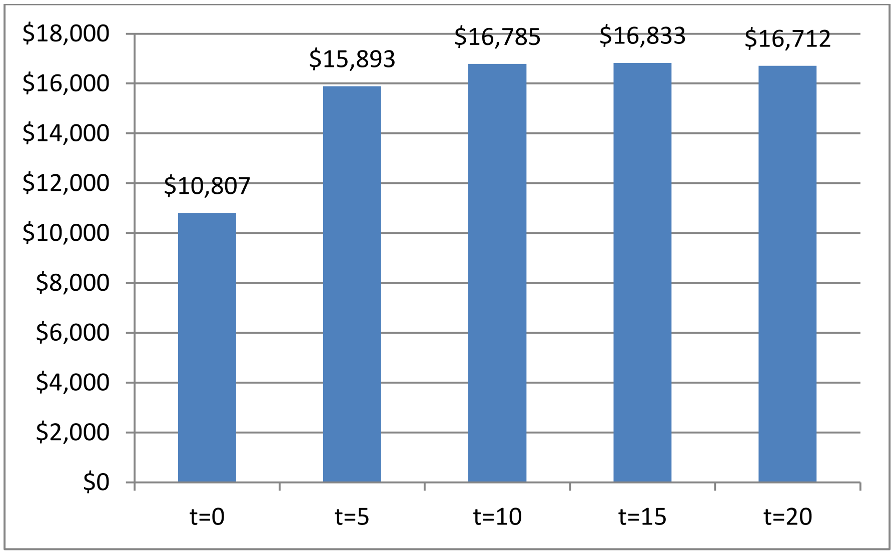


Figure 9 shows the time path of average housing rents. We find that rents rise significantly in the early years as the population grows. This is primarily a result of the developer meeting the demands of consumers who want houses on larger lots, which also tend to be the most profitable for the developer. As Figure 3 above showed, a substantial number of 2-acre lots are developed in the early years and this leads to a sharp increase in average rents. Eventually, rents stabilize and even decline a bit in the final period. By this final period, a number of smaller lots are starting to be developed (see Figure 3); this tends to dampen housing rents.

Figure 9. Weighted Average Annual Housing Rents over Time



Sensitivity Analysis

As in all ABMs, a combination of factors generates the observed model outcomes. In our model, land prices, housing rents, the number of houses, house types (lot size and house size), and the preferences and incomes of consumers who end up located in the area are all endogenous. Final values for these variables are the result of complex interactions among agents and the forces of the marketplace, with important feedback effects between land and housing markets. Also important are underlying fundamentals such as farmland productivity and the preferences of consumers attempting to locate in the community.

In this section, we conduct sensitivity analyses on three key parameters of the model: farmland productivity, consumer preferences for house size versus lot size, and travel costs. Specifically, we run the model under the following conditions:

(1) equal and constant agricultural productivity across all farms equal to the mean productivity in the baseline model runs;

(2) higher agricultural productivity in the “western” region – all farms that have at least one cell (1 square acre) within a distance of five cells from the western border have 10 percent higher productivity than the most productive farm in the baseline model runs; all remaining farms have the same productivity as in the baseline;

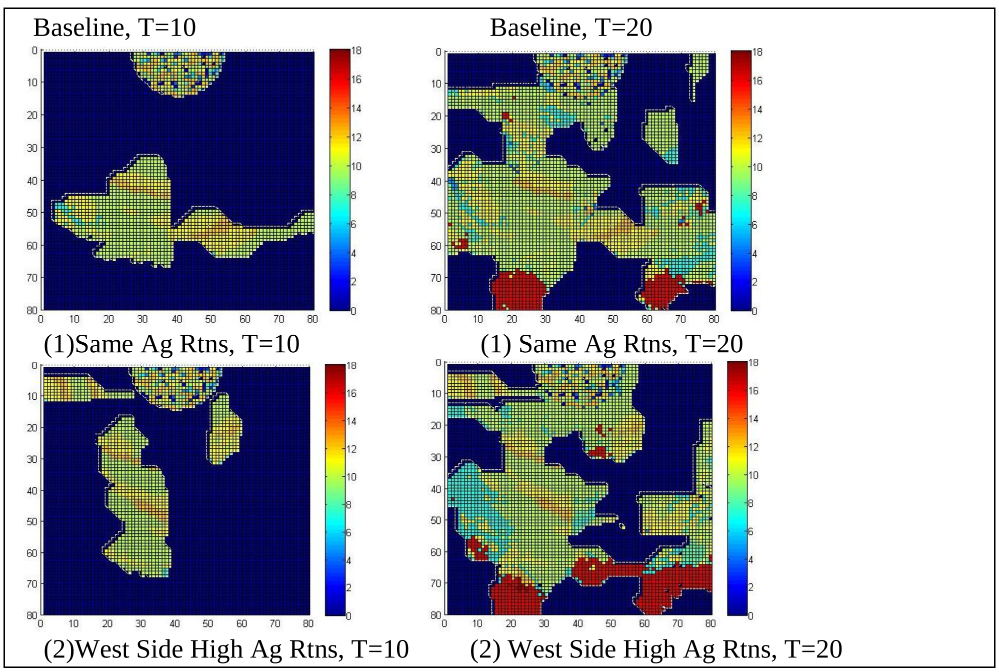
(3) high travel costs – cost per mile of traveling to the SDD is 5 times higher than in the baseline model run;

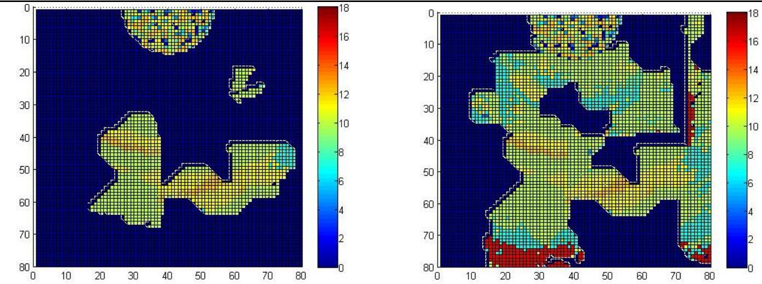
(4) high consumer preference for house relative to lot size, along with a narrower range of consumer preferences – $0.7 \leq \beta \leq 0.9$ (compared with a baseline of $0.1 \leq \beta \leq 0.9$); and

(5) low consumer preference for house size relative to lot size, along with a narrower range of consumer preferences – $0.1 \leq \beta \leq 0.3$ (compared with a baseline of $0.1 \leq \beta \leq 0.9$).

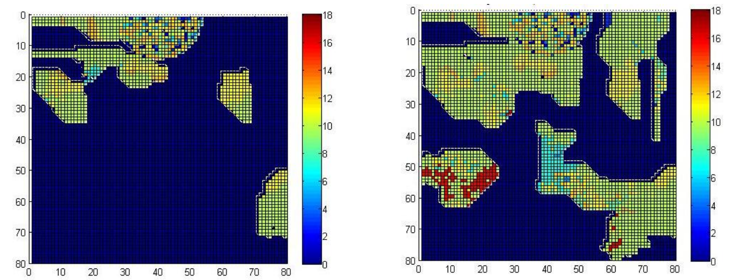
Figure 10 shows the maps for T=10 and T=20 for each of the sensitivity cases, along with the baseline model. Having the same agricultural productivity across all farms (case (1)) tends to push development in the earlier periods northward compared with the baseline. This finding makes sense as removing the variability in agricultural productivity means that travel costs to the SDD play a relatively more important role and this tends to push development northward. The prediction models still play a role, however, which we can see from the fact that development does not move smoothly from north to south over time. Farmers are uncertain about future land prices and are heterogeneous in their predictions about these prices. Thus, farmers view the same land prices differently and form varying expectations of future land prices, which contribute to dispersed patterns of land conversion over time. By T=20, as development pressures have increased with population growth, the spatial patterns in case (1) begin to look quite close to the baseline case.

Figure 10. Spatial Patterns of Development at T=10 and T=20: Baseline and Sensitivity Cases





(3)High Travel Costs, T=10 (3) High Travel Costs, T=20



(4)High beta, T=10 (4) High beta, T=20

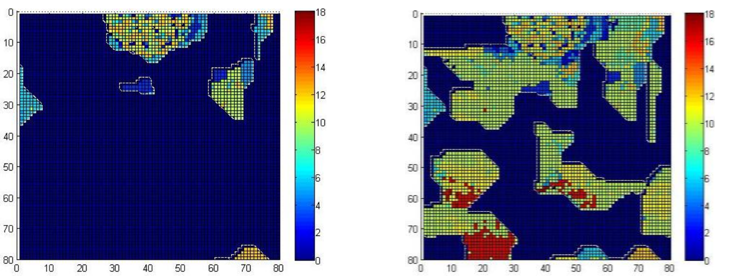
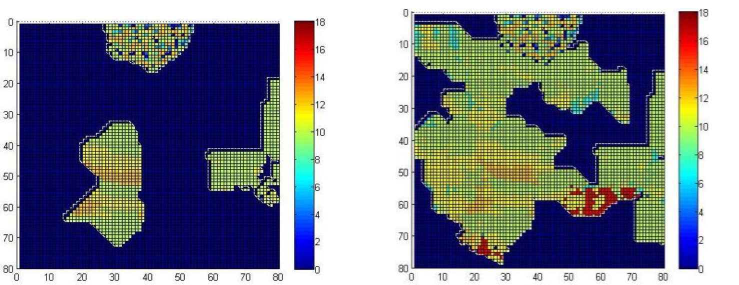


Figure 10 (cont).

(5)Low beta, T=10 (5) Low beta, T=20



In case (2), highly productive farmland in the West tends to stay in agriculture. Even by the final period, farms in that area remain undeveloped, and there is more development in the East compared to the baseline. Density is somewhat higher as well, as would be expected when a large region is not developed. Higher travel costs (case (3)) clearly shift development to the North, closer to the SDD, as would be expected but by T=20, development pressures have led to development in the South, as in the baseline.

Some spatial differences show up in the two 'beta' cases—namely, development shifts slightly to the North in the 'high beta' case (5)—but the main difference in cases (4) and (5) from the baseline is in the types of houses built rather than the location. This shows up more clearly in Figure 11: comparing the red bars (high beta) to the blue bars (baseline) in the figure shows a shift toward houses on smaller lots and a shift toward large houses; comparing the green bars (low beta) to the blue bars (baseline) shows a shift toward houses on larger lots and a shift toward small houses. These changes are as expected—when consumers value the size of the house relatively more than the lot it sits on (high beta), the market tends toward larger houses and smaller lots; smaller houses and larger lots result when consumers value the size of the lot relatively more than the house (low beta). Interestingly, restricting the values for β also reduces the number of houses, and we say more about this below.

Figures 12 and 13 highlight the differences in the density gradients. Because development moves closer to the SDD when travel costs are higher, density is higher in that location than in the baseline and lower farther out. This can be seen in Figure 12 where the red line (case 4) lies above the blue line (baseline) until about 1.7 miles from the SDD then lies below it. The 'high beta' case has a similar effect, as shown in Figure 13, but the 'low beta' case looks very much like the baseline.

Figure 11. Number of Houses in T+20, by Type: Baseline and 'Beta' Sensitivity Cases

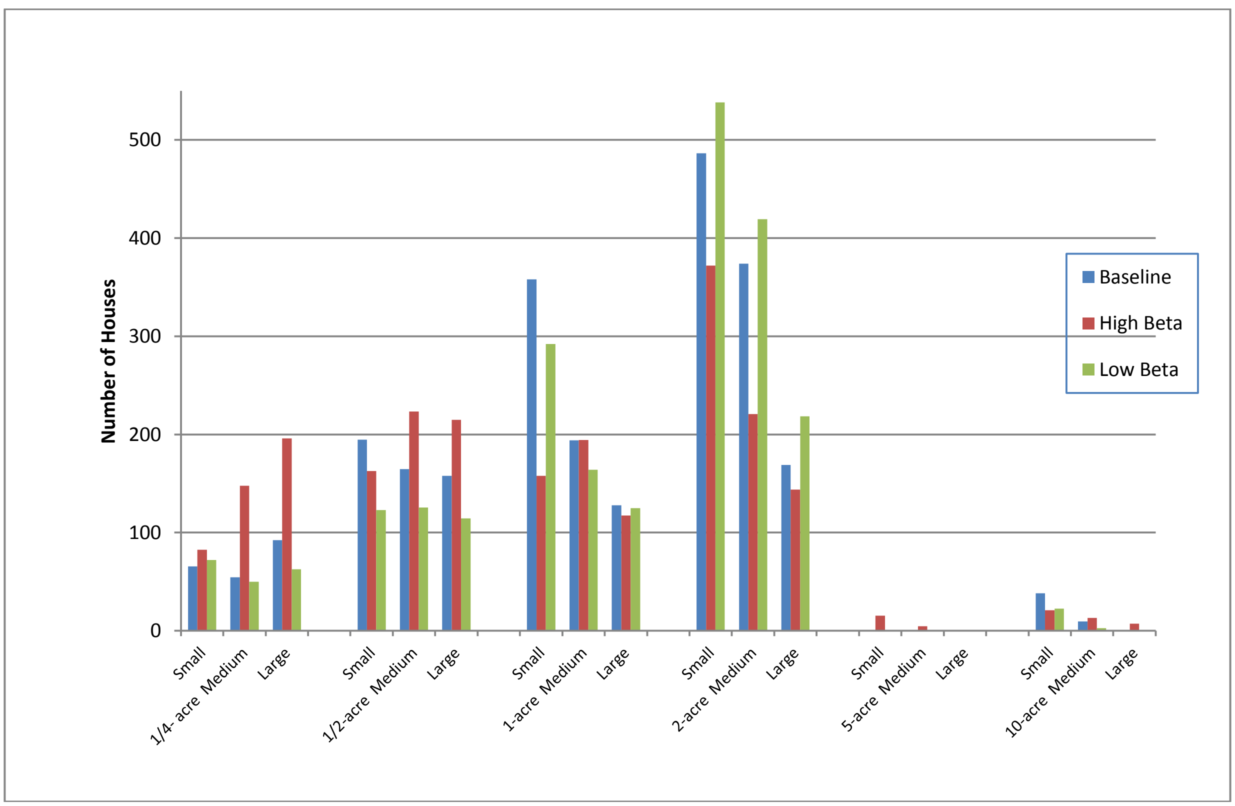


Figure 12. Density Gradient: Baseline and High Travel Cost Case

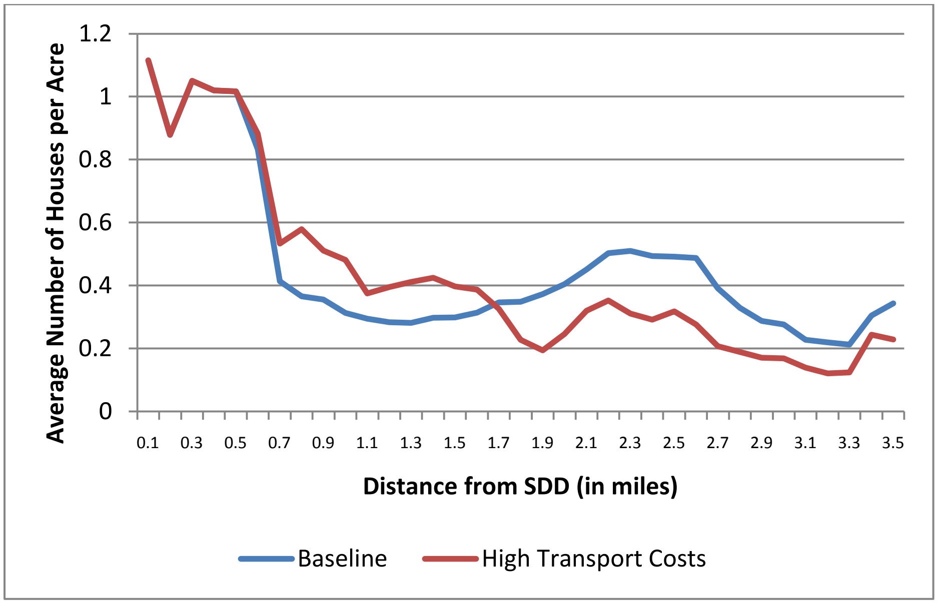


Table 2 shows the total extent of development, both the number of houses and the acreage, in each of the cases. For the sensitivity cases, we also show the percentage difference from the baseline. High travel costs dampen development substantially—the number of houses is 10% below the level in the baseline and the total acres developed is almost 12% lower. When travel costs are higher, fewer consumers can afford to locate in the community and this reduces development. Acreage falls by more than the number of houses as consumers live in houses with smaller lots located closer to the SDD.

Figure 13. Density Gradient: Baseline and Beta Cases

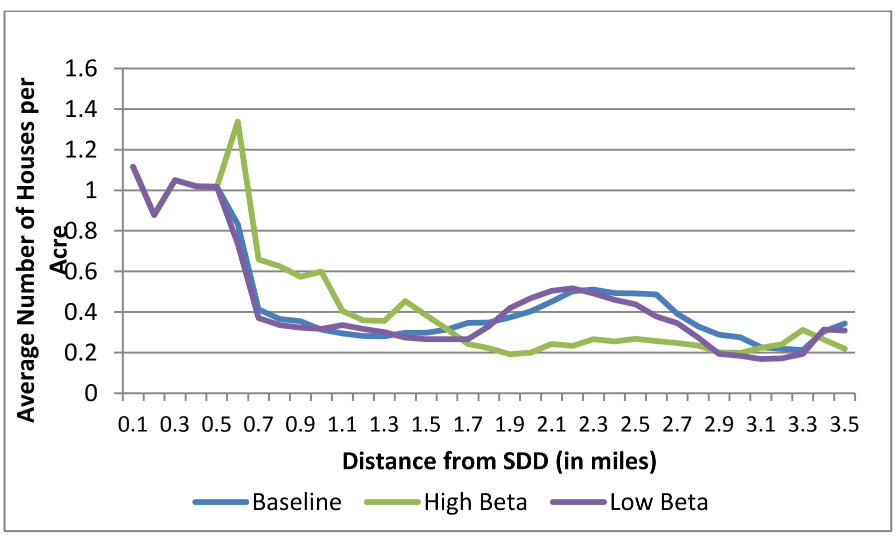


Table 2. Amount of Total Development in T=20: Baseline and Sensitivity Cases

	Houses		*Land*	
	Number	*% Difference from Baseline*	*Acres*	*% Difference from Baseline*
Baseline	2,488		3,539	
High Travel Costs	2,239	-10.0	3,130	-11.6
Same Ag Returns	2,541	2.2	3,676	3.9
West Side Higher Ag Returns	2,521	1.3	3,502	-1.1
High Beta	2,296	-7.7	2,867	-19.0
Low Beta	2,330	-6.3	3,416	-3.5

Narrowing the range of consumer preferences for house size versus lot size—i.e., the parameter β in the utility function—also dampens development. The number of houses in both the 'high beta' and 'low beta' cases is below the baseline. Acreage developed is also below the baseline and in the 'high beta' case, the difference is dramatic—19 percent less acreage is developed in this case. This houses vs acres result for the 'high beta' case is expected as higher values for β indicate a relative preference for house size over lot size. This tends to increase density and lower the acreage used for development. The opposite holds for the 'low beta' case—we see a greater reduction in the number of houses than in acres; again, this is expected as a low value for β indicates a relative preference for lot size over house size.

The two sensitivity cases for agricultural productivity do not lead to markedly different levels of overall development. Less variability in productivity across the landscape tends to increase development, and interestingly, having one highly productive region has only a very small impact on development, increasing the number of houses relative to the baseline but slightly reducing acreage. The latter impact is expected as the highly productive land remains in farming throughout the model periods.

Finally, we examine the economic variables—housing rents, incomes, and land prices—for the sensitivity cases relative to the baseline. The two agricultural productivity cases are very similar to the baseline in all respects thus we do not report those findings here.[17] The ‘high travel costs’ and two ‘beta’ scenarios, however, reveal some interesting results that comport with economic intuition. These results are shown in Table 3.

Table 3. Housing Rents, Land Prices, and Incomes: Baseline and Selected Sensitivity Cases

	Average Annual Housing Rent	Average Land Price (in $/acre)	Average Income
Baseline	$16,712	$2,824	$92,772
High Travel Costs	$15,789	$2,638	$105,291
High Beta	$16,064	$3,466	$98,420
Low Beta	$16,755	$2,969	$99,811

When travel costs are high, we find that housing rents and land prices are lower than in the baseline but incomes are higher. Rents are lower for two reasons: (i) the dampening of overall demand, which showed up in Table 2, and (ii) a change in house type—smaller houses on smaller lots. The lower housing rents have a feedback effect on the land market, dampening average land prices, thus this result also makes sense. Incomes are higher, however, because consumers who locate in the community have to be able to afford the higher travel costs. Incomes in this scenario are the highest of all of the cases and are 13.5 percent above the baseline.

In the two ‘beta’ cases, there is less variation in preferences than in the baseline, i.e., we used a narrower range for the β parameter for both the high and low beta scenarios. When we restrict the preferences, there is more competition among consumers for the available housing. This results in those consumers with relatively higher incomes outbidding consumers with relatively lower incomes, and this explains the higher average incomes for those two cases in Table 3.

[17] The only noticeable difference for the two agricultural productivity cases is the average land price in case (3), where one region in the West is highly productive. The price is slightly above the baseline case. This result makes sense: since some of the land now has higher returns in agriculture, it commands a higher price in the land market and brings up the average land price.

The rents and land prices for the two ‘beta’ cases can also be explained. In the ‘high beta’ case, there is more small lot development than in the baseline (see Figure 3 above), as consumers place relatively more value on house size than lot size. Houses with less land tend to cost less, thus the lower average housing rents in this scenario. On the other hand, smaller lot sizes mean more houses per acre and this increases the value of land for development and raises average per-acre land prices. In the ‘low beta’ case, the opposite holds: the bigger lots preferred by consumers lead to higher housing rents but lower land prices.

Conclusions

We have described a unique agent-based model of housing and land markets and used it to study the dynamic and spatial patterns of development in a hypothetical community on the urban fringe. The model runs show how spatial patterns evolve over time with individual agent decisions creating aggregate land use patterns from the bottom–up, using the fundamentals of microeconomic decisionmaking. These fundamentals mean that agents optimize as in more traditional equilibrium economic models, but the model incorporates much more heterogeneity than those models, including heterogeneity in how agents form expectations of future prices. Some path dependence in outcomes emerges as a result of these expectations, but fundamental economic features—agricultural productivity, consumer preferences and incomes, and developer costs—are also central to the results. We showed this by conducting some sensitivity analyses around key parameters. When we vary parameters defining agricultural productivity, travel costs, and consumer preferences, the predicted number, types, and location of houses conforms to predictions from economic theory. In addition, economic variables such as land and housing prices and consumer incomes are also altered in ways that are consistent with economic intuition.

The model shows that sprawl patterns of development can arise in a monocentric city framework purely from heterogeneity in (i) agricultural productivity across the landscape, (ii) consumers’ housing preferences, and (iii) how expectations of future prices are formed. We find that leapfrog development can occur—i.e., farms farther from the SDD may be converted before those that are closer to the SDD. We also see relatively large average lot sizes and somewhat dispersed development patterns. And these results are obtained in a model without features that have been used in previous economic models to generate sprawl as an equilibrium outcome, such as natural geographic constraints, road congestion, amenities and open space, or zoning. We show that such patterns are possible simply from heterogeneity and bounded rationality on the part of landowners attempting to forecast future land prices.

The model has some limitations in its current form. It is a highly stylized representation of a real landscape. Our purpose was to develop an ABM with a rich level of economic detail and test the importance of some of the model parameters through various sensitivity analyses. Although we feel that the results shed light on the various factors that affect the time path and spatial patterns of development, the specific patterns of land use in the model are not meant to represent any particular reality. Developing an application of the model to a real community is an extension for future research. To do this, it will be important to link different components of the model to real world data and behavior. Along these lines, we would like to more fully explore the how the developer responds to uncertainty in predicting future demands for housing and future profits, and include revisions to the model to reflect this behavior. In addition, accounting more fully for the carrying costs of land that is not developed and of vacant housing in realistic would be a further improvement. Finally, we would like to assess the range of prediction models for land and housing price expectations, and compare those to any available evidence on actual decisionmaking. Sensitivity analyses on the prediction models would an additional way to assess their importance.

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