

Why Don't Lenders Finance High-Return Technological Change in Developing-Country Agriculture?

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Abstract

Most of the literature attributes credit constraints in small-farm developing-country agriculture to the variability of returns to investment in this sector. But the literature does not fully explain lenders' reluctance to finance investments in technologies that provide both higher average and less variable returns. To fill this gap, this article develops an information-theoretic credit market model with endogenous technology choice. The model demonstrates that lenders may refuse to finance any investment in a riskless high-return technology— regardless of the interest rate they are offered—when they are imperfectly informed about loan applicants' time preferences and, therefore, about their propensities to default intentionally in order to finance current consumption.

Key Words: agriculture, asymmetric information, credit, developing country, technology adoption.

JEL Classification Numbers: O12, O16, O33, Q14, D82

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Allen Blackman*

Empirical research has repeatedly demonstrated that borrowing constraints prevent small farmers in developing countries from adopting high-return innovations such as tube wells and modern crop varieties (Feder, Just and Zilberman).¹ Economists have devoted considerable effort to understanding why rural lenders limit small farmers' access to credit (Bell 1988). The recent literature focuses on information asymmetries, specifically, the inability of lenders to observe borrowers' characteristics and actions. But while this literature explains why lenders' may refuse to finance traditional investments, it does not adequately explain why they may refuse to finance the adoption of new high-return technologies. This article seeks to fill this gap.

Existing information-theoretic models fall short of explaining lenders' reluctance to finance technological change because they depend on the implicit assumption that farmers are permanently wedded to traditional technologies. For example, in models such as Carter's that employ Stiglitz and Weiss's seminal framework, default risk arises from the riskiness of agricultural investment projects. Farmers default "unintentionally" when their projects perform so poorly that they are unable to repay their debts. Lenders may ration credit to entire groups of small farmers because they default unintentionally more often than large farmers, and because it is not possible to observe the riskiness of individual farmers' projects. But this explanation does not fully account for lenders' reluctance to finance investments in technologies which raise mean yields or lower yield variability and, therefore, significantly reduce the likelihood of unintentional default.

Existing information-theoretic models of rationing in rural credit markets also rely on informational assumptions that do not generalize to important institutional settings. As noted

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¹ Other factors blamed for slow technological change on small farms include a lack of human capital, risk aversion, imperfections in labor and input markets, unfavorable tenurial arrangements, and the tendency of small farms to fuel productivity growth by improving efficiency rather than adopting new technologies. For some technologies, differences in adoption rates between small and large farmers diminish over time (Feder, Just and Zilberman).

above, models in the spirit of Stiglitz and Weiss assume that credit rationing arises from lenders' inability to observe the characteristics of individual farmers' proposed investment projects. But empirical research suggests that small farmers borrow mainly from informal lenders (Zeller et al., p. 67). As longstanding members of rural communities, such lenders are typically well-informed about their clients' proposed investment projects. The main informational difficulty they face is an inability to observe *ex ante* whether borrowers will actually invest or will instead default "intentionally" to finance current consumption (Bell 1990, pp. 312-13, Srivastava).²

This article develops an information-theoretic model that explains lenders' reluctance to finance small farmers' investments in high-return technologies and that avoids some of the limiting informational assumptions characteristic of the previous literature. It argues that lenders cannot observe loan applicants' time preferences or expenditures and, therefore, have imperfect information about their propensities to abscond with credit to finance current consumption. Because of this information asymmetry, relatively large loans create incentives for individual borrowers to abscond and they select for absconders among the pool of loan applicants. As a result, lenders may refuse to offer *any* relatively large loans regardless of the interest rate they are offered. But small farmers generally require sizable loans to adopt new technologies since adoption entails fixed transactions and learning costs and (on occasion) capital indivisibilities. Hence, asymmetric information about farmers' time preferences effectively prevents lending for investments in high-return technologies.

Empirical research suggests intentional default is a significant problem in rural credit markets. It has been widely blamed for rampant default losses in formal agricultural credit development programs—by one estimate, 40-90% of all funds disbursed (Braverman and Guasch, Sanderatne). In addition, as noted above, intentional default has been characterized as the preeminent threat faced by informal lenders. Evidence aside, one would expect intentional default to be prevalent in small farm agriculture. Since small farms are both firms and households, their use of credit to finance consumption is to be expected and, by extension, intentional default should follow when the return on consumption is sufficiently high. Moreover, small farmers are generally poor and have meager resources to buffer against consumption

² Bell (1988, Section 2.1.1) focuses on precisely this problem. However, Bell assumes that lenders can observe the propensities of individual borrowers to default intentionally. As a result, lenders are able to completely eliminate willful default. This result contradicts abundant evidence (discussed below) of persistent intentional default in both formal and informal rural credit markets.

shocks like medical emergencies and ceremonial obligations. As a result, when such shocks occur, small farmers have strong incentives to consume credit, even if high-return investment projects are available.³

Empirical research also buttresses several other elements of this article's thesis. Experimental studies suggest that small farmers' discount rates are highly variable and generally exceed market interest rates by a considerable margin (Pender). Analysis of loan repayment performance in rural credit markets has shown that, all other things equal, the incidence of default is positively correlated with loan size (Sharma and Zeller, Floro and Yotopoulos). Research on technical change indicates that small farmers require significant credit to finance fixed costs associated with adopting new technologies, even when these technologies would appear to be scale-neutral (Feder, Just and Zilberman). Finally, regarding information asymmetries, abundant evidence of the diversion of 'agricultural' credit to non-farm uses suggests that many lenders cannot monitor or dictate how borrowers use their loans (Braverman and Guasch, p. 1255). Monitoring costs are high because farms are often geographically dispersed and because some purchased inputs such as hired labor and fertilizer are only observable for a short period of time. Lenders are usually unable to prevent the diversion of credit to consumption by 'lending in kind' since farmers can resell the inputs they receive (Von Pischke and Adams).

This article's principal contribution to the policy literature is to explain lenders' reluctance to finance high-return investments that enhance borrowers' capacities to repay debt.⁴ It makes two contributions to the theoretical literature. Most importantly, it develops an information-theoretic model of a rural credit market in which technology choice is endogenous. Nearly all existing models assume borrowers' technologies are exogenously fixed.⁵ In addition,

³ Many agricultural borrowers default not because they have pressing consumption needs but because they have the power and connections to get away with it. However, this type of intentional default is characteristic of large farms not small ones (Braverman and Guasch, p. 1257).

⁴ This paper focuses on lenders' reluctance to finance high-return investments outside the confines of interlinked markets. Bhaduri argues that in interlinked credit markets, landlord/lenders have an incentive to block technological innovation by share tenant/borrowers because it may reduce usury income by more than it increases share rent, an argument that has sparked considerable debate.

⁵ Those few models that do not make this assumption only allow borrowers a simplistic technology choice. For example, in Stiglitz and Weiss (second model) borrowers choose between a safe, low-return technology and a risky, high-return technology. The purpose of this feature is to provide borrowers with an opportunity for moral hazard, not to shed light on the link between finance and technology choice. These models can not explain why lenders do not finance investments in safe, high-return technologies.

this article describes equilibrium credit rationing arising from asymmetric information about borrowers' time preferences.⁶ Because this model allows for endogenous technology choice and highlights moral hazard, it yields unconventional policy prescriptions for mitigating credit rationing.

Methodologically, this article follows a three step graphical approach: borrower and lender optimization problems are specified, these are used to derive indifference curves in contract space, and finally, indifference curves are used to describe equilibria (Jaffee and Russell, Stiglitz, Bose). After presenting these three steps, the article discusses policy prescriptions and conclusions. Most proofs are relegated to an appendix.

Borrowers' Utility Maximization Problem

We assume that risk neutral borrowers maximize simple two period additive utility. In the first period they contract for a loan of size L at gross interest rate R and either invest L or consume it. In the second period they either pay RL and consume the net return on their investment, or they default and pay a cost, K .

All borrowers have access to the same set of investment opportunities, $i \in \{t,n\}$, where "t" is a traditional technology and "n" is a new technology. The payoffs from the investment opportunities, $g^t(L)$ and $g^n(L)$, are positive, strictly concave functions of L (fixed factors may include land or managerial skill). The new technology is characterized by two assumptions:

Assumption 1. The marginal product of credit under the new technology exceeds that of credit under the traditional technology for every level of investment, i.e.,

$$dg^n/dL > dg^t/dL \quad \forall L.$$

Assumption 2. Adopting the new technology involves a fixed cost, "F."

⁶ The literature includes models of credit rationing that arises from lenders' inability to observe borrowers' honesty (Jaffee and Russell), effort (Watson), ability (Clemenzenz), and production risk (e.g., Stiglitz and Weiss, first model).

The second assumption reflects the fact that, as discussed in the introduction, even the adoption of supposedly scale-neutral technologies like modern crop varieties by small farmers involves significant fixed transactions costs and learning costs such as the costs of developing new markets and training hired labor. In addition, I assume,

Assumption 3. F and R are small enough that if sufficient funds are available, investment in the new technology is more profitable than either investment in the traditional technology or inaction, i.e.,

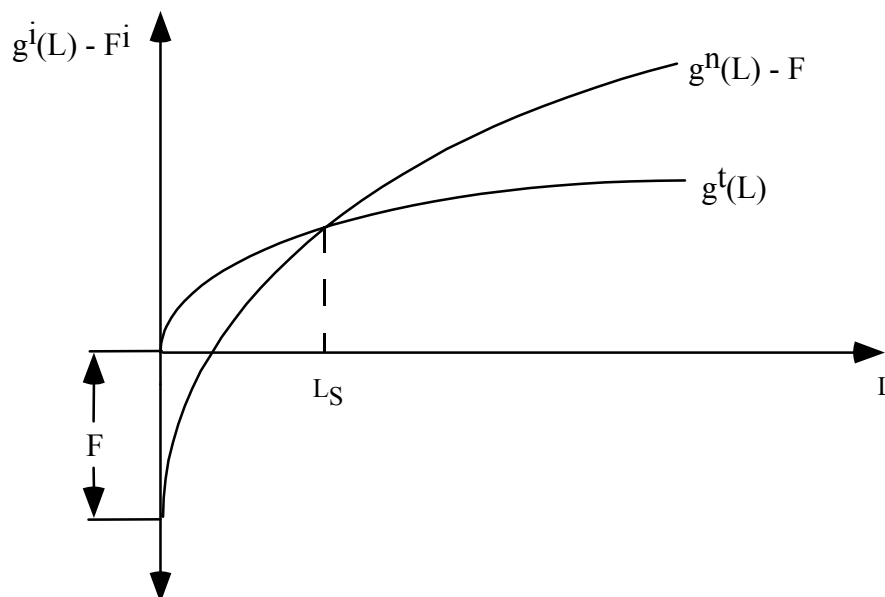
$$\{\max_{(L)} (g^n(L) - RL - F)\} > \{\max_{(L)} (g^t(L) - RL)\} > 0.$$

This assumption simply restricts attention to those cases where farmers prefer to adopt despite the costs involved.⁷ It is needed to ensure a demand for loans to finance adoption so that the model can explain why lenders may refuse to meet this demand. Figure 1 depicts production functions satisfying these three assumptions.

A borrower's choice of whether to invest or consume the loan depends critically on her relative valuation of first and second period consumption, i.e., on whether she has medical expenses, ceremonial obligations, etc. that she needs to finance in the first period. Time preferences are represented by a constant discount factor. To keep the notation simple, instead of multiplying period two consumption by a discount factor, all first period consumption is multiplied by the reciprocal of the discount factor, β . A larger β indicates "impatience"—a preference for period one consumption. A total of 'm' different β 's are randomly distributed among borrowers.

⁷ Assumption 3 restricts the size of R as well as F since, given any two production functions that satisfy Assumptions 1 and 2, if R is sufficiently large, the net return on investment in the new technology, $g^i(L) - RL - F$, will be negative no matter how small F is and, therefore, the traditional technology will dominate the new technology. The restriction on R amounts to a restriction on the parameters $g^i(\bullet)$, F , D (the lender's cost of funds), and $H(\beta)$ (the distribution of discount rates among loan applicants) because, as the equation for the lender's zero profit locus (6) demonstrates, when lenders are restricted to zero-profit by free entry, the zero-profit R is an implicit function of these parameters.

Figure 1. Gross return on new and traditional investment; Switch line, L_S .



Small farmers are rarely able to fully collateralize loans since ill-defined property rights and incomplete markets often limit the marketability of their assets (Besley). Therefore, there is an upper bound on the borrower’s liability to the lender.

Assumption 4. If a borrower’s gross product, $g(L)$, is not sufficient to repay his debt, RL , the borrower pays a fixed cost of default, K . Additionally, the lender confiscates any output the borrower may have produced.

Thus, although default is costly to the borrower, opportunities for moral hazard arise because loans are undercollateralized when $RL > K$. The diagrammatic and mathematical exposition of the model are much simpler given the reasonable assumption that,⁸

⁸ As detailed in Appendix A.2 (i), this assumption ensures that the default locus described in the next section is everywhere negatively sloped.

Assumption 5. K , the cost of default, is at least as great as F , the fixed cost of adoption.

This article focuses on lenders' behavior toward one group of observationally indistinguishable small farmer loan applicants. All loan applicants in this group are assumed to have comparable observable characteristics including production technologies, the fixed cost of default and assets. To simplify the exposition, the assets of farmers within this group are normalized to zero.

The borrower's utility maximization problem is:⁹

$$(1) \quad \max \{ \max_{(i)} g^i(L) - RL - F^i, \beta L - K \}.$$

The borrower's maximization problem breaks down into a choice among the three strategies listed in Figure 2. The return to each strategy—and, as a result, the borrower's choice among them—depends on the terms of the loan contract and on the borrower's β . The next two sections show precisely how the borrower's choice depends on R , L and β . They derive the two loci pictured in Figure 2 that divide the contract space into subsets of contracts that elicit each strategy.¹⁰

Switch Line and Default Locus

As Figure 1 illustrates, the fixed cost associated with adoption implies that, given any contract (R,L) , farmers will obtain a higher net return from the new technology than the old technology when $L > L_S$ where,

⁹ The assumption that borrowers have zero assets together with the assumption that they cannot split their loans between investment and consumption in the first period implies that borrowers who invest consume nothing in the first period and borrowers who abscond consume nothing in the second period. However, these assumptions are made purely for the sake of simple exposition. A model in which assets are positive and capital may be split between uses yields qualitatively identical results (see Blackman 1998).

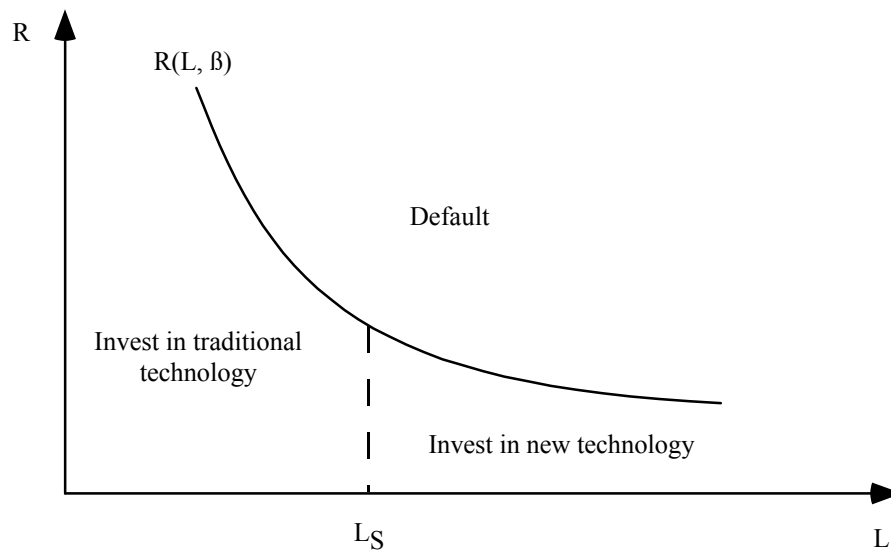
¹⁰ The individual rationality constraint is discussed in Appendix A.1.

Definition 1. L_S is the value of L such that,

$$(2) \quad g^n(L) - F = g^t(L).$$

Equation (2) defines a vertical switch line in contract space that is depicted in Figure 2. Given a contract to the right of the switch line, borrowers will prefer the new technology to the traditional technology.

Figure 2. Borrowers' optimizing strategies; Switch line, L_S ; Default locus, $R(L,\beta)$.



Default contracts are delimited by the locus of contracts yielding the same utility whether the borrower defaults or invests, i.e.,

Definition 2. The default locus, $R(L,\beta)$ is the set of contracts such that,

$$(3) \quad \beta L - K = g^i(L) - RL - F^i$$

where, as discussed above, $i^* = t$ when $L < L_S$ and $i^* = n$ otherwise.

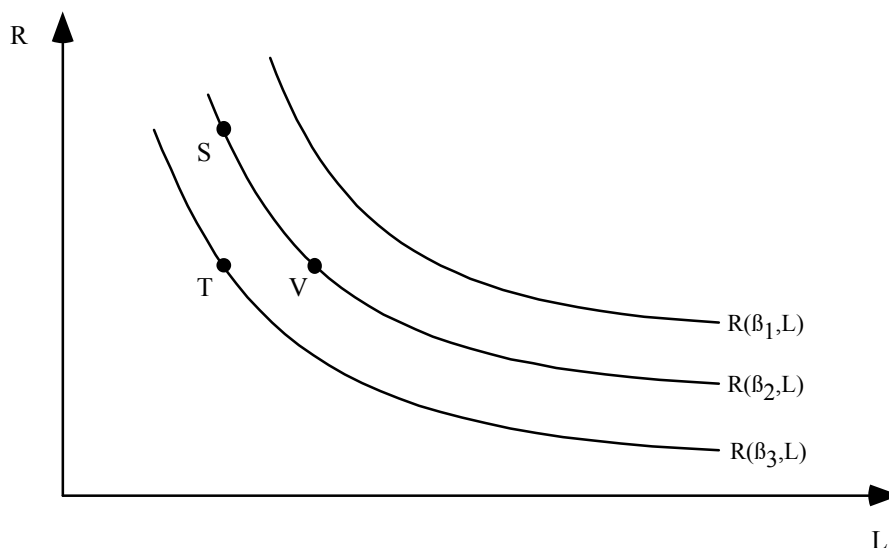
Borrowers default on contracts that lie above $R(L,\beta)$.

Proposition 1. The default locus has the following characteristics:

- (i) it is everywhere negatively sloped
- (ii) increases in β shift it down.

The intuition for a downward sloping default locus is as follows. Farmers offered an increasingly large L given a fixed R will at some point find that the total return on absconding overtakes the total return on investing. This happens because the marginal net return on absconding, β , is constant while the marginal net return on investing, $dg/dL - R$, falls due to diminishing returns. Given a higher fixed R , the switch point occurs at a smaller L since increasing R lowers the return on investment. Similarly, given a higher β , the switch point occurs at a smaller L since increasing β raises the return on absconding. Figure 3 summarizes the impact of changes in β on the default locus and also illustrates that given any contract (say, V) sufficiently impatient borrowers ($\beta > \beta_2$) will choose to default while more patient ones ($\beta \leq \beta_2$) will choose to invest.

Figure 3. Default loci for different β 's ($\beta_3 > \beta_2 > \beta_1$)



Borrowers' Indifference Curves

Borrowers' indifference curves are given by,

$$(4) \quad \max \{ \max g^i(L) - RL - F^i, \beta L - K \} = \bar{U} \quad \forall \bar{U}. \tag{i}$$

where $i^* = t$ when $L < L_S$ and $i^* = n$ otherwise. Each of the borrower's three alternative optimizing strategies—default, invest in the traditional technology, and invest in the new technology—implies a different set of characteristics for indifference curves and these curves may incorporate all three sets of characteristics. To describe indifference curves, the following definition is useful:

Definition 3. L^i_R is the loan size at which $dg^i/dL = R \quad \forall R$.

As illustrated in Figure 4, L^i_R map out a set of downward sloping convex curves.

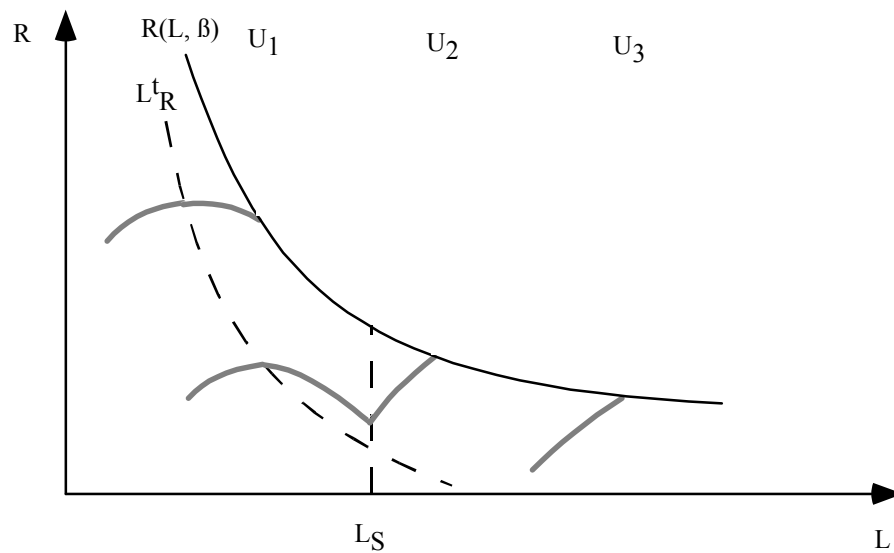
Proposition 2. Borrowers' indifference curves have the following characteristics:

- (i) when default is optimal ($R > R(L, \beta)$), indifference curves are vertical.
- (ii) when investment in the traditional or new technology is optimal ($R < R(L, \beta)$) on each side of the switch line indifference curves are single peaked, positively sloped when $L < L^i_R$, reach a unique maximum at $L = L^i_R$, and negatively sloped when $L > L^i_R$. The global maximum of each curve occurs at L^p_R .

Indifference curves lying completely below the default locus are depicted in Figures 5 and 6 while those that intersect it are depicted in Figure 4.

Intuitively, the slope of an indifference curve, $dR/dL|_{\bar{H}}$, indicates the borrower's marginal willingness to promise to pay for additional credit. Above the default locus where absconding is the optimal strategy, indifference curves are vertical because borrowers are willing to *promise* to pay an infinite interest rate for more credit. This implies that borrowers who abscond will always choose the largest available loan. Below the default locus, $dR/dL|_{\bar{H}}$ is positive if, at a given contract, the marginal product of credit, dg^i/dL , is greater than the marginal cost of credit, R , and is nonpositive otherwise. On each side of L_S , $dR/dL|_{\bar{H}}$ is decreasing in L since, as L increases, the marginal product of credit falls. Note that $dR/dL|_{\bar{H}}$ is discontinuous at the switch line because the marginal product of credit receives a boost when the new technology is adopted. Assumption 3 guarantees that the second hump of each indifference curve is higher than the first.

Figure 4. Borrowers' indifference curves ($U_3 > U_2 > U_1$).



Lender's Zero-Profit Locus

Three sets of assumptions characterize lending. First, to abstract from market power and risk aversion, we assume,

Assumption 6. Lenders are risk neutral, enter freely, and have unlimited access to funds at a constant gross economic cost of “D” per unit.

Second,

Assumption 7. Credit contracts are exclusive, that is, each borrower contracts with only one lender.

Lenders insist on exclusive contracts to protect their claims on borrowers' collateral in the event of default, and to be able to gauge the size of each borrower's debt. As explained below, the latter enables lenders to calculate the probability of repayment.¹¹ Finally, we assume,

Assumption 8. Lenders cannot observe *ex ante* borrowers' time preferences or expenditures, but do know $H(\beta)$, the c.d.f. of $\beta \in (\underline{\beta}, \bar{\beta})$. Furthermore, lenders are not able to dictate borrowers' expenditures.

Given their inability to observe β , lenders cannot plot borrowers' default loci—i.e., cannot tell which contracts induce default—and, therefore, must charge a risk premium on each contract so that, on average, they can cover losses due to default. The size of the risk premium will depend on the probability that each contract will be repaid. Recall that, as Figure 3 illustrates, given any contract (R,L) , relatively patient borrowers will choose to repay while more impatient ones will default. Thus, the probability that a particular contract will be repaid is simply the probability that a random applicant given this contract has a β sufficiently low such that the net return on default is less than or equal to the maximum available net return on investment. The critical value of β is,¹²

Definition 4. $\beta^i(R,L)$: the value of β such that,

(5) $\beta L - K = g^{i*}(L) - RL - Fi^*$,

¹¹ This assumption is not entirely satisfactory since its empirical validity varies across credit markets. In formal credit markets, borrowers sometimes supplement their loans with informal credit (Bell, 1990, p. 318). However, in informal credit markets, contracts are usually exclusive (Siamwalla, pp. 278-79, Aleem, p. 335). Importantly, the poor small farmers that are the focus of this article are usually restricted to informal markets (Zeller et al., p. 67). Although a full analysis of equilibria in markets with non-exclusive contracts is beyond the scope of this article, depending on the assumptions made, such an analysis would likely show that non-exclusive contracting exacerbates the credit rationing described in the next section since with such contracts, lenders do not know borrowers' total indebtedness, are therefore less able to accurately gauge the probability of intentional default, and as a consequence must raise risk premiums and/or restrict lending. Hoff and Stiglitz and Bose reach similar conclusions about the impact of non-exclusive contracting on credit terms and availability (although for different reasons).

¹² Intuitively $\beta^i(R,L)$ is the β of the borrowers who, given (R,L) , are just indifferent between investing L and absconding with it. Graphically, it is the β of the borrowers whose default loci contain (R,L) . For example, in Figure 3, for contract T, $\beta^i(R,L) = \beta_3$.

where $i^* = t$ when $L < L_S$ and $i^* = n$ otherwise.

and the probability that a random loan applicant given contract (R,L) will repay is $H(\beta^i(R,L))$.

Proposition 3. $\beta^i(R,L)$ and, therefore, $H(\beta^i(R,L))$ are decreasing in R and L .

This results from the fact that, as Figure 3 illustrates, increases in R and L make default more attractive relative to investment and, therefore, induce marginal borrowers to default.¹³

For fully collateralized contracts, that is, contracts such that $RL \leq K$, the zero-profit locus will be a horizontal line at $R = D$. For contracts that are not fully collateralized ($L > K/D$ at $R = D$), Proposition 3 implies the zero-profit locus is positively sloped and may bend backward (as in Figure 5). The set of contracts yielding zero-expected-profit is given by,

$$(6) \quad H(\beta^i)RL + (1 - H(\beta^i))K - DL = 0.$$

Differentiating totally, and then substituting out D using (6) yields the slope of the zero-profit locus,

$$(7) \quad \left. \frac{dR}{dL} \right|_{\bar{\pi}} = \frac{-h(\beta^i) \frac{\partial \beta^i}{\partial L} (RL - K) + (1 - H(\beta^i)) \frac{K}{L}}{LH(\beta^i) + h(\beta^i) \frac{\partial \beta^i}{\partial R} (RL - K)}$$

where $h(\beta)$ is the p.d.f. of β . Since $\partial \beta / \partial L$ is negative, the sign of the numerator, $-\partial \pi / \partial L$, is positive. Since $\partial \beta / \partial R$ is negative, the sign of the denominator, $\partial \pi / \partial R$, is ambiguous: increases

¹³ Consider contract T in Figure 3 which is on the default locus of borrowers with $\beta^i(T) = \beta_3$. The probability of repayment is $H(\beta_3)$. Suppose the lender increases L so that borrowers are now offered contract V . V lies outside the default loci of borrowers who prefer investment at T . Now borrowers must have $\beta < \beta^i(V) = \beta_2$ to prefer repayment to default. The probability of repayment is now $H(\beta_2)$ which is less than $H(\beta_3)$. Clearly, the same argument applies when R is increased and the borrower is offered S .

in R have a positive effect on profit by increasing the revenue per dollar repaid (the first term), and a negative effect by decreasing the likelihood of repayment (the second term). If lenders are willing to make any loans such that $RL > K$ then when L is small, the positive effect dominates; otherwise, no interest rate could compensate the lender for making even the smallest such loan. Thus, for L sufficiently small, the zero-profit locus is positively sloped, implying lenders are able to offset default risk associated with increasing L by raising R . For sufficiently large loans, given fairly unrestrictive assumptions on $H(\bullet)$ and $\beta^i(\bullet)$, the negative effect of raising R unambiguously dominates the positive, implying that, at some critical loan size, the zero-profit locus bends backward. As demonstrated in Appendix A.5, sufficient conditions for a backward-bending zero-profit locus (which is not needed for the main result of the article) are: (i) $\beta^i(R, L)$ is concave in L , (ii) the elasticity of $H(\beta^i)$ with respect to L is less than negative 1, and (iii) $H(\beta)$ is log concave.¹⁴

Full Information Equilibrium

To highlight the role of asymmetric information in generating low-technology equilibria, it is helpful to first consider the full information equilibrium. With full information, lenders will be able to plot the default locus of each loan applicant, and will be willing to offer any contract below it at $R = D$. Relatively patient borrowers whose default loci at $R = D$ lie to the right of the switch line will be able to get loans large enough to make adoption worthwhile. Sufficiently impatient borrowers, on the other hand, will not. To describe the full information equilibrium formally, we need the following definitions:

Definition 5. A set of contracts $\{N\}$ is a Nash equilibrium if no additional contracts can be found that are profitable when offered along with $\{N\}$.

¹⁴ The first condition dictates that the marginal product of credit diminishes relatively quickly (see Appendix A.4). The third condition dictates that $\log H$ is concave. It is satisfied by most usual distributions including uniform, normal, logistic, chi-squared, exponential, and Laplace (Bagnoli and Bergstrom). The second condition $[dH/d\beta^i(\bullet) * \partial\beta^i(\bullet)/\partial L * L / H(\bullet) < -1]$ dictates that a ten percent increase in L reduces the probability that a random loan applicant will repay by more than ten percent. It is likely to be met when, in the area of contract space where the zero-profit locus becomes steep: (a) $dH/d\beta^i(\bullet) = h(\beta^i)$ is relatively large because $\beta^i(R, L)$ returns a ‘typical’ value of β ; (b) the absolute value of $\partial\beta^i(\bullet)/\partial L$ [defined in Appendix A.4] is relatively large because the marginal return to investing L is relatively low and R is relatively high; and (c) L is relatively large and $H(\bullet)$ is relatively small.

Definition 6. For each of 'm' different borrower types indexed by $j = (1, 2, \dots, m)$, L_j is the loan size below borrower j 's default locus that maximizes her utility given $R = D$.

Given these definitions,

Proposition 4: The set of contracts (D, L_j) $j = (1, 2, \dots, m)$ is the full information Nash equilibrium.

Proof: By the Definition 6, if borrowers are offered (D, L_j) , they will not be willing to accept any other contract below their default loci. As a result, no other set of contracts is profitable when offered along with (D, L_j) . Q.E.D.

Asymmetric Information Equilibrium

This section shows that, with asymmetric information, the credit market may be characterized by equilibria wherein lenders refuse to provide any observationally indistinguishable small farmers with sufficient credit to finance the adoption of new technologies regardless of the interest rate they are offered. One way this might occur is straightforward. The zero-profit locus may bend backward at a relatively small loan size so that lenders refuse to make any loans large enough to finance adoption. That is, $L^* < L_S$. Intuitively, this means lenders are not able to raise R to offset the default risk associated with large 'adoption' loans because increases in R so severely exacerbate moral hazard.

The remainder of this section develops a less obvious result: even when the zero-profit locus does not bend backward, non-adoption equilibria may still obtain. The formal discussion is somewhat abstract, it is prefaced with an intuitive explanation. Consider the market illustrated in Figure 5 in which lenders would seem to be willing to finance adoption albeit by charging high risk premiums. 'Good' borrowers—those who intend to invest—are confronted with a choice between an 'adoption contract' like A^n (big L but high R) and a 'non-adoption contract' like A^t (small L , lower R). If the risk premium attached to the adoption contract is so high that good

borrowers prefer non-adoption contracts then adverse selection occurs: good borrowers choose non-adoption contracts but ‘bad’ borrowers—those who default—choose adoption contracts. This follows from the fact that bad borrowers have vertical indifference curves (Proposition 2(i)) and will choose the largest loan on the market regardless of R . Thus, large adoption contracts will be unprofitable and lenders will not offer them at any R .¹⁵

This story is perhaps most intuitively told as follows. Imagine a medium sized market town serviced by a dozen competitive lenders. The standard existing contract is \$40 at 20% interest. A new profitable modern crop variety becomes available that, due to the fixed costs associated with adoption, requires a \$100 investment. Unless they can keep the risk premium on \$100 loans low enough so that all patient and impatient borrowers find these loans irresistible, lenders will refuse to make \$100 loans because they know that all the impatient borrowers who intend to default will surely apply for them.

Formally, first note that,

Proposition 5. No asymmetric information pure strategy Nash equilibria exist.

Proof: For any contract *above* the zero-profit locus, there exists a contract on the zero-profit locus that is profitable when offered along with it. For any contract O *on* the zero-profit locus (Figure 6), there will always exist a ‘cream skimming’ contract, M , involving less credit but a lower interest rate that some borrowers who intend to invest prefer to O . Proposition 2(i) ensures that no borrowers who intend to default will choose M . Therefore, M is strictly profitable when offered along with O . Q.E.D.

¹⁵ Note that only contracts that pool absconders and investors are profitable. Lenders are not able to offer separating contracts of the type described in Milde and Riley because they can not break even on contracts that attract only absconders.

Figure 5. Non-adoption equilibrium: $U^0(A^t) > U^0(A^n)$.

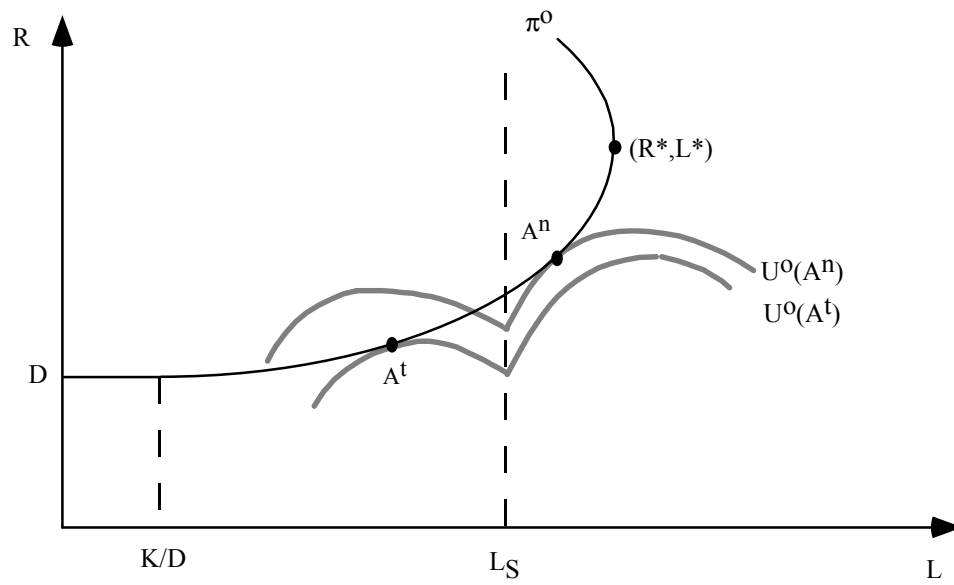
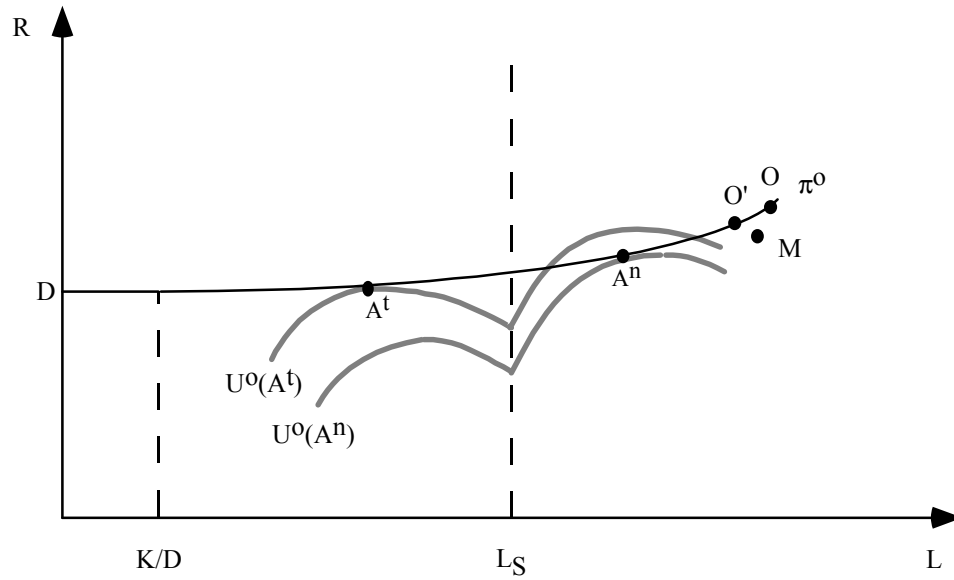


Figure 6. Adoption equilibrium: $U^0(A^n) > U^0(A^t)$.



The phenomenon of the nonexistence of pure strategy Nash equilibria is common in the insurance and credit market literatures. As a result, several alternatives to the Nash equilibrium are commonly used. Clemenz (p. 163), in reviewing the literature on asymmetric information in credit markets, concludes that, of these, the Wilson (or “anticipatory”) equilibrium is “most convincing.”

Definition 7. A set of contracts $\{C\}$ is a Wilson equilibrium if no additional contracts can be found that are profitable when offered along with $\{C\}$ and remain profitable even if all of the loss-making contracts are removed from the market.

The central idea is that one should not rule out a contract like O in Figure 6 as an equilibrium simply because competitors can hypothetically offer cream-skimming contracts like M since, in the long run, contracts like M are loss-makers and competitors, anticipating this, will not actually offer them. M is a loss maker in the long run because after M succeeds in undercutting O and O is removed from the market, M will attract all the absconders.

Proposition 6. The Wilson equilibrium, “ A ”, is the smallest loan on the zero-profit locus demanded by any borrower in the market.

Proof: Contracts above and below the zero-profit locus cannot be Wilson equilibria. For any contract, X , above the zero-profit locus, a loan of the same size on the zero-profit locus is profitable when offered along with X . Contracts below the zero-profit locus are not profitable in the long run after loss making contracts are removed from the market. Since only borrowers who repay prefer smaller loans to larger ones, given any contract O on the zero-profit locus (Figure 6), if any borrower prefers O' , a smaller loan on the zero-profit locus, a competing lender could profitably offer O' and attract only borrowers who repay. Even if O were subsequently withdrawn from the market, profits at O' would never fall below zero since O' is on the zero-profit locus. Q.E.D.

Graphically, contract A is a point of tangency between the zero-profit locus and good borrower’s indifference curves. As shown in Figures 5 and 6, two points of tangency are

possible: “ A^t ,” the tangency point to the left of L_S and “ A^n ,” the tangency point to the right. If A^n maximizes good borrowers’ utility then, in equilibrium, all good borrowers adopt. But if A^t maximizes their utility, then in equilibrium, no borrowers adopt.

Whether A^t or A^n is the Wilson equilibrium depends on the shape of the zero-profit locus in relation to good borrowers’ indifference curves. If the zero-profit locus is relatively steep, A^t , not A^n , will be the global maximum. The next section discusses the determinants of this geometry.

Policy Prescriptions

Figures 5 and 6 make clear that the flatter the zero-profit locus and the larger L^* , the greater the likelihood of an adoption equilibrium.

Proposition 7: The following flatten the zero-profit locus or raise L^* , or both:

- (i) increases in the productivity of the new technology, g^n
- (ii) decreases in the fixed cost of adoption, F
- (iii) increases in the cost of default, K
- (iv) increases the proportion of relatively patient borrowers.

This proposition suggests a number of unconventional policy prescriptions.

While the literature on credit rationing in developing-country agriculture generally focuses on “supply-side” corrective measures such as improving lenders’ information about borrowers, lowering their cost of funds and removing regulatory restrictions, Proposition 7 demonstrates that “demand-side” measures—namely, increasing the productivity of the new technology and reducing fixed adoption costs—can also mitigate credit rationing. These objectives can be accomplished via applied research, technical extension and investments in rural infrastructure. In addition, while the bulk of the literature blames credit rationing on the threat of unintentional default and therefore recommends measures to prevent such default, Proposition 7 suggests credit rationing can be mitigated by policies that discourage willful default. Two means of doing this are to impose especially harsh sanctions (such as long-term exclusion from the credit market and public censure) on demonstrably willful default, and to reduce the proportion of relatively impatient borrowers by, for example, improving preventative health care. Finally,

although a formal treatment is outside the scope of the present article, there are a number of reasons to believe that arrangements wherein groups of borrowers are made jointly liable for repayment would mitigate credit rationing. Group lending may: (i) increase the profitability of new technologies since group members can split fixed adoption costs; (ii) increase the cost of willful default since group members can apply unconventional social and economic sanctions; and (iii) reduce the proportion of patient borrowers since group members (often neighbors or business associates) can screen out particularly impatient borrowers.¹⁶

Conclusion

This article argues borrowers may be unable to obtain loans large enough to cover the set-up costs of adopting new technologies, regardless of the risk premium they offer to pay, for two reasons. First, raising the risk premium may so severely exacerbate incentives to abscond that it actually lowers lenders' expected profits. Second, even when this is not the case, the interest rates on large 'adoption' loans may be so high that good borrowers prefer less expensive non-adoption loans and the applicant pool for adoption loans contains only absconders who always prefer the largest loan on the market. Therefore, lenders restrict loan size to the smallest zero-profit loan demanded by any borrower in the market.

Finally, a word about the implications of this model for developing-country agrarian income distribution. The comparative statics imply that the financial constraints on adoption described here will be less severe for groups of observationally indistinguishable borrowers who pay higher default costs and lower adoption costs. Relatively wealthy, medium- and large-scale farmers fit the bill. They have more extensive collateral and therefore pay higher default costs. Furthermore, they are likely to have an advantage in terms of education, transportation and communication which implies they pay lower adoption costs. Also, it may be that large farmers have an easier time financing adoption because they are perceptibly less prone to divert credit to consumption (i.e., have a distribution of β that is more favorable from the lender's point of view) both because they are less vulnerable to consumption shocks like medical emergencies and

¹⁶ Group lending is not a panacea, however. In some circumstances, it can exacerbate moral hazard (Huppi and Feder).

because they have more savings to buffer against consumption shocks. All this implies a bias in agricultural finance that reinforces technological duality. Wealthy large farmers are able to finance adoption but poor small farmers are not. Moreover, this technological duality perpetuates itself over time—better access to credit leads to adoption which leads to better access to credit, and so on, while limited access to credit leads to technological stagnation which leads to limited access to credit and so on. In short, wealthy farmers get wealthier while poor farmers stay poor. Thus, asymmetric information in rural credit markets contributes to the perpetuation of a low-level equilibrium in small farm agriculture.

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Appendix

A.1. Individual rationality constraint.

Individual rationality dictates that borrowers only accept contracts providing a non-negative return on either investment or default. The net return on default, $\beta L - K$, is non-negative as long as $L \geq K/\beta$. For convenience define the critical loan size to be $\bar{L}(\beta) \equiv K/\beta$. Thus, as long as $L > \bar{L}(\beta)$, any contract will yield a non-negative return. When $L < \bar{L}(\beta)$, however, borrowers will only accept contracts yielding a non-negative return on investment. In this case, the individual rationality locus is identical to the indifference curves described in Appendix A.3 (ii) (for $\bar{U} = 0$) except that for $L < L_S$ it is everywhere negatively sloped. To see this, note that in this range, individually rational contracts are delimited by,

$$g^t(L) - RL = 0,$$

with slope,

$$dR/dL = g^t(L)(\sigma - 1)/L^2,$$

where σ is the elasticity of output with respect to L . This expression is negative since the concavity of $g^t(L)$ implies $\sigma < 1$.

A.2. Proposition 1. Characteristics of the default locus.

Equation (3) may be written:

$$R = \{g^i(L) - F^i - \beta L + K\}/L.$$

Taking derivatives,

$$(i) \quad dR/dL = [g^i(L)(\sigma - 1) + (F^i - K)]/L^2,$$

where σ is the elasticity of output with respect to L . This derivative is negative since the concavity of $g^i(L)$ implies $\sigma < 1$ and Assumption 5 ensures that $K \geq F^i$.

$$(ii) \quad dR/d\beta = -1.$$

A.3. Proposition 2. Characteristics of borrowers' indifference curves.

(i) When default is optimal, the borrower's indifference curves are given by,

$$\beta L - K = \bar{U} \quad \forall \bar{U},$$

which obviously describes a family of vertical lines.

(ii) When investment is optimal, the borrower's indifference curves are given by,

$$g^i(L) - RL - F^i = \bar{U} \quad \forall \bar{U},$$

with slope,

$$\begin{aligned} dR/dL &= (dg^i/dL - R)/L &> &> \\ &= 0 \text{ as } dg^i/dL = R. \\ &< &< &< \end{aligned}$$

The second derivative is,

$$\begin{aligned} d^2R/dL^2 &= [(d^2g^i/dL^2)L - (dg^i/dL - R)]/L^2 \\ &= (d^2g^i/dL^2)/L < 0 \text{ when } dg^i/dL = R. \end{aligned}$$

Thus, on each side of $L_S(\beta)$, indifference curves are single peaked and have a unique maximum at L^i_R where $dg^i/dL = R$. Assumption 3 ensures that the global maximum is at L^i_R .

A.4. Proposition 3. Characteristics of $\beta^i(R,L)$.

Since $H(\beta)$ is monotonically increasing in β , its derivatives have the same sign as those of $\beta^i(R,L)$. Rearranging (5) yields,

$$(A1) \quad \beta^i(R,L) = [g^i(L) - RL - F^i + K]/L.$$

Taking derivatives,

$$\partial \beta^i / \partial L = [g^i(L)(\sigma - 1) + F^i - K]/L^2 < 0.$$

(where σ is the elasticity of output with respect to L) because the concavity of $g^i(L)$ implies $\sigma < 1$ and because $K \geq F^i$ by Assumption 5.

$$\partial \beta^i / \partial R = -1.$$

$$\partial^2 \beta^i / \partial L^2 = [d^2g^i/dL^2 - 2\partial \beta^i / \partial L]/L \leq 0 \text{ if } d^2g^i/dL^2 \leq 2\partial \beta^i / \partial L.$$

$$\partial^2 \beta^i / \partial R^2 = 0.$$

$$\partial^2 \beta^i / \partial R \partial L = \partial^2 \beta^i / \partial L \partial R = 0.$$

A.5. Lenders' zero-profit locus.

The following shows that for $RL > K$, the slope of the zero-profit locus is increasing in L as long as $\partial^2\beta/\partial L^2$ is negative, the elasticity of H with respect to L is less than -1 , and $H(\beta)$ is log concave. Dividing the numerator and denominator of (7) by $H(\beta)$, differentiating with respect to L , and suppressing superscripts for clarity yields,

$$\frac{d}{dL} \left(\frac{dR}{dL} \Big|_{\pi} \right) = \frac{\left\{ - \left[L + \frac{h}{H} \frac{\partial \beta}{\partial R} (RL - K) \right] \left[R \frac{h}{H} \frac{\partial \beta}{\partial L} + \left(\frac{h}{H} \frac{\partial^2 \beta}{\partial L^2} + \left(\frac{\partial \beta}{\partial L} \right)^2 \psi \right) (RL - K) + \frac{K}{H^2} \left(Lh \frac{\partial \beta}{\partial L} + H \right) - \frac{K}{L^2} \right] \right.}{\left. + \left[\frac{h}{H} \frac{\partial \beta}{\partial L} (RL - K) - \frac{(1-H)K}{H} \frac{1}{L} \right] \left[1 + \frac{h}{H} \frac{\partial \beta}{\partial L} L + (RL - K) \left(\frac{h}{H} \frac{\partial^2 \beta}{\partial L \partial R} + \frac{\partial \beta}{\partial L} \frac{\partial \beta}{\partial R} \psi \right) \right] \right\}}{\left[L + \frac{h}{H} \frac{\partial \beta}{\partial R} (RL - K) \right]^2}$$

where,

$$\psi = \frac{d}{d\beta} \left(\frac{h}{H} \right) = \frac{\left(H \frac{dh}{d\beta} - h^2 \right)}{H^2}.$$

The sign depends on the signs of the terms in the numerator. The first term in square parentheses is $\partial\pi/\partial R$ and, therefore, must be positive for L sufficiently small if the credit market exits for loans such that $RL > K$ (otherwise, no interest rate could compensate the lender for making even the smallest such loan). Given the signs of the derivatives of $\beta(R,L)$ presented in Appendix A.4, the remaining terms in square parentheses are negative as long as: (a) $\partial^2\beta/\partial L^2 < 0$; (b) $(Lh \cdot \partial\beta/\partial L + H) < 0$; (c) $(1 + h/H \cdot \partial\beta/\partial L \cdot L) < 0$; and (d) $\psi < 0$. The first condition is met by assumption; the second and third conditions follow directly from the assumption that the elasticity of H with respect to L is less than -1 , and the fourth condition follows immediately from the assumption that H is log concave, i.e., $d^2\log H(\beta)/d\beta^2 < 0$.

A.6. Proposition 7. Policy Prescriptions.

(i) I show that increases in "D": (a) increase the slope of the zero-profit locus; and (b) reduce L^* .

(a) Taking the total derivative of (6) to get $dR/dL|_{\pi}$, taking the derivative of this expression with respect to D , simplifying and suppressing superscripts for clarity yields,

$$\frac{d}{dD} \left(\frac{dR}{dL} \Big|_{\pi} \right) = \frac{1}{LH + h \frac{\partial \beta}{\partial R} (RL - K)}.$$

The denominator is $\partial\pi/\partial R$ and, therefore, must be positive for L sufficiently small if the credit market exists for undercollateralized loans.

(b) (R^*, L^*) is defined by two equations: (1) $\pi = 0$, and (2) $\partial\pi/\partial R = 0$. Dividing the second equation by $H(\beta)$ and differentiating totally yields,

$$\mathbf{J} \begin{bmatrix} \partial R^*/\partial D \\ \partial L^*/\partial D \end{bmatrix} = \begin{bmatrix} L \\ 0 \end{bmatrix}$$

where,

$$\mathbf{J} = \begin{bmatrix} \left(L + \frac{h}{H} \frac{\partial\beta}{\partial R} (RL - K) \right) & \left(\frac{h}{H} \frac{\partial\beta}{\partial L} (RL - K) - (1-H) \frac{K}{L} \right) \\ \left(L \frac{h}{H} \frac{\partial\beta}{\partial R} + \psi \left(\frac{\partial\beta}{\partial R} \right)^2 (RL - K) \right) & \left(R \frac{h}{H} \frac{\partial\beta}{\partial L} + \frac{h}{H} \frac{\partial^2\beta}{\partial L^2} (RL - K) \right) \\ & + \psi \left(\frac{\partial\beta}{\partial L} \right)^2 (RL - K) - \frac{K}{L^2} \\ & + \left(Lh \frac{\partial\beta}{\partial L} + H \right) \frac{K}{L^2 H^2} \end{bmatrix}.$$

By definition, the northwest element of \mathbf{J} , $\partial\pi/\partial R$, is zero at (R^*, L^*) . Since ψ , $\partial\beta/\partial R$ and $\partial\beta/\partial L$ are negative, the southwest and northeast elements are also negative. Thus,

$$|\mathbf{J}| < 0.$$

By Cramer's rule,

$$\partial L^*/\partial D = |\mathbf{J}_2|/|\mathbf{J}| = - \left(L \frac{h}{H} \frac{\partial\beta}{\partial R} + \psi \left(\frac{\partial\beta}{\partial R} \right)^2 (RL - K) \right) L \div |\mathbf{J}| < 0.$$

(ii) I show that increases in the productivity of the new technology: (a) lower the zero-profit interest rate on loans to the right of L_S ; and (b) raise L^* .

(a) Assume that $g^n(L, p)$ is differentiable with respect to an index of productivity, p , and that $\partial g^n/\partial p > 0$ and $\partial^2 g^n/\partial L \partial p > 0$. Totally differentiating (6) yields,

$$\frac{dR}{dp} \Big|_{\pi} = \frac{-h \frac{\partial\beta}{\partial p} (RL - K)}{LH + h \frac{\partial\beta}{\partial R} (RL - K)},$$

to the right of L_S . If the credit market exists for undercollateralized loans, the denominator, $\partial\pi/\partial R$, must be positive for L sufficiently small. The numerator and, therefore, the entire

expression, is negative as long as $\partial\beta^n/\partial p$ is positive. $\partial\beta^n/\partial p$, derived from (A1), is equal to $(\partial g^n/\partial p)/L > 0$.

(b) Totally differentiating the conditions that define (R^*, L^*) (see section A.6ib above) after dividing the second condition by H yields,

$$\mathbf{J} \begin{bmatrix} \partial R^*/\partial D \\ \partial L^*/\partial D \end{bmatrix} = \begin{bmatrix} -\left(h \frac{\partial\beta}{\partial p} (RL - K)\right) \\ -\left(\frac{\partial\beta}{\partial R} \frac{\partial\beta}{\partial p} \psi(RL - K)\right) \end{bmatrix},$$

Both elements of the vector on the right hand side are negative since $\partial\beta^n/\partial p$ is positive and $\partial\beta^n/\partial R$ and ψ are negative. By Cramer's rule,

$$\partial L^*/\partial p = |\mathbf{J}_2|/|\mathbf{J}| = -\left(L \frac{h}{H} \frac{\partial\beta}{\partial R} + \psi \left(\frac{\partial\beta}{\partial R}\right)^2 (RL - K)\right) * \left(-h \frac{\partial\beta}{\partial p} (RL - K)\right) \div |\mathbf{J}| > 0.$$

(iii) I show that decreases in F: (a) lower the zero-profit interest rate on loans to the right of L_S ; and (b) raise L^* .

(a) Totally differentiating (6) yields,

$$\frac{dR}{dF} \Big|_{\pi} = \frac{-h \frac{\partial\beta}{\partial F} (RL - K)}{LH + h \frac{\partial\beta}{\partial R} (RL - K)},$$

to the right of L_S . If the credit market exists for undercollateralized loans, the denominator, $\partial\pi/\partial R$, must be positive for L sufficiently small. The numerator and, therefore, the entire expression is positive as long as $\partial\beta^n/\partial F$ is negative.

(b) Totally differentiating the conditions that define (R^*, L^*) (see section A.6ib above) after dividing the second condition by H yields,

$$\mathbf{J} \begin{bmatrix} \partial R^*/\partial F \\ \partial L^*/\partial F \end{bmatrix} = \begin{bmatrix} -\left(h \frac{\partial\beta}{\partial F} (RL - K)\right) \\ -\left(\frac{\partial\beta}{\partial R} \frac{\partial\beta}{\partial F} \psi(RL - K)\right) \end{bmatrix},$$

where both elements of the vector on the right hand side are positive since $\partial\beta^n/\partial F$, $\partial\beta/\partial R$, and ψ are negative. By Cramer's rule,

$$\partial L^*/\partial F = |\mathbf{J}_2|/|\mathbf{J}| = -\left(L \frac{h}{H} \frac{\partial\beta}{\partial R} + \psi \left(\frac{\partial\beta}{\partial R}\right)^2 (RL - K)\right) * \left(-h \frac{\partial\beta}{\partial F} (RL - K)\right) \div |\mathbf{J}| < 0$$

(iv) I show that increases in K raise L^* . Totally differentiating the conditions that define (R^*, L^*) (see section A.6ib above) after dividing the second by H yields,

$$\mathbf{J} \begin{bmatrix} \partial R^* / \partial K \\ \partial L^* / \partial K \end{bmatrix} = \begin{bmatrix} -\left(h \frac{\partial \beta}{\partial K} (RL - K) + (1 - H) \right) \\ -\left(\frac{\partial \beta}{\partial R} \frac{\partial \beta}{\partial K} \psi (RL - K) - \frac{h}{H} \frac{\partial \beta}{\partial R} \right) \end{bmatrix},$$

where both elements of the vector on the right hand side are negative since ψ and $\partial \beta / \partial R$ are negative and $\partial \beta / \partial K$ is positive ($\partial \beta / \partial K = 1/L$). By Cramer's rule,

$$\partial L^* / \partial K = |\mathbf{J}_2| / |\mathbf{J}| = - \left(L \frac{h}{H} \frac{\partial \beta}{\partial R} + \psi \left(\frac{\partial \beta}{\partial R} \right)^2 (RL - K) \right) * \left(-h \frac{\partial \beta}{\partial K} (RL - K) - (1 - H) \right) \div |\mathbf{J}| > 0.$$

(iv) I show that increases in “ w ”, an index of the proportion of relatively patient borrowers: (a) reduce the slope of the zero-profit locus; and (b) reduce L^* .

(a) Let $H(\beta, w)$ depend on w in the sense of first order stochastic dominance, that is, $\partial H / \partial w \geq 0$ and $\partial^2 H / \partial w^2 \leq 0$. Graphically, an increase in w shifts the c.d.f. of β up and p.d.f. of β to the left. Totally differentiating (6) and dividing the numerator and denominator by H yields,

$$\frac{d}{dw} \left(\frac{dR}{dL} \Big|_{\bar{\pi}} \right) = \left\{ \left(\frac{d\pi}{dR} / H \right) \left(-\tau \frac{\partial \beta}{\partial L} (RL - K) - D \frac{\partial H}{\partial w} / H^2 \right) - \left(-\frac{d\pi}{dL} / H \right) \left(\tau \frac{\partial \beta}{\partial R} (RL - K) \right) \right\} / \left(\frac{d\pi}{dR} / H \right)^2$$

where,

$$\tau = \left(\frac{H \frac{\partial h}{\partial w} - h \frac{\partial H}{\partial w}}{H^2} \right).$$

Of the terms inside the curly brackets in the first expression, the first is positive since $\partial \pi / \partial R > 0$ for L sufficiently small; the second is negative for undercollateralized loans as long as τ is negative (since $\partial \beta / \partial L$ is negative and $\partial H / \partial w$ is non-negative); the third is positive since $\partial \pi / \partial L$ is negative; and the last term is positive as long as τ is negative (since $\partial \beta / \partial R$ is negative). Thus, the entire expression is negative as long as τ is negative. Since $\partial H / \partial w$ is non-negative, the sign of τ depends on the sign of $\partial h / \partial w$. This, in turn, depends on $\beta(R, L)$. For all $\beta(R, L)$ greater than the mean of $h(\beta, w)$, $\partial h / \partial w$ is negative and therefore, τ is negative. When $\beta(R, L)$ is very small, then $H(\beta, w)$ is zero, and again, τ is negative. Therefore, we may assume that τ is negative for all $\beta(R, L)$.

(b) Totally differentiating the conditions that define (R^*, L^*) (see section A.6ib above) after dividing the second condition by H yields,

$$\mathbf{J} \begin{bmatrix} \partial R^*/\partial w \\ \partial L^*/\partial w \end{bmatrix} = \begin{bmatrix} -\left(\frac{\partial H}{\partial w}(RL-K)+K\right) \\ -\left(\tau \frac{\partial \beta}{\partial R}(RL-K)\right) \end{bmatrix}.$$

Both elements of the vector on the right hand side are negative since $\partial H/\partial w$ is positive and τ and $\partial \beta/\partial R$ are negative. By Cramer's rule,

$$\partial L^*/\partial w = |\mathbf{J}_2|/|\mathbf{J}| = -\left[L \frac{h}{H} \frac{\partial \beta}{\partial R} + \psi \left(\frac{\partial \beta}{\partial R}\right)^2 (RL-K)\right] * -\left(\frac{\partial H}{\partial w}(RL-K)+K\right) \div |\mathbf{J}| > 0.$$