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The Informational Role of Spot Prices and Inventories

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Abstract

We examine the role that spot markets and physical inventories play in revealing to uninformed traders the expectations of informed traders. Although many papers investigate potential mechanisms by which futures markets may disseminate such information, the role of spot markets has not been examined in comparable detail. Because the incentive for speculative trading in futures contracts stems from the failure of spot markets to eliminate differences in beliefs regarding future market conditions, the scope for speculative trading in the futures market is therefore determined, but also limited, by the extent to which spot market transactions disseminate private information. Using a rational expectations approach, we show that equilibrium differences in beliefs are determined by specific characteristics of the underlying commodity, including storage costs, the amplitude of unexpected demand and supply shocks, the accuracy of information acquired by informed investors, the numbers of informed and uninformed investors, and the elasticity of demand and supply.

Key Words: futures trading, speculation, inventories, private information

JEL Classification Numbers: D82, D84, G13, G14

Contents

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Overview

In this paper, we explore the role that spot markets and physical inventories play in revealing to uninformed traders the expectations of informed traders. Although a large literature investigates the potential mechanisms by which futures markets may disseminate such information, the role of spot markets has not been examined in comparable detail. Portions of the literature may even give the impression that spot markets fail completely in this regard unless supplemented by futures markets—or by an even larger set of contingent commodity claims.[1] Because the incentive for speculative trading in futures contracts hinges on the failure of spot markets to eliminate differences in beliefs regarding future market conditions, the scope for speculative trading in the futures market is therefore determined, but also limited, by the extent to which spot market transactions disseminate private information.

We examine the extent to which spot prices disseminate the expectations of informed investors to uninformed traders. We also show how the dissemination of information is determined by specific characteristics of the commodity in question, including storage costs, the amplitude of unexpected demand and supply shocks, the accuracy of information acquired by informed investors, the numbers of informed and uninformed investors, and the elasticity of demand and supply for the commodity in question. The empirical implications of our results arise because the difference in beliefs between informed and uninformed traders, and therefore the incentive for speculative trading, is also determined by these characteristics.

Thus, our research objective consists of two parts: (a) to set forth a rational theory of spot market prices that illuminates the factors pertinent to the revelation of information and (b) to

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[1] This point was recognized years ago by Working (1942, 50), who observed: "Anticipations of all manner of developments that are thought to be predictable play a part in determining the price of a future. The error of the common theory has lain merely in supposing that the prices of futures, or of some particular futures, tend to be more strongly influenced by these anticipations than are spot prices."

develop testable implications of our theoretical analysis that point the way to additional empirical research (to be undertaken in a separate paper).

Related Literature

It is widely known that spot prices of storable commodities reveal information about the future expectations of traders. For example, the standard Hotelling (1931) theory of intertemporal equilibrium for an exhaustible resource (like oil) determines the current price of a commodity as a function of future supply of and demand for that commodity. As public expectations regarding *future* supply and demand change, then so too must the *current* price. This theoretical property of spot prices is also believed to work in practice, as exemplified by Feldstein’s (2008) suggestion to bring down the current price of crude oil by enhancing the investment climate for future exploration, as well as by the sometimes dramatic impact on spot prices of periodic revisions to U.S. Department of Agriculture predictions of future crop harvest levels, as reported by Pleven and Moffett (2012).

Our interest lies in a different direction. No doubt, spot prices reflect common expectations of future market conditions, if such a consensus is known to exist. But suppose it does not. Suppose instead that certain investors invest to acquire information that provides more accurate predictions about future conditions, whereas others do not. Assuming that all traders are rational, we ask whether informed traders’ participation in the spot market will reveal their information to those who were previously uninformed. This is not an all-or-nothing proposition; we mean to explore the extent to which private information acquired by informed traders is revealed, and the factors on which this determination rests.

Previous researchers, including Milgrom and Stokey (1982) and Tirole (1982), have demonstrated that, in an economy with complete markets and rational expectations, no difference in beliefs can persist in equilibrium. Any private information initially held by certain traders is disseminated to all through the price mechanism and becomes common knowledge. Such an economy would include not only a complete set of futures markets for each commodity, but a complete set of contingent claims contracts as well, but no speculative trading. Grossman (1977) and Grossman and Stiglitz (1980) demonstrate conditions under which futures markets and equilibrium futures prices effectively disseminate all private information to the market at large, but they also recognize that transaction costs and information costs place limits on the market mechanism that might lead to equilibrium differences in investors’ beliefs.

We demonstrate that even in the absence of futures markets (or any more complete market in contingent claims), rational expectations render spot markets effective at revealing some private information, and the extent of revelation may be large or small depending on characteristics of the commodity in question. The informational role of spot prices may, therefore, vary significantly across the set of traded commodities. We are not aware of any previous literature that has examined this aspect of spot markets, or that has considered how the informational role of spot prices varies across commodities as a predictor of the scope of speculative futures trading.

Rational Expectations Equilibrium in the Spot Market for a Storable Commodity

To frame these questions in a familiar but rigorous context, we adapt and extend Grossman's (1977) analysis of rational expectations equilibrium for a storable commodity, like wheat. The commodity can be produced only during certain parts of the year, but people want to consume it throughout the year. Following Grossman, we assume that the year's harvest, Q, is fixed exogenously and that consumers' demand for wheat in each period depends only on that period's price, according to the function

$$D = D(P_t, w_t) \; for \; t = 1, 2, \tag{1}$$

where P_t represents the period price of wheat and the w_t represents stochastic demand shocks that are assumed to follow independent normal distributions with zero mean and standard deviation given by σ. We assume that $\partial D/\partial P < 0$ and $\partial D/\partial w > 0$.[2]

In addition to consumers, the market contains firms that purchase and store wheat from one harvest to the next, which effects an intertemporal allocation of the harvest. We assume that some of these firms are "informed," meaning that at the opening of the period 1 spot market, they observe w_1 directly and also acquire a signal, θ, that is correlated with the future demand shock w_2. We assume that θ and w_2 are jointly normally distributed with correlation coefficient ρ. Obviously, the quality of informed firms' estimate of future demand increases in ρ. The conditional density of w_2 given θ is also normal and is denoted $f(w_2|\theta)$; informed firms use this distribution in addition to their knowledge of w_1 to make inferences about future demand and future price. Uninformed firms observe neither w_1 or θ, but know the marginal density, $\mu(\theta)$, and

[2] The assumption that supply is fixed is for notational convenience and does not affect the results. Supply shocks can be incorporated explicitly, or (as we have done) simply subsumed in w_1. That is, suppose the harvest is given by $Q+\varepsilon_s$, where ε_s represents a supply shock, and let ε_d represent the shock to demand. If we then define $w_1 = \varepsilon_d - \varepsilon_s$ as the shock to the net demand curve, the model developed in the text follows directly.

observe the first-period price; we therefore assume that they make inferences about future demand and price using P_1 and $\int f(w_2|\theta)\,\mu(\theta)d\theta$.

Using all information at their disposal, each type of firm is assumed to purchase first-period inventories to maximize expected profits.[3] If $C(I)$ represents the cost of holding inventory level I, then each uninformed firm must solve the equation, $\max_I$: $E[\pi^a] = (E[P_2|P_1] - P_1)I - C(I)$, to obtain an inventory supply function, $I^a = S^a(P_1)$, that satisfies the first-order condition

$$E[P_2|P_1] - P_1 = MC(I^a), \tag{2}$$

where $MC(\cdot)$ represents the firm's marginal cost of inventory. Likewise, each informed firm must solve the equation, max_I: $E[\pi^b] = (E[P_2|w_1,\theta] - P_1)I - C(I)$, to obtain an inventory supply function, $I^b = S^b(w_1,\theta,P_1)$, that satisfies

$$E[P_2|w_1,\theta] - P_1 = MC\left(I^b\right). \tag{3}$$

Whereas Grossman (1977) assumed only one firm of each type, we allow m uninformed firms and n informed firms. For the present, we will assume that m and n are determined exogenously. Therefore, total inventories carried over from the first to second period are given by

$$\vartheta = m \times S^a(P_1) + n \times S^b(w_1,\theta,P_1). \tag{4}$$

For the given number of firms (m,n), we now define a rational expectations equilibrium as a pair of mappings $\left(\tilde{P}_1,\tilde{P}_2\right)$ such that[4]

$$P_1(\cdot) \stackrel{\text{def}}{=} P_1^e(w_1,\theta), \qquad P_2(\cdot) \stackrel{\text{def}}{=} P_2^e\left(w_1,w_{2,}\theta\right) \tag{5}$$

and

$$D[P_1^e(w_1,\theta)] = Q - \vartheta^e(w_1,\theta) \tag{6}$$

$$D[P_2^e(w_1,w_2,\theta)] = \vartheta^e(w_1,\theta) \tag{7}$$

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[3] As Tirole (1982) notes, inventories in this model constitute a positive–sum game that may generate an equilibrium difference in beliefs despite the rational expectations of all traders.

[4] Grossman's (1977) proofs of the existence and uniqueness of equilibrium easily generalize to this case, which differs only in terms of the numbers of informed and uninformed firms.

$$\vartheta^e(w_1,\theta) \stackrel{\text{def}}{=} m \times S^a\{E[\tilde{P}_2|\tilde{P}_1 = P_1^e(w_1,\theta)] - P_1^e(w_1,\theta)\} + n \times S^b\{E[\tilde{P}_2|w_1,\theta] - P_1^e(w_1,\theta)\} \quad (8)$$

for all w_1, w_2, and θ, and where

$$E[\tilde{P}_2|w_1,\theta] \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} P_2^e(w_1,w_2,\theta) f(w_2|\theta) dw_2. \quad (9)$$

Assuming only one trader of each type, Grossman (1977) demonstrated that P_1^e fails to fully reveal the private information held by the informed investor, except in some degenerate cases. This remains true when multiple traders are involved, but as we show below, the extent to which private information is revealed varies with the relative numbers of informed and uninformed traders, and depends as well as on fundamental characteristics of the commodity in question. The equilibrium difference in beliefs about $\tilde{P}_2$, which is denoted $\Delta(w_1,\theta)$, is defined as

$$\Delta(w_1,\theta) \stackrel{\text{def}}{=} E[\tilde{P}_2|w_1,\theta] - E[\tilde{P}_2|\tilde{P}_1 = P_1^e(w_1,\theta)]. \quad (10)$$

In particular, by examining the structure of $\Delta(w_1,\theta)$, we are able to discover the extent to which $P_1^e(w_1,\theta)$ reveals the quantity $E[\tilde{P}_2|w_1,\theta]$ to uninformed firms, and to identify the factors that produce more complete revelation.

To proceed, we adopt the same linear demand functions that Grossman employed. Thus, from this point we assume

$$D_t = k + \frac{1}{h} P_t + w_t \quad \text{for } t = 1, 2. \quad (11)$$

Inventory costs depend on whether stocks are held separately by individual firms or pooled in a common storage facility. Grossman assumed separate holdings, with each firm's cost determined by the size of its own inventory according to $C(I) = cI^2/2$.[5] If inventories are pooled, however, the aggregate (industrywide) cost would be given by $C(\vartheta) = c\vartheta^2/2$, where $\vartheta = \sum_j I_j$ represents the combined inventory of all firms.

It seems reasonable to assume that a shared inventory facility would operate like a public utility subject to cost-of-service rate regulation, in which case each firm would be charged the same amount, $c\vartheta/2$, per unit held in storage.[6] Thus, an individual firm's cost can be represented as a function of its own inventory and the stock held by others

[5] One implication of this assumption is that the aggregate cost of holding a stock of a given size falls as the stock is subdivided into more holdings, each of smaller size.

[6] The charge per unit is equal to the facility's average cost of storage.

$$C(I|\vartheta) = \frac{cI^2}{2} + \iota\frac{cI(\vartheta - I)}{2}, \tag{12a}$$

where ι is a variable that indicates whether inventories are shared ($\iota = 1$) or separate ($\iota = 0$).[7] The firm's marginal cost is, accordingly

$$MC(I|\vartheta) = \frac{c}{2}(I + \iota\vartheta). \tag{12b}$$

From equations (2), (3), and (12b), the respective inventory supply functions must satisfy

$$I^a(P_1) = \frac{E[\tilde{P}_2|P_1] - P_1}{c} - \frac{\iota\vartheta^a}{2} \tag{13a}$$

$$I^b(w_1, \theta) = \frac{E[\tilde{P}_2|w_1,\theta] - P_1}{c} - \frac{\iota\vartheta^b}{2}, \tag{13b}$$

where ϑ^x represents total inventories less the amount held by one firm of type x.

The expectation of future price held by informed investors can be computed from equation (7) after first inverting the demand function in equation (11)

$$E[P_2|w_1, \theta] = E[h(I - k - w_2)|w_1, \theta] = h(Q - 2k) - P_1 - y_2, \tag{14}$$

where $y_2 = h(w_1 + E[w_2|\theta])$. The leading terms on the right-hand side of equation (14) are directly observable by uninformed traders. But to read the price expectations of informed investors, uninformed traders also need to know the quantity y_2, which represents the informed investors' view of the sum of demand shocks. To see whether P_1 actually reveals that additional information, combine equations (6) and (8) using the demand and inventory functions given by equations (11) and (13) to obtain

$$k + \frac{1}{h}P_1 + \frac{m}{c}\left(E[\tilde{P}_2|P_1] - P_1\right) - \frac{\iota(1-m-n)}{2}\vartheta - Q = -w_1 - \frac{n}{c}\left(E[\tilde{P}_2|w_1,\theta] - P_1\right), \tag{15}$$

which, after substituting for ϑ and using equation (14) to evaluate $E[P_2|w_1, \theta]$, is equivalent to[8]

[7] Because intermediate values between 0 and 1 can be interpreted as partial pooling, we will permit ι to be any number in the closed interval [0,1].

[8] ϑ is determined by summing across equations (13a) and (13b), which yields $\vartheta = \frac{1}{c\left(1-\frac{\iota(1-m-n)}{2}\right)}\{m \times E[(\tilde{P}_2|P_1) - P_1] + n \times E[(\tilde{P}_2|w_1,\theta) - P_1]\}$.

$$P_1\left(\frac{(1-\phi)}{h}-\frac{(2n+m)h}{c}\right)+\frac{m}{c}E[\tilde{P}_2|P_1]+Q\left(\frac{nh-c(1-\phi)}{c}\right)-k\left(\frac{2nh-c(1-\phi)}{c}\right)=y_1, \quad (16)$$

where

$$y_1=\left(\frac{nh-c(1-\phi)}{c}\right)w_1+\frac{nh}{c}E[w_2|\theta] \quad (17)$$

and $\phi=\frac{\iota(1-m-n)}{2}$.

The left-hand side of equation (16) depends only on known parameters and P_1. Thus, P_1 reveals to uninformed traders the quantity y_1, which is the wrong linear combination of demand shocks; so y_1 deviates systematically from the desired quantity, y_2. The spot price therefore sends a garbled signal to uninformed traders, who cannot, without additional information, read very precisely the expectations of informed traders.

Before examining factors that determine the degree of garbling, it is worth noting that, if uninformed traders are able to observe both P_1 and w_1 (the first-period demand shock) they are much better informed because the value of $E[w_2|\theta]$, and therefore y_2, can then be inferred from equation (16). Only in that event will the private information of informed traders be fully revealed. Although we have assumed that uninformed traders are not able to observe w_1 directly, there are two ways in which they might acquire such information indirectly. First, if the total volume of inventories is announced (say, by an omniscient government agency), then any firm that observes P_1 can use the inventory data to infer w_1 from equations (6) and (11). Alternatively, any firm that happens to be aware of the industrywide inventory cost function and that also knows that its own storage is billed at the average cost of service could infer total inventory (and hence w_1) by simply inverting the cost function. This highlights the pivotal role that information regarding physical stocks and inventory costs plays in leveraging the information revealed by the spot price. Either type of information (i.e., physical stocks or inventory costs) may be sufficient to produce a fully revealing equilibrium.

In practice, neither of these paths toward a fully revealing equilibrium may be easy to achieve. Although government agencies do announce estimated inventory levels for many commodities, these estimates are based on incomplete surveys, published after significant lags, and are typically subject to revision.[9] The other possibility, that uninformed traders manage to accurately read industry aggregates from their individual storage costs, also seems doubtful—and becomes impossible if inventories are kept separate and not pooled (our special case of $\iota = 0$). Where circumstances do permit either of these possibilities, then the spot market equilibrium we model would indeed be fully revealing, and thereby eliminate the incentive to gather private information as well as the incentive to engage in speculative trading in the futures market. For the balance of this paper, and because we believe that the alternative hypothesis holds greater interest, we maintain the assumption that uninformed traders do *not* have accurate inventory data, and therefore are not able to precisely read the expectations of informed traders.

Despite the garbling that occurs under this scenario, some information is nevertheless revealed. By inspection of equation (17), it is apparent that, as $\frac{nh}{c}$ grows large relative to $\frac{1+m+n}{2}$, y_1 converges to y_2. This suggests that the magnitude of the average difference in beliefs may depend systematically on these underlying factors (i.e., inventory costs, the elasticity of demand, and the relative number of informed investors who participate in the market)—a possibility that we examine in more detail below.

Even where garbling occurs, the equilibrium difference in beliefs is, on average, zero because positive and negative differences cancel out.[10] In either case, the discrepancy puts uninformed traders at a disadvantage, with effects that do not cancel out. Therefore, rather than using $E[\Delta(w_1, \theta)]$ to measure the average difference in beliefs, it is better to focus on the mean squared error, which accumulates the absolute difference between informed and uninformed traders

$$DIFF = E[\Delta(w_1, \theta)]^2 = var(y_2|y_1) = \sigma_{y_2}^2\left(1 - r_{y_2 y_1}^2\right), \tag{18}$$

where

[9] The U.S. Energy Information Administration (EIA), for example, releases crude oil and refined product inventory data (for the United States only) with weekly, monthly, and yearly lags, with improved accuracy at the longer lags. Although revisions to the weekly stock data are typically small (1–2 percent), much larger revisions occasionally occur for particular products (e.g., 9 percent, on average, for stocks of ultra-low sulfur distillate in 2006 and 18 percent, on average, for stocks of reformulated motor gasoline in 2007, the latest two years for which EIA has prepared summary reports of such revisions). However, revisions to the reported weekly *change* in petroleum stocks are drastically higher, averaging 52 percent in 2006 and 80 percent in 2007. See Heppner and Breslin (2009) for more detail.

[10] Information available to uninformed bidders is not biased because $E[w_2] = E\big[E[w_2|\theta]\big]$.

$$\sigma_{y_2}^2 \stackrel{\text{def}}{=} var(y_2) \qquad \text{and} \qquad r_{y_1y_2} \stackrel{\text{def}}{=} \frac{cov(y_1y_2)}{\sqrt{var(y_1)}\sqrt{var(y_2)}}. \tag{19}$$

The second equality in equation (18) is based on Grossman's (1977) proof; the third equality is simply a change of notation to highlight the importance of the correlation between y_1 and y_2.[11] For convenience, we refer to the two parts of *DIFF* as the *range of variation* $\left(\sigma_{y_2}^2\right)$ and the *degree of garbling* $\left(1 - r_{y_1y_2}^2\right)$.

The average difference in beliefs can be evaluated using equation (18) and the expressions given previously for y_1 and y_2, which imply the following:

$$var(y_2) = h^2\big(var(\widetilde{w}_1) + var(E[\widetilde{w}_2|\theta])\big) = h^2\sigma^2(1+\rho^2) \tag{20}$$

$$var(y_1) = \left(\frac{nh}{c} - (1-\phi)\right)^2 var(\widetilde{w}_1) + \left(\frac{nh}{c}\right)^2 var(E[\widetilde{w}_2|\theta]) = \sigma^2\left[\left(\frac{nh}{c}\right)^2(1+\rho^2) - \frac{2nh(1-\phi)}{c} + (1-\phi)^2\right] \tag{21}$$

$$cov(y_1y_2) = \sigma^2\left[\frac{nh^2}{c}(1+\rho^2) - h(1-\phi)\right] \geq 0, \tag{22}$$

where we have used the independence of w_1 and w_2, and where the symbol ρ denotes the simple correlation coefficient between θ and w_2, which measures the quality of information available to informed investors. The fact that the demand curve is downward sloping $(h < 0)$ implies that the covariance must be nonnegative. After substituting these terms in equation (18), $DIFF$ takes the form

$$DIFF = h^2\sigma^2(1+\rho^2)\left\{1 - \frac{\left[nh^2(1+\rho)^2 - (1-\phi)ch\right]^2}{[n^2h^2(1+\rho^2) - 2cnh(1-\phi) + c^2(1-\phi)^2][h^2(1+\rho^2)]}\right\}. \tag{23}$$

The Quality of Information Revealed by Spot Prices

In this section, we explore the quality of information that is revealed to uninformed traders in the spot market, as measured by *DIFF*, the average difference in beliefs. At one extreme, *DIFF* = 0 if all information is revealed, an outcome that occurs only if $r_{y_1y_2}^2 = 1$ (cf. equation [17]). At the other extreme ($r_{y_1y_2}^2 = 0$), no information is revealed and the maximal value of *DIFF* = $h^2\sigma^2(1+\rho^2)$, which approaches infinity as the volatility of price fluctuations ($h\sigma$) grows.

[11] Grossman's original proof is not dependent on the number of traders, as shown in our appendix.

For any given commodity, the size of the actual difference in beliefs will fall somewhere between these extremes, depending on the fundamental characteristics of the commodity in question, including inventory cost (c), elasticity of demand (h), volatility of demand shocks (σ), and the quality of information available to informed investors (ρ). In addition, the numbers of firms (m, n) play a direct role. For present purposes, we assume that the numbers of firms are determined exogenously, like the other structural parameters. Later, we relax this assumption and permit the number of informed and uninformed investors to be determined endogenously; this has additional implications for the average difference in beliefs and how it might vary across commodities.

Volatility of Demand

Using equation (23), we evaluate and sign the partial derivatives of *DIFF* with respect to each factor. We begin with the volatility of demand shocks, which has by far the simplest impact because it affects only the range of variation. Indeed, *DIFF* is simply proportional to σ^2 (cf. equation [23]), and we find

$$\frac{\partial DIFF}{\partial \sigma^2} = h^2(1+\rho)^2\left(1 - r_{y_1 y_2}^2\right) \geq 0, \tag{24}$$

which shows that the average difference in beliefs varies directly with the volatility of demand, and more specifically that, for any given degree of garbling ($1 - r_{y_1 y_2}^2 > 0$), the resulting difference in beliefs is magnified by inelastic demand (large $|h|$) and the accuracy of the informed investors' forecasting model (ρ).

Inventory Cost

We turn next to the cost of carrying the commodity in inventory (c), which is also fairly simple because, by inspection of equation (23), it affects *DIFF* only through the degree of garbling, not the range of variation. As shown in the appendix, higher inventory cost increases the degree of garbling, which implies

$$\frac{\partial DIFF}{\partial c} > 0. \tag{25}$$

It also follows immediately from equation (23) that the average difference in beliefs vanishes as inventory cost goes to zero

$$DIFF_{c=0} = 0. \tag{26}$$

Higher inventory costs thus increase the average difference in beliefs by increasing the degree of garbling. Holding all else equal, we may therefore expect larger differences in beliefs to persist in markets for commodities that are more expensive to store (like electricity and natural gas) and during periods in which storage is in short supply.

Forecast Accuracy

The accuracy of informed investors' prediction of the future demand shock is given by ρ, which measures the correlation between θ and w_2. The first component of *DIFF* (range of variation) is, by inspection of equation (23), clearly increasing in ρ. The second component of *DIFF* (degree of garbling) is also increasing in ρ (see the proof in the appendix). Together, these results imply

$$\frac{\partial DIFF}{\partial \rho} > 0. \qquad (27)$$

Thus, the more accurate the informed forecast is, the greater, on average, the difference in beliefs between informed and uninformed traders will be, holding all other factors constant.

Elasticity of Demand

Variations in the slope of the demand curve (h) exert countervailing forces on the two components of *DIFF*. The range of variation is increasing in h—just as it was increasing in the accuracy of the demand forecast (ρ), but the degree of garbling decreases in h, unlike the influence of ρ.[12] As shown in the appendix, the second force dominates and the overall effect is unambiguous

$$\frac{\partial DIFF}{\partial h} < 0. \qquad (28)$$

In terms of demand elasticity, at any given price level, demand becomes more elastic as h increases (toward zero). Therefore, equation (28) implies that the average difference in beliefs, holding all else equal, should be highest for commodities with the least elastic demand.

[12] This difference arises because any change in h impacts the expected level of demand in *both* periods, whereas the accuracy of the demand forecast only pertains to demand in the second period.

Numbers of Traders

Like inventory costs, the numbers of traders (m, n) affect *DIFF* only through the second component (garbling). As shown in the appendix, any increase in the number of informed traders reduces the degree of garbling, whereas any increase in the number of uninformed traders increases the degree of garbling. Therefore, we have

$$\frac{\partial DIFF}{\partial m} \geq 0 \qquad \text{(with strict equality if and only if } \iota = 0) \tag{29}$$

$$\frac{\partial DIFF}{\partial n} < 0 \tag{30}$$

Based on equations (29) and (30), we would expect a larger average difference in beliefs to persist in commodity markets in which relatively few informed traders participate. But we are also able to show (see appendix) that, if the numbers of informed and uninformed firms increase in fixed proportion, the equilibrium difference in beliefs will fall

$$\frac{\partial DIFF}{\partial (m+n)|m/n = constant} < 0. \tag{31}$$

Thus, a greater number of traders will increase the degree of revelation, even if the average trader is no better informed.

All results reported so far assume that the numbers of traders (m, n) are determined exogenously. In the next section, we address the incentive for traders to enter this market, the incentive for them to become informed, and the impact of endogenous decisions on the equilibrium difference in beliefs.

The Return to Private Information and Entry of Informed Traders

To the extent that the revelation of private information is incomplete, informed traders enjoy an advantage relative to uninformed traders and earn higher expected profits

$$E\left[\pi^b(\tilde{w}_1, \tilde{\theta})\right] \geq E\left[\pi^a\left(\tilde{P}_1 = P_1^e(\tilde{w}_1, \tilde{\theta})\right)\right],$$

where the expectation is taken over the joint distribution of w_1 and θ.[13] As long as this difference exceeds the cost of becoming informed, uninformed firms have an incentive to become informed, or additional informed firms have an incentive to enter the market. In either case, as we have already shown, the effect is to reduce the difference in beliefs. Thus, if the profit differential varies directly with the difference in beliefs, and if firms are rational, the entry of informed firms will continue until the difference in expected profits is reduced to the cost of becoming informed.

We characterize here the equilibrium of this process for the special case of $\iota = 0$ (separate inventories), but this case is not particularly unique, except in the simplicity of its derivation.[14] Given $\iota = 0$, the expected difference in profits between informed and uninformed traders is given by[15]

$$DIFF\pi = E[\pi^b - \pi^a] = \frac{1}{2c} DIFF, \tag{32}$$

where $DIFF$ represents the average difference in beliefs, as defined above.

We assume that entry (or exit) of informed traders occurs until the incremental profit accruing to private information falls (rises) to equal the cost of becoming informed, denoted z. Recalling the determinants of $DIFF$ discussed above, this allows us to close the model and determine the equilibrium number of informed traders (n^e) via the zero-profit condition

$$\frac{1}{2c} DIFF(c, \sigma^2, \rho, h, m, n^e) \equiv z. \tag{33}$$

Taking the total differential of equation (33) yields

$$\frac{\partial \frac{1}{2c} DIFF}{\partial c} dc + \frac{1}{2c}\left(\frac{\partial DIFF}{\partial \sigma^2} d\sigma^2 + \frac{\partial DIFF}{\partial h} dh + \frac{\partial DIFF}{\partial \rho} d\rho + \frac{\partial DIFF}{\partial m} dm + \frac{\partial DIFF}{\partial n^e} dn^e\right) = dz, \tag{34}$$

which permits determination of the separate effect of each structural parameter on the equilibrium number of informed traders

[13] The strict equality applies only if either the distribution of w_1 is degenerate or the signal carries no information (θ is uncorrelated with w_1). The proof is straightforward and follows the same argument used by Grossman (1977).

[14] Recall that all comparative static properties of the model hold for all values $\iota \in [0,1]$.

[15] Grossman derived this expression assuming that $m = n = 1$, but his derivation does not depend in any way on the number of firms.

$$\frac{dn^e}{d\sigma^2} = -\frac{\partial DIFF/\partial\sigma^2}{\partial DIFF/\partial n^e} > 0 \quad \text{(i.e., larger shocks} \rightarrow \text{more informed traders)} \tag{35a}$$

$$\frac{dn^e}{dh} = -\frac{\partial DIFF/\partial h}{\partial DIFF/\partial n^e} < 0 \quad \text{(i.e., more elastic demand} \rightarrow \text{fewer informed traders)} \tag{35b}$$

$$\frac{dn^e}{d\rho} = -\frac{\partial DIFF/\partial\rho}{\partial DIFF/\partial n^e} > 0 \quad \text{(i.e., better forecasts} \rightarrow \text{more informed traders)} \tag{35c}$$

$$\frac{dn^e}{dm} = -\frac{\partial DIFF/\partial m}{\partial DIFF/\partial n^e} = 0 \quad \text{(i.e., more uninformed traders} \rightarrow \text{no impact)} \tag{35d}$$

$$\frac{dn^e}{dz} = \frac{1}{\partial DIFF/\partial n^e} < 0 \quad \text{(i.e., costlier forecasts} \rightarrow \text{fewer informed traders)} \tag{35e}$$

$$\frac{dn^e}{dc} = -\frac{(\partial DIFF/\partial c - DIFF/c)}{\partial DIFF/\partial n^e} = \frac{(1-\varepsilon)(DIFF/c)}{\partial DIFF/\partial n^e} \gtrless 0 \text{ as } \varepsilon \gtrless 1, \tag{35f}$$

where ε is the elasticity of the difference in beliefs with respect to inventory cost. Thus, if the difference of opinion is inelastic with respect to inventory cost, costlier inventories mean fewer informed traders.

Because the difference in beliefs provides the incentive for speculative futures trading, the size of that incentive is determined in equilibrium by equation (33), which can be written as

$$DIFF = 2cz. \tag{36}$$

Equilibrium in the spot market therefore implies that the incentive for speculative trading is dependent on just two factors—the cost of inventories and the cost of information—and must be increasing in each. The cost of holding inventories may depend on the physical properties of the commodity in question as well as the extent to which existing storage facilities are already full. The cost of information is determined by the stability of factors that influence demand and supply as well as the transparency of the industry. Both factors may be expected to vary across time and across commodities; this enables an empirical test of the theory. It remains for future research to test the hypothesis that resulting variations in beliefs between informed and uninformed traders cause systematic variations in the extent of speculative trading in futures contracts. Future work should also examine the related hypothesis that, as a result of equilibrium entry of informed traders, variations in the other factors (elasticity and volatility of demand and the quality of information) that would influence the difference in beliefs holding the numbers of traders constant, actually play no role in determining the extent of speculative trading.

Another avenue of research that we are currently pursuing is to identify the impact of partial revelation of information on the level of spot prices and their volatility. We suspect that the same commodity-specific factors that regulate the revelation of information (i.e., demand and supply elasticities, volatility of market shocks, quality of information, and inventory costs) would also influence the extent to which speculation by informed traders tends to alter the behavior of spot prices.

Conclusion

Much attention has focused recently on the degree of financial speculation in various commodity futures markets. In our view, the existence and scope of that activity may be due as much to the characteristics of the underlying commodities as to the characteristics of the traders involved. Commodity characteristics that impede the revelation of information via spot trading—like high inventory costs or opaque public data sources—sustain differences in beliefs that give rise to speculative trading in futures. To the extent that speculative futures trading is perceived to be excessive, the cure for the problem (or at least its cause) may be sought in the underlying spot market.

Many concrete initiatives to enhance the information available to support trading in spot markets can be cited in this regard, especially within the realm of energy markets. Recent efforts by the Joint Organisations Data Initiative (JODI) to produce a transparent and open-access global database on monthly crude oil and refined product stocks and flows is one example,[16] although China is a notable holdout that has not yet elected to release its own inventory data. The International Organization of Securities Commissions' (2012) ongoing consultative report on the functioning and oversight of oil price reporting agencies, prepared in response to the G20 Leaders' Cannes Summit Final Declaration, is another example. And the U.N. Statistical Commission's (2011) recently released report, *International Recommendations for Energy Statistics*, represents an even broader effort to systematically increase the scope, quality, and transparency of data regarding the supply and use of energy. Similar initiatives apply to many nonenergy markets, such as the G20's new reporting program (Agricultural Market Information System) to enhance food market transparency,[17] and the imminent opening in China of a public

[16] Available online at the JODI website: http://www.jodidata.org/.

[17] Available online at the AMIS website: http://www.amis-outlook.org/home/en/.

spot market trading platform for rare earth metals, a market that up to now has remained largely opaque because rare earths have not been traded in public markets.[18]

If governments wish to decrease the amount of speculative trading in futures markets (and it appears that they do), progress reached through initiatives that shine a brighter light on the fundamentals of the underlying spot markets may be an effective complement, and perhaps substitute, for placing broader restrictions (e.g., reduced position limits, higher margin requirements, and outright prohibitions) on futures trading itself. The former approach works by reducing the demand for speculation, whereas the latter can only hope to suppress it.

[18] As reported by Yap (2012).

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Appendix

A. Proof that $DIFF = \sigma_{y_2}^2\left(1 - r_{y_1 y_2}^2\right)$.

Combining the equilibrium conditions in equations (6) and (7) with the linear demand forms in equation (11), we have $I = k + h^{-1}P_2 + w_2 = Q - (k + h^{-1}P_1 + w_1)$, which can be solved for $P_2 = h(Q - 2k - w_1 - w_2) - P_1$. Taking the conditional expectation gives $E[\tilde{P}_2|w_1, \theta] = h[Q - 2k - E(P_1|w_1, \theta)] - \tilde{y}_2$, where $\tilde{y}_2$ is given by equation (14). That describes one side of the "beliefs" that enter into $DIFF$. To get the other side, we note the relationship, $E\left[\tilde{P}_2|P_1\right] = E\{E[\tilde{P}_2|w_1, \theta]|P_1\} = E(\{h[Q - 2k - E(P_1|w_1, \theta)] - \tilde{y}_2\}|P_1)$, where we have substituted for $E[\tilde{P}_2|w_1, \theta]$ from the previous step. After taking the expectation in the last expression, we have $E\left[\tilde{P}_2|P_1\right] = h[Q - 2k - E(P_1|w_1, \theta)] - E(\tilde{y}_2|P_1)$. By definition, the difference in beliefs is computed as the difference between the two, $\Delta(w_1, \theta) = E[\tilde{P}_2|w_1, \theta] - E\left[\tilde{P}_2|P_1\right] = E(\tilde{y}_2|P_1) - \tilde{y}_2 = E(\tilde{y}_2|y_1) - \tilde{y}_2$, where the last step is based on the fact that P_1 and y_1 are one-to-one (which can be inferred from equation [16]).

Recall the definition, $DIFF = E[\Delta(w_1, \theta)]^2$. By a well-known property of the variance of a random variable, we may write $E\{[\Delta(w_1, \theta)]^2|y_1\} = Var[\Delta(w_1, \theta)|y_1] + \{E[\Delta(w_1, \theta)|y_1]\}^2$. But, because $E[\Delta(w_1, \theta)|y_1] = 0$, this implies $E\{[\Delta(w_1, \theta)]^2|y_1\} = Var[\Delta(w_1, \theta)|y_1] = Var[E\{(\tilde{y}_2|y_1) - \tilde{y}_2\}|y_1] = Var(\tilde{y}_2|y_1) = \sigma_{y_2}^2\left(1 - r_{y_1 y_2}^2\right)$, where the next-to-last step is due to the fact that $E[(\tilde{y}_2|y_1)|y_1]$ is nonstochastic. Because $\tilde{y}_2$ and $\tilde{y}_1$ are jointly normally distributed, the last expression does not depend on y_1, and we can write $Var[\Delta(w_1, \theta)] = \sigma_{y_2}^2\left(1 - r_{y_1 y_2}^2\right)$. QED.

B. Proof that $\partial DIFF/\partial c > 0$.

After combining similar terms and converting to a common denominator, equation (23) takes the following form

$$DIFF = \frac{\sigma^2 c^2 h^2 (1-\phi)^2 (R-1)}{n^2 h^2 R - 2cnh(1-\phi) + c^2(1-\phi)^2} \tag{A1}$$

where $R = (1 + \rho^2)$. Taking the derivative with respect to c gives

$$\frac{\partial DIFF}{\partial c} \propto 2\sigma^2 c h^2 (1-\phi)^2 (R-1)[n^2 h^2 R - 2cnh(1-\phi) + c^2(1-\phi)^2]$$

$$-2[c(1-\phi)^2 - nh(1-\phi)][\sigma^2 c^2 h^2 (1-\phi)^2 (R-1)]$$

After dividing by $\sigma^2 c h^2 (1-\phi)^2 (R-1)$, which is positive, this implies

$$\frac{\partial DIFF}{\partial c} \propto 2[n^2 h^2 R - 2cnh(1-\phi) + c^2(1-\phi)^2] - 2c[c(1-\phi)^2 - nh(1-\phi)]$$

$$= 2n^2h^2R - 2cnh(1-\phi) > 0$$

where the inequality is due to $n \geq 1, c > 0, R \geq 1,\ \phi < 0$, and $h < 0$. QED.

C. Proof that $\partial DIFF/\partial n < 0$.

Recall that $(1-\phi)$ is a function of n, with $\frac{\partial(1-\phi)}{\partial n} = \frac{\iota}{2}$. Differentiating equation (A1) with respect to n therefore yields

$$\frac{\partial DIFF}{\partial n} \propto \iota(1-\phi)\sigma^2c^2h^2(R-1)[n^2h^2R - 2cnh(1-\phi) + c^2(1-\phi)^2]$$

$$-2\left[nh^2R - ch\left(\frac{\iota}{2}n + (1-\phi)\right) + \frac{\iota}{2}c^2(1-\phi)\right][\sigma^2c^2h^2(1-\phi)^2(R-1)]$$

After dividing by $\sigma^2c^2h^2(1-\phi)(R-1)$, which is positive, this implies

$$\frac{\partial DIFF}{\partial n} \propto \iota n^2h^2R - \iota cnh(1-\phi) + 2ch(1-\phi)^2 - 2nh^2R(1-\phi)$$

$$= -\big(nh^2R - ch(1-\phi)\big)(1 + m + n + \iota n) < 0.\ \text{QED.}$$

D. Proof that $\partial DIFF/\partial m \geq 0$.

Recall that $(1-\phi)$ is a function of m, with $\frac{\partial(1-\phi)}{\partial m} = \frac{\iota}{2}$. Differentiating equation (A1) with respect to m therefore yields

$$\frac{\partial DIFF}{\partial m} = \iota(1-\phi)\sigma^2c^2h^2(R-1)[n^2h^2R - 2cnh(1-\phi) + c^2(1-\phi)^2]$$

$$-2\left[\frac{\iota}{2}c^2(1-\phi) - \frac{\iota}{2}cnh\right][\sigma^2c^2h^2(1-\phi)^2(R-1)]$$

After dividing by $\sigma^2c^2h^2(1-\phi)(R-1)$, which is positive, this implies

$\frac{\partial DIFF}{\partial m} \propto \iota n^2h^2R - \iota cnh(1-\phi) \geq 0$ (with strict equality if and only if $\iota = 0$). QED.

E. Proof that $\partial DIFF/\partial \rho > 0$.

We demonstrate that $\partial DIFF/\partial R > 0$ and use the fact that ρ is one-to-one with $R = (1+\rho^2)$. First, we differentiate equation (A1) with respect to R

$$\frac{\partial DIFF}{\partial R} \propto \sigma^2c^2h^2(1-\phi)^2[n^2h^2R - 2cnh(1-\phi) + c^2(1-\phi)^2] - n^2h^2[\sigma^2c^2h^2(1-\phi)^2(R-1)]$$

After dividing by $\sigma^2c^2h^2(1-\phi)^2$, which is positive, this implies

$$\frac{\partial DIFF}{\partial R} \propto n^2h^2 - 2cnh(1-\phi) + c^2(1-\phi)^2 > 0. \text{ QED.}$$

F. Proof that $\frac{\partial DIFF}{\partial h} < 0$**.**

Differentiating equation (A1) with respect to h yields

$$\frac{\partial DIFF}{\partial h} \propto 2\sigma^2c^2h(1-\phi)^2(R-1)[n^2h^2R - 2cnh(1-\phi) + c^2(1-\phi)^2] - 2[n^2hR - cn(1-\phi)][\sigma^2c^2h^2(1-\phi)^2(R-1)]$$

After dividing by $\sigma^2c^2h(1-\phi)^2(R-1)$, which is positive, this implies

$$\frac{\partial DIFF}{\partial h} \propto -2cnh(1-\phi) + 2c^2(1-\phi)^2 > 0. \text{ QED.}$$

G. Proof that $\partial DIFF/\partial(m+n) < 0$ *(assuming* m/n *is constant).*

Define $\lambda = \frac{m}{n}$. It follows that $(1-\phi) = 1 - \frac{\iota}{2} + \frac{\iota}{2}n(1+\lambda)$, and $\frac{\partial(1-\phi)}{\partial n} = \frac{\iota}{2}(1+\lambda)$, holding λ constant. Differentiation of equation (A1) with respect to n, but treating λ as constant, then yields

$$\frac{\partial DIFF}{\partial m + n(m/n\,constant)}$$

$$= \iota(1+\lambda)\sigma^2c^2h^2(1-\phi)(R-1)[n^2h^2R - 2cnh(1-\phi) + c^2(1-\phi)^2]$$

$$- 2\left[nh^2R - ch(1-\phi) - \frac{\iota}{2}chn(1+\lambda) + \frac{\iota}{2}(1+\lambda)c^2(1-\phi)\right][\sigma^2c^2h^2(1-\phi)^2(R-1)]$$

After dividing by $\sigma^2c^2h^2(1-\phi)(R-1)$, which is positive, and collecting terms, we have

$$\frac{\partial DIFF}{\partial m + n|m/n\,constant} \propto \iota(1+\lambda)n^2h^2R - \iota(1+\lambda)cnh(1-\phi) - 2nh^2R(1-\phi) + 2ch(1-\phi)^2$$

$$= \iota(1+\lambda)n^2h^2R - \iota(1+\lambda)cnh(1-\phi) - 2nh^2R\left[1 - \frac{\iota}{2}\big(1 - n(1+\lambda)\big)\right] + 2ch(1-\phi)^2$$

$$= nh^2R(\iota - 2) + ch(1-\phi)[2(1-\phi) - \iota n(1+\lambda)]$$

$$= [nh^2R - ch(1-\phi)][\iota - 2] < 0$$

where the last expression is obtained by substitution: $\iota n(1+\lambda) = \iota - 2\phi$. QED.