Prices vs. Quantities Revisited: The Case of Climate Change

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Abstract

Uncertainty about compliance costs causes otherwise equivalent price and quantity controls to behave differently. Price controls – in the form of taxes – fix the marginal cost of compliance and lead to uncertain levels of compliance. Meanwhile quantity controls – in the form of tradable permits or quotas – fix the level of compliance but result in uncertain marginal costs. This fundamental difference in the face of cost uncertainty leads to different welfare outcomes for the two policy instruments. Seminal work by Weitzman (1974) clarified this point and derived theoretical conditions under which one policy is preferred to the other.

This paper applies this principal to the issue of worldwide greenhouse gas (GHG) control, using a global integrated climate economy model to simulate the consequences of uncertainty and to compare the efficiency of taxes and permits empirically. The results indicate that an optimal tax policy generates gains which are five times higher than the optimal permit policy – a \$337 billion dollar gain versus \$69 billion at the global level. This result follows from Weitzman's original intuition that relatively flat marginal benefits/damages favor taxes, a feature that drops out of standard assumptions about the nature of climate damages.

A hybrid policy, suggested by Roberts and Spence (1976), is also explored. Such a policy uses an initial distribution of tradeable permits to set a target emission level, but then allows additional permits to be purchased at a fixed "trigger" price. The optimal hybrid policy leads to welfare benefits only slightly higher than the optimal tax policy. Relative to the tax policy, however, the hybrid preserves the ability to flexibly distribute the rents associated with the right to emit. Perhaps more importantly for policy discussions, a sub-optimal hybrid policy, based on a stringent target and high trigger price (e.g., 1990 emissions and a \$100/tC trigger), generates much better welfare outcomes than a straight permit system with the same target. Both of these features suggest that a hybrid policy is a more attractive alternative to either a straight tax or permit system.

Key words: Climate Change, Policy Under Uncertainty, Price and Quantity Controls, General Equilibrium Modeling

JEL Classification No(s): Q28, D81, C68

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1 Introduction

Seminal work by Weitzman (1974) drew attention to the fact that, in regulated markets, uncertainty about costs leads to a potentially important efficiency distinction between otherwise equivalent price and quantity controls. Despite this well-known observation and its relevance for climate change policy, most of the debate concerning the use of taxes and emission permits to control greenhouse gases (GHGs) has centered on political, legal and revenue concerns. This paper responds to this important omission by examining the efficiency properties of permit and tax policies to reduce global warming.

The basic distinction among policy instruments arises because taxes fix the marginal cost of abatement at the specified tax level (assuming optimal firm behavior). With uncertainty about costs, this generates a range of possible abatement levels and emission outcomes. In constrast, a permit system precisely limits emissions but leads to a range of potential cost outcomes. When coupled with a model of the benefits associated with emission reduction, this divergence in emission and cost outcomes creates a distinction in the expected welfare associated with each policy.

In the case of climate change, part of the cost uncertainty arises due to uncertainty about the level of future baseline emissions. The Intergovernmental Panel on Climate Change (1992) gives a range of CO_2 emission levels in 2025 of between 8.8 and 15.1 GtC.[2] The cost of attaining a particular target, say the 1990 emission level of 7.4 GtC, will obviously fluctuate depending on the level of future uncontrolled emissions.

In addition to the baseline, however, there is considerable uncertainty about the cost of reducing emissions below the baseline. A study by Nordhaus (1993) reports that a $30/tC tax might reduce

[1] Fellow, Quality of the Environment Division, Resources for the Future. Financial support from the National Science Foundation is gratefully acknowledged (Grant SBR-9711607). Richard Morgenstern, Raymond Kopp and Richard Newell, along with seminar participants at the CEA and RFF, provided valuable comments on an earlier draft. The author alone is responsible for all remaining errors.

[2] Gigatons Carbon.

emissions anywhere from 10 to 40%. While some models predict that a $300/tC would virtually eliminate emissions, other models require a tax in excess of $400/tC. This wide range of reduction estimates only compounds the uncertainty about baselines to generate extreme uncertainty about the cost of a particular emission target fifteen or twenty years in the future.

Motivated by the policy implications of these large uncertainties, this paper uses a modified version of the Nordhaus (1994b) integrated climate-economy model in order to analyze alternative policies under uncertainty. In particular, the model incorporates uncertainty about a wide range of model parameters developed in both Nordhaus (1994b) and Pizer (1996). The simulations are then sped up using a technique presented in Pizer (1996) that greatly facilitates computation. Key elements of the model are discussed in the next section while specifics are covered in Appendix A.

Before presenting the multi-period policy results in Section 4, Section 3 presents a simpler one-period analysis. This analysis, following Weitzman, uses marginal cost and benefit estimates derived from the full multi-period dynamic model to examine optimal policy for the single year 2010, ignoring the policy choice in future years. This simpler analysis provides considerable intuition for the full multi-period policy analysis and, surprisingly, closely replicates the optimal multi-period policy results in 2010.

The simulations indicate that an optimal permit policy would begin with a 13 GtC target in 2010, rising gradually over time to accomodate some economic growth. This policy generates $69 billion in expected net benefits versus a no policy, business-as-usual alternative. Importantly, this gain is very sensitive to correctly setting the target. Slightly more stringent targets lead to dramatic welfare losses. Meanwhile, the optimal tax policy starts at $7/tC in 2010 and also rises gradually over time. In contrast, this policy generates $337 billion in net benefits and the gain is much less sensitive to the exact choice of tax level.

As an alternative to both the pure tax and permit policies, a combined hybrid policy is proposed (Roberts and Spence 1976; Weitzman 1978; McKibbin and Wilcoxen 1997). Such a mechanism would involve an initial distribution of tradeable permits, with additional permits available from the government at a specified “trigger” price. This system turns out to be only slightly more efficient than a pure tax system, but allows a flexible distribution of rents associated with emission rights due to its permit system component. Perhaps more importantly for current policy discussions, sub-optimal hybrid policies based on a stringent target and high trigger price have much better welfare outcomes than a sub-optimal permit policy with the same target. Both the improved flexibility and better welfare outcomes make the hybrid policy an attractive alternative to either permits or taxes alone.

2 Background

2.1 Weitzman's Result

The analysis presented in Weitzman (1974) concerns the choice of policy instrument used to regulate a market where either political considerations or market failure require government intervention. A price (tax) or quantity (permit) instrument is at the government's disposal and the question posed by Weitzman is which of the two leads to the best welfare outcome, measured as net social surplus.[3] Importantly, the policy must be fixed *before* any uncertainty is resolved.

With complete certainty concerning costs, price and quantity controls can be used to achieve the same outcomes. For every price, there is a profit maximizing level of production where price equals marginal cost and for every level of production there is an associated marginal cost. Note that while uncertainty about *benefits* leads to uncertain welfare outcomes, it does not lead to uncertainty about the level or marginal cost of production – these are determined by the structure of costs. Therefore, the two instruments (which affect production) can be used to obtain exactly the same production outcome, generating the same set of welfare outcomes, as long as costs are known.

The interesting case arises when *costs* are not certain. Then, fixing the marginal cost through a price instrument leads to an uncertain level of production. Correspondingly, fixing the level of production with a quantity instrument leads to an uncertain marginal cost.

Weitzman's basic result was that price instruments would be favored when the marginal benefit

[3] Here and throughout it is assumed that the quantity instrument is an *efficient* quantity instrument; e.g. a tradeable permit system with negligible transaction costs.

schedule was relatively flat and quantity instruments would be favored when the marginal cost schedule was relatively flat. In particular, he derived an expression for the difference in expected welfare between the price and quantity instrument:

$$\Delta = \frac{\sigma^2}{2C''^2}(B'' + C'') \tag{1}$$

where σ^2 is the variance of the shocks to the marginal cost schedule, C'' is the slope of the (linear) marginal cost schedule and B'' is the slope of the (linear) marginal benefit schedule.[4] Since the marginal benefit schedule is assumed to be downward sloping ($B'' < 0$), the price instrument is preferred when $|B''| < |C''|$ and the quantity instrument is preferred when $|B''| > |C''|$.

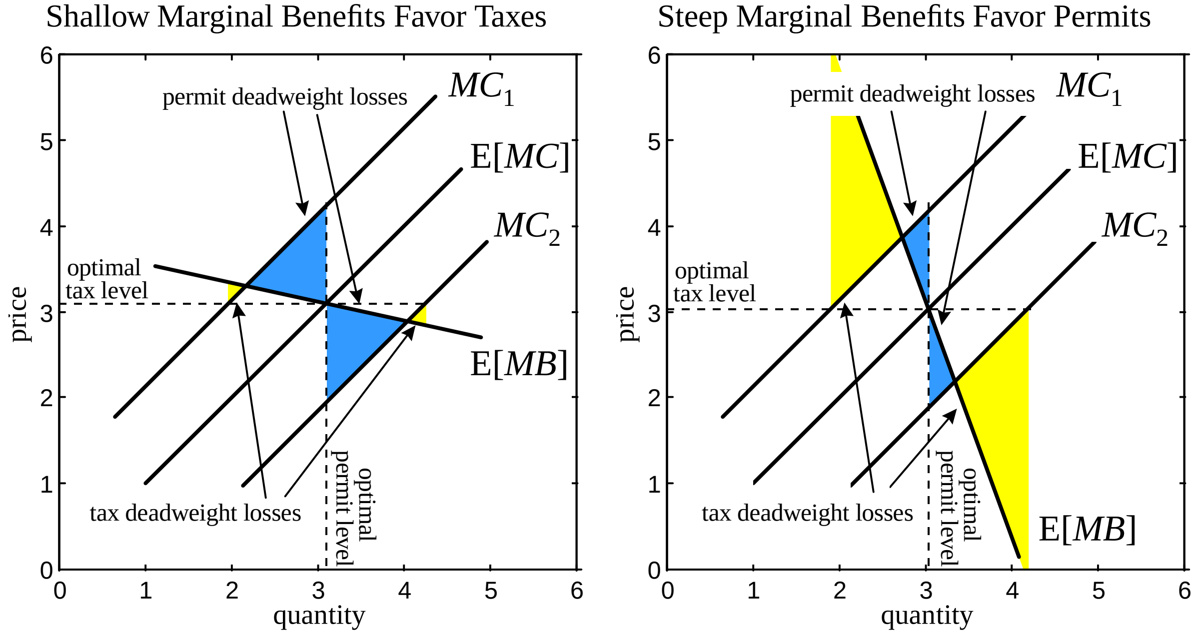
The intuition behind this result is that when marginal benefits are relatively flat ($|B''| < |C''|$), the optimal price *after* uncertainty is resolved remains relatively constant over the range of potential cost outcomes. Therefore, fixing the price before the outcome is known leads to levels of control that are "close" to what would be chosen after uncertainty is resolved. In contrast, when marginal benefits are relatively steep ($|B''| > |C''|$), the optimal quantity is relatively constant over the range of outcomes. In that case, fixing the quantity in advance leads to results close to what would be chosen after uncertainty is resolved. By choosing policies that provide levels of control close to what what would be chosen after uncertainty is resolved, the deadweight loss is minimized. This is shown visually in Figure 1.

2.2 Combining Taxes and Permits

Not long after Weitzman's original article, several authors suggested using a hybrid permit policy (Weitzman 1978; Roberts and Spence 1976) to regulate a market. That is, producers are given the choice of either obtaining a permit in the marketplace or purchasing a permit from the government at a specified trigger price. Such a system works like a permit system by fixing emissions as long as the marginal cost (e.g., the price of the permit) lies below the trigger price and works like a tax system by fixing marginal cost when marginal cost hits the trigger price. When the trigger is set

[4] This result is derived for the case of linear marginal costs and benefits, where uncertainty enters as small shifts to each curve (therefore the slopes B'' and C'' are known with certainty). The uncertainty about costs is assumed to be independent of the uncertainty about benefits.

Figure 1: Deadweight Loss of Taxes Versus Permits[a]



[a] $\mathrm{E}[MB]$ indicates expected marginal benefits, $\mathrm{E}[MC]$ indicates expected marginal costs, and MC_1 and MC_2 indicate alternative cost outcomes.

high, such a combined mechanism functions like a pure permit system (since additional permits are never sold) and when the number of permits is set low, it functions like pure tax mechanism (since additional permits are always sold). By encompassing both tax and permit mechanisms as special cases, the hybrid policy should always perform at least as well as either one.

Recently, a paper by McKibbin and Wilcoxen (1997) proposed just such a hybrid policy to address the problem of global climate change, but not based on these efficiency arguments. Instead, they discuss the merits of a hybrid policy in comparison to a strict 1990 emission target in a certainty context. Based on a trigger price of $10/tC (at which price additional permits would almost certainly be sold), McKibbin and Wilcoxen argue that the hybrid policy would lower costs, improve monitoring and enforcement, and avoid disruptive international capital flows. While the lower cost would come at the expense of a less stringent policy, they argue that a 1990 target is unrealistic. They also point out that a hybrid system would preserve the permit system's ability to flexibly distribute the rents associated with emission rights while, at the margin, providing a revenue incentive for governments to enforce and monitor emissions.[5] Finally, they point out that an efficient international permit system involving international permit trades would lead to potentially large trade distortions.

While their points concerning the benefits of a hybrid policy under complete certainty are important, they are tangential to the question addressed in this paper. That is, what are the expected welfare gains associated with alternative policies under *uncertainty*. More to the point, it turns out that these welfare gains are large enough to potentially dwarf almost all other concerns.

2.3 Model Description

This analysis uses a global integrated climate economy model containing a stylized representation of global economic activity and climate behavior. The stylized approach, based on Nordhaus (1994b), is appropriate given this paper's focus on uncertainty. While additional detail in both the climate and economy modules might improve the results for a particular set of assumptions or provide answers to other questions, it is unlikely that such embellishments would affect the range of predicted aggregate outcomes or the insight concerning optimal policy choice under uncertainty.

Economic behavior in this model involves a single sector of global economic activity. Global capital and labor are combined to produce a generic output each year which is either consumed or invested in additional capital. A representative agent chooses that amount of consumption each period which maximizes her expected utility across time.

Climate change enters the model through the emission of greenhouse gases arising from economic activity. These emissions accumulate in the atmosphere and lead to a higher global mean temperature. This higher temperature then causes damages by reducing output according to a quadratic damage function.

The opportunity to reduce the effect of climate change arises from the use of more expensive, GHG reducing, production technologies. In particular, there is a cost function describing the reduction in output required to reduce emissions by a given fraction. This cost function captures

[5] As pointed out by Goulder, Parry, and Burtraw (1996), losing the revenue from such rents could also have important welfare consequences.

substitution both among and away from fossil fuels. While the cost of reductions in any period are born entirely in that period, the consequences of reduced emissions persist far into the future due to the longevity of greenhouse gases in the atmosphere.

Appendix A describes the model in more detail.

2.4 Key Modeling Assumptions

Repetto and Austin (1997) point out that many of the prediction differences among climate change models can be linked to differences in a few key assumptions. For that reason, it is instructive to highlight how such issues are handled in this model. Two of the assumptions they identify – the use of a computable general equilibrium (CGE) framework and the provision of revenue recycling options – are impossible to explore in this model (due to the fact that it is already a CGE model without a government sector). However, the remaining assumptions considered by Repetto and Austin are related to alternative descriptions of costs and benefits.

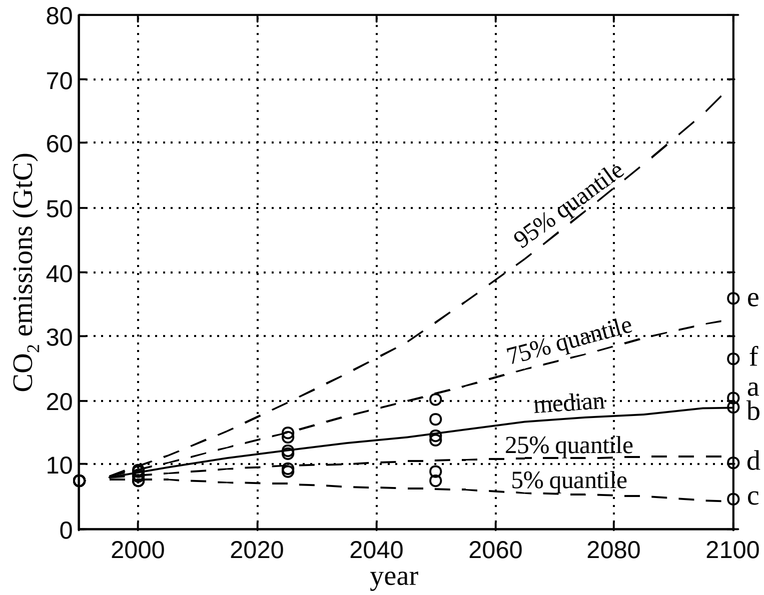
Costs and benefits vary widely in this model due to uncertainty. Still, realized costs and benefits can be traced to two distinct sets of assumptions: those addressing underlying trends and those addressing abatement costs and climate damages directly.

2.4.1 Trends

Trends are essential features in any model with long time horizons since they determine the course of exogenous change. Particularly in the climate change context, assumptions about (1) exogenous population growth, (2) productivity improvements and (3) energy efficiency/carbon content determine the baseline of future uncontrolled greenhouse gas emissions – and by extension the baseline for costs and benefits. This model combines the work of Nordhaus and Yohe (1983), Nordhaus (1994b), Pizer (1996), and Nordhaus and Popp (1997) to characterize each of these trends. Within each state of nature, these three trends are governed by an initial growth rate coupled with an associated slowdown.[6]

[6] The trend in productivity growth is further overlayed with a mean-zero random walk.

Figure 2: Simulated CO_2 Emission Distribution vs IPCC Scenarios



[a] Lines indicate the distribution of CO_2 emission paths generated by the model. These reflect controllable carbon equivalent GHG emissions, scaled by the fraction due to CO_2 (e.g., 86.6%; see p. 71, Nordhaus 1994b). Circles (∘) indicate 1992 IPCC CO_2 emission scenarios (p. 12, IPCC 1992; pp. 101–112, Pepper et al. 1992); letters in right margin refer to individual scenarios.

Table 1: Summary of Baseline Trend Assumptions

	range	source
Global Output ($ trillions, 1995)	24	Nordhaus (1994b)[a]
Emissions/Output Ratio (tC billions/$ trillions, 1995)	0.385	Nordhaus (1994b)[a]
Population Growth (1995)	1.24%	Nordhaus (1994b)[a]
Productivity Growth (1995)	0.16 to 2.46%	Pizer (1996)
Emission/Output Ratio Growth	-0.11 to -2.31%	Nordhaus and Popp (1997)
Population Slowdown Rate	0.27 to 3.31%	Nordhaus (1994b)
Productivity Slowdown Rate (also used to slow the growth in the emissions/output ratio)	0.20 to 2.43%	Nordhaus (1994b)

[a] Adjusted to 1995.

While Table 1 summarizes the range of underlying trend assumptions, the consequent distribution of CO_2 emission scenarios is shown in Figure 2 along with the 1992 IPCC projections. The IPCC forecasts tend to fall between the 25th and 75th percentile in 2025 and between the 5th and 75th by the years 2050 and 2100. The median of the IPCC forecasts remains quite close to the median (50th percentile) of the simulated emission levels throughout the forecast period. This suggests that, relative to the IPCC forecasts, the model in this paper predicts a similar central tendency but with a larger spread. In particular, this paper suggests that future uncontrolled emissions could be much higher than all six of the IPCC forecasts. For several reasons, this is arguably a realistic assessment.

First, it is important to recognize that the IPCC scenarios do not have a probabilistic interpretation. They are subjectively developed scenarios combining a large number of alternative assumptions in six particular combinations. Their two high emission scenarios, for example, alternatively combine high population growth with lower per capita productivity growth, and vice versa. Further, the underlying growth forecasts themselves have little if any probabilistic interpretation.[7]

Second, even analyses that are well-grounded in probability often underestimate the probability of extreme events. This phenomena has been documented in everything from the measurement of physical constants to the forecast of future energy demand (Shlyakhter and Kammen 1992). Such results suggest that forecasts with "thicker tails" (like the distribution in Figure 2) are a more realistic description of the likely outcome distribution.

Finally, the time horizon under consideration – over one hundred years – makes any forecast based on historical data somewhat dubious. Such forecasts implicitly assume that recent historical trends will continue far into the future without appreciable change. For example, based on recent experiences with growth slowdowns, global CO_2 emissions – which tripled over the last fifty years – are forecast to triple again only after one hundred years. It is quite plausible, however, that this assumed slowdown could fail to materialize. If emissions continued to triple every fifty years, this

[7] See Pepper et al. (1992) for further details. Note that early Census population forecasts using similar non-probabilistic techniques often grossly misforecast population. For example, the forecast range given in 1966 (#381, U.S. Department of Commerce, Bureau of the Census) lies completely above the actual population reported in 1989 (#1045).

would essentially agree with the 95% quantile of the predicted emission distribution.

All of these reasons indicate that while the assumptions in Table 1 and emission forecasts in Figure 2 are not above question, they provide a reasonable probabilistic picture of the likely emission outcomes.

2.4.2 Damages

Damages are perhaps the least understood aspect of climate change and at the same time one of the most important. In this model, damages are modeled as a quadratic function of temperature change, in turn determined by GHG concentrations. In particular,

$$\text{fractional reduction in GDP due to damages} = 1 - \frac{1}{1 + D_0/9 \cdot T^2}$$

where T is the change in temperature relative to a pre-industrialization (1860) baseline and D_0 is a parameter describing the GDP reduction associated with a 3° temperature increase.[8]

In addition to the parameter describing the GDP loss from increased temperatures, two other parameters affect the degree of climate damages. First, there is a parameter describing the fraction of emitted GHGs which actually accumulate (versus those which are absorbed in the oceans). Second, there is a parameter describing the change in temperature for a doubling of atmospheric GHG concentrations. Table 2 describes the range of assumed values for all three parameters.

There are also two implicit assumptions which influence the final results concerning taxes and permits. The first is that climate damage is presumed to be a gradually occuring phenomenon (represented by a quadratic function). The fact that damages from increased GHG concentrations rise smoothly and gradually contributes to the relatively flat marginal benefit schedule discussed in Section 3.1. In contrast, a damage function that involved sudden and/or critical phenomena – such as the breaching of a concentration threshold generating dramatically higher damages – would lead to a steeper or perhaps stepwise damage function (though uncertainty about the threshold level would tend to smooth the expected damage function).

[8]This function is almost identical to specifying fractional damages= $-D_0/9 \cdot T^2$ as long as the damages are less than 5% of GDP. The more complex specification simply prevents global GDP from becoming negative.

Table 2: Summary of Climate Damage and Abatement Cost Assumptions[a]

description	probability parameter equals the given value				
	20%	20%	20%	20%	20%
Damage Assumptions:					
fraction of emitted CO_2 which accumulates	0.50	0.59	0.64	0.69	0.78
change in temperature from CO_2 doubling	1.5	2.2	2.9	3.7	4.4
GDP loss[b] for 3 degree rise in temperature	0.0	0.4	1.3	1.6	3.2
Cost Assumptions:					
GDP loss[b] for 100% reductions	2.7	3.4	6.9	8.0	13.3
GDP loss[b] to obtain 1990 emissions in 2010[c]	0.08	0.11	0.21	0.25	0.41

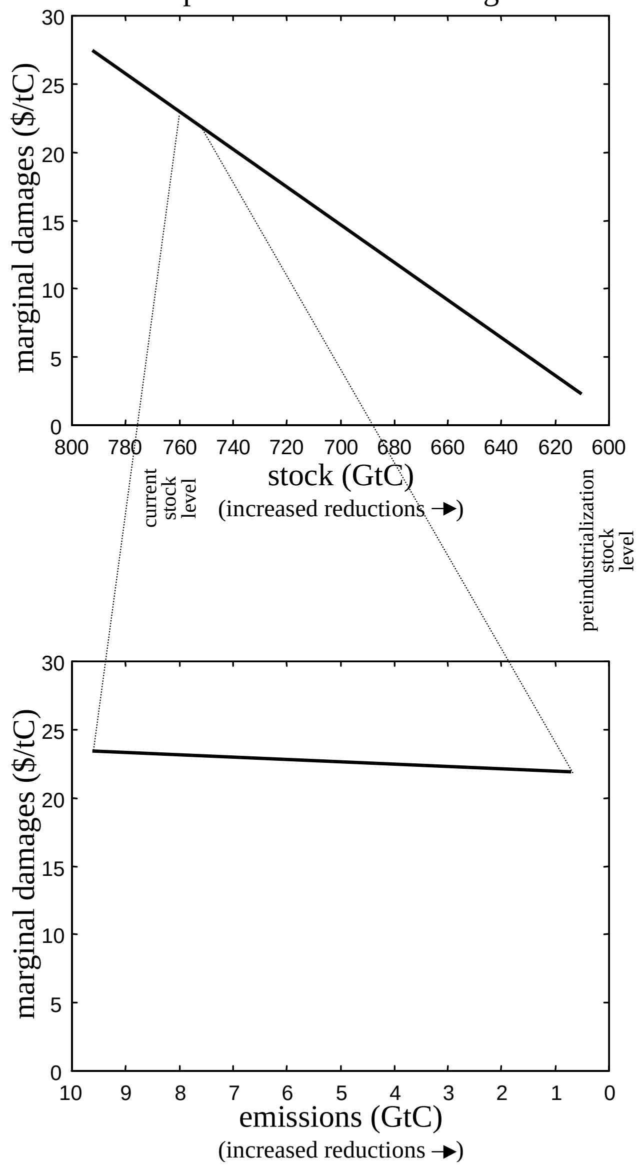
[a] Source: Nordhaus and Popp (1997).
[b] As a percent of global GDP.
[c] Assuming a 30% reduction from baseline.

The second, less controversial, assumption is that damages are related to the *stock* of GHGs in the atmosphere and not the annual *flow*. This contrasts with traditional pollutants, such as particulates, SOx, NOx, etc., whose damages are related to annual flows. Stock pollutants by their nature will have relatively flat benefit curves associated with emission reductions since the reductions in any period have a relatively small effect on the total stock. Even though emissions in one period persist far into the future, the volume of emissions in a single period always remains a tiny fraction of the existing stock. If the total stock changes by a small amount – even if that change persists – the marginal benefit schedule remains essentially constant.

To better understand this phenomenon, imagine a pollutant, like carbon dioxide, where it assumed that damages depend on the stock of the pollutant. Presumably, the damages themselves are not linear (i.e., directly proportional to concentrations) but rise by an increasing amount as the stock rises. The damage from the first ton of CO_2 above the preindustrial baseline is small, while the incremental damage from an additional ton of CO_2 on top of current concentrations is much larger – as depicted in the top panel of Figure 3. In that panel, damages from the first ton of CO_2 above the preindustrial baseline are around \$1/tC but damages from an additional ton of CO_2 once emissions reach the current level of 760 GtC are over \$20/tC.

Now suppose that total annual emissions are currently 10 GtC and, for simplicity, a ton of emis-

Figure 3: Stylized Relationship Between Stock Damages and Emission Damages[a]



[a]This figure highlights the fact that reductions in any one period are a relatively small fraction of the total stock of GHGs. Therefore, for any smooth damage function determined by the GHG stock (in turn implying somewhat linear marginal damages for the GHG stock), the marginal damage function for emissions in any one period is essentially flat. It simply expands a tiny segment of the stock marginal damage schedule over the range of possible policy choices. This ignores the possibility that emission reductions in one period may be associated with emission reductions in other periods, in turn leading to a steeper slope in the future.

sion reductions translates into exactly one ton of reduced stock.[9] Even if emissions are reduced to zero, the stock – and by extension marginal damages – fall only slightly, as shown in the lower panel of Figure 3. This is because annual emissions are only a small fraction of the total stock.

There are three important caveats to this point. The first, noted above, is that damages are assumed to be relatively smooth. If, for example, the marginal damages rose dramatically at around 750 GtC, the graphical argument in Figure 3 would fail. The second is that there are likely to be ancillary benefits associated with GHG reductions. In particular, the reduced burning of fossil fuels promises to reduce concentrations of particulate matter, SOx and NOx, all of which have non-trivial morbidity and mortality benefits.[10]

Finally, it presumes that reductions in different periods are unrelated. If reductions in one period are accomplished only in conjunction with reductions in other periods, such a presumption is wrong. As an extreme example, suppose emission reductions in one period imply equal reductions in all future periods. In this scenario, emission reductions in one period would eventually lead to proportional reductions in the stock after enough periods elapsed. While Figure 3 would be an appropriate rendering of the immediate effect of emission reductions on the pollutant stock, the effect on the stock in future periods would be more dramatic as the associated reductions in future periods took effect.

There are several reasons to think that such a relation is plausible. First, certain decisions about emission reductions – namely investments in research, fixed plant and equipment – affect emissions in many periods. While it is unlikely that marginal adjustments in emissions would be completely restricted by such decisions, some linkages are inevitable. Second, some policies, such as taxes, will lead to increased or decreased controls in every period depending on whether costs turn out to be high or low in each state of nature. In both cases, however, it remains an empirical question whether these effects significantly affect the otherwise flat slope of the marginal benefit schedule.

[9] In reality it translates into something less than a ton of reduced stock in the atmosphere since between 25 and 50% of emissions are quickly absorbed in the oceans.

[10] See, for example, Burtraw and Toman (1997).

2.4.3 Costs

The cost of reducing GHG emissions below the uncontrolled baseline is based on a survey of studies summarized in Nordhaus (1993). These studies consider the cost of reducing GHG emissions through various means, including the use of specific technologies, econometric estimates of fuel substitution, and mathematical programming. The relationship and range of estimates is approximated in Nordhaus (1994b) by a power rule,

$$\text{fractional reduction in global GDP} = b_1(\text{fractional reduction in GHG emissions})^{2.887}$$

where the parameter b_1 takes on the values shown in Table 2.

More important than the particular numbers entering the cost function, however, are the assumptions that (1) marginal costs rise more and more steeply as additonal reductions are undertaken and (2) that the choice of emission level is an annual decision involving an annual cost function. Both of these points are crucial because they affect the relative slope of marginal costs. This, in turn, affects the difference in expected welfare between taxes and permits discussed in Section 2.1.

If, for example, we believe that substantial reductions of GHGs will involve the development of new carbon-free technologies, it seems reasonable that the marginal cost of reducing emissions will eventually flatten once those technologies are brought on-line. This will diminish the argument that marginal benefits are relatively flat compared to costs. Alternatively, many decisions to reduce emissions – such as investment in innovative research or in new capital – are made over horizons of decades rather than annually. This introduces a positive correlation of among costs and potentially reductions in different periods. As pointed out previously, such correlation makes the benefits associated with a reduction decision in one period more valuable than the actual reductions in that period alone. This effect tends to favor quantity controls.

3 Single-Period Policy Simulations

Given the long-term nature of climate change, a policy to reduce GHG emissions inevitably involves decisions over periods of years or decades. However, understanding the differences between GHG tax and permit mechanisms under uncertainty is complicated when policies are viewed as paths for tax and permit levels, rather than single-period, single-valued choices. With that in mind, this section presents welfare results for a single-period policy analysis – reductions in GHG emissions in the year 2010. Since a single dimension captures the range of policies in this context, graphs can be used to view policy consequences. It should be emphasized that while the policy is only implemented in a single period, the measurement of costs and benefits especially includes consequences over a 250 year horizon. Section 4 discusses the multi-period policy results.

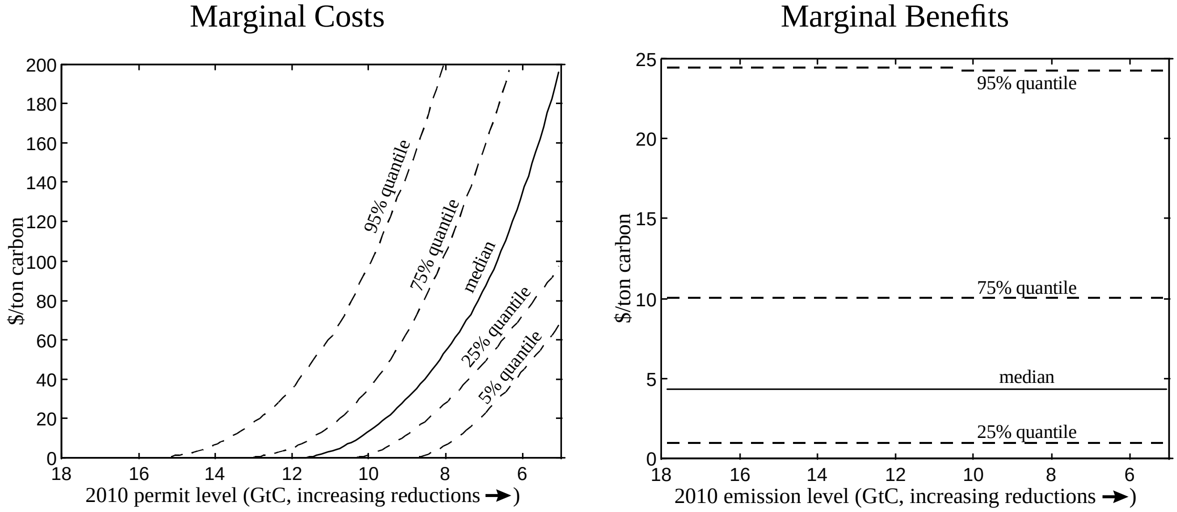
3.1 Marginal Costs and Benefits

While there are a number of strong assumptions in the Weitzman analysis which are inappropriate for examining climate change policy, it remains a sensable starting point. Based on that principle, the obvious question is how the marginal costs and benefits of GHG emission reductions in the year 2010 compare in terms of relative slopes. In order to answer this question, the integrated assessment model outlined in Section 2.3 and described in detail in Appendix A is used to compute costs and benefits.

First, the welfare associated with different levels of emissions in 2010 is computed in net present (2010) value terms for several thousand randomized trials. Marginal benefits are computed by numerically differentiating the derived schedule of benefits. Second, the cost associated with achieving different levels of emissions in 2010 is computed based on the model's cost function for the same set of randomized trials. This cost is similarly numerically differentiated to obtain marginal costs in the year 2010. In some trials and for some levels of emissions, the cost is zero since the given emission level may be higher than actual uncontrolled emissions. Achieving the specified emission level in such cases is costless.

These two calculations result in a distribution of marginal benefits and marginal costs at dif-

Figure 4: Distribution of Marginal Costs and Benefits in 2010[a]

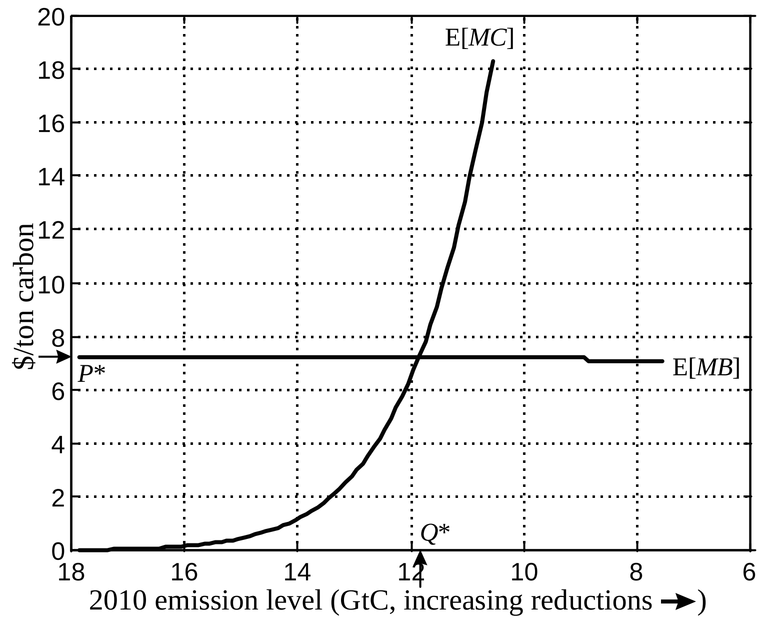


[a]Marginal costs are based on dollar value ($2010) of lost global GDP in order to reduce emissions at or below the indicated level. In those cases where uncontrolled emissions are below the indicated level, the marginal cost is zero. Marginal benefits are based on the dollar value ($2010) of the net present value of forgone damages at the given emission levels. These foregone damages hold constant all future emissions at their baseline level. Values are expressed in $2010 and, due to different discount rates across states of nature, will not be weighted equally when balancing costs and benefits.

ferent levels of emission. Figure 4 attempts to summarize these distributions by showing how the different quantiles of marginal costs and benefits vary over the range of emissions considered. Keeping in mind that 1990 GHG emissions were around 8.5 GtC, the left-hand panel indicates that achieving 1990 emission levels in 2010 would involve a marginal cost of between zero and $180/ton – a very wide range. This large variation occurs for two reasons: (1) marginal costs are assumed to rise steeply given the specified cost function, and (2) the baseline emissions in 2010 are not known with certainty. This panel essentially depicts a distribution of fairly steep curves whose horizontal intercept is unknown.

The right panel, in contrast, indicates constant but still unknown marginal benefits. As suggested earlier, the fact that damages due to GHGs depend on their stock in the atmosphere rather than emissions in any one year, coupled with the fact that GHGs remain in the atmosphere for a very long time, makes marginal benefits insensitive to the level of annual emissions in a single year. For example, in 1995 the stock of GHGs in the atmosphere was 760 GtC (170 GtC above the pre-industrial baseline) and annual emissions were around 10 GtC. The difference in stock between

Figure 5: Expected Marginal Costs and Benefits in 2010[a]



no reductions and 100% reductions in 1995 is only 6% of the stock,[11] making it unlikely that the marginal benefit of the first ton of reductions is much different than the marginal benefit of the last ton.

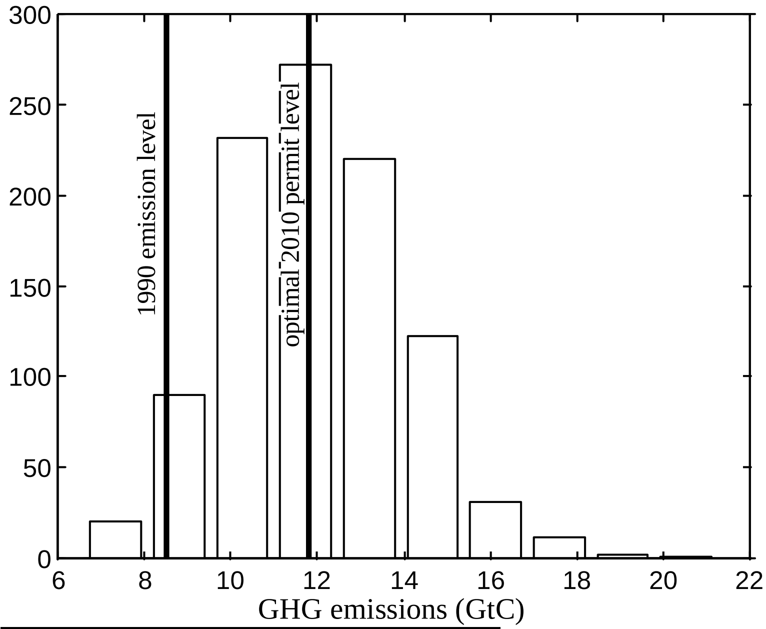
Taking expectations across these marginal costs and benefits yields the schedules shown in Figure 5. Under the assumptions made by Weitzman, the optimal permit level is simply emission level where expected marginal benefits equal expected marginal costs and the optimal tax level is similarly the expected marginal benefit at that intersection. Thus P^* and Q^* indicate the optimal tax and permit policies in 2010 for controlling GHG emissions.

Visually, it is apparent that expected marginal benefits are flatter than expected marginal costs. Calculating the slopes at the intersection, $B'' = -0.0012$ and $C'' = 5.4$. Further, setting $\sigma^2 = \mathrm{var}(MC_{\text{emissions}=11.9\text{GtC}})$ yields $\sigma^2 = 270$. This allows a rough calculation of the welfare gain of taxes over permits using Equation (1): $\Delta = \frac{270}{2 \cdot 5.4^2}(-0.0012 + 5.4) \approx \$25$ billion. Discounting this to 1995 (the base year of the model) with a 6% discount rate generates an estimated gain of $10 billion from using taxes instead of permits – just in the year 2010.

In addition to the welfare difference, it is interesting to compare the optimal quota in 2010 to

[11] This ignores the fact that perhaps only 50% accumulates.

Figure 6: Histogram of Uncontrolled 2010 Emissions Levels

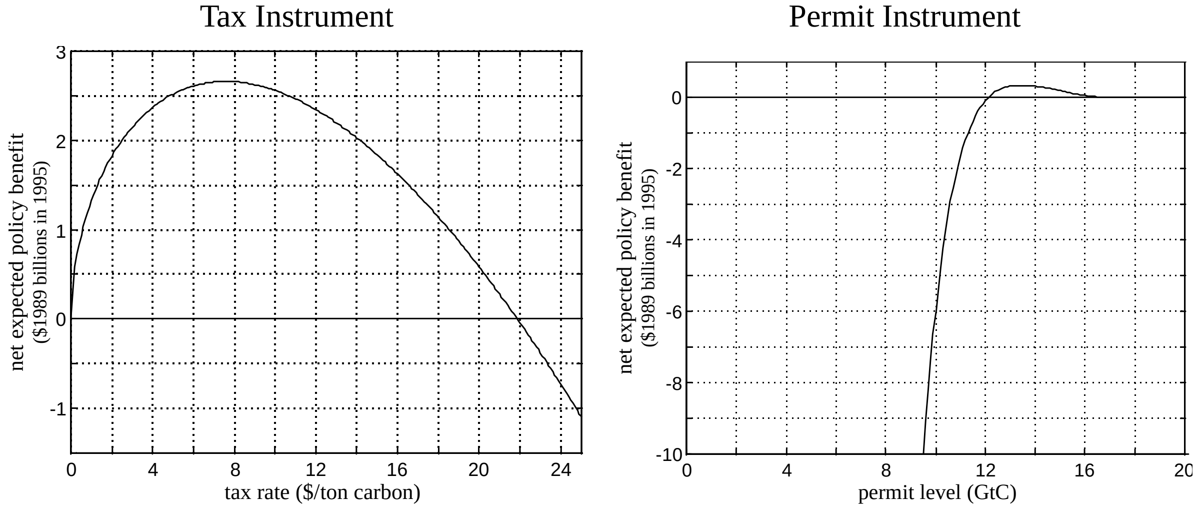


[a]GHG emissions include CO_2 and CFCs. 1990 emissions are based on 7.4 GtC of CO_2 and 1.1 GtC of CFCs (p. 71, Nordhaus 1994b). The 1990 emissions also correspond to data from IPCC(1992), p. 12, using a factor of 1300 to convert tons of CFCs into tons of carbon equivalent global warming potential (GWP) (Lashof and Ahuja 1990). The optimal 2010 permit level is discussed in the text.

the range of actual emissions. Figure 6 shows 1990 emissions (a target under consideration), the optimal 2010 permit level, and the distribution of uncontrolled emissions in 2010. Roughly half of the emission scenarios involve uncontrolled emissions *below* the optimal permit level (versus around 3% below the 1990 level). That is, implementing the optimal permit policy would result in non-binding targets half the time.

Why is the optimal permit policy so loose? Intuitively, uncertainty about baseline emissions is large relative to reductions and a large amount of reductions is costly. Therefore committing to an emission target which is almost surely below all forecasts (such as 1990 emissions levels) will involve extremely high costs in the event of high growth and high baseline emissions. The risk of such high costs, even taking into account the highest estimated benefits, are unjustified in this model and lead one to prefer a less stringent (higher) target.

Figure 7: Welfare Consequences of Pure Tax and Permit Instruments in 2010



3.2 Welfare Consequences of Pure Tax and Permit Mechanisms

While the previous analysis based on Figure 5 provides important intuition and a rough approximation of the welfare consequences of taxes and permits, it fails to capture several important failures of the Weitzman assumptions. While the intuition behind these failures and their individual consequences is discussed in Appendix C, this section summarizes their net effect. In particular, the net welfare gains of alternative tax and permit policies are computed numerically and shown in Figure 7.

These results simply confirm the intuition in Figure 5. Namely the welfare gain from the optimal tax instrument, around $2.5 billion, is much larger than the gain from the optimal permit instrument, around $0.3 billion. Although the rough calculation using Weitzman's formula suggested a difference of $10 billion, there are many reasons why Weitzman's result is not exactly right in this context as discussed in Appenidix C.

Figure 7 also indicates the large risk associated with setting an emission target incorrectly – specifically setting a target too low. While the net benefits of a tax are positive for a wide range of values, from zero to $20/ton C, the net benefits of a permit system rapidly become negative as the target falls below 11 GtC. At the proposed 1990 emission level, 8.5 GtC, the welfare loss is more than $10 billion. This is a consequence of reductions becoming extremely expensive in the high-emission states of the world. Thus, an important conclusion from Figure 7 is that low targets could turn out to be very costly. Importantly, were the world to confront such high costs, in all likelihood the parties to any global climate agreement would agree to relax their commitments. The fact that such a potential even exists could then lead to strategic behavior in the private sector in order to make costs appear high, a point returned to in Section 4.5.

3.3 The Hybrid Permit Policy

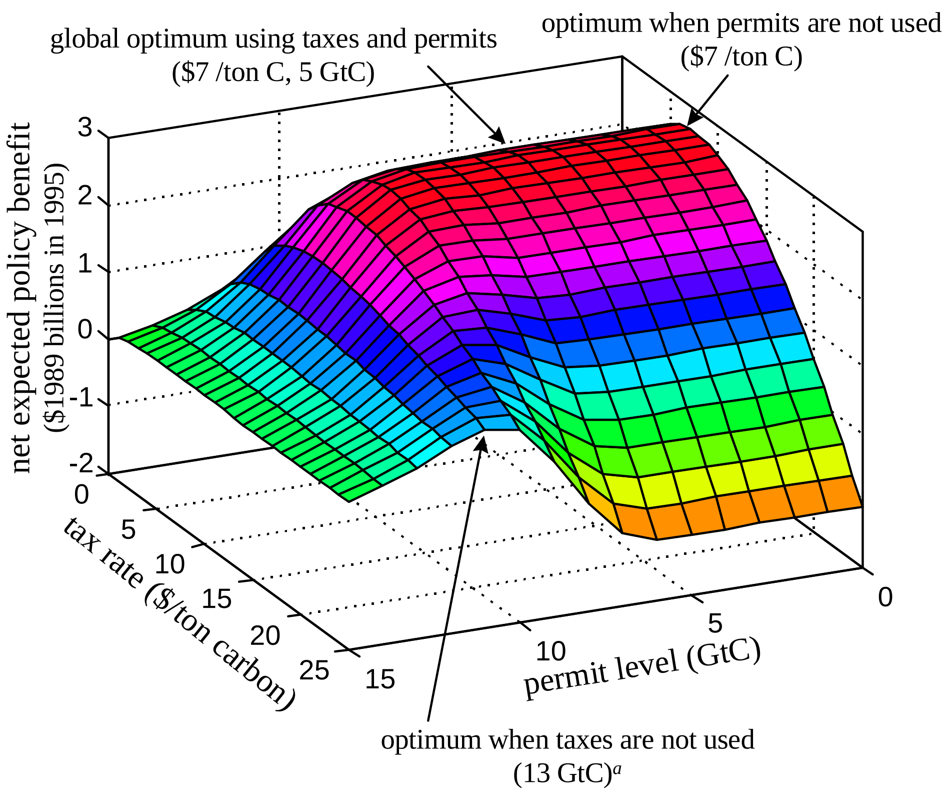
As pointed out by Roberts and Spence (1976), a hybrid permit policy which encompasses permits and taxes and special cases will perform at least as well as either alternative. It is natural to wonder whether it can do much better. Figure 8 shows that, in fact, it cannot; the optimal hybrid policy is only marginally better than the optimal pure tax policy. The global optimum across all permit levels and trigger prices, roughly a 5 GtC emission target and a $7/ton C trigger, is remarkably close to the proposal suggested by McKibbin and Wilcoxen (1997).[12]

The proper way to read Figure 8 is to note that for low permit levels (< 5 GtC; back right edge of figure), the policy is essentially a tax and the shape of the surface is simply an extrusion of the tax curve shown in Figure 7. For extremely high trigger prices (not shown; $> \$50$ tC), the policy is essentially a permit system and follows the permit curve shown in Figure 7. The outline of this shape is just beginning to show along the front right edge of the figure. For higher permit levels and lower trigger prices (essentially the middle of the figure), some combination of permits and taxes is at work, indicated by the odd shape of the surface.

While the optimal hybrid policy appears to be no better than the optimal tax policy, it clearly performs better than the optimal permit policy. In fact, relative to a straight target of 1990 emissions, with its attendant $10 billion loss from Figure 7, *any* of the policies shown in Figure 8 are an order of magnitude better (note the vertical scale only descends to a $2 billion loss). As long as policy debate continues to focus on targets and timetables, rather than taxes, these two comparisons are most relevant.

[12] They advocated 1990 emission levels as the permit volume coupled with a $10/ton C trigger. 1990 global controllable GHG emissions were 8.5 GtC.

Figure 8: Net Expected Policy Benefits of Alternative Hybrid Permit Policies in 2010



[a]The optimal permit level when additional permits are no longer purchased (13 GtC) does not become apparent until the trigger price is set roughly twice as high as shown in the figure (around $50/ton carbon), otherwise the additional permits are purchased in a non-trivial number of states of nature. From the figure, it is evident that the optimal permit level as a function of the trigger price is increasing. At $25/ton carbon, however, the optimal permit level is only 11 GtC. The net expected benefit when the trigger price is high enough to no longer matter and the permit level is 13 GtC is $0.3 billion, roughly one-tenth of the benefit from a straight tax or hybrid permit policy.

4 Multi-period Policy Under Uncertainty

Up to this point the analysis has focused on the costs and benefits of different policies in a single year. The problem of climate change, however, is spread out over decades if not centuries. Policies to combat climate change are therefore likely to be in place for a long time. It is not immediately obvious whether the results comparing different instruments in a single year are immediately applicable to a multiperiod policy (since policies in future periods may change the optimal policy in 2010).

In this section optimal policy paths for taxes, permits and hybrid permit mechanism are explored. In addition to considering the net welfare consequences of optimal policies, results concerning the range of climate outcomes are presented and sub-optimal policies are explored.

4.1 Optimal Permit Policy

To compute optimal policies, the paths of alternative permit, tax and hybrid permit systems were parameterized with six values describing stringency in 2010 (the first year of implementation), 2020, 2040, 2070, 2110 and 2160. Policies in intervening years are based on smooth interpolations,[13] except the stringency in 2160 which is allowed to be discontinous and is held fixed through the end of the simulation (2245). The length of the simulation as well as the spacing of the policy parameters was subjectively chosen to make the policy evaluation in the 2000-2100 interval as accurate as possible, especially the early 2000-2050 period.

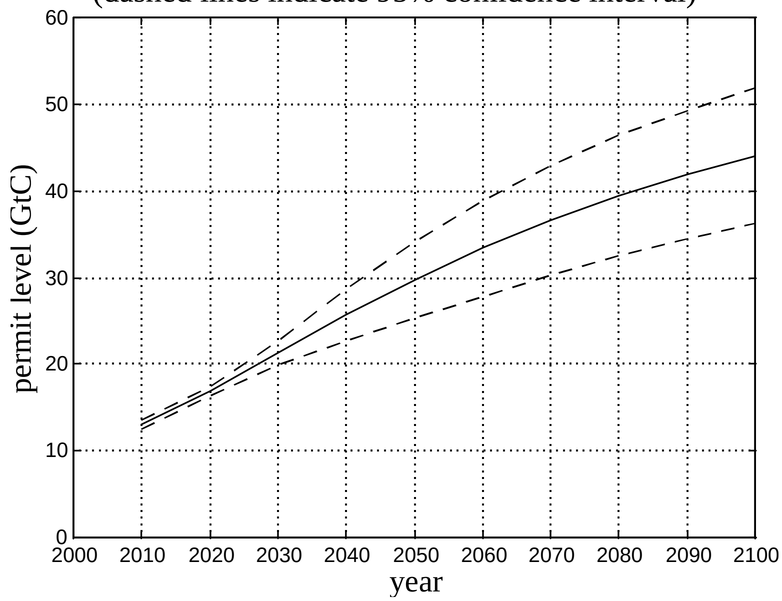
The resulting optimized permit policy, which limits global greenhouse gas emission to a specified level, is shown in Figure 9. The policy is not known with certainty due to sampling error – only 8,000 states of nature were used to estimate the policy (this could be reduced with additional computing power).[14] Interestingly, the optimal permit level of 13 GtC in 2010 is roughly the same as the optimal permit level determined in the one-period analysis. Perhaps this is not so surprising

[13] Policies are interpolated to ten-year intervals using a cubic spline; annual policies are linearly interpolated from the ten-year values. The hybrid policy is always interpolated linearly due to sampling error.

[14] The stringency was optimized over eight sets of 1,000 states of nature, taking on average 30 minutes to converge. These eight sets were then averaged and the standard deviation among the eight estimates was used to compute a standard error for the average.

Figure 9: Optimized Permit Level – Multi-period Policy

(dashed lines indicate 95% confidence interval)



given the fact that initial emission reductions do not substantially affect the GHG stock for many years, at which point the discounted benefits in 2010 will be small.[15]

An important observation is that the proposal to reduce emissions to their 1990 levels (roughly 8.5 GtC) is far below the optimal permit level in these simulations. Further, the optimal permit level rises in the future to accomodate growth in population and per capita productivity. While this kind of policy might appear to ignore the possibility that a carbon-free technology becomes not only commerically available but dominant in the next century, it conversely accomodates the possibility that such a technology never gets off the ground.

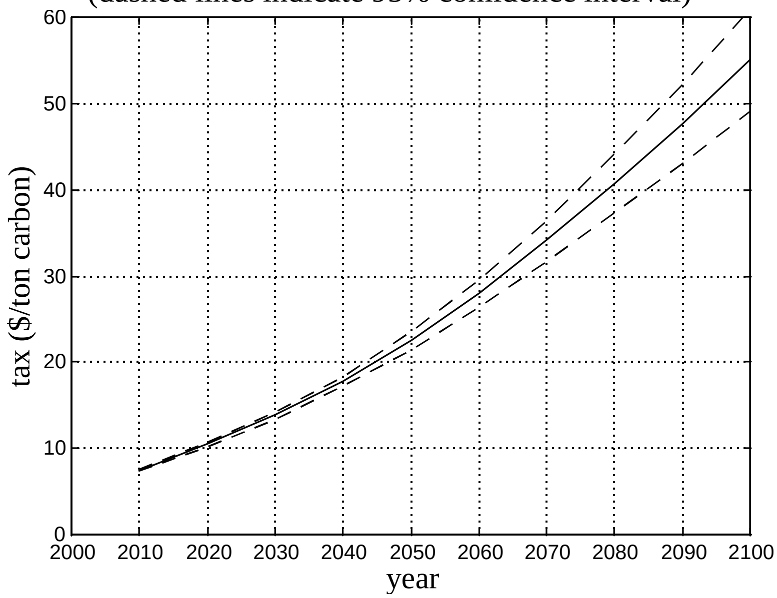
When the optimal permit policy is implemented it improves welfare on average by $69 billion ($1989).[16]

[15]An important assumption in the underlying DICE model is that damages are continuous and have been occuring since the beginning of industrialization. Thus we are already experiencing the consequences of global warming and current emission reductions are predominately concerned with reducing damages for roughly the next 30 years.

[16]This is the net present value of benefits minus costs and has an associated sampling error of $16 billion. This can be compared to annual global output in 1995 which was $24 trillion.

Figure 10: Optimized Tax Level – Multi-period Policy

(dashed lines indicate 95% confidence interval)



4.2 Optimal Tax Policy

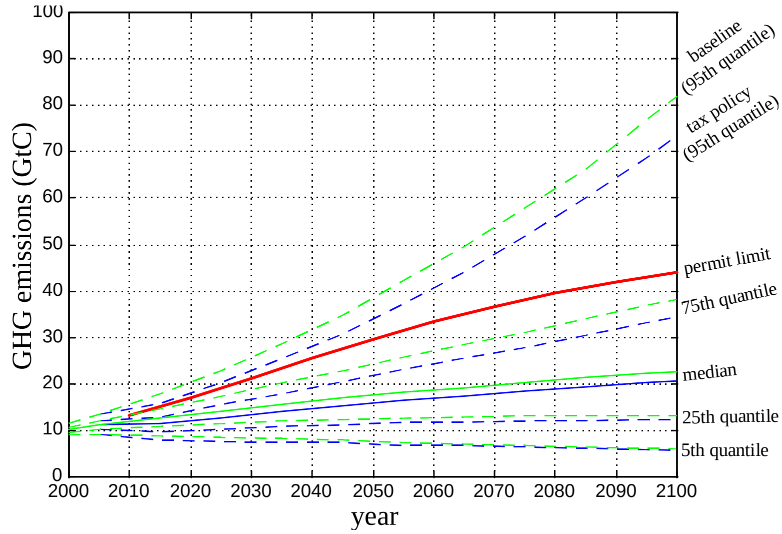
While the permit policy requires that emissions in every state of nature be at or below the specified level, a tax policy instead fixes the marginal cost of emission reductions. Consumers and producers facing a tax on greenhouse gas emissions will reduce emissions until the cost of further reductions equals the cost of paying the tax instead. If costs are particularly low, emissions may be completely eliminated and the marginal cost of the last ton of reductions will lie below the specified tax rate.

Figure 10 shows the tax policy which maximizes expected welfare. The initial tax of $7.35 is close to, but slightly lower than, the tax computed in the one-period analysis. Unlike the optimal permit policy, which rises and relaxes in stringency in order to accomodate growth, the optimal tax policy becomes *more stringent* in the future. This occurs because, unlike a constant permit level, a constant tax automatically encourages proportionally higher emissions as the economy grows. While some increase in emissions is desirable as the economy grows, a proportional increase is not – therefore the tax must increase in stringency while the permit system must be relaxed.[17]

An important difference between tax and permit policies is that taxes do not provide a strict

[17]An alternative interpretation of the rising tax rate is that, since emissions in adjacent periods generate essentially the same consequences, the shadow price of emissions must rise with the interest rate.

Figure 11: Distribution of Emission Levels Under Alternative Policies



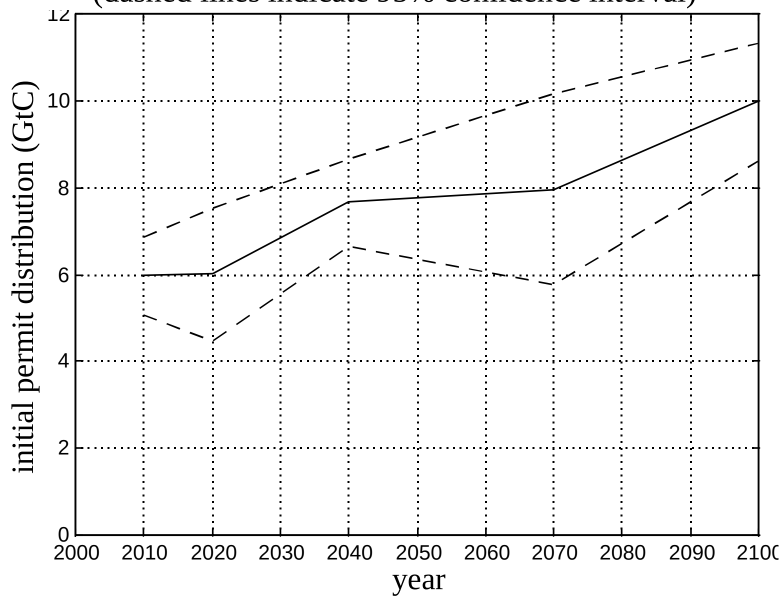
limit on emissions. This, however, should not be viewed strictly as a weakness: relaxing the level of emissions when costs turn out to be particularly high is desirable if the expected marginal benefits of reduction are fairly constant. Figure 11 shows the distribution of resulting emissions with and without the optimal tax policy alongside the optimal permit policy. Note that the optimal permit policy is not binding in over 75% of the states of nature (it lies above the 75th quantile of baseline emissions). Meanwhile the optimal tax policy leads to emissions above the optimal permit level in somewhere between 5 and 25% of the states of nature.

Finally, if the welfare changes associated with the optimal tax policy are averaged across states of nature, the expected welfare improvement amounts to $338 billion – compared to $69 for the permit instrument (the standard error in this case is $21 billion). This represents an expected improvement five times higher than that obtained with the permit system – a large ratio but smaller than the factor of eight determined in the one-period analysis.

Why might this ratio decline when viewed over a longer horizon? As pointed out by Weitzman (1974), discussed later by Stavins (1996), and pointed out in Section 2.4, positive correlation between marginal costs and benefits leads to a preference for quantity instruments (e.g., permits). In the case of a stock pollutant, such correlation is likely over long periods of time due to uncertainty about baseline emissions. In a state of nature with particularly low uncontrolled, baseline

Figure 12: Optimal Initial Permit Distribution for a Hybrid Permit Policy

(dashed lines indicate 95% confidence interval)



emissions, the pollutant stock is lower in the future making marginal damages lower. At the same time, lower emissions make any single period target cheaper to obtain. The reverse occurs in states of nature with high baseline emissions. This positive correlation among costs and benefits induces some preference for quantity controls. Based on the fact that price controls still offer a five-to-one improve over quantity controls, however, the empirical consequence of this effect is still quite low.

4.3 Optimal Hybrid Permit Policy

The trigger price path for the optimal hybrid permit policy turns out to be insignificantly different than the optimal tax level. As suggested in Figure 8, the corresponding optimal target is difficult to determine since the expected benefits are relatively flat over a range of values. This occurs because the marginal effect of the policy is derived only from those states of nature where baseline emissions are above the permit level but marginal costs are below the trigger – this turns out to be a relatively small fraction of the sample. The wide confidence intervals in Figure 12 reveals this difficulty. Even with 8,000 states of nature the optimal target is difficult to determine.[18]

[18]This sampling error could be reduced with additional simulations.

Table 3: Net Benefits of Hybrid Policies with 1990 Emission Targets
(expected NPV in $1989 billions; policies begin in 2010)

trigger price ($/tC)[a]	expected welfare gain	sampling error
5	181	6
10	237	9
20	272	13
30	256	15
50	146	19
70	-14	22
100	-287	26
150	-739	32
no trigger price	-3359	104

[a] Both the trigger price and target are fixed from 2010 forward.

Interestingly, the optimal initial permit level remains close to the 1990 emission level (8.5 GtC) for almost the entire forecast period. Setting the intitial distribution to 8.5 GtC in every period, in fact, has a negligible effect on the welfare as long as the optimal tax level is used as the trigger price (as was true in the one-period case).

4.4 Sub-optimal Policies

The purpose of this paper has been to compare optimal tax, permit and hybrid policies for reducing GHG emissions. However, many of the policy proposals actually under consideration deviate from those discussed in this paper. For that reason, this section considers the welfare consequences of sub-optimal policies. Specifically, the welfare consequences of hybrid policies involving 1990 emission targets coupled with alternative, fixed trigger prices are presented in Table 3.

Not surprisingly, the "no trigger price" option (e.g., a straight permit solution) entails the greatest losses, on the order of $3 trillion. In the one-period simulations, a key result was that low quantity targets generate large welfare losses. This result carries over to the multi-period context. Surprisingly, however, trigger prices as high as $50/tC still generate positive welfare gains even though the optimal price (shown in Figure 10) remains below $50/tC for the next 100 years.

A second important conclusion from Table 3 is that even as the trigger price approaches

$100/tC, the expected welfare gain remains an order of magnitude better than the straight permit approach. At $100/tC, the expected loss is capped below $300 billion – versus $3 trillion under the straight permit system. Even with a tax of $250/tC (not shown), the expected loss is cut in half. This highlights the role of a trigger price as an "escape valve" for adverse cost outcomes. In 2010, there is barely a one-in-four chance that additional permits would be sold at $100/tC. Yet, the expected loss is reduced by a factor of ten. Regardless of whether one is confident about the exact welfare outcomes presented in this paper, the potential for a hybrid permit policy to reduce extremely adverse cost outcomes should be clear.

4.5 Non-efficiency Issues

While the metric used to compare policies in this paper has been economic efficiency, there are obviously many other features associated with different policies which are important. As noted by Stavins (1989), criteria such as feasibility and provision of dynamic incentives matter both to policymakers and for welfare more broadly defined. In this regard, it is useful to note that the hybrid permit policy offers several advantages over the pure tax and permit alternatives.

In the case of permits, it seems unlikely that a world confronted with exceptionally and unexpectedly high compliance costs would continue to abide by a treaty based on a somewhat arbitrary target (e.g., 1990 emissions). Such a possibility opens the door for strategic behavior under a straight permit policy. Namely, firms have some incentive to make costs look high in order to encourage countries to opt out of treaty commitments – that would benefit the firms since they would stand to gain additional emission rights. In order to make costs look high, innovative research might be deferred and older, less efficient plants might be kept on-line longer.

In contrast, a hybrid permit policy provides an automatic relief mechanism to deal with unexpectedly high costs: additional and unlimited permits are sold at the fixed trigger price. Firms have no incentive to misrepresent costs because, regardless of how they represent costs, their only option is to pay for additional permits at the margin. In other words, the certainty of a trigger price is a much better signal to firms to move forward on new investments and research to reduce emissions than an uncertain government adjustment of emission targets.

Regarding taxes, as McKibbin and Wilcoxen (1997) point out, the potential tax burden associated with GHG emissions is enormous. With a gigaton of emissions in the U.S. alone and a $10 tax, this amounts to $10 billion. If the tax rate were $100 – still reasonable given a 1990 emission target – the burden would be $100 billion. Given the U.S. recent experience with the proposed BTU tax, such a policy would appear to be politically infeasible. The hybrid policy, in contrast, would allow any fraction of that burden to be distributed flexibly. By choosing which fraction of the permits to distribute freely and which fraction to auction, policymakers can balance the competing interests of revenue, equity and political feasibility.

5 Conclusion

Discussions of alternative tax and permit mechanisms for combating climate change have generally ignored the fact that the costs and benefits of future reductions are highly uncertain. Such uncertainty can lead to large efficiency differences between the two policies. This paper has explored this question in the context of an integrated climate-economy model capable of simulating thousands of uncertain states of nature.

The resulting welfare analysis indicates that taxes are much more efficient than permits for controlling GHG emissions – by a factor of *five to one* ($337 billion versus $69 billion in net benefits). This derives from the relatively flat marginal benefit curve associated with emission reductions. Relatively flat marginal benefits are partially a product of the quadratic damage function and partially a generic feature of stock pollutants like GHGs.

An important observation in this analysis is the risk involved in setting permit level too stringently. Not only does the optimal permit policy involve lower welfare gains, but setting the permit level incorrectly can lead to massive losses. The tax instrument, in contrast, leads to welfare gains over a much wider range of values.

In addition to pure tax and permit systems, this paper explored the possibility of a hybrid permit system. The hybrid policy involves an initial allocation of permits followed by the subsequent sale of additional permits at fixed trigger price if costs turn out to be high. By making the initial distribution small or the subsequent sale price high, this combined system can be made to mimic either the pure tax or permit system.

The hybrid permit system improves on the optimal tax outcome, but only slightly (by about $2 billion or 0.5%). However, the hybrid policy offers the flexibility of a permit mechanism in terms of distributing the rents associated with emission rights. Further, a sub-optimal hybrid policy with a stringent target and high trigger price generates much better welfare outcomes than a straight permit policy with the same target. Taken together, the improved flexibility and welfare outcomes enhance the credibility of the hybrid policy relative to either the permit or tax policy alone. Such credibility is critical for the encouragement of private sector activities to reduce emissions as well as abatement costs.

A Model Specification

A.1 Economic Behavior

Economic behavior within each state is derived from a representative agent model where consumption must be optimally allocated across time. In a typical model with constant exogenous productivity growth, agent preferences define a steady state to which the economy converges over time. In the presence of random shocks and slowly changing trends, the economy instead converges to a distribution of states (due to the random shocks) which is itself slowly evolving (due to the slowly changing trends). For the moment we ignore these changing trends and focus on a standard stochastic growth model.

The representative consumer in this model exhibits constant relative risk aversion τ with respect to consumption per capita. Utility is separable across time, discounted at rate ρ and weighted each period by population. With the further assumption that preferences satisfy the von Neuman-Morgenstern axioms, the consumer's optimization problem can be written as

$$\max_{\substack{C_t \\ t\in\{0,1,\dots\}}} \mathrm{E}\left[\sum_{t=0}^{\infty}(1+\rho)^{-t}N_t\frac{(C_t/N_t)^{1-\tau}}{1-\tau}\right] \tag{A.1}$$

where C_t is consumption in period t and N_t is population. That is, the consumer maximizes expected discounted utility where each period's utility is population weighted. This consumption program $\{C_0, C_1, \dots\}$ is subject to the resource constraints describing production

$$Y_t = (A_t^* N_t)^{1-\theta} K_t^{\theta} \tag{A.2}$$

and capital accumulation

$$K_{t+1} = K_t(1-\delta_k) + Y_t - C_t \tag{A.3}$$

where Y_t is aggregate output, $A_t^* N_t$ is effective labor input and K_t is the capital stock. A_t^* is a measure of productivity distinct from capital but not completely exogenous, as discussed later. The parameter θ summarizes the Cobb-Douglas production technology given in Equation (A.2) and δ_k reflects the rate of capital depreciation in the capital accumulation equation (A.3). Finally, there is a transversality condition for a balanced growth steady state,

$$\rho > (1-\tau) \times (\text{asymptotic growth rate of } A_t^*) \tag{A.4}$$

This is always satisfied by assuming zero growth asymptotically.

Even with exogenous constant growth models for N_t and A_t^*, the dynamic optimization problem given by Equations (A.1–A.3) is difficult if not impossible to solve analytically.[19] However, choosing

$$\Delta \ln(K_{t+1}/N_{t+1}) = \alpha_1 + \alpha_2(\ln(K_t/N_t) - \ln(A_t^*)) \tag{A.5}$$

and

$$C_t = K_t(1-\delta_k) + Y_t - K_{t+1} \tag{A.6}$$

– where α_1 and α_2 are functions of the parameters $(\rho, \tau, \theta, \delta_k)$ – yields a close approximation of optimal consumer behavior around the balanced growth steady state. This technique of approximating optimal dynamic behavior has its origins in the real business cycle literature beginning with Kydland and Prescott (1982). It is also related to the technique of feature extraction discussed by Bertsekas (1995).[20]

Intuitively, Equation (A.5) approximates behavior around a balanced growth steady state. At such a steady state, $\ln(K_t/N_t) - \ln(A_t^*)$ is constant and $\Delta \ln(K_{t+1}/N_{t+1}) = (\text{growth rate of } A_t^*) = \alpha_1 + \alpha_2(\ln(K_t/N_t) - \ln(A_t^*)) =$ constant. If some unforseen shock moves the economy away from the equilibrium value of $K_t/(A_t^* N_t)$ and α_2 is negative, e.g., the steady state is stable, then the economy will move back toward the steady state. In particular, when $K_t/(A_t^* N_t)$ is too high, capital accumulation will slow. If $K_t/(A_t^* N_t)$ is too low, capital accumulation will increase. Importantly, even if the growth rate of A_t^* is not constant, this approximation performs well as long as expected productivity growth changes gradually.

[19] Long and Plosser (1983) derive an analytic solution for the case of $\delta_k = 1$ and $\lim \tau \to 1$ (log utility).

[20] See Appendix A of Pizer (1997) for a simple derivation of expressions for α_1 and α_2.

A.2 Long-term Growth, Climate Behavior and Damages

This section explains the remainder of the state-contingent model – specifically the evolution of A_t^* and N_t. This includes exogenous growth projections, climate behavior and damages from global warming (based primarily on the DICE model, Nordhaus 1994b). Exogenous labor productivity A_t is modeled as a random walk in logarithms with an exponentially decaying drift. That is,

$$\log(A_t) = \log(A_{t-1}) + \gamma_a \exp(-\delta_a t) + \sigma_a \epsilon_t \tag{A.7}$$

where γ_a is the initial growth rate, δ_a is the annual decline in the growth rate, σ_a is the standard deviation of the random growth shocks and ϵ_t is a standard NIID random shock. This means that productivity growth begins with a mean growth rate of γ_a (around 1.3%) in the first period and eventually declines to zero. In addition, random and permanent shocks change the level of productivity every period. The standard error of these shocks is σ_a.

Net labor productivity A_t^* is distinguished from this exogenous measure A_t by the fact that A_t^* describes the amount of output available for consumption and investment – after output has been reduced by control costs and climate damages. To that end, A_t^* is expressed as A_t multiplied by a factors describing these two phenomena:

$$A_t^* = \left(\frac{\overbrace{1 - b_1 \mu_t^{b_2}}^{\text{control costs}}}{\underbrace{1 + (D_0/9) \cdot T_t^2}_{\text{damages}}} \right)^{\frac{1}{1-\theta}} A_t \tag{A.8}$$

μ_t is the fractional reduction in greenhouse gas emissions at time t (the "control rate") versus a business as usual/no government policy baseline, while b_1 and b_2 parameterize the cost of attaining these reductions. Since b_1 and b_2 are both positive, additional rates of control involve reductions in net productivity. T_t is the average surface temperature relative to pre-industrialization in degrees Celsius and D_0 is the fractional loss in aggregate GDP from a 3° temperature increase. For temperature changes less than 10°, this is essentially a quadratic damage function.[21] Additional details about the control cost and damage functions can be found in Nordhaus (1993) and Nordhaus (1994a), respectively.

[21] Over larger ranges, the damage function becomes S-shaped.

Population is modeled in the same way as exogenous productivity but without the stochastic element:

$$\log(N_t) = \log(N_{t-1}) + \gamma_n \exp(-\delta_n t) \tag{A.9}$$

where γ_n is the initial growth rate and δ_n is the annual decline in the growth rate. Note that these models predict zero growth asymptotically, though this may occur centuries in the future.[22]

The remaining portion of the model explains the link between economic activity (measured as aggregate output Y_t) and warming (measured as the average surface temperature T_t). The first step is linking output to emissions:

$$E_t = \sigma_t(1 - \mu_t)Y_t \left(\frac{A_t}{A_t^*}\right)^{1-\theta} \tag{A.10}$$

where E_t is emission of controllable greenhouse gases,[23] σ_t is an exogenous trend in emissions/output, and μ_t is the rate of emissions reductions induced by the policymaker. The expression $Y_t \left(\frac{A_t}{A_t^*}\right)^{1-\theta}$ reflects raw output prior to the effects of climate damages and control costs. The model of σ_t is, as with labor productivity and population, based on exponentially decaying growth:

$$\log(\sigma_t) = \log(\sigma_{t-1}) + \gamma_\sigma \exp(-\delta_a t) \tag{A.11}$$

where γ_σ is the initial growth rate of emissions/output (a negative number) and δ_a is the annual decline in the growth rate. Note that the annual decline in the emissions/output growth rate is the same as the annual decline in labor productivity growth (δ_a).

Emissions of greenhouse gases accumulate in the atmosphere according to:

$$M_t - 590 = \beta E_{t-1} + (1 - \delta_m)(M_{t-1} - 590) \tag{A.12}$$

where M_t is the atmospheric concentration of greenhouse gases in billions of tons of carbon equivalent. β is a measure of the retention rate of emissions. Low values of β indicate that emissions do

[22] For example, the range of parameters used in the simulations (with $\delta_{a/n} \in (0.25\%, 2.5\%)$) leads to a halving of the growth rates every 20 to 200 years.

[23] See discussion of controllable versus uncontrollable greenhouse gases in Nordhaus (1994b), page 74. For the most part, controllable greenhouse gases are CO_2 and CFCs and uncontrollable greenhouse gases are everything else.

not, in fact, accumulate while a value of unity would mean that every ton of emitted greenhouse gases remains in the atmosphere. The parameter δ_m plays the role of a depreciation rate: it is assumed that greenhouse gases in the atmosphere above the pre-industrialization level of 590 billion tons slowly decays. This decay reflects absorption of greenhouse gases into the oceans which are assumed to be an infinite sink.

Above average concentrations of greenhouse gases in the atmosphere lead to increased radiative forcings, a measure of the rate of transfer between solar energy produced by the sun and thermal energy stored in the atmosphere. This is modeled according to

$$F_t = 4.1 \times \log(M_t/590)/\log(2) + O_t \tag{A.13}$$

where F_t measures radiative forcings in units of watts per meter squared. The specification is such that a doubling of greenhouse gas concentrations leads to a roughly four fold increase in forcings (since 590 is the concentration before industrialization). O_t in this relation represents radiative forcings due to other uncontrollable greenhouse gases and is assumed exogenous to the model:

$$O_t = \begin{cases} 0.2604 + 0.0125t - 0.000034t^2 & \text{if } t < 150 \\ 1.42 & \text{otherwise} \end{cases} \tag{A.14}$$

Increased forcings lead to temperature changes according to

$$T_t = T_{t-1} + (1/R_1)\left[F_t - \lambda T_{t-1} - (R_2/\tau_{12})(T_{t-1} - T^*_{t-1})\right] \tag{A.15}$$

$$T^*_t = T^*_{t-1} + (1/R_2)(R_2/\tau_{12})(T_{t-1} - T^*_{t-1}) \tag{A.16}$$

where T_t is the surface temperature and T^*_t is the deep ocean temperature, both expressed in changes relative to pre-industrialization levels in degrees Celsius. Note that if $M_t = 590$ and $O_t = 0$ (e.g., pre-industrialization), T_t and T^*_t will equilibriate to zero. The parameter λ describes the equilibrium change in surface temperature for a given change in radiative forcings. In particular, based on (A.13) and (A.15), a doubling of the concentration of greenhouse gases in the atmosphere will lead to a $4.1/\lambda$ rise in surface temperature in the long run. This parameter $4.1/\lambda$ is a measure of the temperature sensitivity of the atmosphere.[24]

[24] λ by itself is referred to as the climate feedback parameter.

The parameters R_1, R_2 and τ_{12} describe the thermal capacity of the surface atmosphere and deep oceans and the rate of energy transfer between them, respectively.

A.3 Social Welfare

A distinguishing feature of this analysis is the use of an econometrically estimated parameter distribution describing uncertainty in the economic model. However, the consumer's objective function given by Equation (A.1) makes no allowance for uncertainty about the preference parameters ρ and τ which are fixed from his or her perspective. In order to encompass uncertainty about preferences, it is necessary to step back and imagine a social planner who would like to maximize the objective given in (A.1) but is unsure of the parameters. Since a policy change which raises the expected utility for one set of parameters may lower the expected utility for another set, the social planner will need to specify a social welfare function to compare gains and losses across states of nature. This social welfare function provides a single objective specifying how changes in utility measured with different preferences are aggregated.[25] It is important to recognize that although parameter values in the representative agent model can be inferred from observed consumer behavior, there is no information available to estimates parameters in a social welfare function. Such information would be revealed only by observing the behavior of an actual social planner. Instead, we must rely on social choice theory and our own sense of fairness to specify the relation.

It is useful to note that the common approach in the climate change literature skirts this issue of preference aggregation by reporting a range of policy prescriptions based on a range of possible preferences and states of nature. For example, Cline (1992) presents benefit-cost analyses for 92 different cases (Tables 7.3 and 7.4). Dowlatabadi and Morgan (1993) integrate out much of the uncertainty in their analysis, but still present results for 48 scenarios. Chapter 8 of Nordhaus (1994b) gives one of the few examples where even preference uncertainty is integrated out, yielding a single welfare metric and a single policy recommendation. In a similar analysis, however, Nordhaus and Popp (1997) choose to fix preferences because of the difficulties with preference aggregation.

[25] E.g., providing a negative loss function across states of nature for the social planner.

Regardless, these authors ubiquitously observe that uncertainty about time preference has large consequences for optimal policy choice.[26] Moreso, in fact, than uncertainty about climate sensitivity and damages. It therefore behooves us to seriously consider how to aggregate over uncertain preferences in the most reasonable way.

In this analysis social welfare is specified as an average of utility measured in each state of nature by Equation (A.1), *rescaled*. The rescaling serves to equate the marginal social welfare of one additional dollar of current consumption in all states of nature. While arbitrary, some adjustment is necessary to prevent the resulting policy prescription from being sensitive to the choice of units in the model.[27] Social welfare can then be written as

$$SW(x) = I^{-1} \sum_{i=1}^{I} u(x, i) \tag{A.17}$$

where $u(x, i)$ is rescaled utility in state i with outcome x and $SW(x)$ is the social welfare associated with x. The rescaling is such that

$$u(+\$1 \text{ in initial period}, i) - u(\emptyset, i) = u(+\$1 \text{ in initial period}, j) - u(\emptyset, j) \qquad \forall i, j$$

That is, a policy corresponding to an extra dollar of consumption in the initial period is assumed to have the same utility gain in every state relative to a no policy ($\emptyset$) baseline.

This social welfare function has its origins in the literature on social choice. Harsanyi (1977) shows that in defining social welfare over lotteries, if individual preferences satisfy the von Neumann Morgenstern axioms then social welfare must have this weighted average form. Otherwise, social preferences will fail to mimic individual preferences over lotteries involving only that individual. This functional form can also be derived from the assumption of cardinal unit comparibility, as discussed by Roberts (1980). More flexible forms require additional assumptions about level or scale comparibility. Our choice of welfare functions is therefore less arbitrary than it might have originally appeared: a more flexible form requires both integrating out uncertainty from the representative agent's perspective (to satisfy Harsanyi's point) and more stringent assumptions about

[26] See discussion in Arrow, Cline, Maler, Munasinghe, and Stiglitz (1996).
[27] An explanation of this point is given in Appendix C of Pizer (1997).

the level of comparibility (to satisfy Robert's point)[28]

B Measuring Uncertainty

Estimates of uncertainty in the model come from two sources: econometric analysis and subjective assessment. The model involves nineteen different parameters. Six are parameters describing observable economic activity:

- pure time preference ρ,
- risk aversion τ,
- output-capital elasticity θ,
- productivity growth γ_a,
- variation in productivity growth σ_a^2, and
- depreciation δ_k.

A joint distribution for these parameters is estimated with historical data. The remaining thirteen describe emissions:

- emissions rate growth γ_σ,

climate change:

- CO_2 retention rate β,
- temperature sensitivity $4.1/\lambda$,
- CO_2 decay rate δ_m,
- thermal capacities and conductivities R_1, R_2 and τ_{12},

control costs and damages:

- cost function parameters b_1 and b_2,
- fractional loss of GDP for 3° temperature rise D_0,

and long-term growth trends

- population growth γ_n,
- productivity slowdown and slowdown in the growth rate of emissions/output δ_a,
- population slowdown δ_n.

Uncertainty about these parameters is based on subjective analysis.

The econometric analysis of the six economic parameters is based on post-war U.S. data.[29]

[28] Additional levels of comparability are especially difficult with the the constant coefficient of relative aversion (CRRA) form in Equation (A.1) where the parameter $\tau \gtrless 1$. Under these assumptions, utility is alternatively bounded from above or below.

[29] See Chapter II of Pizer (1996).

Series describing aggregate investment, capital services, output and prices are fit to the model described by Equations (A.2),(A.3) and (A.5). The posterior parameter distribution which arises from this analysis is summarized in Table B.1.

Nordhaus (1994b)[30] develops a distribution for the remaining parameters based on a two-step subjective analysis. The first step involves testing his model's sensitivity to each parameter being changed, one at a time, to a more extreme value. Those parameters which produce the largest variance in model output are then further scrutinized. A discrete, five-value distribution is developed for seven of these thirteen variables. The other six are fixed at their best guess values. The distribution of the seven uncertain parameters is summarized in Table B.2. Values of the six fixed parameters as well as initial conditions for the model are given in Table B.3.

C Factors Which Complicate the Weitzman Analysis

In this section the factors which complicate Weitzman's (1974) original analysis are discussed. While these factors do not affect the basic intuition behind his result – that a flatter marginal benefit curve favors taxes – they do qualify it. In particular, the intersection of the expected marginal benefit and expected marginal cost curve can no longer be used to determine the optimal tax policy when there are non-linear marginal costs and non-additive shocks. As noted by Weitzman and others (Stavins 1996), correlation also changes the optimality result, with positive correlation among costs and benefits favoring permits and negative correlation favoring taxes. Other factors such as truncation and discounting further complicate the simple graphical analysis in Section 3.

C.1 Non-linearities and Non-additive Shocks

A simple graphical comparison of expected marginal benefits and expected marginal costs allows one to quickly derive the optimal quantity control. This works because these marginal measures are with respect to the quantity measure *and* expectations are taken holding the quantity fixed.

[30] Chapters 6 and 7.

Table B.1: Marginal distributions of uncertain economic parameters
(narrow bars indicate values used in simulations without uncertainty)

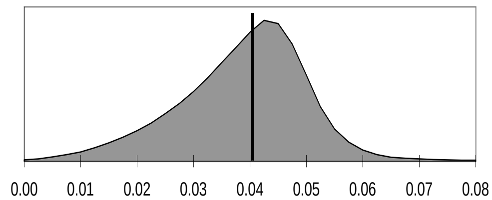
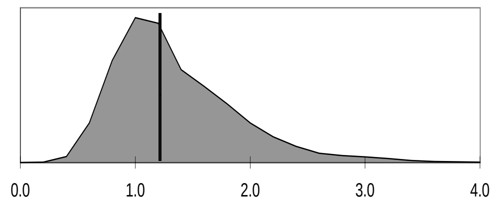
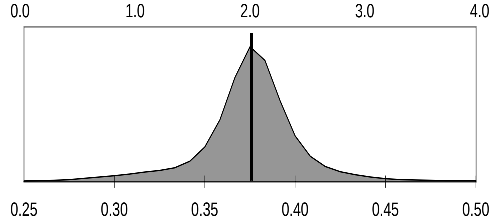
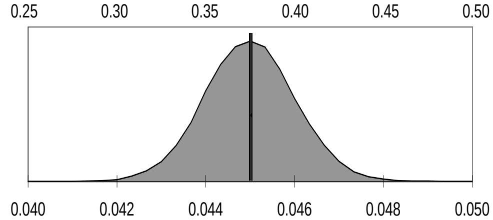
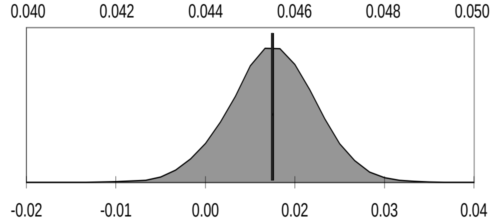
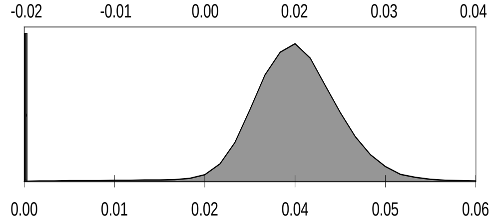
description	symbol	equation	distribution
pure rate of time preference	ρ	(A.1)	
coefficient of risk aversion	τ	(A.1)	
output-capital elasticity	θ	(A.2)	
rate of capital depreciation	δ_k	(A.3)	
initial productivity growth rate	γ_a	(A.7)	
standard error of productivity shocks	σ_a	(A.7)	

Table B.2: Discrete distributions of uncertain climate/trend parameters
(narrow bars indicate values used in simulations without uncertainty)

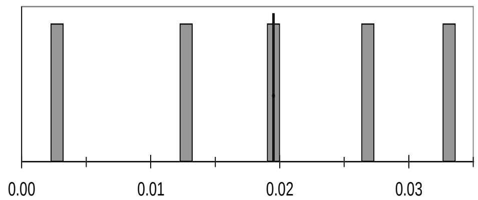
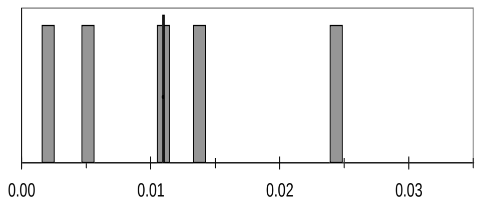
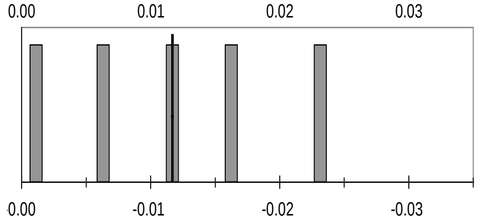
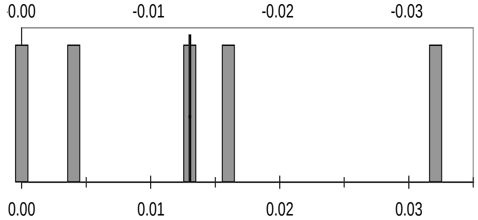
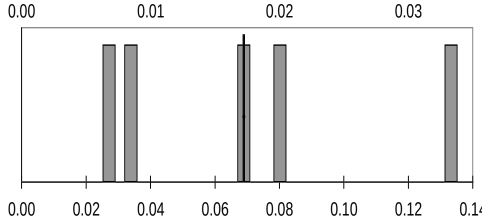
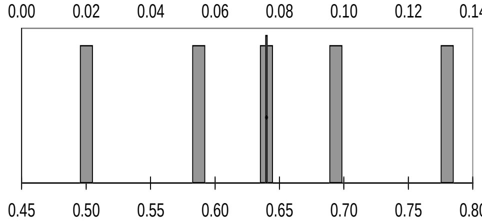
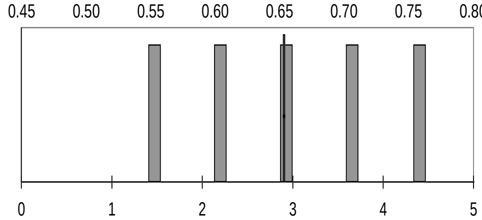
description	symbol	equation	distribution
annual decline of population growth rate	δ_n	(A.9)	
annual decline of productivity growth rate	δ_a	(A.7),(A.11)	
initial growth rate of CO_2 per unit output	γ_σ	(A.10),(A.11)	
damage parameter (% loss of GDP for 3° temperature rise)	D_0	(A.8)	
cost function parameter	b_1	(A.8)	
retention rate for CO_2 emissions	β	(A.12)	
temperature sensitivity to CO_2 doubling (in °C)	$4.1/\lambda$	(A.13),(A.15)	

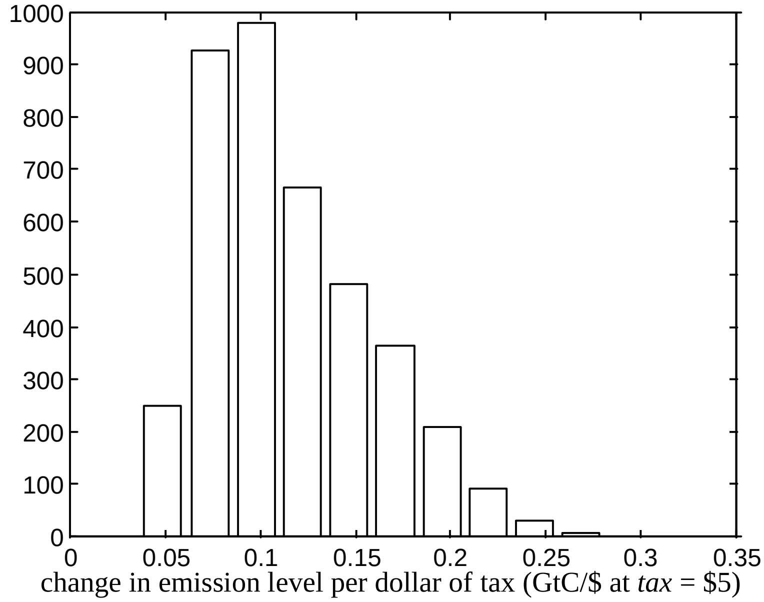
Table B.3: Description of fixed parameters[a]

parameter	symbol	equation	units	value
cost function parameter	b_2	(A.8)		2.887
decay rate of atmospheric CO_2	δ_m	(A.12)		0.00833
1/thermal capacity of atmosphere	$1/R_1$	(A.15)	$^\circ$C-meter^2/watt-year	0.048
thermal conductivity b/w atmosphere and oceans	R_2/τ_{12}	(A.15),(A.16)	watt/$^\circ$C-meter^2	0.44
1/thermal capacity of deep oceans	$1/R_2$	(A.16)	$^\circ$C-meter^2/watt-year	$\frac{0.002}{0.44}$
1995 global population	N_0	(A.9)	millions of people	5590
initial population growth	γ_n	(A.9)		0.0124
initial rate of CO_2 emissions per unit of output	σ_0	(A.11)	billion tons CO_2 per \$1989 trillions	0.385
1995 global capital stock	K_0	(A.2),(A.3)	\$1989 trillions	79.5
1995 global output	Y_0	(A.2),(A.3),(A.10)	\$1989 trillions	24.0
1995 atmospheric concentrations of CO_2	M_0	(A.12)	billions of tons of C equivalent	763.6
1995 surface temperature[b]	T_0	(A.8),(A.15),(A.16)	$^\circ$Celsius	0.763
1995 deep ocean temperature[b]	T_0^*	(A.15),(A.16)	$^\circ$Celsius	0.117

[a] All fixed parameters are from Nordhaus (1994b). The parameters that do not depend on time are from Nordhaus' Table 2.4. Initial values for temperature, CO_2 concentrations, and output in 1995, as well as the initial annual growth rate for population, are based on the Nordhaus base case simulation. The 1995 capital stock is adjusted upward to reflect differences in the definition of capital as well as underlying parameter values. The decay rate of atmospheric CO_2 is divided by ten to convert from a decennial to annual rate. The annual thermal capacity of the ocean and atmosphere are from the second line of Nordhaus' Table 3.4b.

[b] Temperatures are measured as deviations from the pre-industrialization level, circa 1900.

Figure C.1: Distribution of $\frac{dQ}{d(tax)}$



In contrast, the optimal price control must be derived by taking the expectation of Δbenefit$/\Delta$tax and Δcost$/\Delta$tax, holding the tax level fixed, and finding the tax level where *these* two marginal measures are equal. This is not generally revealed by a simple diagram such as Figure 5. It is revealed, however, if the slope of the marginal cost schedule is constant and known. In that case, the condition for the optimal tax reduces to the intersection of the expected marginal cost and benefit curves:

$$\begin{gathered}
\mathrm{E}\left[\frac{\Delta\text{benefit}}{\Delta\text{tax}}\right] = \mathrm{E}\left[\frac{\Delta\text{cost}}{\Delta\text{tax}}\right] \\
\mathrm{E}\left[\frac{\Delta\text{benefit}}{\Delta\text{quantity}}\frac{\Delta\text{quantity}}{\Delta\text{tax}}\right] = \mathrm{E}\left[\frac{\Delta\text{cost}}{\Delta\text{quantity}}\frac{\Delta\text{quantity}}{\Delta\text{tax}}\right] \\
\mathrm{E}\left[\frac{\Delta\text{benefit}}{\Delta\text{quantity}}\frac{1}{\text{MC slope}}\right] = \mathrm{E}\left[\frac{\Delta\text{cost}}{\Delta\text{quantity}}\frac{1}{\text{MC slope}}\right] \\
\mathrm{E}\left[\frac{\Delta\text{benefit}}{\Delta\text{quantity}}\right]\frac{1}{\text{MC slope}} = \mathrm{E}\left[\frac{\Delta\text{cost}}{\Delta\text{quantity}}\right]\frac{1}{\text{MC slope}} \\
\mathrm{E}\left[\frac{\Delta\text{benefit}}{\Delta\text{quantity}}\right] = \mathrm{E}\left[\frac{\Delta\text{cost}}{\Delta\text{quantity}}\right] \\
\mathrm{E}\left[MB\right] = \mathrm{E}\left[MC\right]
\end{gathered} \tag{C.18}$$

This condition, that the slope of the marginal cost curve is constant, will be violated if the marginal cost curve is non-linear or there are non-additive shocks. Under these conditions, the slope of the marginal cost curve cannot be factored outside the expectation, as shown in (C.18). To verify this condition, the slope of the marginal cost curve can be examined across states of nature and at different tax levels to see if it remains constant. Figure C.1 shows the distribution of slopes for a single tax level of $5/tC.[31]

Given these violations, it will not be possible to determine the optimal tax level from Figure 5 precisely, though it may still provide a rough approximation.

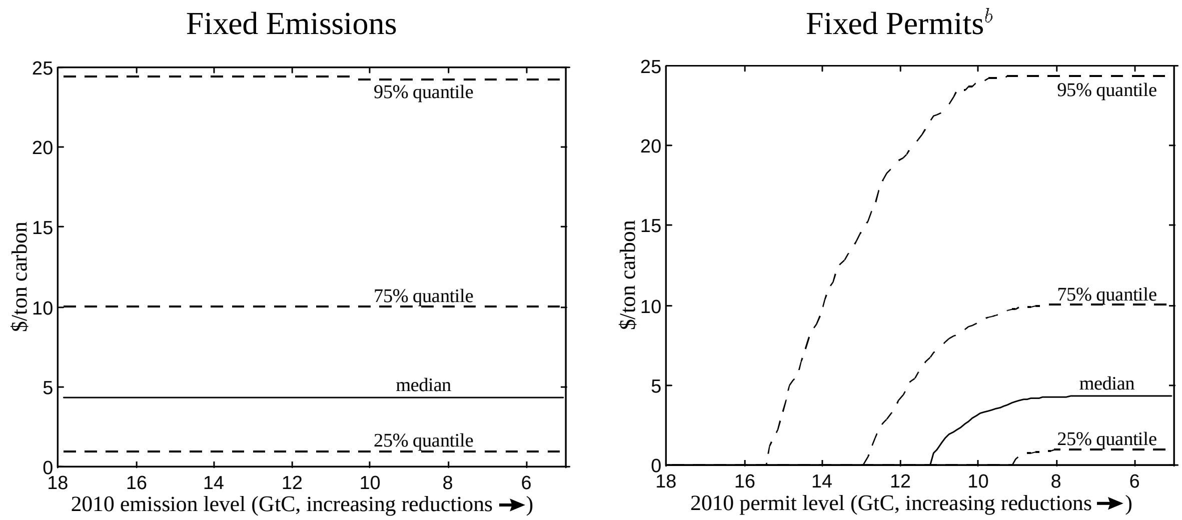
C.2 Truncation

An important point ignored by the Weitzman analysis is whether a particular quantity control is binding in every state of nature. Figure 6, for example, reveals that a quota of 12 GtC in 2010 would lie above the actual emission level roughly half the time. In these states of the world, it is inappropriate to count any benefits from the policy since the policy has no consequence. That is, in order to properly compare the expected marginal cost of a 12 GtC permit policy to its benefits, marginal benefits should only be counted when the policy is binding.

Figure C.2 shows the consequence of this calculation. The left panel (the same as Figure 4) shows the distribution of marginal benefits when emissions are fixed at – rather than limited to – different levels. This calculation ignores the actual level of emissions in 2010. In contrast, the right panel shows the marginal benefit associated with limiting emissions to the specified level (as would occur with a permit system). In those states of nature where uncontrolled emissions are below the value on the x-axis, they remain unchanged and there is no marginal benefit. Therefore at high permit levels (18 GtC) that lie above uncontrolled emission levels in every state, the marginal benefit is zero. Moving to the right, marginal benefits initially rise as additional states become contributing benefits. Eventually, the marginal benefit schedule must slope downwards as the marginal benefit in every state where benefits occur is declining.

[31] From Figure 5 it is clear that the slope also changes at higher permit/tax levels.

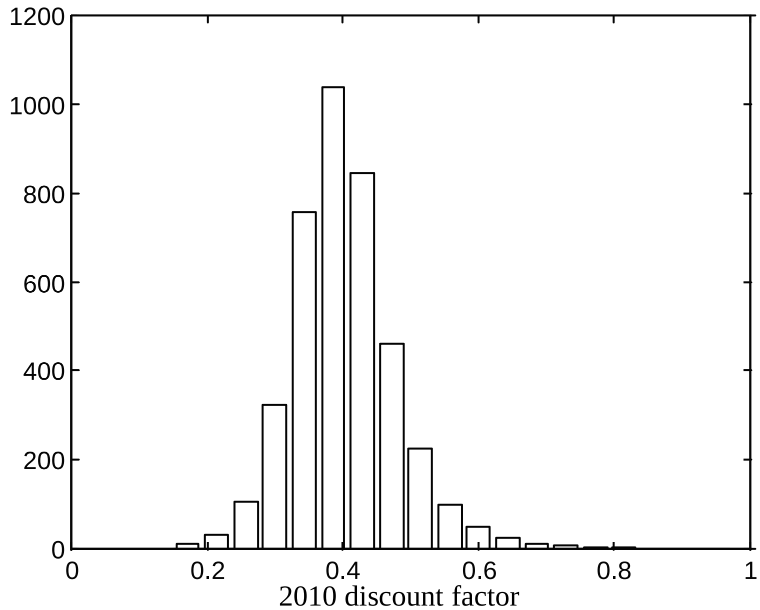
Figure C.2: Distribution of Marginal Benefits in 2010[a]



[a]These marginal benefits are based on the discounted (to 2010) value associated with emission reductions in the year 2010 only. Emissions in other periods are held at their uncontrolled levels. Values are expressed in $2010 and, due to different discount rates across states of nature, will not be weighted equally when balancing costs and benefits.

[a]The left panel indicates the range of marginal benefits obtained by varying the level of 2010 emissions as shown, ignoring the forecast level of uncontrolled emissions. The right panel indicates the range of marginal benefits when permits are used to control the level of emissions at *or below* the permit level. That is, in cases where uncontrolled emissions are below the indicated level, there is no marginal benefit since emissions are not, in fact, being reduced. As shown in Figure 6, uncontrolled emissions in 2010 are unlikely to be more than 18 GtC. The marginal benefit when the permit level is set to 18 GtC is therefore zero.

Figure C.3: Distribution of Discount Factors in 2010



C.3 Discounting

Discounting is an important issue which is ignored in the simple static case. Even when considering policy in a single period (2010), it is necessary to wonder whether costs and benefits should be measured in 2010 or 1995. This is relevant because different states of nature will involve different discount rates. Specifically, those states with higher growth involve more discounting relative to states of nature with low growth. This follows the basic intuition that extra dollars are more valuable when one is poor than when one is rich.

The static analysis in Section 3 is based on dollar welfare measures in 2010. In contrast, the dynamic analysis in Section 4 is based on dollar welfare measures in 1995. Surprisingly, there is little difference in the initial policy outcome, suggesting the descrepancy is small. Figure C.3 shows the range of discount factors observed in 2010 relative to 1995.

References

Arrow, K. J., W. R. Cline, K.-G. Maler, M. Munasinghe, and J. E. Stiglitz (1996). Intertemporal equity and discounting. In J. Bruce, H. Lee, and E. Haites (Eds.), *Climate Change 1995: Economic and Social Dimensions of Climate Change*. Cambridge: Cambridge University Press.

Bertsekas, D. P. (1995). *Dynamic Programming and Optimal Control*, Volume 1. Belmont, MA: Athena Scientific.

Burtraw, D. and M. Toman (1997). The benefits of reduced air pollutants in the U.S. from greenhouse gas mitigation policies. Discussion Paper 98-01, Resources for the Future.

Cline, W. (1992). *The Economics of Global Warming*. Washington: Institute of International Economics.

Dowlatabadi, H. and M. Morgan (1993). A model framework for integrated studies of the climate problem. *Energy Policy 21*(3), 209–221.

Goulder, L., I. W. Parry, and D. Burtraw (1996). Revenue-raising vs. other approaches to environmental protection: Thecritical significance of pre-existing tax distortions. NBER Working Paper No. 5641.

Harsanyi, J. C. (1977). *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. New York: Cambridge University Press.

Intergovernmental Panel on Climate Change (1992). The 1992 IPCC supplement: Scientific assessment. In J. Houghton, B. Callander, and S. Varney (Eds.), *Climate Change 1992: The Supplemental Report to the IPCC Scientific Assessment*, pp. 1–22. Cambridge University Press.

Kydland, F. E. and E. C. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*(6), 1345–1370.

Lashof, D. A. and D. R. Ahuja (1990). Relative contributions of greenhouse gas emissions to global warming. *Nature 344*, 529–531.

Long, J. B. and C. I. Plosser (1983). Real business cycles. *Journal of Political Economy 91*, 39–69.

McKibbin, W. J. and P. J. Wilcoxen (1997). A better way to slow global climate change. Brookings Policy Brief No. 17, the Brookings Institution.

Nordhaus, W. D. (1993). The cost of slowing climate change. *Energy Journal 12*(1), 37–65.

Nordhaus, W. D. (1994a). Expert opinion on climate change. *American Scientist 82*(1), 45–51.

Nordhaus, W. D. (1994b). *Managing the Global Commons*. Cambridge: MIT Press.

Nordhaus, W. D. and D. Popp (1997). What is the value of scientific knowledge? an application to global warming using the PRICE model. *Energy Journal 18*(1), 1–45.

Nordhaus, W. D. and G. W. Yohe (1983). Future paths of energy and carbon dioxide emissions. In N. R. Council (Ed.), *Changing Climate*, pp. 87–152. Washington: National Academy Press.

Pepper, W., J. Leggett, R. Swart, J. Wasson, J. Edmonds, and I. Mintzer (1992). Emission scenarios for the IPCC: An update; assumptions, methodology and results. Detailed documentation in support of Chapter A3 of *Climate Change 1992: The Supplementary Report to the IPCC Scientific Assessment*; eds. J.T. Houghton, B.A. Callander and S.K. Varney. Cambridge: Cambridge University Press, 1992.

Pizer, W. A. (1996). *Modeling Long-Term Policy Under Uncertainty*. Ph. D. thesis, Harvard University.

Pizer, W. A. (1997). Optimal choice of policy instrument and stringency under uncertainty: The case of climate change. Working Paper #97-17, Resources for the Future.

Repetto, R. and D. Austin (1997). The costs of climate protection: A guide for the perplexed. World Resources Institute, Washington.

Roberts, K. (1980). Interpersonal comparability and social choice theory. *Review of Economic Studies 47*, 421–439.

Roberts, M. J. and M. Spence (1976). Effluent charges and licenses under uncertainty. *Journal of Environmental Economics and Management 5*, 193–208.

Shlyakhter, A. I. and D. M. Kammen (1992). Estimating the range of uncertainty in future development from trends in physical constants and predictions of global change. working paper 92–06, Center for Science and International Affairs, John F. Kennedy School of Government, Harvard University.

Stavins, R. N. (1989). Harnessing market forces to protect the environment. *Environment 31*(1), 5–7,28–35.

Stavins, R. N. (1996). Correlated uncertainty and policy instrument choice. *Journal of Environmental Economics and Management 30*(2), 218–232.

U.S. Department of Commerce, Bureau of the Census (various issues). *Current Population Reports*. Series P-25, Population estimates and projections.

Weitzman, M. L. (1974). Prices vs. quantities. *Review of Economic Studies 41*(4), 477–491.

Weitzman, M. L. (1978). Optimal rewards for economic regulation. *American Economic Review 68*(4), 683–691.