Intergenerational Discounting
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Decreasing Discount Rate and Time Consistent Valuation

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$$SWF_t(c_t) = E\left[\int_t e^{-\rho(\tau-t)} u(c_\tau) d\tau \,\middle|\, c_t\right] \longrightarrow r_{t\to\tau}(c_t) = \rho - \frac{1}{\tau - t} \ln \frac{E\left[u'(c_\tau) \middle| c_t\right]}{u'(c_t)}$$

- Because the SWF has a constant RPPP, the pricing formula derived from it yields a time consistent valuation (TCV) method.
- By TCV, I mean that I can use backward induction to value projects.

An illustration

t=0 t=1 t=2

growth=2%

growth= x~N
Ex==2%, σ=20%
x observed at t=1

Preferences: ρ=0, η=2.

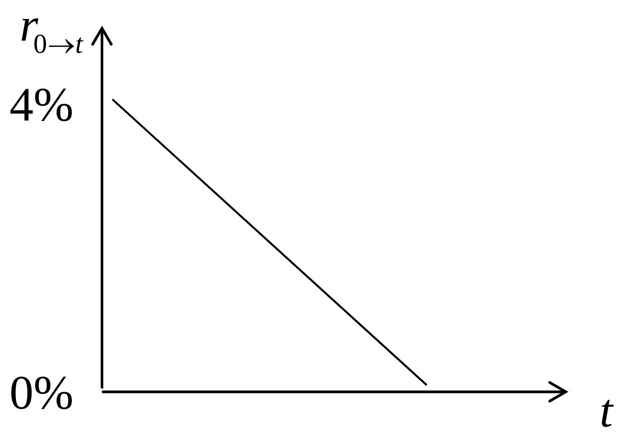
Use of pricing formula:

$$r_{t\to\tau}(c_t) = \rho - \frac{1}{\tau - t}\ln\frac{E\left[u'(c_\tau)\middle|c_t\right]}{u'(c_t)}$$

$$r_{0\to 1} = 4\%$$

$$r_{0\to 2} = 0\%$$

$$r_{1\to 2}(x) = 2x$$



Valuation at t=0 of $1 at t=2

- Direct method: $NPV_{0\to 2} = 1\times e^{-r_{0\to 2}} = 1.$

- Valuation by backward induction:
  - Value at t=1 in state x of $1 at t=2: $NPV_{1\to 2}(x) = 1\times e^{-r_{1\to 2}(x)} = e^{-2x}.$
  - Value at t=1 prior to observing x (Arrow-Lind): $Ee^{-2x} = e^{4\%}$
  - Present value of this at t=0: $e^{-r_{0\to 1}}e^{4\%} = \$1$