

# Emissions Targets and the Real Business Cycle

*Intensity Targets versus Caps or Taxes*

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## **Abstract**

For reducing greenhouse gas emissions, intensity targets are attracting interest as a flexible mechanism that would better allow for economic growth than emissions caps. For the same expected emissions, however, the economic responses to unexpected productivity shocks differ. Using a real business cycle model, we find that a cap dampens the effects of productivity shocks in the economy on all variables except for the shadow value of the emissions constraint. An emissions tax leads to the same expected outcomes as a cap but with greater volatility. Certainty-equivalent intensity targets maintain higher levels of labor, capital, and output than other policies, with lower expected costs and no more volatility than with no policy.

**Key Words:** emissions tax, cap-and-trade, intensity target, business cycle

**JEL Classification Numbers:** Q2, Q43, Q52, H2, E32

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## Contents

<b>Introduction</b> .....	<b>1</b>
<b>Deterministic Model</b> .....	<b>4</b>
Emissions Cap.....	9
Emissions Tax.....	9
Summary and Comparison.....	10
<b>Numerical Model with Stochastic Productivity Shocks</b> .....	<b>11</b>
Numerical Solution and Simulation Method .....	11
Results for the Deterministic Case.....	13
Results with Stochastic Productivity .....	16
Sensitivity Analysis: Productivity Growth .....	22
Sensitivity Analysis: Developing-Country Volatility and Risk Aversion .....	23
<b>Conclusion</b> .....	<b>25</b>
<b>References</b> .....	<b>28</b>

## **Emissions Targets and the Real Business Cycle: Intensity Targets versus Caps or Taxes**

Carolyn Fischer and Michael Springborn\*

### **Introduction**

Even though consensus has grown on the need for dramatic reductions in anthropogenic emissions of greenhouse gases (GHGs), which contribute to global climate change, considerable debate continues on which policies would best serve that goal. Many academics argue for carbon taxes as the most efficient domestic and global mechanism [1], but few governments are seriously considering a carbon tax as a primary policy for slowing GHG emissions. Many countries, including those of the European Union, have committed to or are proposing caps on GHG emissions. Other countries, including Canada, China, and India, have announced plans to pursue intensity targets, which are also the basis for some prominent proposals to include developing countries in a global framework [2]. These targets would index emissions allowance allocations to economic output, the idea being that a flexible mechanism would better allow for economic growth (e.g., [3]).

How much of a boon is this flexibility? From a policy design standpoint, one could equivalently assign caps that follow a growth path or assign declining intensity targets or carbon taxes to meet a cap. Therefore, a growth path is not an inherent feature of intensity targets, nor is a fixed emissions path a defining characteristic of emissions caps. Furthermore, when the ultimate goal is reducing overall emissions and stabilizing atmospheric concentrations, any policy would have to be ratcheted over time. However, in the face of uncertain economic growth, the policies offer different qualities. Holding expected allocations constant, intensity and emissions targets are likely to provoke different economic responses to unexpected productivity

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shocks. This paper explores the impacts of such economy-wide emissions regulations on the business cycle.

A long literature in environmental economics, beginning with Weitzman's seminal 1974 paper [4], has compared price and quantity instruments for regulating emissions. More recently, researchers have begun to also compare intensity-based instruments. Several of these latter works, including Newell and Pizer [5] and Quirion [6], follow the partial equilibrium approach of Weitzman. Others have taken a general equilibrium approach, focusing on the role of tax interactions [7,8], the role of multisector and international trade [9,10],<sup>1</sup> or both [11]. Given that uncertainty about economic growth and the macroeconomic transition effects of carbon policy is driving interest in indexed emissions targets, surprisingly few studies address these aspects directly. Much of the previous theoretical analysis of intensity targets and alternative instruments has focused on variance in abatement and compliance costs as the critical metric. This literature, including contributions by Kolstad [12], Quirion [6], Pizer [3], Jotzo and Pezzey [10], and Sue Wing and co-authors [13] is reviewed by Peterson [14] who observes that a common thread is the importance of the correlation between GDP and emissions in determining whether abatement cost uncertainty is lower under an intensity target. This paper takes a broader approach, characterizing the response in a set of macro-level variables to economy-wide emissions regulations via price, quantity, and intensity instruments, operating in the context of an uncertain business cycle.

In contrast to the preceding prices-versus-quantities literature, we use a dynamic stochastic general equilibrium (DSGE) model to compare the dynamic effects of these policy choices under productivity shocks. We specify a dynamic Robinson Crusoe economy, with choices over consumption, labor, capital investment, and a polluting intermediate good. We consider three policies for constraining emissions from the polluting factor: an emissions cap, an emissions tax, and an intensity target that sets a maximum emissions-output ratio. The economy is subject to uncertain shocks to overall productivity. We start with a simple approach to characterizing the response by solving analytically for the steady state following a single, permanent shock; this is our "SS" model. To implement the full real business cycle, "RBC" model, we specify a productivity factor that evolves according to a first-order autoregressive

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<sup>1</sup> Jensen and Rasmussen [30] consider using a general equilibrium model of the Danish economy and find that allocating emissions permits according to output dampens sectoral adjustment but imposes greater welfare costs than grandfathered permits.

process, which includes an i.i.d. random shock each period. To solve the RBC model numerically, we parameterize the model with plausible values from the macroeconomics literature.

Our analysis and an unpublished work by Heutel [15] are the first attempts of which we are aware to examine climate policy in an RBC framework – that is, in a DSGE model with uncertainty over future productivity. Heutel’s focus is on the optimal dynamic tax or quota policy, which adjusts each period in response to income and price effects. Heutel finds that price effect dominates, driving increased emissions levels and prices during economic expansions. Our approach differs in that we compare the performance of three instruments (tax, cap, and intensity target) in each set to achieve an exogenous and fixed level of expected emissions reduction. We conduct a cost-effectiveness analysis conditional on a given abatement target. Whereas we account for labor market responses to policy and productivity shifts and abstract from considering direct damages from emissions, Heutel sets aside labor fluctuations to concentrate on the interesting dynamics of the optimal *endogenous* policy.<sup>2</sup> We incorporate labor for two main reasons. First, since labor market impacts are often highlighted in environmental policy debates, labor is a critical outcome variable in its own right. Second, as we will further discuss in the results below, the dynamic impulse response of labor to a productivity shock in the full RBC model is, uniquely, not single-peaked. Our analytical results for variable levels in the SS model and expected variable levels in the RBC model tell the same story. Implementation of any of the three instruments leads all variable levels to fall, except under the intensity target policy where labor remains unchanged from the no policy setting. This particular consistency occurs because adjustments in response to the intensity target policy in consumption and production exactly offset within the labor optimality condition. In a comparison of levels under the three instruments, we find that deterministic outcomes under the cap and tax policies are identical and, aside from emissions, lower than those of the intensity target. Thus, given an identical emissions reduction constraint, total output is higher with the intensity target than with the cap or tax. This arises because additional production under the intensity target earns additional permits, increasing the returns to production. Consequently, the emissions intensity target must be set below the emissions intensity observed under the cap and tax policies.

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<sup>2</sup> Other modeling differences lie in the representation of abatement opportunities.

Considering volatility, the SS model reveals that the sensitivity of output to a particular productivity change is dampened by the cap. Similarly, when stochastic productivity shocks are incorporated in the RBC analysis, the cap policy leads to the lowest levels of volatility for each variable and therefore minimal variation in production and utility as well. The tax policy has the opposite effect. Optimal investment under the tax policy is much more sensitive to deviations in the productivity factor than under any other policy. Not surprisingly then, the volatility of each variable, and ultimately production and utility is greatest under the tax. Meanwhile, the sensitivity to shocks under the intensity target is unchanged from the no policy case.

### Deterministic Model

Although the issues at play involve economic growth and uncertainty, much of the intuition regarding the policy differences can first be derived from a simple, deterministic model without growth, by looking at the steady-state responses to different emissions policies and degrees of a permanent productivity change. Consider a simple Robinson Crusoe economy. Let  $C$  be the consumption good,  $K$  be capital,  $L$  be labor,  $l$  be leisure, and  $M$  be a polluting intermediate good. The representative agent gets utility  $u(C, l)$  from consumption and leisure. Total production  $Y$  is a function of capital, labor and polluting inputs  $F(K, M, L)$ , adjusted by a productivity factor  $\Theta$  with an expected value of 1, where  $Y = \Theta F(K, M, L)$ . Capital depreciates at rate  $\delta$  and is augmented with investment  $I$ , so  $K_{t+1} = I + (1 - \delta)K_t$ . Total output is allocated between consumption, investment and intermediate inputs ( $C + I + M \leq Y$ ), and time is allocated between leisure and labor ( $l = 1 - L$ ). Emissions are assumed to be proportional to the use of  $M$  and units of emissions are chosen such that the quantity of emissions is equal to  $M$ .<sup>3</sup> For the remainder of the analysis we will refer to the level of the intermediate polluting good and the level of emissions interchangeably. The emissions constraint requires that  $M \leq A_t(Y)$ , where  $A_t(\cdot)$  is the permit allocation, which may vary over time and with output.

We assume the specific functional forms of log utility and Cobb-Douglas constant returns to scale technology:

$$u = \ln C_t + \omega \ln(l_t)$$

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<sup>3</sup> We abstract from economic growth, and we also ignore the implications of improvements in abatement technology. We will relax this assumption when considering an extension incorporating growth in our sensitivity analysis.

$$F(K_t, M_t, L_t) = K_t^\alpha M_t^\gamma L_t^{1-\alpha-\gamma}$$

The Lagrangian for the constrained utility maximization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \left\{ \begin{array}{l} \left( \frac{1}{1+r} \right)^t [\ln C_t + \omega \ln(1-L_t)] \\ + \lambda_t [\Theta_t K_t^\alpha M_t^\gamma L_t^{1-\alpha-\gamma} - C_t - M_t - K_{t+1} + (1-\delta)K_t] \\ + \phi_t [A_t(\Theta_t K_t^\alpha M_t^\gamma L_t^{1-\alpha-\gamma}) - M_t] \end{array} \right\}, \quad (1)$$

where  $r$  represents the discount rate,  $\lambda_t$  is the shadow value of the national income identity, and  $\phi_t$  is the shadow value of the emissions constraint. Note that within this planning problem, any policy-generated revenues are conserved within the system as lump-sum transfers and wash out of the income constraint. A further simplification will be to let the effective shadow value of emissions be defined as  $\hat{\phi}_t \equiv \phi_t / \lambda_t$ , that is, the nominal shadow value normalized by the marginal value of income.

The first-order conditions produce six equations for the six variables in each time period:

$$C_t: \quad \lambda_t = \frac{1}{(1+r)^t C_t} \quad (2)$$

$$K_t: \quad \alpha \Theta_t K_t^{\alpha-1} M_t^\gamma L_t^{1-\alpha-\gamma} (1 + \hat{\phi}_t A_{t,Y}) = \frac{\lambda_{t-1}}{\lambda_t} + \delta - 1 \quad (3)$$

$$M_t: \quad \gamma \Theta_t K_t^\alpha M_t^{\gamma-1} L_t^{1-\alpha-\gamma} (1 + \hat{\phi}_t A_{t,Y}) - (1 + \hat{\phi}_t) = 0 \quad (4)$$

$$L_t: \quad \frac{\omega C_t}{1-L_t} = (1-\alpha-\gamma) \Theta_t K_t^\alpha M_t^\gamma L_t^{-\alpha-\gamma} (1 + \hat{\phi}_t A_{t,Y}) \quad (5)$$

$$\lambda_t: \quad \Theta_t K_t^\alpha M_t^\gamma L_t^{1-\alpha-\gamma} = K_{t+1} - (1-\delta)K_t + C_t + M_t \quad (6)$$

$$\phi_t: \quad M_t = A_t(Y_t) \quad (7)$$

where  $A_{t,Y}$  represents the derivative of  $A_t$  with respect to  $Y$ .

Further substituting and rearranging, we determine expressions for capital, emissions, and consumption as shares of output and labor in terms of the labor-leisure ratio



$$k_t \equiv \frac{K_t}{Y_t} = \frac{\alpha_t(1 + \hat{\phi}_t A_{t,Y})}{\frac{C_t(1+r)}{C_{t-1}} - (1-\delta)} \quad (8)$$

$$m_t \equiv \frac{M_t}{Y_t} = \frac{\gamma(1 + \hat{\phi}_t A_{t,Y})}{1 + \hat{\phi}_t} \quad (9)$$

$$c_t \equiv 1 - m_t - k_{t+1} \frac{Y_{t+1}}{Y_t} + (1-\delta)k_t \quad (10)$$

$$z_t \equiv \frac{L_t}{1-L_t} = \frac{(1-\alpha-\gamma)(1 + \hat{\phi}_t A_{t,Y})}{\omega c_t} \quad (11)$$

with output being determined in equilibrium with the policy constraint, Equation (7). Note that  $z=L/(1-L)$  is a monotonic, increasing, and convex function of  $L$ .

Alternatively, rearranging (9), we solve for the shadow value of emissions:

$$\hat{\phi}_t = \frac{\gamma / m_t - 1}{1 - A_{t,Y} \gamma / m_t} \quad (10)$$

Note that the shadow value will depend on the emissions rate and any adjustment in allowance allocations associated with each policy. If these are constant, as we will see they are by definition for the intensity target, then the shadow value is likewise constant over time.

Let us now abstract from the path dynamics and focus on the steady state, with  $C_{t+1} = C_t = C$ , etc. (steady-state levels will be denoted by the absence of a time index) and the shadow values growing at the rate of time preference. (The Lagrange multipliers  $\lambda_t$  and  $\phi_t$  are *present value* multipliers; when solving for steady-state values, the *current value* multipliers will be constant, as will the ratio of the present value multipliers,  $\hat{\phi}$ .) Let  $\hat{\beta} \equiv 1 / (r + \delta)$ . Steady-state equilibrium levels are given by

$$k = \hat{\beta} \alpha (1 + \hat{\phi} A_Y) \quad (11)$$

$$m = \gamma \frac{(1 + \hat{\phi} A_Y)}{1 + \hat{\phi}} \quad (12)$$

$$c = 1 - m - \delta k \quad (13)$$

$$z = \frac{(1 - \alpha - \gamma)}{\omega c} (1 + \hat{\phi} A_\gamma) \quad (14)$$

With these general results for the SS model, we now can use some simple comparative statics to evaluate the effects of specific emissions policy choices.

### **No Policy**

As an initial benchmark, consider the absence of an emissions policy. Without any regulation, we can drop the constraint on emissions, so  $\phi = 0$ . Simplifying the above equations, we have  $k = \hat{\beta}\alpha$ ,  $m = \gamma$ ,  $c = 1 - \gamma - \hat{\beta}\delta\alpha$ , and

$$z = \frac{1 - \alpha - \gamma}{\omega(1 - \gamma - \hat{\beta}\delta\alpha)} \quad \text{or} \quad L = \frac{1 - \alpha - \gamma}{1 - \alpha - \gamma + \omega(1 - \gamma - \hat{\beta}\delta\alpha)}. \text{ Solving for production, then, we get}$$

$$Y = \Theta^{\frac{1}{1-\alpha-\gamma}} (\hat{\beta}\alpha)^{\frac{\alpha}{1-\alpha-\gamma}} \gamma^{\frac{\gamma}{1-\alpha-\gamma}} L, \text{ from which the percentage response to a change in the productivity factor is } \frac{d\{Y\}/Y}{d\Theta/\Theta} = \frac{1}{1-\alpha-\gamma}; \text{ that is, the elasticity of output is greater than one.}$$

Note that in the absence of an emissions policy, the steady-state GDP shares of consumption, capital, emissions are invariant to the productivity variable, as is the share of time allocated to labor versus leisure. Therefore, with the exception of labor, their levels will all vary in a positive manner with permanent productivity changes, proportional to  $\Theta^{\frac{1}{1-\alpha-\gamma}}$ . Meanwhile, total labor supply in the steady state is uniquely indifferent to the productivity parameter, since the effect of increased marginal productivity of labor is exactly offset by the falling marginal value of income,  $\lambda$  (see Equations (2) and (5)).<sup>4</sup>

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<sup>4</sup> These results, and the similar ones that follow, emerge from the chosen functional forms of utility and output; with Cobb-Douglas functions, a constant share of income (or input expenditures) is devoted to each good (factor). Since a change in productivity does not change the relative value of a dollar of consumption and leisure (or capital, labor and emissions), it does not change these shares.

### Intensity Target

Consider next an intensity target of  $\mu$  per unit of output, so  $A(Y) = \mu Y$ . We assume a binding target, which implies  $m = \mu < \gamma$ . Furthermore, in equilibrium,  $A=M$ .

Simplifying the steady-state equation for the emissions share, we get  $m = \gamma \frac{(1 + \hat{\phi}\mu)}{1 + \hat{\phi}} = \mu$ ,

from which we derive the effective shadow value of the emissions constraint:

$$\hat{\phi} = \frac{\gamma - \mu}{\mu(1 - \gamma)} \quad (15)$$

which we notice is independent of the productivity factor.

Substituting into the remaining steady-state equations, we get  $k = \hat{\beta}\alpha \frac{(1 - \mu)}{(1 - \gamma)}$ ,  
 $c = 1 - \mu - \delta\hat{\beta}\alpha \frac{(1 - \mu)}{(1 - \gamma)} = (1 - \gamma - \delta\hat{\beta}\alpha) \frac{(1 - \mu)}{(1 - \gamma)}$ , and  $z = \frac{(1 - \alpha - \gamma)}{\omega c} \left( \frac{1 - \mu}{1 - \gamma} \right) = \frac{1 - \alpha - \gamma}{\omega(1 - \gamma - \delta\hat{\beta}\alpha)}$ .

Thus, we observe again that steady-state consumption, capital, and emissions shares of GDP are invariant to permanent productivity changes (the latter by definition). Their levels are then all procyclical, in the sense of responding in the same direction as the change in the productivity factor. Labor supply is also invariant, both to productivity changes and to the policy stringency, since the effects filter through the change in the marginal productivity of labor (to produce final output and additional permits) and the marginal value of income, which offset. Consequently, we observe the same sensitivity of steady-state output to productivity factor changes as with no policy:  $\frac{d\{Y\}/Y}{d\Theta/\Theta} = \frac{1}{1 - \alpha - \gamma}$ .

Notably, capital as a share of output is *increasing* with the stringency of the emissions constraint, which will stand in contrast to the other policies. The reason is that additional investment and production also produce additional emissions allocations. The rate of consumption also increases with policy stringency, since the capital buildup does not absorb all of the decrease in the polluting intermediate good:  $\frac{dc}{-d\mu} = \frac{1 - \gamma - \delta\hat{\beta}\alpha}{(1 - \gamma)} > 0$ .

### Emissions Cap

With an emissions cap,  $M$  is fixed. In this case,  $A(Y) = \bar{M}$ , so  $A_Y = 0$ . The key steady-state conditions then reduce to  $k = \hat{\beta}\alpha$ ,  $m = \frac{\gamma}{1 + \hat{\phi}}$ ,  $c = 1 - \frac{\gamma}{1 + \hat{\phi}} - \delta\hat{\beta}\alpha$ ,  $z = \frac{1 - \alpha - \gamma}{\omega c}$ , and  $m = \bar{M}/Y$ . We see that the capital share is constant and identical to the no-policy case, also implying it is strictly lower than that under the intensity target. Labor supply also carries the same relationship to the consumption rate as in the no-policy case.

On the other hand, we also see that the effective shadow price of emissions is no longer independent of the productivity variable, but rather procyclical:

$$\hat{\phi} = \frac{\gamma\Theta F}{\bar{M}} - 1 \quad (16)$$

In other words, an increase in productivity, which would otherwise increase emissions, raises the price of emissions permits to maintain the cap. As a result, consumption as a share of GDP reacts in a procyclical manner, since the cap prevents additional output from being used as more of the intermediate good:  $c = 1 - \bar{M}/Y - \delta\hat{\beta}\alpha$ .

Meanwhile, labor supply then becomes countercyclical, to compensate for the inability to expand emissions:  $L^* = \frac{1 - \alpha - \gamma}{1 - \alpha - \gamma + \omega(1 - \bar{M}/Y - \delta\hat{\beta}\alpha)}$ . The increase in the marginal productivity of labor from a positive productivity change, dampened under the cap constraint, is no longer strong enough to offset the decrease in the marginal value of income, so labor falls under the cap.

Substituting these values and solving for production, we get  $Y = \Theta^{\frac{1}{1-\alpha}} (\hat{\beta}\alpha)^{\frac{\alpha}{1-\alpha}} \bar{M}^{\frac{\gamma}{1-\alpha}} L^{\frac{1-\alpha-\gamma}{1-\alpha}}$ .

Overall, steady-state production under the cap is less sensitive to a given permanent productivity shock than in the preceding scenarios, both since labor supply is countercyclical and since  $\frac{d\{\Theta^{\frac{1}{1-\alpha}}\}}{d\Theta} < \frac{d\{\Theta^{\frac{1}{1-\alpha-\gamma}}\}}{d\Theta}$ .

### Emissions Tax

Suppose that instead of emissions trading, we have a fixed price, as with a carbon tax, with the revenues rebated in lump-sum fashion to the representative consumer. Let this price be fixed, so  $\hat{\phi} = \tau$  (i.e., the tax is fixed in terms of the marginal value of income). The new problem is similar to that of the emissions cap, in which the permits are allocated lump-sum, with

$A_M = A_K = A_L = 0$ , and the equilibrium value of that lump-sum transfer is  $\hat{\phi}A$ . But in this case, the equilibrium value of the lump-sum allocation equals the emissions tax revenues; that is,  $\tau A = \tau M$ .

The key steady-state conditions then reduce to  $k = \hat{\beta}\alpha$ ,  $m = \frac{\gamma}{1+\tau}$ ,  $c = 1 - \frac{\gamma}{1+\tau} - \delta\hat{\beta}\alpha$ , and  $z = \frac{1-\alpha-\gamma}{\omega c}$ . With the emissions price fixed, labor supply and the GDP shares of consumption, capital, and emissions are all invariant to productivity changes, as in the no-policy and intensity target scenarios.

### **Summary and Comparison**

A summary of analytical results is presented in Table 1 so that the policy effects can be seen side-by-side. First, it is useful to compare outcomes under certainty, with  $\Theta = 1$ . In this case, we notice that the emissions tax achieving the same emissions as the cap will replicate all the same prices and quantities as the cap. The intensity target, on the other hand, has important differences: the capital share is higher than with the other policies or no policy (since  $(1-\mu)/(1-\gamma) > 1$ ), and the labor allocation is also higher (since  $\gamma > m$  when emissions are constrained), remaining at no-policy levels. Given the same total emissions target, then, with the other factors of production being larger, it must be that total output is higher with the intensity target than with the cap or tax. As a consequence, the emissions intensity target must be lower than the emissions rate under the other policies to achieve the same level of total emissions.<sup>5</sup> We also observe that the consumption rate is higher with the intensity target than with no policy, but it is unclear whether it is higher than with the cap or tax policies (since  $\gamma > m$  but  $\mu < \gamma$ ).

Other differences arise in response to innovations in the productivity parameter. Under the emissions cap, obviously, emissions are fixed, and output is less responsive to a change than the other policies because of a countercyclical effect on labor supply and emissions intensity.

An important caveat in thinking about the effect of productivity shocks is that the steady-state analysis considers a permanent productivity shock, as opposed to transitory ones. As we will see in the next section, while much of the intuition from these fundamental comparisons remains valid, some of the particular results do not hold along a path with stochastic

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<sup>5</sup> These results echo those in static models, such as Fischer [31] and Fischer and Fox [11].

productivity. For example, in the SS model, a permanent change in productivity has the same effect on output, in percentage terms, in all but the emissions cap policy. The other steady-state variables remain constant as a share of output; their levels are then procyclical and respond to productivity changes in the same percentage terms as output. When shocks are transitory, however, their cumulative effect is also manifested in the capital stock responses, which in turn influence the reactions of the other variables. We now turn to a numerical version of the model, incorporating a stochastic process into the overall productivity factor.

**Table 1. Comparison of Analytical Results**

	<i>No Policy</i>	<i>Intensity Target</i>	<i>Emissions Cap</i>	<i>Emissions Tax</i>
$m$	$\gamma$	$\mu$	$\frac{\bar{M}}{Y}$	$\frac{\gamma}{1+\tau} = \frac{\bar{M}}{Y}$
	With a binding target, $\mu < \gamma$ and $\bar{M}/Y < \gamma$ . Since $k^{IT} > k^{tax} = k^{cap}$ and $L^{IT} > L^{tax} = L^{cap}$ then, for equivalent emissions, Intensity Target must use less emissions per unit of output: $\mu = m^{IT} < m^{tax} = m^{cap} < \gamma$ .			
$k$	$\hat{\beta}\alpha$	$\hat{\beta}\alpha \frac{(1-\mu)}{(1-\gamma)}$	$\hat{\beta}\alpha$	$\hat{\beta}\alpha$
	Emissions Cap and Tax do not affect the capital share, but Intensity Target increases it.			
$L/(1-L)$	$\frac{1-\alpha-\gamma}{\omega(1-\gamma-\hat{\beta}\delta\alpha)}$	$\frac{1-\alpha-\gamma}{\omega(1-\gamma-\delta\hat{\beta}\alpha)}$	$\frac{1-\alpha-\gamma}{\omega(1-m-\delta\hat{\beta}\alpha)}$	$\frac{1-\alpha-\gamma}{\omega(1-m-\delta\hat{\beta}\alpha)}$
	Intensity Target leaves labor supply unchanged from No Policy, but Cap and Tax reduce it equally ( $L^{IT} = L^{NP} > L^{tax} = L^{cap}$ ).			
$c$	$1-\gamma-\hat{\beta}\delta\alpha$	$(1-\gamma-\delta\hat{\beta}\alpha) \frac{(1-\mu)}{(1-\gamma)}$	$1-m-\delta\hat{\beta}\alpha$	$1-m-\delta\hat{\beta}\alpha$
	All policies raise consumption shares above No Policy, but unclear if Intensity Target raises it more.			
$\frac{d\{Y\}/Y}{d\Theta/\Theta}$	$\frac{1}{1-\alpha-\gamma}$	$\frac{1}{1-\alpha-\gamma}$	$\frac{\Theta^{\frac{1}{1-\alpha}}}{1-\alpha-(1-\alpha-\gamma)\left(\frac{\partial L/L}{\partial Y/Y}\right)}$	$\frac{1}{1-\alpha-\gamma}$
	Only the Cap changes the responsiveness of output to a permanent productivity change.			

## Numerical Model with Stochastic Productivity Shocks

### Numerical Solution and Simulation Method

Because of the nonlinear form of the first-order conditions, specifically the intertemporal Euler and labor equations, we use a numerical method to calculate a first-order approximation to

the equilibrium conditions. To begin, we parameterize the model using standard calculations from the real business cycle (RBC) literature and our own analyses (see Table 2). For production parameters we start with King, Plosser, and Rebelo's [16] (hereafter KPR) calculation of mean annual share of GNP to labor (verified with current data). We decompose the total capital share of output in our model into energy inputs,  $M$  (to represent the intermediate polluting good), and all other nonenergy capital,  $K$ . The baseline share of energy to output is set equal to the mean ratio of annual energy expenditures to GDP. Finally, the share of nonenergy capital to output is set equal to one minus the labor and energy shares. The utility parameter, discount factor, and depreciation rates all reflect standard RBC model assumptions.

The productivity factor is given by  $\Theta_t = \exp(z_t)$ , where  $z_t$  evolves according to a stationary, first-order autoregressive process,

$$z_t = \eta z_{t-1} + \varepsilon_t \quad (17)$$

and where  $\varepsilon_t$  is an i.i.d. normal random variable, drawn once each period, with a mean of zero and standard deviation  $\sigma$ . Parameters of the productivity factor process approximately follow Prescott [17] and much of the subsequent macroeconomic literature.

Given these parameter values, we linearize the efficiency conditions by taking a first-order Taylor approximation around the steady-state levels of our variables. Using a standard eigenvalue decomposition method, we then solve for decision functions that take state variables ( $K$  and  $\Theta$ ) at the beginning of the period and return optimal levels of  $C$ ,  $M$ ,  $L$ , and capital investment.<sup>6</sup>

To characterize the long-run central tendency and volatility of variables for each policy-scenario combination, we simulate 1,000 realizations, each 100 years in length. In each simulation, the initial capital stock is set to its steady-state level for the particular policy setting, and the initial productivity factor is set to one. However, for our preferred welfare comparisons between policies we modify the assessment in two ways. First, since we are concerned with the *transition* between a policy-free starting point and the new policy environment, we run each simulation from an initial capital stock level as given by the unconstrained steady state. Second, we examine relative utility across a range of shorter time horizons and discount rates. In all

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<sup>6</sup> Note that this is a constrained optimum subject to the relaxation of linearizing the equilibrium conditions, and hence the decision rules, around the steady state.

simulations, the economy is subjected to a new shock each period, after which optimal decisions are made over the choice variables.

**Table 2. Summary of Simulation Parameter Values and Sources**

	<i>Parameter</i>	<i>Level</i>	<i>Source</i>
$1 - \alpha - \gamma$	Share of output going to $L$	0.58	Mean annual ratio of total employee compensation to GNP (KPR for 1948–1985, same result calculated for 1970–2001 using data from NIPA [18])
$\gamma$	Share of output going to $M$	0.09	Mean ratio of total energy expenditures to GDP (1970–2001), data from EIA [19]
	Conventional share of output going to total capital (in models without $M$ )	0.42	Calculated as one minus the share to $L$
$\alpha$	Share of output going to $K$	0.33	Conventional share to total capital less share to energy capital
$\omega$	Utility parameter	0.2	From KPR, chosen indirectly by specifying steady-state hours worked (0.20) based on the average fraction of hours devoted to market work in 1948–1985
$\beta$	Discount factor	0.95	From KPR, consistent with the observed average real return to equity, 1948–1981
$\delta$	Depreciation rate	0.096	Calculated assuming an investment-output ratio of 25% and a capital stock-output ratio of 2.6
$\eta$	Autocorrelation parameter	0.81	Annual analog of the quarterly rate of 0.95 [17]
$\sigma$	Standard deviation of random parameter $\varepsilon_t$	0.014	Annual analog of the quarterly level of 0.007 [17]

As a robustness check, we also modify the model with a labor-enhancing productivity factor and perform the same analysis in the context of exogenous growth in the baseline. The results, viewing the variables as shares of output along the growth path, are essentially identical to those in the no-growth case, so we concentrate our reporting on the latter case.

### **Results for the Deterministic Case**

We begin by numerically solving for steady-state values in the deterministic case ( $\Theta = 1$ ), which reproduces the analytical approach above with no shocks. After calculating the benchmark case of No Policy, we consider the three policy scenarios – Intensity Target, Emissions Cap, and Emissions Tax – and solve for the level of stringency such that all meet the same emissions reductions from the benchmark case in the deterministic steady-state. We choose a reduction target of 20 percent, stylized on the well-known European Union target of a 20 percent reduction



(from 1990 levels) by 2020, and the similar 20 percent reduction targets (from 2005 by 2021) in the recent Waxman-Markey and Kerry-Lieberman legislative proposals in the 111<sup>th</sup> Congress in the United States.

The results are reported here and in Tables 3 and 4. The policy simulations produce GDP reductions of 2.1 to 3.3 percent, and consumption reductions of 0.3 to 1.1 percent. To put these magnitudes in perspective, they are somewhat larger than those found by static computable general equilibrium (CGE) models for comparable targets (e.g. [20, 21]). In part, CGE models, having more detailed representation of energy sources and industries, allow more substitution opportunities that may lower overall costs.

In the absence of uncertainty, there is no difference between the cap and the tax, as one would expect. The intensity target, on the other hand, requires a more stringent intensity level than the other policies, and it also results in a 17 percent higher permit price. On the other hand, consistent with the analytical results, it generates no decrease in employment and increases capital as a share of output; as a result, the GDP decline is a third smaller than with the other policies. Although the consumption share does not rise as much as with the cap or tax, total consumption falls only 0.3 percent from no policy, as compared to 1.1 percent with the cap or tax.

To characterize the welfare costs of achieving emissions reductions we calculate, from a no policy baseline, the percentage reduction in consumption needed to replicate utility levels under each policy instrument (holding labor fixed)—a standard approach in the RBC literature (e.g. [22, 23]). For the steady-state case, this “welfare cost” metric is presented in the final column of Table 4. Comparative welfare results demonstrate that focusing solely on steady-state analysis can be misleading. When we consider a single period at the new steady state under each policy, the welfare costs of complying with the emissions reduction goal with the intensity target are less than those with the cap or the tax policy.<sup>7</sup> However, in our preferred welfare comparison, where we consider the transition dynamics (from a no policy starting point) to that new steady state, we find that this ordering does not hold. Since the new steady-state capital level for the cap and tax is lower than for the intensity target under the cap and the tax there is a longer period of elevated consumption combined with relaxed investment and labor along the

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<sup>7</sup> Utility levels exclude damages from emissions, but since emissions are equal across the policy scenarios, that doesn't change the relative evaluation.

transition to the new steady state. From a present value of utility (PVU) perspective, the cap and tax then dominate the intensity target.

In welfare cost terms, under a mid-run horizon (30 years) the percentage decrement in annual consumption from the no policy case to replicate the PVU under each policy (accounting for the transition) is 0.09% for the tax, 0.10% for the cap and 0.25% for the intensity target. This welfare cost (and PVU) dominance of the cap and the tax over the intensity target holds for all time horizons considered (1-100 years) and for any discount rate between 1-25%. This outcome is not necessarily intuitive since the single-period, steady-state utility is higher under the intensity target than the two other constraints. One might expect that given a low enough discount rate the intensity target would eventually dominate other policies under a PVU analysis. However, this is not the case, since the optimal policy function also adjusts as the discount rate changes in such a way as to decrease the difference in steady-state, single-period utility levels.

There is one minor difference in the transition properties of the cap and tax. Once the cap is imposed, the new steady-state level for  $M$  is achieved immediately. The tax, which is set to achieve the same level for  $M$  at the deterministic steady state, results in excess transition emissions slightly above the cap level, while the capital stock is above the steady state. However these excess emissions under the tax start at a maximum of 1% of emissions under the cap and the deviation attenuates from there. In a PVU analysis, outcomes under the cap and tax are virtually the same—a very slight advantage for the tax disappears when we value excess transition emissions above the cap at a marginal damage cost equal to the tax rate. While excess transition emissions also occur under the intensity target, they are quite small (a maximum of 0.1% above emissions under the cap) and valuing them at a marginal damage cost equal to the tax rate does not qualitatively change the intensity target's PVU-subordinance to the cap and tax.

Recall from the analytical SS model results (Table 1) that whether the consumption share under the intensity target was greater than for the cap and tax policies was ambiguous. Given our model parameters, we see that the intensity target consumption share is lower, since the proportional increase in production, relative to the cap or tax, outweighs the same in consumption.

**Table 3. Deterministic Steady-State Consumption, Capital, and Emissions Shares**

	$c$	$L/Y$	$k$	$m$
No Policy	0.697	0.923	2.22	0.0900
Intensity Target	0.709	0.943	2.26	0.0735
Cap	0.712	0.951	2.22	0.0745
Tax	0.712	0.951	2.22	0.0745

**Table 4. Steady-State Levels in the Deterministic Case, with Percentage Changes Relative to No Policy**

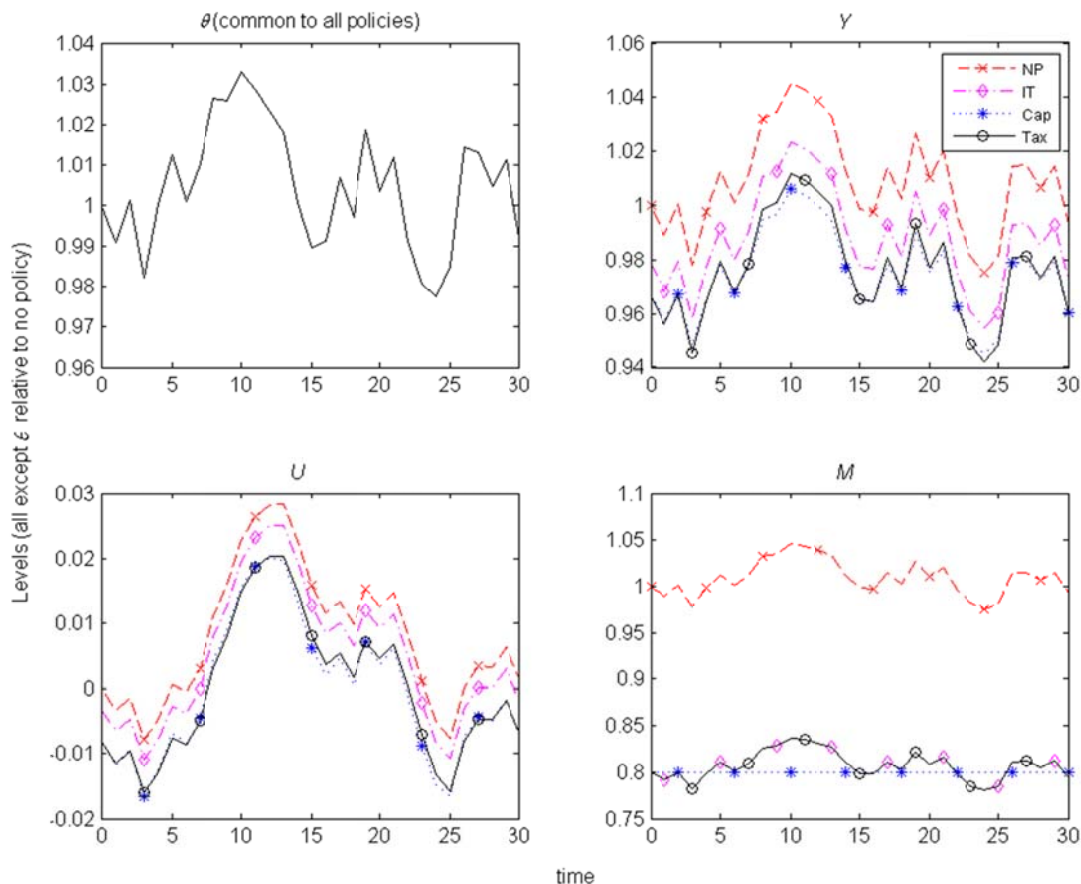
Policy	Variable						<i>welfare cost</i>
	<i>C</i>	<i>L</i>	<i>K</i>	<i>M</i>	<i>Y</i>	<i>U</i>	
No Policy (NP)	0.609	0.806	1.94	0.079	0.87	-0.825	
<i>change from NP</i>	<i>0%</i>	<i>0%</i>	<i>0%</i>	<i>0%</i>	<i>0%</i>		<i>0</i>
Intensity Targ.	0.607	0.806	1.93	0.063	0.86	-0.828	
<i>change from NP</i>	<i>-0.32%</i>	<i>0.00%</i>	<i>-0.3%</i>	<i>-20.0%</i>	<i>-2.1%</i>		<i>0.32%</i>
Cap	0.602	0.803	1.88	0.063	0.84	-0.833	
<i>change from NP</i>	<i>-1.1%</i>	<i>-0.43%</i>	<i>-3.3%</i>	<i>-20.0%</i>	<i>-3.3%</i>		<i>0.82%</i>
Tax	0.602	0.803	1.88	0.063	0.84	-0.833	
<i>change from NP</i>	<i>-1.1%</i>	<i>-0.43%</i>	<i>-3.3%</i>	<i>-20.0%</i>	<i>-3.3%</i>		<i>0.82%</i>

### Results with Stochastic Productivity

Next, to evaluate the effects of uncertainty and volatility in the productivity parameter, we solve for the optimal linearized decision functions, presented in Table 5. These functions map the state variables ( $K$  and  $\Theta$ ) into investment, consumption, and labor choices. The decision rules are calculated in terms of proportional deviation from steady state (PDSS).<sup>8</sup> For example, the PDSS of the capital stock in period  $t+1$  under no policy is given by  $K'_{t+1} = 0.8594 * K'_t + 0.3372 * \theta'_t$ . The decision functions were used to conduct 1,000 stochastic 100-year simulations for each emission policy. In Figure 1 we present example output under the four policies for a 30-period segment of one simulation. The stochastic productivity factor path is shown in the first panel, and the remaining panels depict the response in production, polluting input, and utility.

<sup>8</sup> For example, if the steady-state level of capital is given by  $K^s$ , then  $K'_t = (K'_t - K^s)/K^s$ .

**Figure 1. Variable Outcomes under No Policy (NP), Intensity Target (IT), Cap, and Tax, Given Path of Productivity Factor  $\theta$ .**



*Note:* Levels are normalized by the NP steady-state level for Y, M, and U.

Our findings on the long-run central tendencies and volatility under each policy are summarized in two key statistics for each variable, reported in Table 6. First, we present the mean of the simulation means (i.e., we take the mean of each simulation time path and then take the mean over all 1,000 simulations). Comparison with Table 4 shows that the variable central tendencies are virtually identical to the deterministic steady-state levels, as expected. Second, we report the mean simulation standard deviation (in percentage terms) as a measure of expected volatility for any given realization of productivity shocks (i.e., for any time path).

**Table 5. Decision Functions for Choice Variables in Terms of Proportional Deviation from Steady State**

No Policy	$K'_{t-1} =$	0.8594	$*K'_t +$	0.3372	$*\theta'_t$
Intensity Targ		0.8594		0.3372	
Cap		0.8534		0.3198	
Tax		0.8588		0.3438	
No Policy	$C'_{t-1} =$	0.5762	$*K'_t +$	0.4755	$*\theta'_t$
Intensity Targ		0.5762		0.4755	
Cap		0.5762		0.4981	
Tax		0.5734		0.4756	
No Policy	$L'_{t-1} =$	-0.0472	$*K'_t +$	0.1378	$*\theta'_t$
Intensity Targ		-0.0472		0.1378	
Cap		-0.0548		0.1118	
Tax		-0.0475		0.1406	

**Table 6. Simulation Central Tendencies and Variability**

Policy	Statistic	Variable							<i>welfare cost</i>
		<i>C</i>	<i>L</i>	<i>K</i>	<i>M</i>	$\theta$	<i>Y</i>	$U_P - U_{NP}^{***}$	
No Policy	msm*	0.609	0.806	1.94	0.079	1	0.87	0	0
	msstd**	2.50%	0.27%	3.09%	3.32%	2.25%	3.32%	0.0242	
Intensity Target	msm	0.607	0.806	1.93	0.063	Same	0.86	-0.00322	0.32%
	msstd	2.50%	0.27%	3.09%	3.32%		3.32%	0.0242	
Cap	msm	0.602	0.803	1.88	0.063	Same	0.84	-0.00810	0.81%
	msstd	2.43%	0.22%	2.86%	0.00%		2.94%	0.0239	
Tax	msm	0.602	0.803	1.88	0.063	Same	0.84	-0.00813	0.81%
	msstd	2.52%	0.27%	3.14%	3.34%		3.40%	0.0244	

\*Mean of simulation means (msm): the mean over 1,000 simulations of the 100-year simulation mean.

\*\*Mean of simulation standard deviations (msstd): the mean over 1,000 simulations of the simulation standard deviations, in percentage terms (except for last column)

\*\*\*( $U_{NP} - U_P$ ) is the deviation from the utility under no policy, msstd's are levels for  $U_p$

The expected levels in the RBC model tell the same story as the analytical results for variable levels in the SS model and the deterministic case. Implementation of any of the three instruments leads all variable levels to fall except under the intensity target policy, where labor remains unchanged from the no policy setting. This particular consistency occurs because adjustments in response to the intensity target policy in consumption and investment exactly offset within the labor optimality condition. As expected from the deterministic numerical analysis, we find that expected levels under the cap and tax policies are identical and lower than those of the intensity target. Thus, given an identical emissions reduction constraint, total output is higher with the intensity target than with the cap or tax. Consequently, we again see that the emissions intensity target must be set below the emissions rate observed under the cap and tax policies.

Recall that utility at the deterministic steady state is the same under a cap or tax, and lower than for utility under an intensity target (see Table 4). These results are essentially maintained in the dynamic setting with stochastic productivity shocks (see Table 6). Even though the average sacrifice in utility for a period (the mean of simulation means) from adopting the cap policy (lowest volatility) is slightly smaller than for the tax policy (highest volatility), we are not able to reject that the means are equal using the nonparametric Wilcoxon signed rank test ( $p = 0.20$ ).<sup>9</sup>

Since optimal capital stock levels are lower under emissions constraints, there is a period of transition from the initial no policy state. As in the deterministic case, utility under the cap and tax policies is greater over this period of transition because investment levels are deflated to a larger extent than under the intensity target. The effect of this investment “holiday” is strong enough that the intensity target performs the worst from an expected PVU perspective. Taking this investment holiday into account, the annual welfare cost of each policy (in consumption-reduction PVU-equivalence terms for a mid-run 30-year horizon as above) is essentially

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<sup>9</sup> Recall that we do not account for the damages from emissions directly in the utility function under the assumption that average emissions under each policy will be approximately equal and that some intertemporal variation is not consequential given our focus on a stock pollutant. To challenge this assumption we look at potential differences in average emissions rate for each simulation between the three policies. For each policy pair (intensity target-tax, intensity target-cap, tax-cap) we calculate the difference in the average emissions rate for each of 1,000 simulations. Since these differences will naturally center around zero, we concentrate on the *variance* of this difference across all simulations expressed as a proportion of the cap to normalize the units. We find that this variance for the intensity target-tax comparison is essentially zero (less than  $10^{-9}$ ) and for the other two pairings is also quite small (approximately  $10^{-5}$ ).

unchanged from the dynamic deterministic setting: 0.09% for the tax, 0.10% for the cap and 0.25% for the intensity target. This welfare cost (or expected PVU) dominance of the cap and the tax policies again holds across the wide range of time horizons (1-100 years) and discount rates (1%-25%) considered. Consistent with the observation that there is greater flexibility under the tax to take advantage of elevated capital levels over the transition period, we find that the expected PVU under the tax is statistically significantly greater than for the cap ( $p < 0.001$ ) for any time horizon greater than eight periods. However, as in the deterministic case, this small PVU advantage of the tax policy over the cap policy no longer holds when the marginal damages of the transition emissions in excess of the cap are valued at the tax rate.

Considering volatility, in general, in both the single permanent shock (SS) and repeated transitory shock (RBC) settings, the variables of interest (emissions, consumption, capital, and labor) are procyclical under each policy; that is, they move in the same direction as the level of the productivity shock. The exceptions are emissions under the cap, which are fixed, and labor. Labor is invariant to shocks in the SS setting, except under a cap, in which case it is countercyclical. In perhaps the starkest divergence between the two settings, the RBC response of labor is procyclical for all policies. This result is explored further below.

Otherwise, the SS results are qualitatively maintained in the RBC setting. In the SS model the sensitivity of output to a particular productivity shock is dampened by the cap. Similarly, from the RBC analysis, Table 6 reveals that the emissions cap, which by definition has the least volatility in emissions, also has the least volatility in all the other variables, including average standard deviations that are 11 to 14 percent less for output, 18 percent less for labor supply, 7 to 9 percent less for capital, and 3 percent less for consumption. When productivity is high, the shadow value of the fixed emissions constraint becomes greater, putting the brakes on the economy, and when productivity is low, the effective permit price drops, easing up on the economy.

The tax policy has the opposite effect in the RBC setting. Optimal investment under the tax policy is more sensitive to productivity factor deviation than under any other policy. This is evident in the optimal linear decision functions for choice variables from Table 5. The coefficient representing the effect of deviation in the productivity factor on next period's capital is largest for the tax. This sensitivity to stochastic productivity is born out dynamically in simulations: the volatility of each variable, and ultimately production and utility, is greatest under the tax (see Table 6). The intensity target, on the other hand, does not change the sensitivity of the economy to productivity shocks: the decision functions for no policy and intensity target are identical and lead to a level of volatility that lies between the cap and tax.

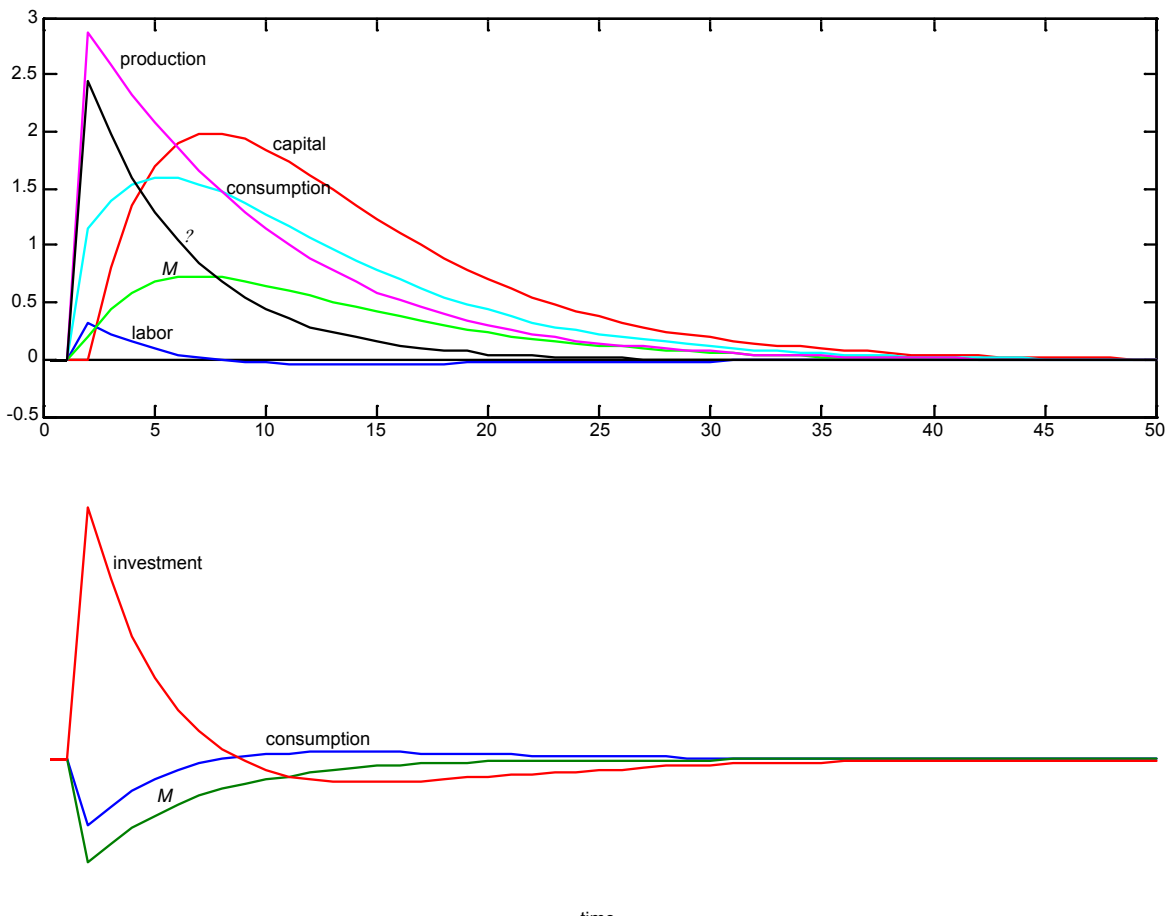
A salient feature of generalizing the SS model to a setting of repeated transitory shocks is that the optimal decision in a time period is taken with respect to the current capital stock as well as the current level of the productivity shock. The capital stock is essentially continually divergent from the steady state and reflects the cumulative response to the series of shock levels encountered up to the present period. Since investment is procyclical, a positive deviation from the steady state roughly reflects a history that, on balance, featured positive productivity shock levels.

Given that background, we now return to the question of why the SS model shows no response or a countercyclical response to a productivity shock while the RBC model results in a procyclical labor response. The RBC decision function for all policies shows that the optimal labor choice is increasing in positive deviations in the *current* productivity level (procyclical) but is *decreasing* in capital stock deviations; that is, the residual effect of *past* productivity levels (see the last decision function in Table 5). (The latter effect occurs because elevated capital stocks invoke elevated consumption, which reduces the marginal value of income and hence the marginal benefit of labor.) However, even once we consider the indirect effect (through capital) of a one-time shock on labor in the RBC model, the immediate effect is still procyclical. In Figure 2 we depict the RBC model response to a one-time, transitory productivity shock (see the path of  $\Theta$ ). In the top panel, while labor clearly follows the direction of the shock, note that the long-term response eventually becomes negative as the procyclical direct effect of the deviation in productivity decays faster than the negative indirect effect of the capital stock.

In the bottom panel of Figure 2 we see what drives the labor effect through an examination of choice variables as shares of output. Recall from the SS model (see Equation (13)) that labor is either countercyclical because the consumption share  $c$  is procyclical (cap policy) or invariant to the shock because  $c$  is constant (all other policies). In contrast, under the long-horizon, transitory shock setting of the RBC model, the consumption share falls while the investment share rises in response to a positive shock. The bottom panel of Figure 1 shows this relationship for the intensity target policy, though a similar relationship holds for each policy we consider. When shocks are transitory, a positive shock leads to a greater relative response in investment versus consumption (though consumption is elevated). In the tension between a marginal productivity of labor increase and marginal value of income decrease that determines the labor response to shocks, it is the former that dominates in the RBC model, leading to a procyclical response, at least in the short run.



**Figure 2. Example Response to One-Standard-Deviation Productivity Shock under Intensity Target Policy.**



Top panel: impulse responses in percentage deviation from steady state. Bottom panel: percentage deviation of output shares from steady state.

**Sensitivity Analysis: Productivity Growth**

Recall from the baseline results discussed above that even though the intensity target is preferred to the cap and tax in terms of the steady-state utility level, when we consider the PVU over various time horizons, starting from the steady state under no policy constraint, the tax is preferred; it is closely followed by the cap. Thus transitions toward a new steady state during which investment is diminished can be important. Although our baseline model abstracts from productivity growth, it is reasonable to suppose that such growth might influence the nature of the transition and therefore affect how instruments perform. To explore this possibility, we

incorporate labor-augmenting, technological progress into the model:

$$F(K_t, M_t, L_t) = K_t^\alpha M_t^\gamma (\rho^t L_t)^{1-\alpha-\gamma},$$

where  $\rho$  is equal to one plus the growth rate of labor productivity, which we set to equal 3.47 percent. This level achieves an intended 2 percent rate of overall growth ( $1.0347^{(1-\alpha-\gamma)} = 1.02$ ) which is the average per capita growth rate over the past 50 years [24]. The only other parameter adjustment is to the rate of depreciation, which falls from 0.096 to 0.076 when accounting for a 2 percent rate of overall growth. We then solve for the balanced growth path (BGP) where, in the deterministic case, all variables except for labor and emissions grow at the constant rate of  $1-\rho$  (i.e., 0.0347). To ensure existence of the BGP, it is necessary to assume that abatement technology improves at a rate equal to overall growth – that is, emissions per unit of  $M$  fall over time at the rate of growth. We address this strong assumption, and the possibility of avoiding it, in our discussion of future research directions below.

As expected, incorporating productivity growth shortens the transition, in this case from the no policy BGP to the new BGP for each policy. However, the ordering based on expected PVU remains unchanged from that of the no-growth setting. The result is also robust to the same range of time horizons (1-100 years) and discount rates (1-25%) considered in the no-growth setting.

Overall, after economic growth is incorporated into the model, decision functions show that choice variables are less sensitive to capital deviations and, except for labor, more sensitive to deviations in the productivity factor. In other words, the direct effects of innovations to the productivity factor are greater while the indirect effect of all past productivity deviations on investment and consumption, as manifested in the capital stock, is diminished. The intuition for this result is that accounting for growth effectively discounts the future marginal value of income (shadow value of the income constraint). However, the degree of these differences is minor. Other than a diminished transition and a small degree of convergence in the mean present value of utility across instruments, there is no significant change in qualitative results vis-à-vis the no-growth setup.

### ***Sensitivity Analysis: Developing-Country Volatility and Risk Aversion***

Given the systematic differences in the volatility of key variables between policies, it is natural to ask to what degree this second-order stochastic relationship translates into a direct preference on expected utility grounds, given preferences with some degree of risk aversion. In

particular, to what degree might the cap policy uniquely generate a benefit in terms of reduced volatility? Barlevy [25] provides a useful survey of the benefits of economic stabilization and the welfare costs of business cycles. The importance of these deviations from stable growth is debatable; arguments range from Lucas's [26] conclusion that they are a small concern to Storesletten et al.'s [27] estimation that lifetime consumption costs of volatility are as high as 7.4 percent for individuals without savings.

Recall that although the cap policy features the lowest volatility, its utility advantage over the tax for a given period on average was not significant (Table 6) and not sufficient to outweigh the advantage of the tax over the transition to a new steady state. Failure to find a significant stabilization benefit to the cap policy might reflect low variability in innovations to the shock process, low risk aversion in the assumed utility function, or both. We explore the effect of an increase in the standard deviation of productivity factor innovation process (Equation (16)), which also reflects the standard manner in which RBC models for developing countries typically differ in their parameterization (e.g., [28]). The issue of volatility and stabilization is particularly important for developing and emerging economies, including major players in the climate debate like China and India. Pallage and Robe [29, abstract] argue that “in many poor countries, the welfare gain from eliminating volatility may in fact exceed the welfare gain from an additional percentage point of growth forever.”

Using the midrange estimate from Neumeyer and Perri [28], based on their analysis of Argentinian data as a case study, we adjust the baseline level of  $\sigma$  from 0.014 to 0.0204. As in the baseline setting, given transitions, the PVU dominance of the cap and the tax over the intensity target is robust to the same range of time horizons and discount rates considered above. Simply raising the variance of innovations to the productivity shock process fails, in this case, to generate much stronger evidence of a strong stabilization benefit to the cap.

Next we consider the sensitivity of our results to the degree of risk aversion over consumption. Note that our measure of utility over consumption,  $\ln C$ , is a special case of the constant relative risk aversion specification,  $C^{1-\psi}/(1-\psi)$ , where the coefficient of relative risk aversion,  $\psi$ , is set to 1. We consider an alternative parameterization with increased risk aversion, setting  $\psi$  to 2. Contrary to initial expectation, elevating risk aversion over consumption in this manner fails to produce a stabilization benefit to utility under the cap. Utility orderings for the instruments based expected PVU are unchanged and consistent over the range of time horizons and discount rates discussed above. An explanation for this effect, at least in part, is found in examining the surprising effect on labor volatility.

Under increased risk aversion over consumption, there is an increased incentive to avoid fluctuations from the steady state in general and to direct fluctuations in income away from consumption and into investment. Thus the decision functions show a decrease in the sensitivity to deviations in the productivity factor and a corresponding increase in sensitivity to capital deviations. Given that optimal labor deviations move opposite to capital deviations, the volatility of labor is increased. This shift is particularly strong for the cap policy, where the inflexibility of choice over  $M$  already drives a high relative sensitivity to capital fluctuations. Ultimately, this constraint under increased consumption risk aversion leads to a reversal of our earlier finding of the cap policy as a stabilizing force: labor volatility under the cap policy is actually slightly greater than under the alternatives. Since the baseline utility measure over labor also includes a degree of risk aversion, it is not surprising that consumption stabilization benefits under the cap may be eroded.

## Conclusion

Stabilizing greenhouse gas concentrations in the atmosphere will require dramatic reductions in global carbon emissions. The choice among policies should be informed both by their expected cost-effectiveness and by how they respond to unexpected events along the path. We find that although a cap and a tax can produce equivalent outcomes in expectation, a cap-and-trade program reduces economic volatility, compared with all other policies and no policy, and a tax enhances volatility. The cap functions as an automatic stabilizer, since the shadow price of the emissions constraint increases with unexpected increases in productivity and decreases with unexpected economic cooling.

We find that an intensity target does indeed encourage greater economic growth than a cap or a tax, since the allocation of additional permits serves as an inducement for additional production. Furthermore, it seems neither to dampen nor to exacerbate aspects of the business cycle. Although emissions do remain volatile, for a stock pollutant like GHGs, the timing of emissions is not generally important. Most of the differences in volatility seem to be rather small, given our parameters and policy targets; the notable exception may be labor, which demonstrates more than 50 percent greater variance under all other policies relative to the cap in our baseline scenario.

Depending on one's perspective and priorities, there is reason to prefer each of the possible instruments considered here. The intensity target achieves the emissions reduction at the lowest welfare cost in the steady state, with no reduction to the labor force. The emissions tax achieves the emissions goal with the lowest direct welfare cost, though it is superseded by the

cap if the marginal damages of the excess transition emissions are comparable to the tax rate. These results are robust to considerations of developing-country levels of volatility in productivity and heightened risk aversion. Finally, the cap achieves the reduction with a slightly higher welfare cost than the tax, but it ensures the cut is achieved without lag, resulting in higher welfare if these additional reductions are valued, and the cap also features a lower level of labor variance than all other policies considered. However, this labor stabilization result does not hold when the volatility of productivity factor innovations is raised to a level representative of emerging economies. All of these policies deviate from optimal policy, in which both emissions prices and quantities should adjust (procyclically) to productivity shocks [13]. Although the emissions cap fixes quantities, both the tax and the intensity target feature fixed emissions prices.

In practice, those distinctions may be less important in a more realistic, decentralized policy setting. The intensity target may not have the same production incentive effect unless actors themselves receive additional allowance allocations in proportion to their output, as with tradable performance standards or output-based allocation. However, it does retain the feature of allowing emissions levels to rise in an expansion. Meanwhile, commonly proposed cost-containment features like banking, borrowing, or price caps tend to make the emissions cap behave over time more like a tax.

In focusing on the core properties of the instruments themselves we have also set aside broader policy interactions, such as with pre-existing tax distortions that might arise with the taxation of labor or capital to fund a public good. Such interactions raise the possibility that revenue generating instruments—the tax and, given permit auctioning, the cap—would have different effects on real wages and welfare when revenues are recycled than with lump-sum transfers or output-based allocation. These interactions have been explored in general equilibrium models also looking at intensity-based instruments or capital investment decisions (e.g., [9, 21, 32]), but may merit further investigation in a real business cycle context. While we have highlighted differences in the properties of various instruments, for some comparisons, deviations were small. In reality it could be the case that institutional or political constraints will swamp such differences. We have also not considered the potential role of market failures within the market-based instruments examine here. Such design elements should be considered in weighing the macroeconomic trade-offs of the different policies. Furthermore, our insights are drawn from a model of a single, closed-economy. While our baseline parameterization of the business cycle reflects economic uncertainty from an open economy (the U.S.), explicit treatment of international linkages via the effects of trade on the business cycle and international permit markets are an important line of future inquiry.

Although we have explored extensions to the basic model that incorporate productivity growth, developing country volatility and increased risk aversion, we have abstracted from population growth. Within the confines of the model the policy constraint could be thought of equivalently as either an absolute cap or a per-capita cap. However, in applying intuition from this analysis to a world with population growth, the constraint modeled here is better thought of as fixing per-capita emissions.

In future work we intend to extend the analysis using a more computationally intensive but flexible backward induction solution approach to relax certain model constraints on the results presented here. Because our solution technique involves approximation of decision rules around the steady-state, the characterization of transitions from a status quo starting point towards the new equilibrium or region is subject to some degree of approximation error. This approach also precludes the consideration of policy anticipation, ratcheting policy stringency over time, and more realistic models of abatement efficiency growth. The steady-state technique is not suited for anticipation of the onset of a policy by economic agents, which would affect the dynamics of the transition path. A dynamic policy ramp, where emissions constraints are ratcheted over time, is better captured by a nonsteady-state approach. Finally, when extended to consider the role of economic growth, the linearization technique requires strong assumptions about the rate of improvement in abatement technology – namely, that it is equal to the rate of productivity growth. Next steps to advance this analysis should include decoupling productivity and abatement technology and providing greater flexibility in policy format and agent expectations overall.

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