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Climate Policy Design with Correlated Uncertainties in Offset Supply and Abatement Cost

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RESOURCES FOR THE FUTURE

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Abstract

Current and proposed greenhouse gas cap-and-trade systems allow regulated entities to offset abatement requirements by paying unregulated entities to abate. These offsets from unregulated entities are believed to contain system costs and stabilize allowance prices. However, the supply of offsets is highly uncertain and may be correlated with other sources of uncertainty in emissions trading systems. This paper presents a model that incorporates both uncertainties in the supply of offsets *and* in abatement costs. We numerically solve a dynamic stochastic model, with parameters relevant to the U.S. climate debate, under a variety of parameter settings, including a system that includes allowance price controls, risk aversion, and competitive offset purchasing. We find that as uncertainty in offsets and uncertainty in abatement costs become more negatively correlated, expected abatement plus offset purchase costs increase, as does the variability in allowance prices and emissions from the regulated sector. These results are amplified with risk sensitivity, larger annual offset limits, and competitive offset purchasing. Imposing an allowance price collar substantially mitigates cost increases as well as the variability in prices, while roughly maintaining expected environmental outcomes. In contrast with previous literature we find a collar may also mitigate emissions variability.

Key Words: climate change, offsets, cap-and-trade, price collars, stochastic dynamic programming

JEL Classification Numbers: Q54, Q58, C61

Contents

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Harrison Fell, Dallas Burtraw, Richard Morgenstern, and Karen Palmer*

I. Introduction

Paying for emission reductions from entities outside of the regulated sectors in an emissions trading program, commonly referred to as emission offsets, has been seen as a way to reduce overall compliance costs. As such, formal channels to provide offset credits have been incorporated into the first two phases of the European Union emissions trading scheme for carbon through the Clean Development Mechanism and Joint Implementation programs. Sub-national cap and trade programs, including the Regional Greenhouse Gas Initiative and the pending California program, also allow for various forms of offsets. Recently proposed U.S. cap and trade legislation aimed at reducing CO_2 emissions have included even more extensive uses of offsets; allowing regulated entities to cover as much as one-third of their emissions with offsets from exempted sources, both domestic and international. While offsets have considerable potential as a mechanism to help meet domestic emissions goals at a low cost, various concerns surround their widespread use including difficulties in establishing emission baselines and demonstrating that offset-credits truly represent additional emission reductions (Fischer 2005, Michaelowa and Purohit 2007, and Rosendahl and Strand 2009). Concerns have also been raised over how generating offset credits from projects in developing countries will impact the long-run prospects for environmental regulations in those countries (e.g., Akita 2003 and Narain and Veld 2008). In this paper we introduce yet another potential problem with offsets, namely their inherently uncertain supply.

While recent legislation is clear in setting aggregate and, in some cases individual, maximum allowed use of offsets, their *actual* use is related to the market supply of offsets. In a globally interrelated economy, it is likely that at a time of high U.S. GDP growth and high demand for offsets, suppliers of offsets including exempted domestic and international sources would be less interested in selling offsets because of the relatively high opportunity cost of resources. For example, high GDP growth may elevate the demand for offsets, but coincidentally

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the return to bringing land into commercial production may also be high, which could lead to a decline in the supply of offsets from sequestered carbon in forests. Hence, there could be a negative correlation between the demand for and supply of offsets. A negative correlation of this type could add to other sources of abatement cost uncertainty and increase price volatility as well as total costs.

This paper examines the implications of heavy reliance on offsets as a policy tool in the context of a cap-and-trade system where abatement costs are inherently uncertain. We do so by adding uncertainty in offset supply to a model with underlying uncertainty in abatement cost. We parameterize the model with values relevant to recent climate legislation to explore the effect of this added offset uncertainty on abatement costs, cumulative emissions, offset purchases, and emission allowance price paths. Of particular interest is the possible role of a price collar – a mechanism that constrains both upside and downside allowance price swings – in limiting adverse consequences in the face of added uncertainty due to offsets.

Overall, when there is no price collar we find the range of possible allowance price outcomes can be greater with offsets than without, calling into question the ability of offsets to provide stability to a trading market. The expected NPV of costs increases with an increase in the persistence of shocks or the negative correlation between abatement cost and offset supply uncertainties. In this context a collar mechanism can play a constructive economic role in reducing costs as well as the variance in allowance prices. Interestingly, the case for a price collar is even greater with a heavy reliance on offsets than without it.

Moreover we find in some situations collars actually reduce the variance in emissions outcomes, a departure from the previous literature that has examined collars without offset supply uncertainty. Nonetheless the introduction of a collar is likely to change the expected emissions outcome, so we evaluate a collar with expected emissions equivalent to the level achieved without a collar and find the outcome depends directly on the spread of the collar, i.e. the likelihood that the collar binds. In addition to the results concerning the use of price collars in emission trading systems with uncertain offset supply, we also consider the impacts of risk aversion, varying offset limits, and competitive offset purchasing environments. We find that with a risk averse agent, compliance costs can dramatically increase if offset supply shocks are both persistent and highly (negatively) correlated with shocks to abatement costs. Similarly, we find that increasing offset purchasing limits and/or moving to a more competitive offset purchasing regime can exacerbate the compliance cost increases associated with increasing offset supply shock persistence and increasing offset and abatement cost shock correlations.

The organization of this paper is as follows: In Section II we outline the basic model, including a discussion of uncertainty, offset purchasing and the price collar. Section III describes the numerical solution algorithm, including the model parameterization. Section IV presents the results for the net present value (NPV) of costs and offset purchases, cumulative emissions and offsets, and the emission price paths. Section V presents some relevant alternative model specifications and parameter sensitivity results. The final section offers concluding observations.

II. Model Setup

This section presents the basic reduced form, dynamic-stochastic abatement cost model. This exercise is not intended to provide comprehensive estimates of abatement costs and emissions outcomes. Rather, it offers a tractable way to compare outcomes of multiple policies and parameter settings with important parameters keyed toward values relevant to the U.S. climate policy debate. Additionally, the analysis does not quantify the benefits of emission reductions. Benefits are ignored here because, even though the variance of cumulative emission outcomes may vary across different policies and parameter settings we examine, expected values of cumulative emissions are quite similar. The near-linear nature of typical abatement benefit functions used in climate change models (see Pizer 2002) and the slow atmospheric decay of CO_2 imply that different policies and parameter settings with similar expected cumulative emissions will have similar expected abatement benefits.[1]

With respect to emission regulation policies, we focus our attention on quantity-based policies (i.e. cap-and-trade) and hybrid policies such as price collars, which combine elements of quantity policies and price policies (i.e., emission taxes). In analyzing the policies, we begin with a representative firm framework with monopsonistic offset purchases. In this framework we consider the contribution of offsets to performance of the program without and with a price collar. We extend this framework to consider a multi-agent system where firms compete in oligopsonistic offset purchasing. Given that the major thrust of this paper is the consideration of behavior under uncertainty, we further consider outcomes when firms are risk-sensitive.

[1] Ancillary benefits from greenhouse gas emissions can be substantial, but they depend on geographic location that is beyond the framework of this analysis.

The Firm's Compliance Decision

In reality, proposed quantity-based instruments are operationally carried out under cap-and-trade systems in a multifirm setting. However, we justify the use of the representative firm based on the result that the emission outcome of a competitive cap-and-trade market is equivalent to that of a centralized planner (i.e., a representative firm) under the goal of minimizing abatement costs (see Rubin 1996 and Cronshaw and Kruse 1996). With respect to offsets, we preserve the equivalence of our representative agent framework and a multifirm competitive market outcome if we assume that for the multifirm model offsets enter the market via an intermediary monopsonistic offset purchaser, such as an independent government corporation as suggested by Purvis, Kopp and Stevenson (2009), that is purchasing offsets to minimize system-wide costs (abatement costs plus offset purchase costs) and provides the offsets to the market at cost.[2] A single aggregating offset-purchasing agency may provide a variety of functions in addition to purchasing offsets for the covered sectors. It may serve as the evaluator of the credibility of emissions reductions, use its purchasing power to direct investments toward nations that are taking steps toward making and fulfilling commitments within an international climate policy regime, and, more generally, direct revenue in a way that complements international aid policy. Although considering these additional roles of an offset purchasing agency is important, their consideration is beyond the scope of this paper.[3]

To begin our analysis of the compliance decision for the representative firm, we first specify a convex abatement cost function $C(a_t)$, similar to those used in Newell and Pizer (2005) and Hoel and Karp (2002), of the form $C(a_t) = \frac{c_t}{2}\left(\bar{q}_t + \theta_t - q_t\right)^2$. In this specification c_t represents the slope of the marginal abatement cost function, $\bar{q}_t$ is the expected profit-maximizing level of emissions in the absence of a regulation (i.e. expected baseline emissions), θ_t is a shock to the baseline emissions, and q_t is the chosen level of emissions (i.e., $a_t = \bar{q}_t + \theta_t - q_t$). We also assume a linear supply of offsets, z_t, such that $z_t = \gamma_t P_t^z$, where P_t^z is the price of offsets.

[2] An additional consideration with a single offset purchaser is how offsets are distributed to the covered firms. There are multiple ways in which this could be accomplished, including auctioning and free allocation. Given the representative firm framework here we do not consider the offset rationing problem in this paper.

[3] It is also possible that the aggregators could be private firms. Though we do not consider such a situation see Fulton and Vercammen (2009) for more on this form of offset provision.

We introduce two sources of abatement cost uncertainty: uncertainty in baseline emissions and uncertainty in offset supply. The uncertainty to baseline emissions, as noted above, is represented by θ_t. Since emission levels are positively correlated with economic activity and since output measures, such as GDP, have been found to be highly persistent in numerous empirical studies, we assume that the shocks to baseline emissions are temporally correlated. As in Newell and Pizer (2003), this correlation is invoked by modeling the evolution of θ_t as an AR(1) process:

$$\theta_t = \phi_1 \theta_{t-1} + \varepsilon_{1t} \tag{1}$$

where $0 < \phi_1 < 1$ and ε_{1t} is a random variable. Uncertainty enters offset supply through γ_t as:

$$\begin{aligned} \gamma_t &= \bar{\gamma}_t + \mu_t \\ \mu_t &= \phi_2 \mu_{t-1} + \varepsilon_{2t} \end{aligned} \tag{2}$$

where $\bar{\gamma}_t$ is the unconditional mean of the slope (γ_t) of the offset supply curve, and μ_t is the additive element of uncertainty, with $0 < \phi_2 < 1$ and ε_{2t} is a random variable.

There are many potential reasons why the offset supply curve may deviate from expectations and, thus, many sensible ways one could model μ_t. In our representation of this deviation, we model μ_t in such a way to account for two particularly salient features likely to be associated with offset markets. First, as with the shocks to baseline emissions, we allow μ_t to be positively correlated over time. Persistence in the offset supply states would seem likely if, for instance, there is a time lag between application and approval for new offset projects and/or if there are contractual obligations that inhibit offset suppliers' abilities to remove their projects from the offset system.

The second feature we account for in the uncertainty of offset supply is the potential correlation between the uncertainty in baseline emissions (θ_t) and the uncertainty in offset supply, as indicated by uncertainty in the slope of the offset supply curve (γ_t). Correlation between offset uncertainty and shocks to domestic economic activity, which are associated with θ_t realizations, seems particularly relevant in a global economy context, yet to our knowledge this correlation has not been considered in other models. For example, correlation between γ_t and θ_t could occur if an increase in economic activity in the United States increases the opportunity cost of providing offsets by increasing the demand for the offset projects' alternative uses. An increase in the demand for alternative uses of offset-providing infrastructure would in turn decrease the quantity of offsets available at each offset price. This implies a negative correlation between γ_t and θ_t. We allow for correlation between the baseline emissions uncertainty and offset supply uncertainty through ε_{1t} and ε_{2t} such that:

$$(\varepsilon_{1t}, \varepsilon_{2t}) \sim N(0, \Sigma) \text{ and } \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_1^2 \end{bmatrix} \quad (3)$$

where $\sigma_{12} = \sigma_{21}$, and thus the conditional correlation between γ_t and θ_t given μ_{t-1} and θ_{t-1} is $\rho = \sigma_{12} / \sigma_1 \sigma_2$.

Given the abatement cost function, offset supply function, and uncertainty structures, we can now set up the dynamic optimization problem for the risk-neutral firm under an emissions regulation problem that allocates y_t emission allowances each period and allows allowance banking/borrowing (B_t):

$$\max_{q,z} \sum_{t=1}^{T} \beta^{t-1} \left[-\frac{c_t}{2} (\bar{q}_t + \theta_t - q_t)^2 - P_t^z z_t \right] = \max_{q,z} \sum_{t=1}^{T} \beta^{t-1} \left[-\frac{c_t}{2} (\bar{q}_t + \theta_t - q_t)^2 - \frac{z_t^2}{\gamma_t} \right] \quad (4)$$

subject to (1) - (3) and:

$$B_{t+1} = R_t B_t + y_t + z_t - q_t \quad (5)$$

$$B_t \geq B_{\min,t}; \text{ and } z_t \leq z_{\max} \quad (6)$$

where β is the discount factor equal to $1/(1 + r)$ and r is the discount rate, B_t is the bank position at t, and R_t is an intertemporal trading ratio.[4] Constraint (5) states that the bank level next period is equal to the bank level this period, adjusted by the trading ratio, plus what the firm receives in allocations and purchases in offsets less what they emit. Constraint (6) provides the boundary conditions, where $B_{\min,t}$ and $z_{\max}$ are predetermined values set by the regulator. Other state variables, such as c_t and $\bar{q}_t$ are assumed to evolve in a known and exogenous manner. The Bellman equation form of the maximization is:

$$V_t(B_t, \theta_t, \gamma_t) = \max_{q,z} \left(-\frac{c_t}{2} (\bar{q}_t + \theta_t - q_t)^2 - \frac{z_t^2}{\gamma_t} + \beta E \left[V_{t+1}(B_{t+1}, \theta_{t+1}, \gamma_{t+1}) \mid B_t, \theta_t, \gamma_t, q_t, z_t \right] \right) \quad (7)$$

where E is the expectations operator at t conditional on the state and control variables at t and subject to the constraints and dynamic equations given above. The first order conditions for optimality are then:

$$\frac{\partial V_t}{\partial q_t} = c_t (\bar{q}_t + \theta_t - q_t) - \beta E_t \left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \right] = 0 \quad (8)$$

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[4] Note that we use "bank" to refer to the cumulative position of over or under compliance at any time t. Thus, the the bank value B_t can take on a positive or negative (assuming borrowing is allowed) value.

$$\frac{\partial V_t}{\partial z_t} = -\frac{2z_t}{\gamma} + \beta E_t\left[\frac{\partial V_t}{\partial q_t}\right] - \lambda_{1t} = 0 \qquad (9)$$

$$\frac{\partial V_t}{\partial B_t} = \beta R_t E_t\left[\frac{\partial V_t}{\partial B_{t+1}}\right] + \lambda_{2t} \qquad (10)$$

where λ_{1t} and λ_{2t} are the shadow values corresponding to the upper bound of z_t and lower bound of B_t, respectively in (6).

From (8) and (10), we derive the following Euler equation:

$$c_t\left(\overline{q}_t + \theta_t - q_t\right) = \beta E_t\left[R_{t+1}c_{t+1}\left(\overline{q}_{t+1} + \theta_{t+1} - q_{t+1}\right) - \lambda_{2t+1}\right] \qquad (11)$$

Equation (11) presents a modified Hotelling rule, similar to that given in Rubin (1996). The relationship states that the expected marginal cost of abatement, corrected for the intertemporal trading ratio and possibility of a binding bank constraint, will rise at the firm's discount rate. Note also that in this form, if the uncertainty parameters are such that there are possibilities of binding bank constraints, $E_t[-\lambda_{2t+1}] > 0$, the solution to the representative firm problem could invoke some precautionary savings even with our assumed linear marginal costs and risk-neutrality.[5] Thus, if borrowing is constrained and shocks are sufficiently large, we should expect to see larger bank levels in expectation as ρ becomes more negative because this would lead to a greater probability of having simultaneously high baseline emissions and low offset supplies – situations where borrowing-constrained firms would like to borrow, but cannot.[6]

From (9) and (8) we also see that when the constraint on offset purchases is not binding, the cost minimizing monopsonist will purchase offsets such that:

$$c_t\left(\overline{q}_t + \theta_t - q_t\right) = {}^{2z_t}\!/_{\gamma_t} = 2P_t^z \qquad (12)$$

Equation (12) states that the representative agent should purchase offsets until the marginal cost of abatement is equal to twice the price of offsets. This result is a familiar prescription for monopsonistic behavior when the firm cannot price discriminate and supply

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5 This point was also noted briefly in Schennach (2000) with respect to emission trading schemes that allow no borrowing.

6 In all of our models we are implicitly considering a case of market incompleteness similar to that discussed in the Bewley-type macroeconomic savings models. This is assumption is not without some validity as discussed Fan et al. (2011). Additionally, some proposed climate bills, such as S. 2877 (U.S. Congress 2009c), introduced by Senators Collins (R-ME) and Cantwell (D-WA) in the 111th Congress, severely restrict the activity of speculators that could provide hedging options for firms.

curves are linear. Note that a multi-firm competitive market model where individual agents are purchasing offsets at a single market clearing price would not obtain such a result because the agents would not account for the pecuniary externalities of offset purchases. We return to this point in the subsequent section when we set up the oligopsonistic offset purchasing in the next section. Finally, from a practical standpoint of determining policy functions for this dynamic optimization, note that (12) shows us the decision for offset purchases (z_t) is described by the state variables θ_t and γ_t and the control variable q_t. Thus, determining the optimal q_t is sufficient to find the optimal z_t.

Price Collars

The problem described above is for a standard quantity-based emissions regulation, such as a cap-and-trade program with banking and limited borrowing. Recently, hybrid mechanisms, which combine elements of a price policy and a quantity policy, have garnered considerable attention. In this analysis, we consider a particular hybrid policy commonly referred to as a price collar. Collar mechanisms work by imposing a price floor, P_t^f, and a price ceiling, P_t^c, on emission allowance prices (i.e. marginal abatement costs) in a cap-and-trade system. The price ceiling is enforced by selling an unlimited number of additional allowances into the market at P_t^c. The price floor could be implemented in one of two ways. First, if emission allowances are auctioned, the regulator could set a minimum price for the auction, often referred to as a reserve price, which would serve as the floor. Alternatively, the government could have an open offer to buy and retire allowances from covered entities at P_t^f. For modeling simplicity, we consider the latter price floor design.[7]

With a price collar, the bank transition equation becomes:

$$B_{t+1} = R_t B_t + y_t + z_t + y_t^c - y_t^f - q_t \qquad (13)$$

where y_t^c is the quantity of allowances bought from the regulator at the price ceiling and y_t^f is the quantity of allowances sold at the price floor. The resulting Bellman equation with price collars for the representative firm framework is given as:

[7] The price floor implemented via an auction floor is more complicated to model because with an auction and banking or borrowing the representative firm would have to choose allowances purchased as well as emission levels. This adds an additional control variable to the optimal control problem and thus complicates matters. However, since we are considering only abatement costs without endogenous technological investment, results under the two possible price floor designs will be the same.

$$V_t\left(B_t,\theta_t,\gamma_t\right)=\max_{q,z,y^c,y^f}\begin{pmatrix} -\dfrac{c_t}{2}\left(\bar{q}_t+\theta_t-q_t\right)^2-\dfrac{z_t^2}{\gamma_t}-P_t^c y_t^c+P_t^f y_t^f \\ +\beta E\left[V_{t+1}\left(B_{t+1},\theta_{t+1},\gamma_{t+1}\right)\mid B_t,\theta_t,\gamma_t,q_t,z_t,y_t^c,y_t^f\right]\end{pmatrix} \tag{14}$$

subject to (13), (1) – (3), (6) and non-negativity constraints on y_t^c and y_t^f. When the non-negativity constraints on collar purchases or sales are not binding, the first order conditions of (14) imply:

$$c_t\left(\bar{q}_t+\theta_t-q_t\right)=P_t^i \tag{15}$$

$$P_t^i=\beta E_t\left[\partial V_{t+1}/\partial B_{t+1}\right] \tag{16}$$

for $i = c$ or f. Equation (15) ensures that marginal costs of abatement will not rise above the price ceiling or fall below the price floor. Equation (16) states that, when the ceiling is triggered, additional allowances y_t^c will be purchased until the expected marginal value of an additional unit of banked allowances at time t equals the price ceiling. Likewise (16) states that when the price floor is in effect, allowances will be sold at the floor price until the expected marginal value of the bank equals the price floor.

With respect to offset purchases under a price collar system, if the price ceiling is in effect, we would expect the same offset purchasing decision as given in (12). However, when the price floor is triggered, the offset purchasing decision becomes more complicated. Assume that for (16) to hold at the price floor ($i = f$) that next period's bank value must be such that $B_{t+1} = B^*$. By rearranging (13) under this condition we have $y_t^f = y_t-\left(B^*-R_tB_t\right)-q_t+z_t$, where q_t solves (15). If $y_t-(B^*-R_tB_t)-q_t > 0$, then any offsets purchased by the regulator in period t and sold to the firm at price P_t^z will just be sold back to the government at P_t^f. We assume the government retires the offsets, as well as allowances, that it buys at the price floor; i.e., we assume the government does not use the allowances or offsets bought at the price floor to reduce future abatement requirements.[8] Hence, the payment to the firms for the offsets purchased at the price floor does not affect total system costs since it simply represents a transfer of funds, but the purchase of the offsets themselves does increase the system-wide costs. Therefore, offsets purchased when the price floor is in effect *and* $y_t-(B^*-R_tB_t)-q_t > 0$ increase system-wide costs. If the offset aggregator is purchasing offsets with the goal of minimizing system-wide

[8] Similarly, we assume that additional allowances sold at the price ceiling will not result in a reduction in future allocation of allowances. For systems with this structure see Murray et al. (2008).

costs, no offsets should be purchased if the price floor is binding and $y_t - (B^* - R_tB_t) - q_t > 0$. By similar reasoning, if $y_t - (B^* - R_tB_t) - q_t < 0$, it is clear that system-wide costs are minimized if offsets are purchased up to the point where no allowances are bought back by the government at the price floor ($y_t^f = 0$) while the price floor is in effect. The solution algorithm and the results presented below proceed based on these price collar implementation rules, with accompanying offset provision rules.

III. Numerical Solution Algorithm

Ideally, one would like to solve emissions and offset provision rules as a function of the continuous state variables and other known model parameters. Given the temporal and cross correlation of uncertainty variables θ_t and μ_t and boundary constraints on bank levels and offset purchases, an analytic optimal control solution for the emissions and offset purchases is not forthcoming. We therefore solve the optimal emissions and offset provision rules numerically through a backward recursion process over a discretized state space. The steps to this numerical solution technique are given below.

First, we descritize the state space. It should be noted that the time paths of c_t, y_t, $\bar{q}_t$, and $\bar{\gamma}_t$ are assumed to be known; thus the discretization is needed only for the state variables B_t, θ_t, and μ_t. For B_t, we use N_B discrete bank levels such that bank levels are evenly distributed between $B_{min,t}$ and zero and between zero and $B_{max,t}$. For the random and exogenously determined state variables θ_t and μ_t we use N_θ and N_μ discrete values evenly spaced between between ± 5 times the standard deviation of ε_{1t} and ε_{2t}, respectively.

Next, we solve for the probability transition matrix P that gives us the probabilities of moving from each possible state in period t to each possible state in $t + 1$, where a state is defined by the B_t, θ_t, and μ_t realization. This in turn allows us to form expectations about the future value function, V_{t+1}, at time period t. In forming P, note that the evolution of the bank state is deterministic given emissions and offset choice. Therefore, the probability matrix can be written in terms of moving from states (θ_t, μ_t) to states (θ_{t+1}, μ_{t+1}). The transition matrix will be determined by the specifications of (1) - (3).

After discretizing the state space and calculating P, we begin the backward recursion step by imposing a nonnegativity terminal condition on the bank state ($B_{T+1} \geq 0$).[9] Cost minimization

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[9] The bank state B_t can be thought of as the bank level at the start of the period, before an emissions level and offset provision are chosen. Therefore, B_{T+1} is the bank level at the end of the final period of the regulation.

will occur when the terminal condition is binding ($B_{T+1} = 0$) for state realizations where optimal emissions in the final period are less than the unconstrained emissions (i.e., $q_T < \bar{q}_t + \theta_t$). Therefore, the optimal emissions quantities in the last period for all bank and shock states are known. Additionally, since the offsets provision decision, z_t, is a function of optimal q_t, final period optimal z_T's are also known. Given, z_T and q_T for each bank-shock state combination, the values of $V_T(B_T, \theta_T, \mu_T)$ are known.[10] Knowing the values of the final period value function, the probability transition matrix, and the state dynamics, we can step back one period to T-1 and determine the optimal q_{T-1}, and thus z_{T-1}, for each (B_{T-1}, θ_{T-1}, μ_{T-1}) state. These optimal decisions, along with the discounted $V_T(B_T, \theta_T, \mu_T)$, again give us values for V_{T-1} (B_{T-1}, θ_{T-1}, μ_{T-1}). We continue this backward stepping procedure to period t = 1, where we get V_1 (B_1, θ_1, μ_1).

The matrix of values for V_1 (B_1, θ_1, μ_1) gives us the expected discounted net present value of system-wide costs at time t = 1 for every initial (B_1, θ_1, μ_1) combination. In addition to getting a matrix of values for the value function at each time step for each possible state, the backward recursion process also collects the optimal emissions and offset provision choices for each state in each time period. These optimal decision matrices can then be used in simulation analyses. The simulation analyses are conducted by first simulating multiple draws of the shock paths for θ_t and μ_t. Then, given an initial bank value $B_1 = 0$ and the optimal emissions decision matrices, emission and offset paths are determined, which form the basis for our expected cumulative emissions and cumulative offsets estimates. In addition, the emission paths coupled with the shock paths allow us to calculate multiple emission price paths (where emission price equals marginal abatement cost) and total abatement cost estimates. The results from these simulation exercises are presented below.

Model Parameterization

Many parameters used to parameterize the model are keyed directly to the recent climate change legislation introduced by Reps. Henry Waxman (D-CA) and Edward Markey (D-MA), H.R. 2454 (U.S. Congress 2009a) and to the analysis of H.R. 2454 by the Energy Information Administration (EIA 2009). However, as stated above, this research should be seen not as a substitute for the EIA analysis but rather as a tractable way to compare policies and parameter settings not possible in EIA's deterministic analysis of the bill. The parameters are summarized in Table 1.

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[10] The value function $V_t(B_t, \theta_t, \mu_t)$ is equivalent to the value function $V_t(B_t, \theta_t, \gamma_t)$ given that $\bar{\gamma}_t$ is deterministic. We therefore use the value functions interchangeably.

To begin, we set the terminal period of the model at T = 39 to simulate H.R. 2454's 2012 – 2050 timeframe. The emission allowances, y_t, are taken directly from H.R. 2454 as laid out in Section 702 of the bill. We set the minimum bank level at $-y_{t+1}$ and the interest rate paid on banked and borrowed permits at $r_{bank} = r_{borr.} = 0$.[11] No limit is given for the maximum bank level, but for discretization we set it at 45 giga-metric tons of CO_2 equivalent ($GmtCO_2e$), which was sufficiently high to not be binding for all simulations conducted. With respect to offsets, we use the 2 $GmtCO_2e$ annual maximum limit given in H.R. 2454.[12] For the expected baseline emissions path, $\bar{q}_t$, we use the baseline emissions path given in EIA (2009) for 2012 – 2030, EIA's analysis period, and extend the trend of EIA's baseline emissions from 2025 – 2030 for the remaining years of our analysis. As stated above, we assume an exogenous decline in the slope of the marginal abatement cost curve (c_t) over time. Unfortunately, to our knowledge, there is no direct corollary to this decline rate that has been estimated empirically or via simulation. We set the decline rate, g_c, at what we feel is a modest -1.25% annual rate. Given these parameter values , we set the initial value of c_0 = \$63/$mtCO_2e$ per $GmtCO_2e$ to approximate the price path in EIA (2009) for EIA's low discount rate case, which sets the discount rate to 0.05, with no uncertainty ($\theta_t = \mu_t = 0,\ \forall\ t$). For the price collar case, we set the initial values of the price floor and price ceiling, P_1^f and P_1^c, at \$10/tCO_2 and \$28/tCO_2, respectively, and increased both at a rate of 5 percent annually. This setting is similar to that described in the Kerry-Boxer bill, S. 1733 (U.S. Congress 2009b).

For the expected offset supply curve slope, $\bar{\gamma}_t$, we use the offset supply curves provided in EIA (2009). EIA provides offset supply schedules for every fifth year from 2010 to 2050, broken down by sector and location (i.e. industry-based, domestic vs. international).[13] We aggregate the predictions that are categorized as "offset applicable"; these include the "Non-Covered US Sector," "US Sequestration," and "International" offsets. After making several unit conversions, we derive aggregate offset supply schedules for every fifth year through 2050 (i.e. for years 2010, 2015, 2020, ...). These offset supply schedules imply non-linear supply curves.

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[11] H.R.2454 charges interest on all permits borrowed beyond next period's allowances. However, even without interest charged on borrowed permits, we did not see a simulation realization with borrowing in excess of next period's allocation, so we exclude the allowance borrowing interest rate provision.

[12] H.R. 2454 puts specific limits on the maximum number of international offsets and domestic offsets, but we make no such distinction here. We also have conducted runs with the annual offset maximum limit at 4$GmtCO_2e$. These results are available upon request from the authors.

[13] The offset supply schedules provide a quantity of offsets at a given price for a range of prices. The schedules are quoted for every fifth year starting in 2010.

To get an estimate of the slope for each of these years, we connect a line from the origin to the EIA-predicted offset price-quantity pair for the given year.[14] This gives us offset supply slopes for every fifth year starting at 2015. To get an expected offset supply slope for every year, we fit a smoothed curve to the slope estimates derived in the previous step. A plot of the initial offset supply slopes and the smoothed series is given in Figure 1. The smoothed estimates form our $\overline{\gamma}_t$ values.

For the parameterization θ_t, we allow for a highly correlated series with $\phi_1 = 0.9$. This high correlation is in line with national GDP series, which are highly correlated to baseline emission schedules. We set $\sigma_1^2 = 0.10$ implying a long-run variance for θ_t of approximately 0.5 $GtCO_2e$, roughly one-third of the final period's allocation.

For the parameters of the offset supply shock dynamics given in (2) and (3), we use a range of values to explore how various levels of shock persistence and levels of correlation with baseline emissions shocks affect model outcomes. For ϕ_2 we use values of 0.0, 0.4, and 0.8. Similarly, we look at correlations of ρ = 0.0, -0.4, -0.8.

IV. Results

This section discusses the results for expected net present value (NPV) of costs (abatement costs plus offset purchase costs), domestic emission outcomes, offset quantities, and emission price paths under a variety of parameter settings and for emission regulation policies with and without price collars. Results presented for expected NPV of costs are based directly on the backward recursion determination of V_1, with the initial bank equal to zero and $\theta_0 = \mu_0 = 0$, as explained above. We remind the reader that for the collar mechanisms, these costs do not include costs of purchasing additional allowances at the price ceilings or reductions in costs due to allowance sales at the price floor. Emission outcomes, offset quantities, price path information, and coverage intervals for the NPV of costs are derived from simulation analysis.[15] We generate 10,000 time paths for θ and μ based on the discretized values of these variables and the derived probability transition matrix. Given these realizations of the uncertain variables and the optimal emission and offset decisions solved in the backward recursion algorithm, we generate 10,000

[14] In EIA (2009), emission allowance price forecasts are only given out to 2030. For years beyond 2030, we simply extend the five percent annual price growth trend of the EIA's price forecast.

[15] The 95 percent coverage interval include the upper and lower values of the variable in question that span 95 percent of the simulated trials.

emissions and offset paths. These emission paths form the basis of our emission price paths, since emission price is assumed to be equal to the marginal cost of abatement. We also obtain NPV of cost estimates from these simulated paths that form the basis of the cost outcome intervals presented below.

Without Price Collars

The upper-left panel of Table 2 presents the NPV of costs and 95 percent coverage intervals (in parentheses) when offsets are purchased by a non-price-discriminating monopsonist. The values cover a range of combinations of ϕ_2 , which positively increases the persistence of shocks to offset supply (γ_t) through the additive term (μ_t), and ρ, which indicates the (negative) correlation between μ_t and the shock to baseline emissions (θ_t) The inclusion of uncertainty in either offset supply increases expected costs compared to the situation with no offset supply uncertainty. The results also show that as the persistence of μ_t increases (ϕ_2 increases) or the negative correlation between μ_t and θ_t increases in magnitude (ρ decreases), expected NPV of costs increase. This increase in costs is non-negligible, resulting in a 13 percent cost increases from the no offset uncertainty persistence and no uncertainty correlation setting ($\phi_2 = \rho = 0$) to the high persistence and high correlation setting (ϕ_2 =0.8, ρ = -0.8).

There are primarily two reasons for this pattern of costs across persistence and correlation parameterizations. The first reason is essentially resulting from the convex cost function. As ϕ_2 increases, the long-run variance of μ_t increases and thus the tails of its unconditional distribution gain probability mass. Similarly, as ρ becomes more negative, the probability of jointly-occurring extreme baseline emission shocks and offset supply shocks increase. With a convex abatement cost function, equally increasing the probability weight on extreme high and low abatement outcomes increases expected abatement costs. Thus, increasing the probability of extreme offset availability states through increasing ϕ_2 and/or increasing the probability of jointly-occurring extreme baseline emission states and offset availability states through ρ becoming more negative increases expected abatement costs and drives an increase in overall NPV expected total costs.

The second reason for this pattern of costs is associated with the offset constraints. In upper right panel of Table 2 we present the average cumulative offset purchases across the different persistence and correlation parameterizations, with 95 percent coverage intervals given in brackets below. Note that even with low persistence in offset supply shocks and low correlation between offset supply and baseline emissions shocks (low ϕ_2 and ρ combinations), expected cumulative offsets purchased around 63 GmtCO_2e, which is over 80 percent of the maximum allowable cumulative offsets of 78 $GmtCO_2e$ (2 $GmtCO_2e$/year x 39 years). Since, in expectation, the cumulative offset limit is met even with low absolute values of ϕ_2 and ρ, increasing the magnitude of these parameters, which increases the probability of being in low offset cost states *and* high offset cost states, effectively only lowers expected cumulative offset purchases by more frequently realizing states with high offset cost. The reduction in offset purchases results in increased abatement and consequently increases NPV of costs.[16] This effect can also be noticed in the increasing spread of the 95 percent coverage intervals. All parameter settings have similar lower bounds of cost, but the upper bound of costs increase dramatically as the absolute values ϕ_2 and ρ increases.

The differences in costs across ϕ_2 and ρ parameterizations are also driven to some extent by the way the uncertainties and correlations between them alter intertemporal allowance allocation decisions. As discussed above, the optimal trajectory of marginal abatement costs over time is dependent on expectations about possible borrowing limit constraints being met in future periods. More specifically, if the parameterization of the uncertainties are such that it will make more likely that the borrowing limit will be met next period, as is the case when ϕ_2 and ρ are increasing in magnitude, current bank levels must increase to maintain the relationship described (11). Indeed, Figure 2 shows us that the expected bank level for the case where $\phi_2 = \rho = 0$ is generally lower than that for the case where $\phi_2 = 0.8$ and $\rho = -0.8$. The increased banking brought about in response to the increase in offset supply shock persistence and increased magnitude of the negative correlation between the offset supply shock and baseline emissions shock increases overall expected costs as forming the larger bank requires additional early-period compliance.

Also, note that without offsets costs are considerably higher than even the highest upper bound of the 95 percent coverage intervals for the various persistence and correlation parameterizations. This result highlights the importance of offsets in reducing system costs, even with its potentially uncertain provision.

In looking at the results on cumulative offset purchase and cumulative domestic emissions (the upper right and lower left panels of Table 2, respectively), we see that unlike a standard quantities based regulation, the availability of offsets will not guarantee a specific

[16] Being in a low offset cost state does lower total system costs (abatement cost plus offset purchase costs) even when at z_{max} because the offset limit is reached at a lower cost. However, these savings do not compensate for the additional costs incurred from being in the high offset cost states more frequently.

quantity of emissions from the covered industries. Indeed the range of possible cumulative emission outcome, as shown in the 95 percent coverage values, can be quite large. We also see variation in cumulative emission and offset purchases across ϕ_2 and ρ parameterizations, with lower cumulative emissions and cumulative offset values occurring in cases where both ϕ_2 and ρ are relatively large in magnitude. The reasoning for this pattern follows from the explanation given above of the NPV of costs pattern over the ϕ_2 and ρ values. Again, since, in expectation, per period offsets limits are met in most periods even for low absolute values of ϕ_2 and ρ, increasing the magnitude of these parameters, which increases the probability of being in both high and low offset cost states, effectively only lowers offset purchases through being in the high offset cost states more frequently. If offset purchases are low, cumulative emissions must also decrease to meet emission goals outlined in the y_t schedule, which in turn lowers the expected value of cumulative emissions.[17]

Finally, we are also interested in how the persistence of the offset supply uncertainty and its correlation with the baseline emissions uncertainty affects allowance price variation, as denoted by marginal abatement cost in our representative firm model. To compare price path variation across parameter settings in a succinct manner we provide root mean square error (RMSE) calculations in the lower right panel Table 2. We calculate RMSE for each setting as:

$$RMSE = \frac{1}{sims}\sum_{i=1}^{sims}\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(P_{it} - \bar{P}_t\right)^2} \qquad (17)$$

where *sims* is the number of simulations (10,000), P_{it} is the marginal abatement cost of simulation *i* in time period *t*, and $\bar{P}_t$ is the expected price path at time *t* taken as the average across all simulated price paths at *t*. We find, as expected, increasing persistence and correlation in the sources uncertainty increase variability in prices. As a point of comparison, for the case with no offsets included at all we get a $RMSE = 22.9$. Thus, we find that if offset supply uncertainty is highly persistent and/or there is a high degree of negative correlation between offset supply uncertainty and general macroeconomic uncertainty the inclusion of offsets can actually *increase* price variability compared to a case without offsets. Such a result calls into question the ability of offsets to provide more stability to an allowance trading market.

[17] Without per period offset limits, expected cumulative emission values across all parameter settings for the case with no price collars will be essentially the same. This is because for large absolute values of ϕ_2 and ρ offsets above the per period cap will enter during the more likely low offset cost states, counter-acting the lower emissions required during the high offset cost states.

With Price Collars

As mentioned above, hybrid price and quantity instruments, such as allowance price collars have been considered in several recently proposed pieces of climate legislation. We model a price collar with an initial price ceiling at P_1^c = \$28/tCO_2e and an initial price floor at P_1^f = \$10. Both the floor and ceiling rise over time at a rate equal to the assumed discount rate of five percent. These collar values are approximately the same as those used in S.1733 (U.S. Congress 2009b). NPV of abatement costs, cumulative emission and offset purchases, and price *RMSE* results from different persistence and correlation parameterization with this collar imposed are given in Table 3. Additionally, Table 3 also includes the percentage of periods with binding price floors and price ceilings. The frequency of a binding price floor (ceiling) is calculated as the total number of simulated prices equal to the price floor (ceiling) divided by the total number of simulated prices ($T \cdot sims$ = 39,000 simulated prices).

Looking at the NPVs of costs under the price collar cases (upper left panel of Table 3) we find, as expected, that costs are reduced relative to their non-collar counterparts. This cost reduction is particularly dramatic for the cases of high persistence in the offset supply shock and a high degree of negative correlation between the shocks. These cost reductions are in part due to the fact that this particular collar has a more relevant price ceiling. Initial allowance prices without a collar are in the \$23 - \$24/tCO_2e range. Thus, the collar considered is not evenly distributed about the initial prices. This results in the ceiling binding more frequently than the floor, as can be seen by the bottom two panels of Table 3. With a more relevant price ceiling, more allowances will enter the system compared to the cases without collars and adding these additional allowances lowers the abatement costs. In addition the price collar reduces the range of potential abatement outcomes, as seen in equation (15). This reduction in the range of possible abatement outcomes also reduces abatement cost given the convex abatement cost curve considered here.

In comparing cost results across ϕ_2 and ρ combinations, as with the no-collar cases we again see that increasing the persistence of the offset supply shock for a given correlation coefficient increases the costs. However, as we increase the magnitude of the negative correlation between offset supply uncertainty and baseline emission uncertainty costs actually fall. This result is again due to the specific collar considered here with the more relevant price ceiling. In addition to binding more frequently, the bottom two panels of Table 3 indicate that for a given ϕ_2 value, the probability of the ceiling binding increases only slightly as the uncertainty sources become more negatively correlated. In these instances, the upper bound of costs for each of the ϕ_2 and ρ combinations will be similar, as can be seen in 95 percent coverage intervals for the NPV of abatement costs given in Table 3. At the same time, since the floors are rarely binding, the lower-bound costs are smaller for more negative ρ under a given ϕ_2. That is, the price ceiling effectively caps the upper bound of costs, but the floor does not effectively restrict lower bound costs in this particular setting. The result is falling expected costs for a given ϕ_2 as the correlation between uncertainties becomes more negative. While this result is clearly not generalizable to other price collar forms, it does highlight the difficulties in predicting how costs will change under complex systems of inter-related uncertainties and hybrid instruments.[18]

With respect to the cumulative emissions when price collars are applied, the expected cumulative emissions are slightly higher than the ϕ_2 and ρ counterparts without the price collar. As noted above, this is as expected since the price ceiling is binding much more frequently than the price floor, and therefore, on net, more allowances are added to the system at the price ceiling than removed with a binding price floor. However, for combinations of ϕ_2 and ρ where both values are relatively large in absolute terms, the coverage intervals for cumulative emissions are narrower with price collars than without price collars. In obtaining this result we note the intervals have significantly greater lower bounds and slightly greater upper bounds. The finding that price collars can actually reduce the variance in emission outcomes compared with a case without collars is a direct contradiction of previous price collar analyses that have not considered offsets (e.g. Philibert 2008; Burtraw, et al. 2009; Fell and Morgenstern 2010). The explanation for the potential reduction in the variance of cumulative emissions is quite straightforward. In instances where offset supply is low, emission targets must be met with additional abatement. With price collars, the reduction in emissions will trigger the price ceiling and add allowances in the system, shifting the mass of the cumulative emissions distribution upward, thereby increasing the lower bound of emission outcomes and narrowing the spread of possible emission outcomes.

With respect to the comparison of offset purchases between the no-collar and collar scenarios, the expected cumulative purchases of offsets are either the same across the two scenarios or slightly larger for the no-collar case. That offset purchasing can be less with price collars is as expected, since, when binding, both the floor and the ceiling reduce the purchases of offsets relative to the case of no collar.

Finally, in looking at the *RMSE* values presented in Table 3, we see that the price collar considered here reduces the price variation for all ϕ_2 and ρ specifications relative to the no-collar

[18] Note also that without offsets, the price ceiling binds in all periods. That is, the price collar effectively becomes a tax with an initial value of \$28/mtCO_2 and rising at five percent annually.

counterparts. This is as expected since the collar obviously reduces the range of possible price outcomes.

Price Collars With Emissions Equivalence

The price collars based on S.1733 more frequently bind as a price ceiling than as a floor, and consequently act as a "safety-valve" (i.e., a hybrid instrument with only a price ceiling). Therefore, we also considered price collars with expected cumulative net emissions (domestic emission less offsets) at the same level as the cumulative allowance allocation of 132.2 GmtCO_2e. Given the often perceived flat marginal damage curve for pollutants such as CO_2, and assuming offsets provide true emission reductions, these collars can be thought to keep expected environmental damages the same as the no-collar cases. To implement this we kept the initial width of the collar the same as in S.1733 (i.e, $18) and used the same growth rates for the price ceiling and floors (i.e, five percent), but adjusted the initial levels of the collars to achieve expected emission equivalence.[19]

The results of this exercise are displayed in Table 4. As before, we present the NPV of costs, offset purchases, domestic emissions, RMSE of allowance prices, and percentage of periods with binding price collars. We also include the initial price ceiling level required to achieve the equivalent expected emissions results.[20] Again, we see the addition of collars lowers costs relative to the no-collar case. However, when achieving the equivalent expected emissions the collars reduce costs by only about one to two percent compared to the no-collar cases. In terms of domestic emissions, we see that these collars only reduce the range of emission outcomes by a small amount compared to the no-collar case, and involve roughly equivalent offset purchases as the no-collar cases. The results are not that surprising given that the collars are rarely binding and thus are rarely utilized (see "% Binding Ceiling" and "% Binding Floor" panels in Table 4).

In each scenario the initial collar values are adjusted to achieve emissions equivalence. In looking at the initial price ceilings levels, we see that the collars slightly increase (i.e., the price floor and ceiling increase at each time period) as the magnitude of the correlation between the shocks increases for a given level of offset supply shock persistence. As the persistence of the

[19] Expected net emissions are calculated as the mean of the cumulative domestic emissions less offsets purchased as determined from the simulation runs.

[20] The initial floors can be calculated by subtracting $18 and the price collars grow at an annual rate of five percent, as in the previous collar cases.

offset supply shocks increase, however, the collars shift upward in a much more noticeable manner. This result is primarily due to the ability to bank allowances and the increased long-run variance of the offset supply shock as the persistence increases. With the increased long-run variance there is a larger range of offset availability. When extreme low-cost offset states are realized, the firm can choose to bank additional allowances for future use as well as reduce some of the total allowances in the system by selling allowances back to the regulator at the price floor. On the other hand, when extreme high-cost offset states are realized, such that the price ceiling is triggered, additional allowances will strictly enter the system. Thus, for a given collar specification, a larger range of offset availability outcomes combined with a higher probability of being in extreme events will lead to an increase in total cumulative emissions. In order to maintain the expected net emissions equivalency, collars must shift up as the range of offset availability outcomes and the probability of extreme cost outcomes increases, thereby making it more difficult for additional allowances to enter the system and more beneficial to sell allowances at the price floor.

In general, these results highlight the importance of collar design in reducing costs. More specifically, if the collars are to be designed such that the expected environmental integrity of the system is to be maintained, collar spreads will have to be reduced to obtain meaningful cost reductions. Additionally, though the collar design from S.1733 is not drastically different from the set of expected equivalent emission collars, the cost results are considerably different. Given the limited information that the regulator may have about the forms of uncertainty in the system and sensitivity of the results to the collar specifications, it will be difficult to assess *a priori* effectiveness of alternative collar designs.

V. Additional Specification and Sensitivities

In addition to our primary modeling framework presented above, we also explore several alternative specifications and model parameterizations to gauge the sensitivity and generalizability of the results. We relax our monopsonistic offset purchaser assumption and consider a model where the representative firm has some degree of risk sensitivity. We also alter the maximum annual offset purchasing limit. There are no collars in any of these scenarios.

Multi-Firm Offset Purchasing

We relax the assumption of a monopsonistic offset purchaser and consider a model with oligopsonistic competition for offsets. To keep the focus on offsets, we ignore trading of allowances across firms and focus on the dynamic problem with offset purchasing as the sole means a firm has to add to its allocation. Representing the sum of all offsets purchased by firms other than firm *i* as $\sum_{j\neq i} z_{jt} = z_{-it}$, the Bellman for firm *i* can be written as:

$$V_{it}\left(B_{it},\theta_{it},\gamma_t\right) = -\frac{c_{it}}{2}\left(\overline{q}_{it}+\theta_{it}-q_{it}\right)^2 - \frac{z_{it}+z_{-it}}{\gamma_t} z_{it} + \beta E_t\left[V_{it+1}\left(B_{it+1},\theta_{it+1},\gamma_t\right) \mid B_{it},\theta_{it},\gamma_t,q_{it},z_{it},z_{-it}\right] \tag{18}$$

subject to the same constraints as (4), but scaled to the individual firm level. First order conditions of (18) will again lead to the Euler equation given in (11). However, the optimal offset purchasing condition changes to the following best response function:

$$z_{it} = \tfrac{1}{2}\left(\gamma_t c_{it}\left(\overline{q}_{it}+\theta_{it}-q_{it}\right) - z_{-it}\right) = \gamma_t\left(c_{it}\left(\overline{q}_{it}+\theta_{it}-q_{it}\right) - P_t^z\right) \tag{19}$$

If we further assume that all $i = 1,\ldots, N$ firms are identical and use the relationship that $P_t^z = \frac{1}{\gamma_t}\sum_{i=1}^{N} z_{it}$ along with (19), then we find the relationship between the firm's marginal cost of abatement and offset price as:

$$c_{it}\left(\overline{q}_{it}+\theta_{it}-q_{it}\right) = \frac{N+1}{N} P_t^z \tag{20}$$

Equation (20) encompasses our earlier result shown in (12) with $N = 1$, and shows that as *N* increases, the pecuniary externality created by purchasing more offsets is increasingly ignored and the offset purchasing result collapses to what one would expect under a competitive outcome where firms would equate marginal costs of abatement to offset prices.[21]

We solve this problem using the same backward recursive method described above, but the assumption that offsets are purchased according to the rule given in (20). Since we are particularly interested in comparing the monopsonistic purchasing case to a competitive outcome, in our simulation we consider the multi-firm offset purchasing framework under conditions where *N* is 10,000, large enough to assure that adding additional firms does not alter the results. Additionally, all parameters given in Table 1 are scaled up proportionally to reflect the value of an individual firm in this multi-firm context. Table 5 presents the NPV of abatement costs plus offset purchase costs, cumulative domestic emissions and offset purchases, and price *RMSE* results for the multi-firm offset purchasing case. The cost, emission and offset purchasing values are the aggregate values for the 10,000 identical firms.

[21] An offset provides an additional unit of emissions for firms. We would, thus, expect competitive firms to abate up to the point where the marginal cost of abatement is equal to the price of emissions, which in this case would be the price of offsets. See (12).

Qualitatively, the patterns for the costs, emissions, offset purchases, and *RMSE* values across ϕ_2 and ρ parameterizations are the same as those for the monopsonistic case. In terms of values, we see that the abatement plus offset purchase costs are greater for the multi-firm case versus the monopsonistic. This is as expected since in the multi-firm offset purchasing case the firms do not fully consider pecuniary externality associated with increased offset purchases and thus purchase a larger quantity of offsets at a higher price. However, we do not see large cost, emission, or offset purchasing differences between the multi-firm and monopsonistic offset purchasing systems. This is largely a product of the fact that even under the monopsonistic case, the cumulative offset purchases are near the maximum limit, given the 2GmtCO_2e annual offset purchase limit imposed in these runs. Although not presented here, we do indeed find greater differences in outcomes and sensitivities to uncertainty persistence and correlations for the multi-firm case compared to the monopsonistic case when the maximum offset purchase limit is increased.

It should also be noted that unlike the monopsonistic offset purchasing case, NPV of costs slightly decrease in the multi-firm offset purchasing case when we move from no offset supply uncertainty to the case with offset supply uncertainty and $\phi_2 = \rho = 0$. This result is largely a product of the expected offset supply curve slopes used in this analysis. As can be seen in Figure 1, the slopes of the offset supply curves are quite small in the early years of the program. Therefore, increased offset supply uncertainty is primarily increasing the probability of more offset availability in these years. With the increased reliance on offsets in the multi-firm offset purchasing scenario, this increase in probability for low-cost offsets in the early years of the program approximately make up for the additional cost of uncertainty for the case where persistence and correlations are extremely small.

Risk Sensitivity

The optimal policy rules for the control variables, and consequently the cost of the regulation, can be greatly affected by the agents' sensitivity to uncertain outcomes. Consequences of risk sensitivity has been considered to some degree in emission regulation analysis with respect to its impact on investment decisions and relationship between compliance choices and initial allocations (e.g. Fan et al. 2011, Hanneman 2009, Stranlund 2009, Ben-David et al. 2000). However, less attention has been given to this issue in dynamic models of emission regulations that include forms of self-insurance such as allowance banking. One would expect that, in efforts to self-insure against high-cost outcomes risk-averse firms to carry larger positive banks than risk-nuetral firms, analogous to the savings models in the macroeconomic literature.

With increased self-insurance through allowance banking, expected costs of the policy will increase as firms must build these banks through overcompliance.

In this paper, we introduce risk sensitivity with discounting in a recursive manner as developed in Epstein and Zin (1989) and Hansen and Sargent (1995). The inclusion of risk sensitivity in this fashion leads to an alteration of the Bellman equation in (7) to:

$$V_t\left(B_t,\theta_t,\gamma_t\right)=\max_{q,z}\left(\begin{array}{l}-\frac{c_t}{2}\left(\overline{q}_t+\theta_t-q_t\right)^2-\frac{z_t^2}{\gamma_t}\\-\beta\frac{2}{\sigma}\log\left(E\left[\exp\left(\frac{-\sigma}{2}V_{t+1}\left(B_{t+1},\theta_{t+1},\gamma_{t+1}\right)\mid B_t,\theta_t,\gamma_t,q_t,z_t\right)\right]\right)\end{array}\right) \quad (21)$$

subject to the allowance banking dynamics given in (5) and the uncertainty dynamics given in (1) and (2). In this formulation, σ is the coefficient of risk aversion where σ=0 implies risk neutrality and a more negative σ implies a more risk averse agent (Tallarini 2000).

Again, we solve the problem with risk aversion through the backward recursion solution method over the descritized state-space as described in Section III, but with the Bellman equation changed to (21) instead of (7). All parameterizations used in this risk aversion problem, other than σ, are the same as those given in Table 1.

Table 6 gives the expected NPV of abatement costs plus offset purchase costs, with 95 percent coverage intervals given in brackets below, for various ϕ_2 and ρ parameterizations and over a range of risk aversion parameters σ. As expected, for any given ϕ_2 and ρ parameterizations, decreasing σ (i.e., making the firm more risk averse) increases abatement costs. This is again driven by the fact that the more risk averse firm will bank more in the face of uncertainty to protect against high cost outcomes. The risk aversion cases also accentuate the sensitivity of the expected cost outcomes to the persistence in the offset uncertainty parameter and correlation between the sources. For instance, when σ = -0.01 we see that moving from the parameterization with $\phi_2=\rho=0$ to ϕ_2 = 0.8 and ρ = -0.8 increases expected cost by 60 percent as compared to a 13 percent increase in expected abatement costs between these two ϕ_2 and ρ parameterization for the risk neutral case. As can be seen in Figure 3, this difference in expected costs is driven by the by the relatively large amount of allowance banking that occurs in the high-persistence, high-correlation case (ϕ_2 = 0.8, ρ = -0.8) compared to the no-persistence, no-correlation case ($\phi_2=\rho=0$). Finally, because risk aversion motivates precautionary banking there is a positive probability that allowances could go unused in the last period. In practice this occurs rarely and the volume of unused allowances is small because the firm begins to rapidly draw down the bank in the final few periods.

Alternative Offset Caps

As discussed above, with the imposed 2 $GmtCO_2e$ cap on annual offset purchases, along with our assumed allowance allocation paths and expected baseline emission paths, the expected cumulative offset purchases are near the total cumulative allowed cap on offsets across all ϕ_2 and ρ parameterizations considered. This purchasing near the cap in part reduces the apparent variation in outcomes across different ϕ_2 and ρ parameterizations. We thus solved additional models under larger offset purchasing caps.

Table 7 reports expected NPV of abatement costs plus offset purchase costs, with 95 percent coverage intervals given in brackets below, for various ϕ_2 and ρ parameterizations and annual offset purchase caps. Of course, increasing the offset purchasing cap lowers costs, but it also increases the relative cost of having more persistent offset supply shocks and negatively correlated offset supply and baseline emission shocks. For instance, when the annual offset purchase limit is 4$GmtCO_2e$, moving from the parameterization with $\phi_2 = \rho = 0$ to $\phi_2 = 0.8$ and $\rho = -0.8$ increases expected cost by 33 percent, compared to a 13 percent increase in expected abatement costs between these two ϕ_2 and ρ parameterization for the case where the offset purchase limit 2$GmtCO_2e$ annually. Though not shown, it should also be noted that increasing the annual offset-purchasing limit also slightly increases the difference in the expected cost between the multi-firm offset purchasing model and the monopsonstic offset purchasing model. For example, when the annual offset purchase limit is 4$GmtCO_2e$ and $\phi_2 = 0.8$ and $\rho = -0.8$, the expected NPV of abatement costs plus offset purchase costs increase for the multi-firm offset purchase case with N = 10,000 is $663.4 billion dollars, roughly 2.3 percent higher than the monopsonistic offset purchasing case. From Tables 2 and 5, we see when the offset purchase limit is 2$GmtCO_2e$ and under the $\phi_2 = 0.8$ and $\rho = -0.8$, the multi-firm offset purchasing model leads to costs only one percent higher than that of the monopsonistic offset purchasing model.

VI. Conclusion

Recently proposed U.S. federal legislation aimed at reducing greenhouse gas emissions (e.g., U.S. Congress 2009a, 2009b), allow regulated entities to cover a significant portion of their emissions through the use of carbon offsets. Although offsets ostensibly provide a low-cost means to meet emission reduction goals, uncertainty remains about the quantity of offsets that will be available in any given time period. Furthermore, little is known about the relationship of this offset supply uncertainty to other sources of abatement cost uncertainty. In this paper, we present a model with uncertainty in abatement costs and uncertainty in offsets. We explore a range of possible parameters to determine how persistence in the uncertainties and correlation between the uncertainties affects total costs, emission outcomes, offset purchases, and emission price paths. This parameter exploration is conducted under two emission regulation policies: a quantity-based regulation with banking and limited borrowing and a quantity-based regulation with banking, limited borrowing, and an emissions' price floor and ceiling (i.e., a price collar).

Using a parameterization of the model keyed toward recently proposed federal cap and trade legislation H.R. 2454 (U.S. Congress 2009a), we find that for a quantity-based regulation without emission price limitations (i.e., no price collar), the expected NPV of abatement costs plus offset purchase costs increases as both the persistence to the shock and the negative correlation between offset uncertainty and baseline emissions uncertainty increase. This result is due, in part, to the specification of a convex cost curve and the imposed annual offset-purchasing quantity limits. For the quantity-based regulation without a price collar, we find that the range of possible cost, cumulative emissions, and price paths can be quite large in situations where both the offset supply shock is highly persistent and the negative correlation between offset uncertainty and baseline emissions uncertainty is relatively large in magnitude. In fact, we find that under specific persistence and uncertainty source correlation settings the variability in prices can actually be *greater* with offsets than without offsets. Additionally, for the cases without price limitations we find the potential for substantial price variability in the early periods because of the relatively high probability of limited offset supply availability. A wide range of possible prices in the early periods may be particularly worrisome for policy makers and market participants as great price variability will surely open the system to considerable criticism.

With the inclusion of a price collar patterned after the recent Senate bill, S.1733 (U.S. Congress 2009b), we find that for the same persistence and correlation parameters the expected NPV of costs is lower and the range of cost outcomes is narrower than without a collar, while expected emissions increase only slightly with the collar compared with the no-collar cases. Importantly, we also find that the range of possible cumulative emission outcomes can actually be smaller with a price collar versus a no-collar policy if the offset supply shock is highly persistent *and* the negative correlation between offset uncertainty and baseline emissions uncertainty is relatively large. For cases with low persistence of offset uncertainty or small negative correlation among the sources of uncertainty, we obtain the standard result that price collars increase the range of possible cumulative emission outcomes. In addition, our results show that cumulative offset purchases decrease slightly with the price collar compared with the no-collar cases. As expected, the range of possible price paths is smaller than with no-collar policies. However, when we design collar systems aimed at achieving the same expected net emissions as the no-collar cases, we find many of the positive features of the cost collar diminish, although this appears to hinge on the likelihood that the collar (either the price floor or ceiling) is binding. Given that these collars are close to those proposed in S.1733, it is clear that the effectiveness of collars will be highly dependent upon their specific design.

We also consider several alternative model specifications, including the case where offsets are purchased in a multi-firm, as opposed to a monopsonistic, environment, where firms are risk sensitive, and where the annual offset-purchasing limit increases. We find a slight cost increase when we consider the case where offsets are purchased in a more competitive market. However, we do find much more cost sensitivity when we allow for risk aversion or increase annual offset-purchasing limits. More specifically, we find that the sensitivity of the cost outcomes to changes in offset uncertainty persistence or small negative correlation among the sources of uncertainty is amplified if either the firms display risk aversion or if the annual offset purchasing limit is increased.

Overall, although we are able to show several potential problems associated with emission regulation programs heavily dependent on offsets, which may have their own political or environmental difficulties, we recognize that price collars are only one of the possible ways of addressing the situation. Additional work is clearly needed to examine how different offset procurement schemes may be better suited to harness the cost saving potential of offsets, while reducing the inherent uncertainties involved with using a market-based mechanism to contain system costs.

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Tables and Figures

Table 1. Model Parameterization Summary

Parameter	Value	Definition	Justification
T	39	Terminal period	Consistent with current climate legislation for 2012 – 2050
β	0.952	Discount factor	Assumes commonly used 5% discount rate
r_{bank}	0.0	Interest on banked permits	From H.R. 2454
$r_{borr.}$	0.0	Interest charged on borrowed permits	From H.R. 2454
ϕ_1	0.9	AR(1) parameter for baseline emissions shock	Based on regression results of historic U.S. CO_2 emissions
σ_1^2	0.10	Variance of error term in baseline emissions shock	Set to make long-run variance of the shock near 0.5 $GmtCO_2$
σ_2^2	0.01	Variance of random error term in offset supply shock	Set at low value because of relatively low values of $\bar{\gamma}_t$
g_c	-0.0125	Rate of decline for slope of marginal abatement cost curve	Modest rate set to reflect technological innovation in abatement costs
c_0	\$63/$mtCO_2e$ per $GmtCO_2e$	Initial slope of marginal abatement cost curve	Set to approximate the emissions price path of EIA's H.R. 2454 analysis for low discount case
z_{max}	2 $GmtCO_2e$	Maximum offset provision	From H.R. 2454
$B_{min,t}$	$-y_{t+1}$	Minimum bank level	From H.R. 2454
y_t	-	Allowance allocation	From H.R. 2454
$\bar{q}_t$	-	Expected baseline emissions	From EIA's H.R. 2454 analysis
$\bar{\gamma}_t$	-	Expected slope of offset supply curve	Based on offset supply schedules and emission prices in EIA's H.R. 2454 analysis
P_1^c	\$28/$mtCO_2e$	Initial period maximum allowance price with price collar implemented	From S. 1733
P_1^f	\$10/$mtCO_2e$	Initial period minimum allowance price with price collar implemented	From S. 1733

Table 2. No-Collar Model Results

Expected NPVs of Costs

ϕ_2 \ ρ	0	-0.4	-0.8
0	871.6 [398, 1558]	875.3 [388, 1597]	878.4 [378, 1617]
0.4	879.7 [402, 1587]	885.8 [384, 1642]	890.8 [375, 1682]
0.8	953.7 [403, 1806]	971.4 [373, 1992]	985.7 [353, 2125]

No Offset Supply Uncertainty: 868.4

No Offsets: 2888.7

Expected Offset Purchases

ϕ_2 \ ρ	0	-0.4	-0.8
0	63.7 [58, 69]	63.7 [58, 69]	63.8 [58, 68]
0.4	63.6 [56, 72]	63.6 [57, 72]	63.6 [58, 72]
0.8	61.2 [45, 76]	61.4 [45, 76]	61.3 [47, 75]

No Offset Supply Uncertainty: 62.8

No Offsets: 0

Expected Domestic Emissions

ϕ_2 \ ρ	0	-0.04	-0.08
0	195.9 [191, 201]	195.9 [191, 201]	196.0 [191, 200]
0.4	195.8 [188, 203]	195.8 [189, 203]	195.8 [190, 202]
0.8	193.4 [176, 207]	193.6 [177, 207]	193.7 [179, 206]

No Offset Supply Uncertainty: 195.0

No Offsets: 132.2

RMSE

ϕ_2 \ ρ	0	-0.04	-0.08
0	20.05	20.64	21.19
0.4	20.32	21.38	22.06
0.8	22.88	25.22	27.08

No Offset Supply Uncertainty: 19.8

No Offsets: 22.9

Notes: Expected NPVs of Costs include abatement and offset purchase costs and are in billions of 2007 dollars. Expected offset purchases and expected domestic emissions are in GmtCO_2e. Bracketed numbers are the 95 percent coverage intervals as determined by the simulations.

Table 3. Price Collar Model Results

Expected NPVs of Costs

ϕ_2 \ ρ	0	-0.04	-0.08
0	821.1 [418, 1169]	819.9 [412, 1168]	818.5 [414, 1170]
0.4	824.5 [420, 1177]	822.7 [413, 1173]	820.0 [409, 1177]
0.8	853.7 [426, 1197]	843.3 [408, 1213]	833.3 [390, 1214]

No Offset Supply Uncertainty: 813.7
No offsets: 978.2

Expected Offset Purchases

ϕ_2 \ ρ	0	-0.04	-0.08
0	63.5 [58, 68]	63.5 [59, 68]	63.4 [59, 67]
0.4	63.3 [56, 70]	63.3 [57, 70]	63.3 [57, 69]
0.8	60.6 [44, 74]	60.7 [45, 74]	61.0 [47, 73]

No Offset Supply Uncertainty: 62.5
No offsets: 0

Expected Domestic Emissions

ϕ_2 \ ρ	0	-0.04	-0.08
0	197.2 [190, 209]	197.4 [191, 209]	197.4 [191, 209]
0.4	197.2 [189, 209]	197.2 [189, 209]	197.6 [191, 208]
0.8	195.7 [181, 209]	196.5 [184, 209]	197.0 [187, 209]

No Offset Supply Uncertainty: 196.5
No offsets: 180.7

RMSE

ϕ_2 \ ρ	0	-0.04	-0.08
0	16.8	17.1	17.4
0.4	16.9	17.3	17.6
0.8	17.2	17.7	18.7

No Offset Supply Uncertainty: 16.3
No offsets: 0.0

Percent Binding Ceiling

ϕ_2 \ ρ	0	-0.04	-0.08
0	8.8%	9.9%	10.3%
0.4	9.3%	10.1%	11.4%
0.8	13.0%	14.6%	15.3%

No Offset Supply Uncertainty: 8.6%
No offsets: 100%

Percent Binding Floor

ϕ_2 \ ρ	0	-0.04	-0.08
0	0.6%	0.6%	0.7%
0.4	0.6%	0.7%	0.7%
0.8	0.6%	0.7%	0.8%

No Offset Supply Uncertainty: 0.5%
No offsets: 0.0%

Notes: Expected NPVs of Costs include abatement and offset purchase costs and are in billions of 2007 dollars. Costs do not include costs of purchasing additional allowances at the ceiling or sales of allowances at the price floor. Expected offset purchases and expected domestic emissions are in GmtCO$_2$e. Bracketed numbers are the 95 percent coverage intervals as determined by the simulations. Percent Binding Ceiling and Percent Binding Floor, are the total percentage of periods that had a marginal abatement cost equal to the price ceiling and floor, respectively.

Table 4. Price Collar Model Results with Expected Emissions Equivalency

Expected NPVs of Costs

ϕ_2 \ ρ	0	-0.04	-0.08
0	870.6 [396, 1541]	873.5 [388, 1567]	876.0 [379, 1598]
0.4	878.2 [399, 1580]	882.8 [381, 1623]	886.2 [365, 1660]
0.8	946.7 [386,1774]	957.3 [341, 1886]	963.5 [299, 1989]

No Offset Supply Uncertainty: 868.5
No offsets: 2838.4

Expected Offset Purchases

ϕ_2 \ ρ	0	-0.04	-0.08
0	63.4 [58, 68]	63.4 [59, 68]	63.4 [59, 67]
0.4	63.2 [56, 70]	63.2 [57, 70]	63.3 [57, 69]
0.8	60.7 [44, 74]	60.8 [45, 74]	61.1 [46, 73]

No Offset Supply Uncertainty: 62.5
No offsets: 0

Expected Domestic Emissions

ϕ_2 \ ρ	0	-0.04	-0.08
0	195.6 [186, 203]	195.6 [186, 203]	195.6 [186, 202]
0.4	195.4 [185, 204]	195.4 [185, 204]	195.5 [186, 203]
0.8	192.9 [178, 206]	193.1 [180, 206]	193.3 [182, 204]

No Offset Supply Uncertainty: 194.7
No offsets: 132.2

RMSE

ϕ_2 \ ρ	0	-0.04	-0.08
0	17.4	17.7	17.9
0.4	17.6	18.0	18.3
0.8	18.2	18.8	19.1

No Offset Supply Uncertainty: 17.3
No offsets: 16.1

Percent Binding Ceiling

ϕ_2 \ ρ	0	-0.04	-0.08
0	3.1%	3.5%	3.8%
0.4	3.4%	3.9%	4.4%
0.8	4.0%	4.9%	5.8%

No Offset Supply Uncertainty: 3.0%
No offsets: 9.3%

Percent Binding Floor

ϕ_2 \ ρ	0	-0.04	-0.08
0	1.3%	1.5%	1.6%
0.4	1.3%	1.4%	1.3%
0.8	1.6%	1.7%	1.8%

No Offset Supply Uncertainty: 1.2%
No offsets: 7.5%

Initial Value of Price Ceiling

ϕ_2 \ ρ	0	-0.04	-0.08
0	31.65	31.65	31.68
0.4	31.84	31.85	31.89
0.8	33.20	33.33	33.34

No Offset Supply Uncertainty: 32.16
No offsets: 58.34

Notes: Price collars rise at the rate of interest, with a fixed initial width of $18. Initial value of price ceiling has been adjusted to equalize total emissions at 132.23 GmtCO$_2$e. Expected NPVs of Costs include abatement and offset purchase costs and are in billions of 2007 dollars. Costs do not include costs of purchasing additional allowances at the ceiling or sales of allowances at the price floor. Expected offset purchases and Expected Domestic Emissions are in GmtCO$_2$e. Bracketed numbers are the 95 percent coverage intervals as determined by the simulations. Percent Binding Ceiling and Percent Binding Floor, are the total percentage of periods that had a marginal abatement cost equal to the P^c and P^f, respectively.

Table 5. Multi-Firm Offset Purchasing Model Results

Expected NPVs of Costs

ϕ_2 \ ρ	0	-0.4	-0.8
0	877.5 [404, 1563]	881.1 [397, 1599]	884.4 [379, 1643]
0.4	885.5 [400, 1588]	891.6 [390, 1643]	896.7 [376, 1688]
0.8	960.1 [407, 1846]	977.9 [376, 2005]	992.3 [348, 2097]

No Offset Supply Uncertainty: 877.6
No offsets: 2888.7

Expected Offset Purchases

ϕ_2 \ ρ	0	-0.4	-0.8
0	64.5 [59, 70]	64.5 [60, 69]	64.5 [60, 69]
0.4	64.3 [57, 71]	64.3 [57, 71]	64.4 [58, 71]
0.8	62.0 [46, 75]	62.0 [46, 75]	62.4 [48, 75]

No Offset Supply Uncertainty: 63.8
No offsets: 0

Expected Domestic Emissions

ϕ_2 \ ρ	0	-0.04	-0.08
0	196.7 [192, 202]	196.7 [192, 201]	196.7 [192, 201]
0.4	196.5 [189, 204]	196.5 [190, 203]	196.6 [190, 203]
0.8	194.2 [178, 208]	194.2 [178, 207]	194.6 [180, 207]

No Offset Supply Uncertainty: 196.0
No offsets: 132.2

RMSE

ϕ_2 \ ρ	0	-0.04	-0.08
0	20.4	20.9	21.7
0.4	20.7	21.7	22.6
0.8	23.4	25.9	27.4

No Offset Supply Uncertainty: 19.8
No offsets: 22.9

Notes: Expected NPVs of Costs include abatement and offset purchase costs and are in billions of 2007 dollars. Expected offset purchases and expected domestic emissions are in $GmtCO_2e$. Bracketed numbers are the 95 percent coverage intervals as determined by the simulations.

Table 6. NPV of Costs with Risk Aversion

ϕ_2, ρ	$\sigma = -1x10^{-5}$	$\sigma = -0.001$	$\sigma = -0.01$
$\phi_2 = 0, \rho = 0$	872.2 [404, 1551]	873.0 [414, 1538]	940.6 [516, 1547]
$\phi_2 = 0, \rho = -0.8$	879.1 [383, 1610]	880.1 [398, 1590]	954.8 [515, 1615]
$\phi_2 = 0.8, \rho = 0$	954.5 [408, 1820]	957.2 [422, 1794]	1378.8 [832, 2142]
$\phi_2 = 0.8, \rho = -0.8$	987.0 [351, 2115]	994.5 [374, 2071]	1506.8 [833, 2430]

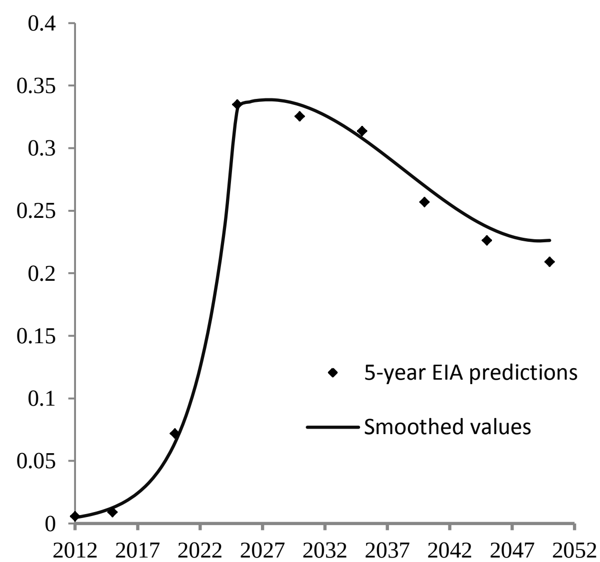
Notes: Values are NPVs of abatement costs plus offset purchases in billions of 2007 dollars. The modeling framework is based on the monopsonistic offset purchasing model without price collars and with risk sensitivity.

Table 7. NPV of Costs with Offset Purchasing Limit Sensitivity

	Maximum Annual Offset Purchases ($GmtCO_2e$)			
ϕ_2, ρ	2.5	3	3.5	4
ϕ_2 = 0, ρ = 0	684.8 [329, 1222]	572.7 [293, 1024]	513.4 [282, 875]	484.1 [275, 783]
ϕ_2 = 0, ρ = -0.8	692.8 [319, 1283]	582.4 [289, 1074]	525.2 [276, 918]	497.4 [268, 846]
ϕ_2 = 0.8, ρ = 0	774.5 [328, 1579]	667.1 [296, 1388]	610.1 [280.3, 1373]	584.0 [280, 1281]
ϕ_2 = 0.8, ρ = -0.8	817.3 [326, 1765]	719.5 [247, 1735]	670.4 [267, 1469]	649.2 [272, 1561]

Notes: Values are NPVs of abatement costs plus offset purchases in billions of 2007 dollars. The modeling framework is based on the monopsonistic offset purchasing model without price collars.

Figure 1. Expected Slope of Offset Supply Curve ($\bar{\gamma}_t$) Values



Notes: Five-year EIA predictions show the slope derived using EIA offset supply projections and EIA predicted offset price, for every 5th year of the program. We used both exponential and cubic fits to calculate the smoothed curve, which we took as our parameter values in the model.

Figure 2. Expected Bank Paths

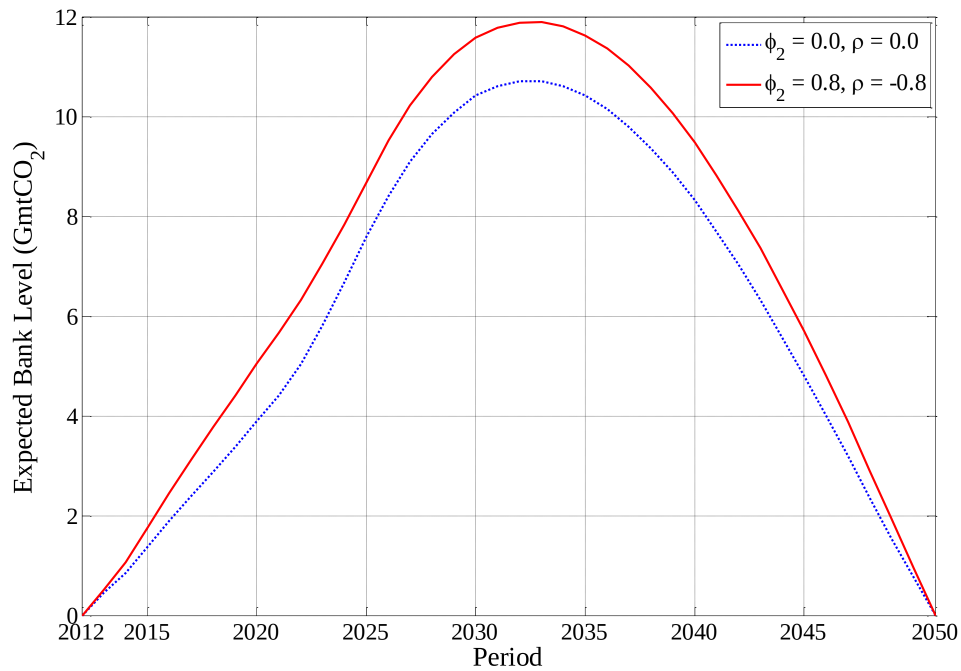


Figure 3.Expected Bank Paths under Risk Sensitivity (*σ* = -0.01)