Optimal Investment in Clean Production Capacity

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Abstract

For the mitigation of long-term pollution threats, one must consider that both the process of environmental degradation and the switchover to new and cleaner technologies are dynamic. We develop a model of a uniform good that can be produced by either a polluting technology or a clean one; the latter is more expensive and requires investment in capacity. We derive the socially optimal pollution stock accumulation and creation of nonpolluting production capacity, weighing the tradeoffs among consumption, investment and adjustment costs, and environmental damages.

We consider the effects of changes in the pollution decay rate, the capacity depreciation rate, and the initial state of the environment on both the steady state and the transition period. The optimal transition path looks quite different with a clean or dirty initial environment. With the former, investment is slow and the price of pollution may overshoot the long-run optimum before converging. With the latter, capacity may overshoot.

Key Words: pollution accumulation, clean technology, capacity investment

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Contents

List of Figures

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1 Introduction

In ongoing debates over how to mitigate long-term pollution threats, there is common agreement that the adoption of more environmentally friendly technologies is crucial. Environmental economists generally have focused on the creation of appropriate economic incentives for pollution control that would induce technology switching as well as reduced consumption of polluting goods and other mitigating responses. Key questions involve the optimal timing and use of investments in clean technologies. There is, however, less understanding of how this process would occur in practice.

In this paper we construct a model that is simple but nonetheless allows us to consider in some detail the interplay of two dynamic processes, the process of environmental degradation or improvement, and the process of developing clean production capacity. The model incorporates both tradeoffs between consumption benefits and environmental damages, and tradeoffs between investment and operating costs for clean production capacity versus the alternative of reducing pollution-creating consumption directly. Consideration of these tradeoffs allows us to explore how the time path of the pollution shadow price evolves, as well as the path of clean capacity creation and utilization. In particular, we find that the optimal steady state can be path dependent: specifically, it may depend on whether clean capacity is used to mitigate an immediate environmental problem or to forestall a future problem.

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A number of other papers have explored such issues. In some respects our analysis resembles those devoted to exploring the creation of backstop resource production capacity in the face of natural resource exhaustion (see Switzer and Salant 1986 and Oren and Powell 1984). Our analysis goes beyond these frameworks by bringing in pollution decay, thereby introducing a renewable resource aspect to the problem. A number of papers have considered problems of pollution accumulation and investment with uncertainty and irreversibility (Kolstad 1996; Ulph and Ulph 1997; Narain and Fisher 2000). Although our analysis is deterministic, we focus more explicitly and directly on path dynamics versus steady-state properties. Feichtinger et al. (1994) and Toman and Withagen (2000) incorporate the possibility of nonconvexities and threshold effects in the pollution damage function, but they do not focus on the creation of clean production capacity. Wirl and Withagen (2000) consider a problem similar to ours, but they limit attention to clean technologies with low variable operating costs; we consider technologies that are costly to create and operate.

By understanding the socially efficient path in this deterministic setting, we can lay the foundation for incorporating additional complexities and policy constraints. Since we do not consider market failures other than the pollution externality, our planning problem can be easily decentralized with an optimal series of emissions taxes. In essence, we focus on the optimal path of those taxes and how it depends on the state of the environment at the time the policy takes effect. The research thus addreses part of the ongoing debate about what portfolio of policies best support socially efficient technology transitions for such problems as climate change, accumulative pollutants like methyl bromide and other ozone depletors, or the protection of water bodies from accumulative pollutants.

2 Model without backstop

Suppose we have a commodity that initially is produced only with a dirty technology, which emits pollution as a by-product. Let q represent this dirty production, with marginal cost c and proportional emissions.[1] Let U be utility from consumption of the good with the following properties: $U'(q) > 0$ for all $q > 0$, $U'(0) = \infty$, $U'(\infty) = 0$ and $U'' < 0$.

The stock of pollution S increases with emissions q and assimilates at a rate of $\alpha > 0$.[2] The instantaneous net growth in the stock is

$$\dot{S}(t) = q(t) - \alpha S(t). \tag{1}$$

Let $D(S)$ be the damage flow from the stock of pollution with the following properties: $D'(S) > 0$ for $S > 0$, $D(0) = 0$, $D'(0) = 0$ and $D'' > 0$. As discussed in Toman and Withagen (2000), the assumption of additively separable consumption utility and pollution damage is not innocuous but very convenient analytically. The social objective is to maximize the integral of discounted utility derived from consumption minus production costs minus damage:

$$\int_0^\infty e^{-\rho t} \left(U(q(t)) - cq(t) - D(S(t)) \right) dt, \tag{2}$$

subject to (1), $q(t) \geq 0$, $S(t) \geq 0$, and $S(0) = S_0$, and where ρ is the social discount rate.

Let us define the current-value Hamiltonian (dropping the "(t)" for brevity):

$$H(S(t), q(t)) = U(q) - cq - D(S) + \psi(q - \alpha S). \tag{3}$$

Define $\tau \equiv -\psi$ to express what would be a negative shadow value of pollution as a shadow cost. In view of the Inada conditions on the instantaneous utility function, consumption will be

[1]We could assume an emissions rate other than 1, but effectively we are normalizing the pollution stock by the emissions rate.

[2]This assumption is made largely for simplicity; we expect our results would apply more generally to assimilation functions $\alpha(S)$ satisfying $\alpha'(S) \geq 0$. The case of $\alpha'(S) < 0$ is more complex, as shown in Toman and Withagen (2000).

positive at all instants of time. The necessary conditions are

$$U'(q+k) = c + \tau \tag{4}$$

and

$$\dot{\tau} = (\alpha + \rho)\tau - D'(S). \tag{5}$$

In a steady state, $\dot{S} = 0$ and $\dot{\tau} = 0$, implying

$$\tau = \frac{D'(S)}{\alpha + \rho}, \tag{6}$$

which is an upward-sloping function of the pollution stock.

From $\dot{S} = 0$ it follows that $q = \alpha S$. Rearranging the first-order condition with respect to q yields

$$\tau = U'(\alpha S) - c, \tag{7}$$

which is a strictly downward-sloping function of the pollution stock.

Thus, these two functions intersect only once and there exists a unique steady state that is asymptotically stable. Let S^D be the steady-state stock of pollution in this only-dirty production (no-backstop) case, satisfying

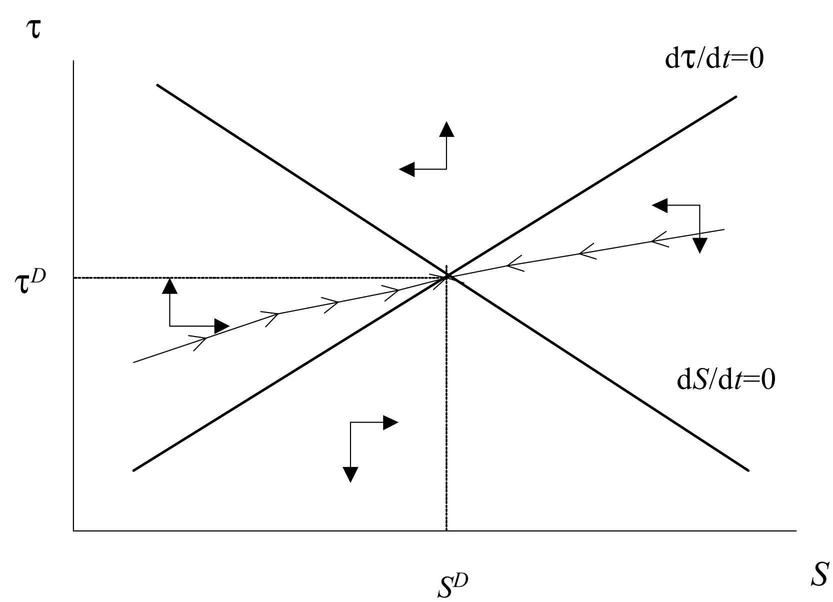
$$U'(\alpha S^D) - c = \frac{D'(S^D)}{\alpha + \rho}. \tag{8}$$

Define $\tau^D = D'(S^D)/(\alpha + \rho)$. Figure 1 draws the phase diagram for the steady state in the no-backstop case.

The comparative statics are straightforward. We have

$$\begin{aligned}&[\alpha U''(\alpha S)(\alpha + \rho)S - D''(S)]dS = \\ &[c - U''(\alpha S)(\alpha + \rho)S - U'(\alpha S)]d\alpha + [c - U'(\alpha S)]d\rho + (\alpha + \rho)]dc\end{aligned} \tag{9}$$

Figure 1: Steady State without Backstop



Hence, in view of the properties of the functions involved, higher production costs entail a smaller stock of pollutants in the steady state. The rate of time preference has a reverse effect. The impact of higher assimilation is ambiguous.

3 Model with backstop

Now suppose a nonpolluting backstop technology exists, but it requires costly investment in production capacity and also has higher production costs. Assume the commodity is uniform and consumers cannot distinguish which technology produced it. Total production then consists of production with the dirty technology, q, and production with the clean technology, k.

Let K be installed clean capacity of which $k \leq K$ is used; clean production is not required always to operate at capacity. The marginal cost of clean production is assumed to be more costly to use than the dirty one, else it would be used in the absence of pollution. We also assume that the marginal cost of clean production is less than the social marginal cost of only dirty production.

Thus, we define the marginal cost of clean production as b, such that

$$c < b < c + \tau^D. \qquad \text{(Assumption 1)}$$

The interpretation of the second assumption is straightforward. The right-hand side gives the marginal costs of an additional unit of production of the dirty commodity, namely the direct production costs plus the marginal pollution costs, in the absence of a backstop. If the marginal cost of producing a marginal unit according to the new technology were not less, it would not be optimal to develop clean production. The exception would be when the initial stock of pollutants is high, encouraging the use of clean technologies during the transition to the steady state. This possibility is discussed later.

We are also considering clean technologies whose capital and operating costs are higher than those of the dirty technology. This restriction means that clean production is only economically viable when the pollution externality is sufficiently large; furthermore, it allows for the possibility that dirty and clean production will co-exist in equilibrium. Wirl and Withagen (2000) consider a case in which the clean technology has no (and thereby lower) operating costs, but requires a fixed investment cost; an example of this type would be solar power. Therefore these cost assumptions are not innocuous. By disallowing lump-sum costs, we rule out "limit cycles" of expanding and contracting clean capacity due to nonconvexities.

Investing in additional capacity I incurs a cost $f(I)$, which is assumed to be positive and convex: $f(0) = 0$, $f(I) > 0$ for all $I \neq 0$, $f'(0) = 0$, $f'(I) > (<)\ 0$ for $I > (<)\ 0$, and $f'' > 0$. For technical reasons, we initially define investment costs and disinvestment savings. An important assumption is that

$$f'(0) = 0; \qquad \text{(Assumption 2)}$$

although somewhat unrealistic, it is a major analytical simplification. It could be justified if we think of our model as approximating the choice over a broad range of clean technologies, some of which have low installation costs (e.g., in-house process improvements) but limited capacity.

Let δ be the rate of depreciation in this capital stock. Therefore, the instantaneous change in capacity is

$$\dot{K}(t) = I(t) - \delta K(t). \tag{10}$$

We assume that the initial stock of the backstop technology equals zero: $K(0) = 0$.

The planner chooses $q(t)$, $k(t)$, and $I(t)$ to maximize for $t \in [0, \infty)$ the discounted value of consumption of the good net of production and pollution costs:

$$\int_0^\infty e^{-\rho t} \left(U(q(t)) - cq(t) - bk(t) - f(I(t)) - D(S(t)) \right) dt \tag{11}$$

subject to (1), (10), $S(0) = S_0$, $K(0) = 0$, $q(t) \geq 0$, $k(t) \geq 0$, $I(t) \geq 0$, and also the capacity constraint: $k(t) \leq K(t)$.

The current-value Hamiltonian for this two-state optimal control problem is

$$\begin{aligned} &\mathcal{H}(S, K, q, k, I, \psi, \varphi) = \\ &U(q + k) - cq - bk - f(I) - D(S) + \psi[q - \alpha S] + \varphi[I - \delta K]. \end{aligned} \tag{12}$$

Incorporating the capacity constraint, we get the following Lagrangian:[3]

$$\mathcal{L}(S, K, q, k, I, \psi, \varphi, \lambda) = \mathcal{H}(S, K, q, k, I, \psi, \varphi) - \lambda[k - K] \tag{13}$$

To describe the optimum, we have the first-order complementary slackness conditions with respect to the control variables. In each of the following pairs, one of the equations must hold with

[3] For optimal control problems with inequality constraints, see Léonard and Long (1992).

equality:

$$q \geq 0, \quad U'(q+k) \leq c+\tau; \tag{14}$$

$$k \geq 0, \quad U'(q+k) \leq b+\lambda; \tag{15}$$

$$f'(I) = \varphi. \tag{16}$$

We define $\tau = -\psi$, as before, and obtain the remaining necessary conditions for an optimum:

$$\dot{\tau} = (\alpha+\rho)\tau - D'(S), \tag{17}$$

$$\dot{\varphi} = (\delta+\rho)\varphi - \lambda, \tag{18}$$

$$\lambda[K-k] = 0. \tag{19}$$

This yields the following preliminary results. First, from the maximization of the Hamiltonian, we find that if there is production of the dirty commodity, marginal utility ("price") of the good equals social marginal costs of production, inclusive of the cost of the pollution externality ($c+\tau$). Second, use of the backstop technology requires equality of marginal utility of the commodity and the marginal production costs, consisting of direct marginal production costs (b) and the shadow value of capacity. Note that since the marginal costs of output (b and $c+\tau$) are independent of the output rates (k and q, respectively), output rates will reflect corner solutions except at transition points.

To have positive capacity expansion in any period, (16) says that the marginal cost of investment must equal the shadow value of added capacity. That value is determined by the present value of future capacity constraints to clean production, discounted by the rates of time preference and depreciation.[4]

[4]Lemma 4 will show that the shadow value of capacity is positive as long as capacity is constrained at some point now or in the future.

According to (17), the shadow external cost of output rises at the rate of interest and assimilation, less the current (embodied) rate of marginal damages. The higher current marginal damages reduce the value to postponing reductions; therefore, the rise in external costs slows.

4 Steady state

In the steady state, $\dot{S} = \dot{K} = 0$. Denoting the steady-state values by "*", we have

$$q^* = \alpha S^*; \tag{20}$$

$$I^* = \delta K^*. \tag{21}$$

Thus, in any steady state with dirty production, emissions just equal the environment's capacity to assimilate and capacity investment just replaces depreciation. Clearly, $I^* > 0$ unless $K^* = 0$, which we show below is not the case.

In the steady state, since K and I are constant, it follows from (16) that φ is constant. With a constant S we can solve the differential equation for τ. If τ goes to infinity, then consumption goes to zero, which is suboptimal. Nor should τ go to $-\infty$. Therefore τ goes to a constant. Thus, from $\dot{\tau} = \dot{\varphi} = 0$, we get

$$\tau^* = \frac{D'(S^*)}{\rho + \alpha}; \tag{22}$$

$$\lambda^* = (\rho + \delta) f'(\delta K^*). \tag{23}$$

We state the following lemmata to characterize the steady state:

Lemma 1 $q^* > 0$.

Proof. Suppose $q^* = 0$. From (14), then $U'(k) \leq c + \tau^*$. From (20), then $S^* = 0$. This implies that $\tau^* = 0$. This violates the first-order conditions, since from (15), $U'(k) \geq b > c$. ■

This lemma rests on the assumption that $\alpha > 0$: with assimilation, some dirty production must exist in the steady state. If dirty production ceased and the environment were to become completely clean, society could save costs by switching some production to the dirty technology, as the burden imposed by the extra emissions would be less than the savings in operating costs.

Lemma 2 $K^* > 0$.

Proof. Since $q^* > 0$, (14) implies that $U'(q^* + k^*) = c + \tau^*$. In the steady state we have a constant K^*. Then also I^* is constant and so is φ^*. Hence $\varphi^* = f'(I^*) = \lambda^*/(\delta + \rho)$. It follows from (15) that $U'(q^* + k^*) \leq b + (\delta + \rho) f'(I^*)$. Suppose that $K^* = 0$. Then $k^* = 0$ and $I^* = 0$. Hence $c + \tau^* \leq b$. However, in this steady state with no backstop, we have $S^* = S^D$ and $\tau^* = \tau^D$. This violates the initial assumption that $b < c + \tau^D$. ■

Lemma 3 $k^* = K^*$.

Proof. Suppose not. Then $\lambda^* = 0 = \varphi^* = K^*$, which would violate Lemma 2. ■

Thus, an immediate consequence of Lemma 2 is that no excess capacity exists in the steady state. Otherwise, one could save on investment costs by disinvesting or allowing depreciation to remove unused capacity.

Now we can compare the steady states in the respective cases by means of a graph. From the first-order conditions for both production types, we get

$$U'(K^* + \alpha S^*) = c + \frac{D'(S^*)}{\rho + \alpha}; \quad (24)$$

$$U'(K^* + \alpha S^*) = b + (\rho + \delta) f'(\delta K^*). \quad (25)$$

Let us consider these equations in detail, omitting * when there is no danger of confusion.

Consider (24) first. If $S = 0$, then K satisfies $U'(K) = c$ (because $D'(0) = 0$). If $K = 0$, then S follows from $U'(\alpha S) = c + D'(S)/(\rho + \alpha)$. The locus of intermediate points is shown in Figure 2 as the curve "q f.o.c."

Consider (25). If $S = 0$, then K satisfies $U'(K) = b + (\rho + \delta) f'(\delta K) > c$. If $K = 0$, then S follows from $U'(\alpha S) = b$. The locus of corresponding intermediate points is shown in Figure 2 as the curve "k f.o.c."

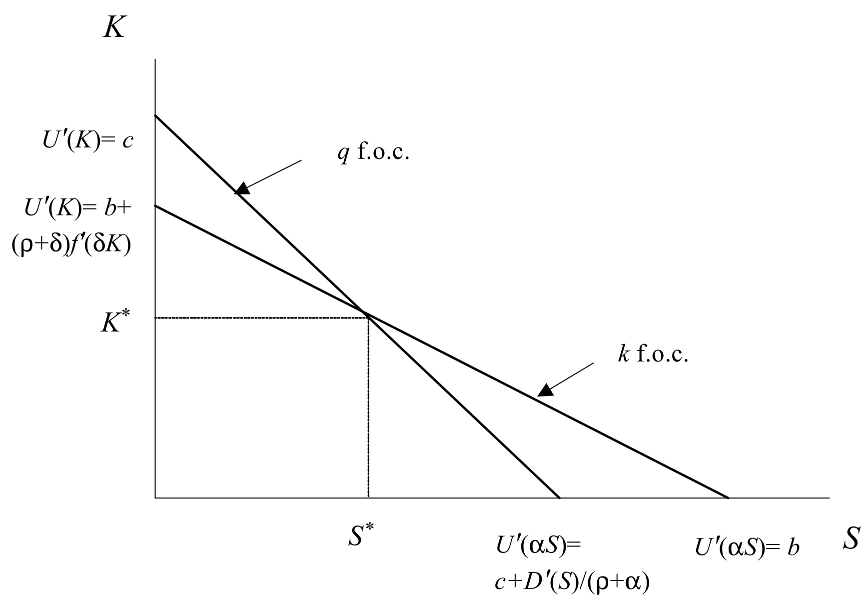
Deriving the respective slopes of these curves, we can see that "q f.o.c." is steeper than "k f.o.c.":

$$\left.\frac{dK}{dS}\right|_{\frac{\partial H}{\partial q}=0} = \frac{D''/(\rho+\alpha) - \alpha U''}{U''}, \tag{26}$$

$$\left.\frac{dK}{dS}\right|_{\frac{\partial H}{\partial k}=0} = \frac{-\alpha U''}{U'' - (\rho+\delta)\delta f''}. \tag{27}$$

Note that $D''/(\rho + \alpha) - \alpha U'' > -\alpha U''$ and $U'' - (\rho + \delta)\delta f'' < U'' < 0$.

Figure 2: Steady State with Backstop



Thus, the two curves (drawn in Figure 2 as lines) cross, and cross only once. A steady state exists, and it is unique and (at least locally) asymptotically stable.

As noted above, with $c + \tau^D > b$, some clean production exists in the steady state. In terms of the figure, if that condition did not hold, the "k f.o.c." line would lie wholly inside "q f.o.c." We also assume that $c < b$, so some dirty production must also exist. Otherwise, "q f.o.c." line would lie wholly inside "k f.o.c." and no intersection would occur.

5 Characterization of optimal paths

From the optimality conditions, we can characterize the paths to the steady state. First, we can establish some bounds on investment and capacity along an optimal path.

Lemma 4 *If there exists t_1 such that $I(t_1) > 0$, then $I(t) > 0$ for all $t < t_1$.*

Proof. By definition, $\varphi \geq 0$. Suppose $\varphi(t) = 0$ for all $t < t_1$ and $\varphi(t_1) > 0$. Then, according to (18), $\dot{\varphi}(t_1^-) = -\lambda(t_1) > 0$, which contradicts $\lambda(t) \geq 0$. ■

Thus, the shadow value of capacity is positive as long as capacity is constrained at some point now or in the future. Until capacity is constrained, according to (18) the shadow price of capacity, and thereby the marginal investment cost, rises at the rate of interest and depreciation. Capacity expansion occurs to smooth subsequent expansion costs over time (since $f'' > 0$). Once clean capacity is constrained, the rate of expansion slows.

Next we place an upper bound on K. Consider the problem of maximizing social welfare in the absence of the dirty technology:

$$\max \int_0^\infty e^{-\rho t}[U(\tilde{K}(t)) - b\tilde{K}(t) - f(\tilde{I}(t)]dt,$$

subject to $\dot{K}(t) = \tilde{I}(t) - \delta\tilde{K}(t)$, $\tilde{K}(0) = \varepsilon > 0$ (since we have assumed $U'(0) = \infty$). Given that the initial capital stock is small, using the same methods as before, it is straightforward to show that for all $t \geq 0$ we have $\tilde{K}(t) \leq \tilde{K}^*$, where $\tilde{K}^*$ is the capacity level in a steady state without dirty production, solving $U'(\tilde{K}^*) = b + (\rho + \delta)f'(\delta\tilde{K}^*)$.

Lemma 5 $\tilde{K}(t) \leq \tilde{K}^*$ *for all* t.

Proof. If $q(t) = 0$ for all $t \geq 0$, then $K(t) \leq \tilde{K}^*$. Suppose for some t, $q(t) > 0$. If $k(t) = \tilde{K}(t)$, then marginal utility is lower, implying lower $\lambda(t)$, and lower $\varphi(s)$ for $s < t$, which means $I(s) < \widetilde{I}(s)$ and then $K(s) < \tilde{K}(s)$ for $s \leq t$. Thus, $K(t) \leq \tilde{K}(t) \leq \tilde{K}^*$ ■

With polluting production available, demand for capacity can only be lower. There is never any economic justification to expand capacity beyond what would be used without dirty production.

The following will also prove useful:

Lemma 6 $\tau(t) < b - c$ *implies* $\dot{\tau}(t) > 0$.

Proof. Suppose for some $t_1 \geq 0$, we have $\tau(t_1) < b - c$. $\dot{\tau}(t_1) > 0$ if and only if $\tau(t_1) > D'(S(t_1))/(\alpha + \rho)$. Given the hypothesis of the lemma from (15), $k(t_1) = 0$; then $\dot{S}(t_1) > 0$ if and only if $\tau(t_1) < U'(\alpha S(t_1)) - c$. We are in a region with $\tau(t_1) < b - c < \tau^D$, such that $k(t_1) = 0$ and the phase diagram of Figure 1 applies. It is easily seen that if we are in a region with $\dot{\tau}(t_1) \leq 0$, we will stay in that region forever, which is not optimal. ■

In principle, there are eight possible regimes, as listed in this table.

Regime	K	k	q
I	0	0	0
II	0	0	+
III	k	+	0
IV	k	+	+
V	+	0	0
VI	+	0	+
VII	$> k$	+	0
VIII	$> k$	+	+

However, only a few regimes are possible for an optimal path.

Regime I implies no production at all, which cannot occur along an optimum, since we assume $U'(0) = \infty$.

Regime II has only dirty production and no capacity building. It cannot occur along an optimal path that ultimately involves clean production, as it would violate Lemma 4, that some investment smoothing always occurs with $f'(0) = 0$.

Regime III has only clean production occurring at capacity. It may be possible when starting with a dirty environment, as discussed in section 6.2.

Regime IV has both dirty production and clean production at capacity.

Regime V implies no production at all, which cannot occur along an optimum, since, as with Regime I, we assume $U'(0) = \infty$.

Regime VI has only dirty production but clean capacity available. As shown in section 6.1, this regime can occur when starting from a relatively clean environment, smoothing investment costs in anticipation of needing clean production.

Regime VII implies excess capacity with no dirty production. This regime cannot occur along an optimum. Since $K > k > 0$ and $q = 0$, (15) implies that $U'(k) = b > U'(K)$. Thus, at this level of clean production, $K > k > \tilde{K}^*$, which violates Lemma 5. It is clearly suboptimal to create capacity that would never be used.

Regime VIII has dual production and excess capacity. It cannot occur when starting with a below-equilibrium pollution stock, the scenario discussed in section 6.1, since in that case k increases and q decreases monotonically, and excess capacity is never developed. It could *theoretically* occur when starting from a dirty environment, when clean capacity needed early on takes longer to depreciate than pollution takes to assimilate. However, we can safely—though not easily—exclude regime VIII in this case as well, and do so in the Appendix.

The analysis leads to the conclusion that there are only three regimes to be considered as candidates for an optimal path when $\delta > 0$ and $f'(0) = 0$: III, IV, and VI. Moreover some transitions are ruled out.

Lemma 7 *There is no direct transition possible from regime III to regime VI in an optimum.*

Proof. In regime III we have $U'(K) \leq c + \tau$ and $U'(K) = b + \lambda$, so $\tau \geq b - c$. In regime VI we have $U'(q) = c + \tau$ and $U'(q) \leq b$, so $\tau \leq b - c$. Suppose there is a transition from regime III to regime VI at time $\bar{t}$. Then, since $\dot{\tau} \geq 0$ as long as $\tau \leq b - c$ (lemma 6), we have $\tau = b - c$ along regime VI. Therefore, S is overdefined: if $S = \widehat{S}$, defined by $D'(\widehat{S})/(\alpha + \rho) = b - c$, and $q = \alpha\hat{S}$ along regime VI, then $U'(\alpha\hat{S}) \neq b$, a contradiction. ■

Lemma 8 *There is no direct transition possible from regime VI to regime III in an optimum.*

Proof. Suppose there is a transition from regime VI to regime III at time $\bar{t}$. Consumption is continuous over time, implying that at the transition point, $U'(K) = U'(q) = b$ and $\lambda = 0$. Define $\bar{K}$ by $U'(\bar{K}) = b$. $\bar{K} > \tilde{K}^*$, which violates lemma 5 for any $\delta > 0$. Alternatively, $I(\bar{t}-) \geq \delta\bar{K}$ and $I(\bar{t}+) \leq \delta\bar{K}$. Therefore $\lambda(\bar{t}) \geq (\rho + \delta) + f'(\delta K^*) > 0$, a contradiction. ■

With those regimes and lemmata established, we can describe the equilibrium path. Suppose the initial stock of pollutants is small and investment costs are low. Then it is optimal to have an initial interval of time when the backstop technology is put in place but not actually used (regime VI). All consumption comes from the conventional technology. After some time a transition is made to the simultaneous use of both technologies (regime IV)

Suppose the initial stock of pollutants is huge. Then there will be an initial, short, interval of time where there is simultaneous use of both technologies. This is the case because it has been assumed that initially there is no backstop technology installed and $U'(0) = \infty$, so some consumption from the old technology is needed at the outset. In the second stage all production may be carried out in the backstop (regime III). This cannot go on forever, and eventually there will be simultaneous use of both technologies (regime IV).

6 Optimal shadow price trajectories

In this section we describe the optimal shadow price trajectories under different circumstances. It turns out that it is important to make a distinction between small and high initial stocks of pollutants.

6.1 Small initial stock of pollutants

This case of a small initial stock of pollutants is similar to the optimal overshooting model of Switzer and Salant (1986). Their problem of optimal investment in backstop capacity for an exhaustible resource is analogous to a scenario without decay of the pollution stock or depreciation of the backstop. We discuss a more general case.

The path of the shadow value of pollution (and thereby the social "price" of the good) follows a trajectory of four distinct phases: dirty production with clean investment, dual production, overshooting, and convergence.

Dirty Production

If the initial stock of pollutants is small, it is clearly optimal not to use the backstop technology immediately, since the shadow price of pollution is still small. During this initial period, when $\tau < c - b$, there is only dirty production and τ rises along with the pollution stock. Any existing capacity will not be utilized as long as $U'(q) < b$. However, this does not imply that there are no investments in the clean technology. Since $f'(0) = 0$ and $f'' > 0$, the existence of *any* positive level of investment at *any* time in the future along the optimal path will trigger some current investment. One always wants to take advantage of negligible costs at low levels of investment.

Dual Production

The first phase is followed by a period in which there is dual production. Once $c + \tau \geq b$, the existing backstop technology comes online and is used at capacity. At the start of this period, there is a downward jump in dirty production. For that reason the stock of pollution will continue to increase in the beginning of this period, but at a slower rate. The shadow price of pollution continues to rise, and investment also continues.

Overshooting

For relatively low rates of capacity depreciation, the shadow value of pollution will actually overshoot its steady-state value. In other words, the environment may have to grow dirtier before it can become cleaner. To demonstrate this, we prove

Lemma 9 *There exists* $t_1 > 0$ *such that* $\tau(t_1) \geq b - c$*.*

Proof. If the price never rises above the marginal cost of producing an initial unit with the clean technology, no rents would ever be generated to offset investment costs. This result follows from the maximization of the Hamiltonian. Suppose that $c + \tau(t) \leq b$ for all t. Then $\lambda(t) \leq 0$ for all t. Obviously, $\lambda < 0$ is impossible, and $\lambda = 0$ implies from (18) that $\dot{\varphi} = \rho\varphi$, implying that $\varphi \longrightarrow \infty$, which is a contradiction. ■

From the steady-state conditions (24) and (25), it follows that

$$\tau^* = b + (\rho + \delta) f'(\delta K^*) - c. \tag{28}$$

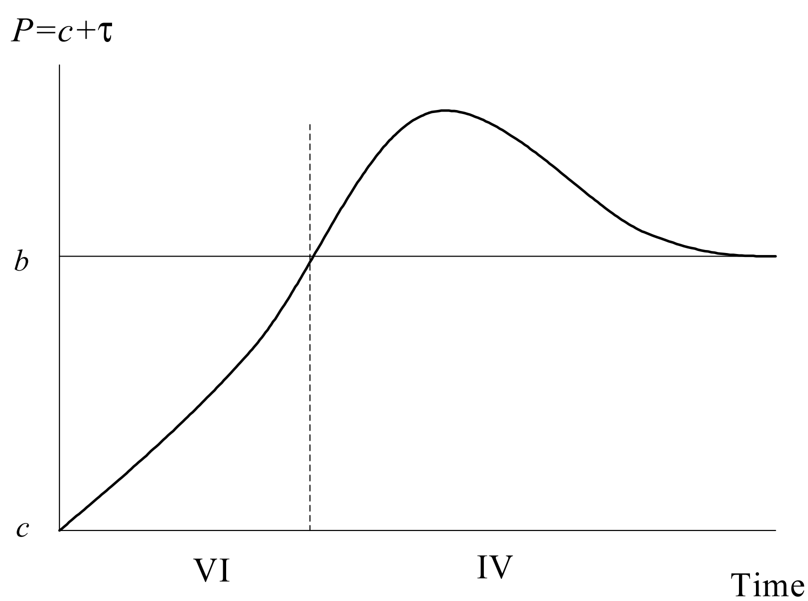
Note that as $\delta \to 0$, the right side of this equation converges to $b - c$, given our assumption of $f'(0) = 0$. It is possible that after the second phase there will be a third phase in which the shadow price of pollution overshoots its steady-state value.

Lemma 10 *If* $\delta = 0$*, there exists* $t_1 > 0$*, such that* $\tau(t_1) \geq \tau^*$.

Proof. This result falls out of Lemma 9 and (28), since in this case the steady-state shadow value of pollution equals that threshold value: $\tau^* = b - c$. ■

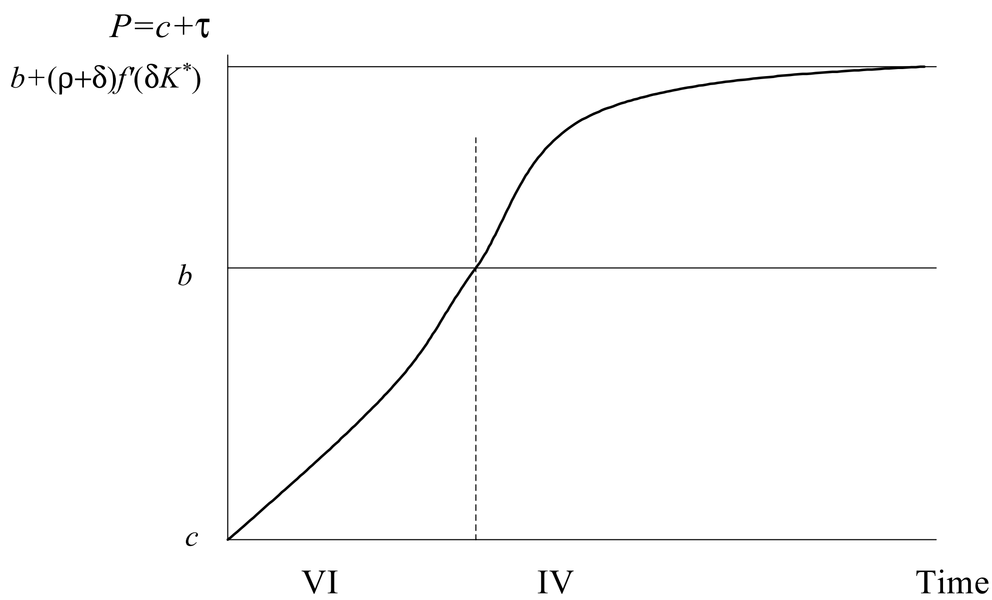
The economic intuition behind Lemma 10 is that in the special case of zero backstop capital depreciation, the socially optimal price of the dirty good must at some point rise higher than the variable cost of clean technology (b) if there is to be investment in creating clean capacity. By continuous dependence on parameters, overshooting also will occur with positive but moderate depreciation rates.

Figure 3: Price Path with No Depreciation



With higher depreciation rates, however, the shadow price may rise monotonically, since high steady-state investment is needed to maintain capacity. This case is portrayed in the figure 4.

Figure 4: Price Path with High Depreciation



An implication of shadow price overshooting is pollution overshooting.

Lemma 11 *S overshoots S^*.*

Proof. Suppose that $S_0 = S^*$ (the optimal steady-state value), and that along the optimum $S(t) \leq S^*$ for all $t \geq 0$. Then $\tau(t) \leq \tau^*$ for all $t \geq 0$. Since the initial stock of capital is zero, we must then have $q(t) \geq q^*$, and pollution will initially grow higher than the steady-state stock, a contradiction. ■

In other words, for the shadow value of pollution to be higher at its peak than in the steady state, it must be that the stock of pollution is greater than the steady-state level at some point between there and the steady state.

Convergence

In a final phase the shadow price of pollution starts converging to the steady state. If clean capacity depreciates rapidly enough, the pollution stock may rise monotonically along with the shadow price of pollution. If τ overshoots its steady-state value, it must necessarily hit a peak and then decline as it converges. In this case, the pollution stock must also peak and then decline to its steady-state level.

Lemma 12 *If $\tau(t)$ peaks at $t = t_1$, then $S(t)$ peaks at $t = t_2 > t_1$.*

Proof. Consider any pairs of points $t_x < t_y$ such that $\tau(t_x) = \tau(t_y)$. Since t_x is on the rise to the peak and t_y is on the decline to the steady state, t_x is closer to a period of higher pollution stocks and t_y closer to the steady state of lower pollution stocks. Thus, to have the present discounted value of marginal pollution damages equal at those two points, it must be that $S(t_y) >> S(t_x)$. Taking an infinitely small interval ϵ, since $S(t_1 - \epsilon) >> S(t_1 + \epsilon)$, it must be that S continues to rise after t_1. Since we know it must peak in order to decline to the steady-state stock, we get $t_1 < t_2 < \infty$. ■

6.2 Large initial stock of pollution

Suppose now that one inherits a pollution stock that is relatively large, such that $S(0) > S^*$ along an optimal path. For example, one may begin from a no-policy steady state, S^N, such that $U'(\alpha S^N) = c$. Then one starts to implement both the environmental pricing policy and the capacity investments simultaneously, as the environment had become far too dirty in the absence of intervention. Although the steady state does not vary according to the initial pollution stock (assuming $\delta > 0$), the transition can look quite different.

Starting with a large pollution stock, we have a sequence of several possible stages: dual production and rapid investment, clean production alone, dual production with capacity overshooting, and convergence. If we relax the assumption that $c + \tau^D > b$, the last stage could involve only dirty production.

Dual Production and Rapid Investment We know from the previous section that with a very dirty initial environment, $\tau_0 > \tau^D > b - c$, clean investment and production is immediately justified. First there will be an initial period of simultaneous clean and dirty production, with much of the weight of environmental recovery carried by curtailment of total (mostly dirty) consumption as its social cost is internalized. Along the path, then, we will see reduction in pollution and shadow price and a rapid buildup in clean capacity. Until (and unless) sufficient capacity exists to displace dirty production, we start and remain in regime IV.

Clean Production Once enough capacity is built, the economy may pass into a regime III path ($k = K > 0 = q$), relying on clean production to allow environmental recovery. During this time, $c + \tau(t) > U'(K(t))$, and the latter is bounded by $U'(K(t) \geq b + (\rho + \delta) f'(\delta \tilde{K}^*)$. Passing through this regime depends on the extent of initial pollution and how slowly the environment recovers compared with how quickly capacity can be built.

Capacity Overshooting and Convergence Clean capacity may increase monotonically to the steady state, its growth slowed as the environment recovers and some dirty production is reintroduced. But it is also possible that clean capacity will overshoot the steady state.

This is easily seen if we assume (contrary to our previous hypothesis) that $c + \tau^D \leq b$, such that dirty production just satisfies demand at the steady state with social costs internalized. Starting from a relatively clean environment, no clean capacity would be built. With a dirty environment, however, $c + \tau^0 > b$, signaling for clean capacity to be built. Later it is depreciated away as the economy converges toward the steady state.

In our analysis the assumption $b < c + \tau^D$ ensures joint utilization of both clean and dirty production in the steady state. Nevertheless, by extension of the argument with $b = c + \tau^D$, one can see that clean capacity will rise and then fall if the initial environment is very dirty.

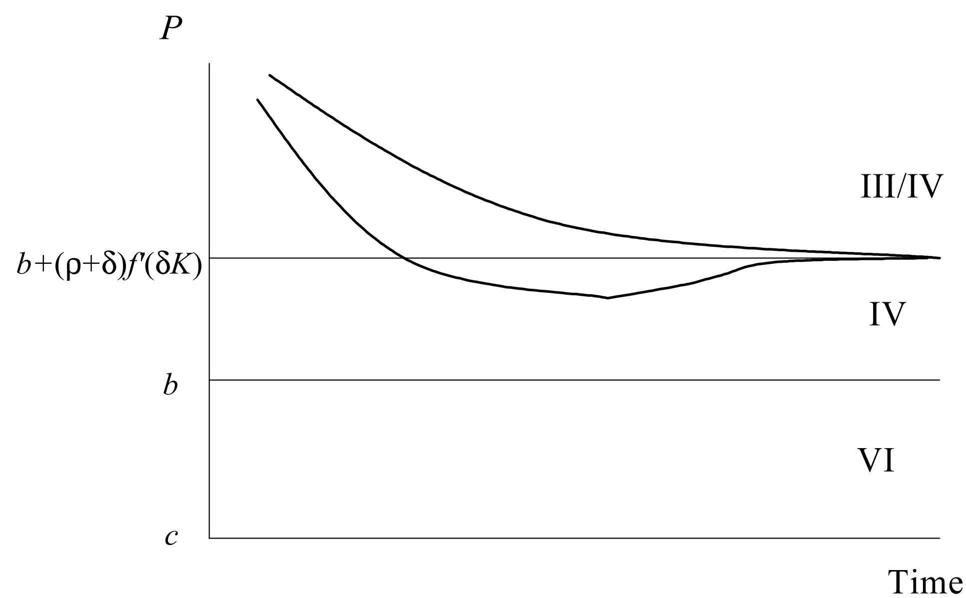
Another interesting wrinkle arises in the special case of $\delta = 0$ (no capacity depreciation). In this case, little clean capacity will be built if the environment starts out relatively clean: our previous K^*. But if the environment starts out dirty, then a lot of capacity will be built ($K^{**} > K^*$) that permanently displaces dirty production in the steady state. From (24), then $S^{**} < S^*$. With no depreciation, that excess capacity represents a kind of technological "lock-in" that affects the final equilibrium. Starting from a dirty environment, the steady state has more total output and less pollution than if one starts with a clean environment!

Capacity overshooting, unlike pollution overshooting, can occur with high or low depreciation rates. Low depreciation rates mean that capacity that was installed to speed environmental recovery lingers afterward, allowing for net disinvestment when pollution is sufficiently assimilated. High depreciation rates make investment more like a variable cost of clean production. Consider the extreme case of a very high δ. Starting from S^N, capacity is immediately put in place to displace some or all of the dirty production. That in turn causes the pollution stock to decline, which induces investment (and thereby capacity) to decline monotonically toward the steady state once $c + \tau < b + (\delta + \rho) f'(\tilde{K}^*)$. Thus, for high rates of depreciation, the path may follow regime IV throughout or pass from III to IV.

Theoretically, it may be possible for pollution to overshoot the steady state as well. We speculate that pollution undershooting could occur in the low depreciation case: clean capacity that was installed when the environment was very dirty lingers a long time, continuing to displace dirty production even once marginal damages are smaller. With $k > k^*$ and declining slowly, we have $q < \alpha S^*$ for longer, possibly long enough to allow τ (and implicitly, but not simultaneously, S) to fall below the steady-state value. With net depreciation of clean capacity, dirty production rises again, converging to the steady state. However, at no time in this process can we be in regime VIII, having dual production and excess capacity.

Figure 5 portrays possible paths when starting from a dirty pollution stock.

Figure 5: Optimal Paths with Large S_0



7 Conclusion

In this paper we have developed as simple a model as possible that still captures the dynamics of pollution accumulation or decay and capital accumulation for clean technology. We look mainly at the case in which both clean and dirty production coexist in the socially optimal steady state. If the clean technology has a dominant cost advantage over the dirty technology and one's pollution damage costs are internalized, then of course the clean technology will displace the dirty technology as soon as capacity can be accumulated (leaving aside other possible market failures related to technology diffusion that are not addressed in this paper). And if utilizing the clean technology makes economic sense only in a highly polluted environment, it naturally follows that this technology will not have a future once the environment recovers.

Although the steady state in our model is invariant with respect to initial conditions (given depreciable capacity), the transition paths are very different. With a clean initial environment, clean capacity is built up gradually, and it is possible (if capital depreciates slowly) that the environmental shadow price and pollution stock will overshoot long-run levels. With a dirty environment, clean capacity will be built up rapidly and may overshoot its long-run level. Both cases reflect inherent tradeoffs between environmental protection through curtailed consumption, rapid investment, or both.

Dynamic optimization models with two or more state variables always are somewhat technically vexing, and ours is no exception. We obtain several of our results by making a crucial but somewhat unsatisfactory assumption that the incremental cost of the first unit of investment is negligible (so perfect investment smoothing is possible). Extension of our results could be obtained by relaxing this assumption and using numerical simulations (parameterized to actual industries) to study the resulting investment dynamics. Simulations also could be used to look at different fixed and variable costs.

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Appendix

This section shows that regime VIII cannot hold for any positive length of time.

Lemma 13 *Regime VIII cannot hold for a positive time interval, if $b - c$ or if α is small.*

Proof. Suppose for $t_2 > t_1 \geq 0$, regime VIII holds in the interval $[t_1, t_2]$. Then along the interval we have $\lambda = 0$ and hence $U'(q + k) = b = c + \tau$. If so, $\tau = \hat{\tau} \equiv b - c$ and $\dot{\tau} = 0$ along the interval, implying $\dot{S} = \dot{q} = 0$. It follows that k is constant, say $\widehat{k}$. Furthermore, $\widehat{k}$ solves $U'(\alpha\widehat{S} + \widehat{k}) = b$, where $\widehat{S}$ solves $D'(\widehat{S}) = (\rho + \alpha)(b - c)$. Since $U'(\alpha\widehat{S} + \widehat{k}) < b + (\rho + \delta)f'(\delta\widetilde{K}^*)$, for α and $b - c$ that are not too large, that solution implies $\widehat{k} > \widetilde{K}^*$, which violates Lemma 5. ■

The result also holds more generally, although it is more complicated to prove. Regime VIII cannot be a steady state, since it violates Lemma 3. Nor can one start with regime VIII because $K_0 = 0$, so it would have to be entered from another state (III, IV, or VI, since all other regimes have been excluded). Suppose the interval starts at t_1. The interval ends at t_2 when the constraint binds (since other variables are unchanged), implying $K(t_2) = \widehat{k}$, $\dot{K} < 0$ and $I < \delta\widehat{k}$. Between t_1 and t_2 we have $\lambda = 0$, so $\dot{\varphi} = (\rho + \alpha)\varphi > 0$, implying from (16) that $\dot{I} > 0$. We show that one cannot switch into this regime from any other.

If we are switching from regime VI, for $t < t_1$ we have $\lambda = 0$ and $\dot{\varphi} = (\rho + \alpha)\varphi > 0$, implying now that $\dot{I} > 0$ for all $t \leq t_2$. To enter VIII, where $K > \widehat{k}$, it must be that $I(t) > \delta\widehat{k}$, for some $t \leq t_1$; but if $\dot{I} > 0$ throughout, then $I(t_2) > \delta\widehat{k}$, contradicting the requirements to end the interval.

If we areswitching from regime IV or III, then because of the continuity of K, $K(t_1) \geq \widehat{k}$, since the capacity constraint becomes slack by definition in VIII. Suppose $\widehat{k} = K(t_1)$; then for slackness to occur, it must be that $\dot{K} > 0$ and $I > \delta\widehat{k}$. Hence there must be $\bar{t} \in (t_1, t_2)$ such that $I(\bar{t}) = \delta\widehat{k}$ and $\dot{I} < 0$. But between t_1 and t_2 we have $\lambda = 0$, so $\dot{\varphi} = (\rho + \alpha)\varphi > 0$, implying from (16) that $\dot{I} > 0$. This contradicts $\dot{I} < 0$. In other words, one could save costs over the interval by investing only enough to maintain $K = \widehat{k}$.

The only possibility then is a discontinuous jump to $\widehat{k} < K(t_1)$ (with q jumping up). However, one can more generally rule out that τ, once it exceeds $b - c$, can return all the way to that level. Consider the problem when slack is not allowed. The necessary conditions are (14), (16), (17) and $\dot{\varphi} = (\rho+\alpha)\varphi - (U'(q+K) - b)$. It might be optimal to have $U'(q+K) < b$ for an initial period of time, during which the backstop is built up but not really necessary and investments are increasing. But once the inequality is reversed, it will remain the case forever. Since the necessary conditions for the modified problem are also sufficient, and since regime IV is maintained irrespective of the stock at the beginning of the interval, it must also be the case that regime IV, once entered, is maintained in the solution of the original problem. This would contradict IV preceding VIII.