Optimal spatial control of biological invasions

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Abstract:

This study examines the spatial nature of optimal bioinvasion control. We develop a spatially explicit two-dimensional model of species spread that allows for differential control across space and time, and we solve for optimal spatial-dynamic control strategies. The qualitative nature of optimal strategies depends in interesting ways on aspects of landscape and invasion geometry. For example, reducing the extent of exposed invasion edge, through spread, removal, or strategically employing landscape features, can be optimal because it reduces long-term containment costs. Optimal invasion control is spatially and temporally "forward-looking" in the sense that strategies should be targeted to slow or prevent the spread of an invasion in the direction of greatest potential long-term damages. These spatially-explicit characterizations of optimal policies contribute insights and intuition to the largely nonspatial literature on controlling invasions and to understanding control of spatial-dynamic processes in general.

Keywords: invasive species; spatial-dynamic processes; spatial spread; reaction-diffusion; management; cellular automaton; eradication; containment; spatial control; integer programming

1. Introduction

Much of the economic research on bioinvasion management frames the issue as a pest control problem, in which the population density of the invader is controlled. This literature has generally focused on the aggregate pest population, without consideration of its spatial characteristics (Pannell [1], Deen *et al.* [2], Saphores [3]). But a critical feature of invasion problems is that they unfold over time and space and are thus driven by spatial-dynamic processes, rather than by simpler dynamic processes. Existing analytical work generally abstracts away from the spatial features of invasions, focusing on *when* and *how much* to control (Eisworth and Johnson [4] and others reviewed in Olson [5] and Epanchin-Niell and Hastings [6]). There is less understanding about *where* to optimally allocate control efforts or the effect of spatial characteristics of the invasion or landscape on optimal control choices.

This paper develops a bioeconomic model of bioinvasions that incorporates a spatial-dynamic spread process and that allows various aspects of space to be characterized explicitly. We examine optimal policies over a range of bioeconomic parameters, spatial configurations, and initial invasion types. The more interesting results show how the geometry of the initial invasion and landscape influences the qualitative characteristics of optimal policies. Optimal solutions often utilize landscape features or alter the shape of the initial invasion in order to reduce the length of exposed invasion front, thereby reducing long term control costs. Optimal policies also exhibit classic forward-looking behavior that not only anticipates impacts over time, but also looks forward over space to slow and steer the invasion front away from the direction of greatest potential damages, or in the direction where the costs of achieving control are low.

2. Related Literature

In its most general form, a spatial spread process may be characterized with a partial differential equation (PDE) over continuous time and space. There is a paucity of literature in economics, mathematics or optimization theory on characteristics of optimally controlled PDE-based state equation systems. The most general is the elegant work by Brock and Xepapadeas [7; 8] who derive modified Pontryagin conditions for the optimal control of a renewable resource governed by continuous PDE state equations. In parallel work, Sanchirico and Wilen [9; 10; 11] model a spatially-linked renewable resource system in continuous time and discrete space. Discretizing space allows the general PDE system to be converted into a system of ODEs of dimension equal to the number of patches. Costello and Polasky [12] characterize the fisheries system as discrete in time and space, allowing them to use dynamic programming to analytically characterize features of the equilibrium of a meta-population fishery system in the presence of stochasticity.

This literature on optimal renewable resource management (fisheries) under diffusion provides only limited insights into understanding optimal bioinvasion management, for several reasons. First, fisheries problems generally involve interior harvest solutions, whereas controls for bioinvasions include critically important corner solutions, such as system-wide eradication. Including eradication complicates the solution since eradication eliminates damages and further control options in finite time. A complete solution thus requires comparing finite and infinite horizon solutions over the parameter space (see Wilen [13]). Second, while the steady state equilibrium is the more interesting part of renewable resource problems, the approach path is arguably more important for bioinvasion problems. Solving for approach paths often requires numerical methods which face an enhanced curse of dimensionality for spatial-dynamic problems, generally limiting the size of the problem that can be analyzed. Third, it is desirable to characterize space in realistic ways, including different shapes of the initial invasion and the landscape, which generally precludes analytical derivation of solutions. The end point conditions for spatial-dynamic problems have only recently been articulated (by Brock and Xepapadeas [8]) and are difficult to incorporate in numerical solutions for all but the most simple of spatial structures.

Recent research examining spatially explicit optimal control of bioinvasions utilizes numerical methods with discrete space representations of spread processes. Albers et al. [14], Bhat et al. [15], Blackwood et al. [16], Ding et al. [17], Finnoff et al. [18], Hof [19], Huffaker et al. [20], Potopov and Lewis [21], and Sanchirico et al. [22] find optimal solutions for special cases, often by focusing only on the steady state, simple landscapes, and interior (non-eradication) solutions, or by tackling reduced dimension problems. We are unaware of work that solves for fully optimal spatial-dynamic solutions (including transition paths and a full range of control options) in large dimension problems that allow for general characterizations of spatial features of the landscape and invasion.

3. A Spatial-dynamic Model of Bioinvasions

We develop and solve a spatially-explicit, deterministic, discrete space-time model that allows for growth and spread of a species and differential control over both time and space. We focus on the situation in which an invasion has arrived, established itself, and been discovered within the focal landscape. Upon discovery, the initial invasion has some arbitrary character (e.g., size, shape, location) that may depend upon seeds having been introduced by animals (e.g., birds), wind, or other mechanism of initial invasion. We then ask how this (general and arbitrarily-shaped) one-time invasion should be managed beginning when it is discovered, in order to minimize the total discounted costs and damages incurred from the invasion.

Although invasion spread can follow a variety of processes, we focus here on deterministic local dispersal.[1] We model the landscape as a lattice or grid of cells (patches) that are linked by dispersal. Patches are either invaded or uninvaded, and in the absence of control, the invasion spreads from invaded patches to adjacent uninvaded patches in each time period, approximating a constant rate of radial spread.[2] Without control, the invasion spreads to fill the entire focal landscape whose explicit boundaries represent ecological or physical limits of the species' potential range of contiguous spread. We incorporate two types of invasion control: clearing of invaded patches and preventing spread from invaded to uninvaded patches. Each discrete control action has an associated cost, and combinations of control actions can be used to eradicate, contain, slow, or redirect the spread of the invasion.

3.1 Spread mechanism

Assume the landscape is a grid of square cells that comprises the total potential extent of contiguous invasion. Each cell is labeled by its row *i* and column *j* in the landscape grid, and each cell can take on one of two states: invaded ($x_{i,j} = 1$) or uninvaded ($x_{i,j} = 0$). In the absence of intervention, the species spreads from invaded cells to adjacent, uninvaded cells in each time period, based on rook contiguity. Thus, if cell (i,j) were invaded at time *t*, cells (i,j), $(i,j+1)$, $(i,j-1)$, $(i+1,j)$, and $(i-1,j)$ would be invaded in the next time period. In each subsequent time step, all

[1] This is done for tractability and because much can be learned from even this simplest case. We discuss how our results might generalize to alternative spread processes, such as stochastic, rare, long-distance dispersal events and repeat invasions.

[2] Our choice of modeling patches as invaded or uninvaded abstracts from detailed population dynamics, but still captures species' constraints to growth, an approach that is commonly employed in meta-population models [23;24]. Our choice of linear rate of spread is the pattern that is predicted for species that disperse primarily based on random, local movements [25].

cells sharing a contiguous border with an invaded cell also become invaded. The choice of grid cell size and time interval are closely linked, because the model assumes that the invasion spreads into adjacent uninvaded space at a rate of one cell per unit time.

3.2 Economic model

The invasive species causes damages proportional to the area invaded, with marginal (and average) damages equaling *d* per cell invaded. The cost of preventing establishment of the invasion in a patch depends linearly on the propagule pressure from adjacent invaded cells. Costs of excluding invasion from a cell thus increase with the number of adjacent (rook contiguous) invaded cells and equal *invaded_neighbors*b*, where *b* is the cost of preventing invasion along each boundary and *invaded_neighbors* is the number of invaded adjacent cells ($0 \leq$ *invaded_neighbors* ≤ 4). Once a cell has been invaded, it remains invaded unless the invasion is removed from the cell at a cost *e*. The cost of clearing thus depends linearly on the area cleared. For a cleared cell to remain uninvaded in the following time periods, control must be applied to prevent reinvasion at a cost *invaded_neighbors*b*. If the entire landscape has been cleared, there are no subsequent control costs.

To parameterize this model, economic parameters must be scaled to match the biological model. Specifically, damages and costs are tied to the size of the cell, and the discount rate must be scaled to match the unit of time. Separately parameterizing removal costs *e* and spread prevention costs *b* allows flexibility in specifying control costs based on species characteristics.

3.3 Optimization set-up

Optimal control of the invasion requires minimizing the present value of the sum of control costs and invasion damages across space and time. We formulate the optimal spatial-dynamic invasion control problem as follows:

Minimize: $\sum_{t \in T, t>0} \beta_t * \left(\sum_{(i,j) \in C} x_{i,j,t} d + \sum_{(i,j) \in C} y_{i,j,t} e + \sum_{(i,j,k,l) \in N} z_{i,j,k,l,t} b\right)$ (1)

subject to:

$$x_{i,j,0} = \underline{x}_{i,j} \quad \forall (i,j) \in C \tag{2}$$

$$y_{i,j,0} = 0 \quad \forall (i,j) \in C \tag{3}$$

$$z_{i,j,k,l,0} = 0 \quad \forall (i,j,k,l) \in N \tag{4}$$

$$x_{i,j,t} \geq x_{i,j,t-1} - y_{i,j,t} \quad \forall (i,j) \in C, t \in T, t \geq 1 \tag{5}$$

$$x_{i,j,t} \geq x_{k,l,t-1} - z_{i,j,k,l,t} - y_{i,j,t} \quad \forall (i,j,k,l) \in N, t \in T, t \geq 1 \tag{6}$$

$$x_{i,j,t} \in \{0,1\} \quad \forall (i,j) \in C, t \in T \tag{7}$$

where

$(i, j) \in C$ indexes cells by row *i* and column *j*, and *C* is the set of all cells in the landscape

$(i, j, k, l) \in N$ indexes pairs of neighboring cells, where $(i, j) \in C$ is the reference cell, $(k, l) \in C$ is one of its neighbors, and *N* is the set of all neighboring cell pairs

$t \in T$ indexes time, where $T = \{0,1,2,..., T_{\max}\}$

$x_{i,j,t} \in \{0,1\}$ is the state of cell (*i*,*j*) at time *t*, where $x_{i,j,t} = 1$ if the cell is invaded and $x_{i,j,t} = 0$ otherwise

$y_{i,j,t} \in \{0,1\}$ is a binary choice variable indicating if invasion is removed from cell (*i*,*j*) at time *t*, where $y_{i,j,t} = 1$ if the cell is cleared and $y_{i,j,t} = 0$ otherwise

$z_{i,j,k,l,t} \in \{0,1\}$ is a binary choice variable indicating if control efforts are applied along the border between cell (*i*,*j*) and cell (*k*,*l*) at time *t* to prevent spread from cell (*k*,*l*) to cell (*i*,*j*), where $z_{i,j,k,l,t} = 1$ if the border is controlled and $z_{i,j,k,l,t} = 0$ otherwise

$\underline{x}_{i,j} \in \{0,1\}$ is the initial state (*t*=0) of invasion for cell (*i*,*j*)

β_t is the discount factor at time t (t>0), where $\beta_t = (1+r)^{1-t}$ and r is the discount rate

d is the damage incurred per time period for each cell that is invaded

e is the cost of removing invasion from a cell

b is the cost per time period of preventing invasion from a neighboring cell

Equation (2) establishes the initial state of the landscape by defining which cells are invaded at t=0. Equations (3) and (4) specify that control efforts do not begin until the first time period. Condition (5) states that a cell invaded in the previous time period remains invaded in the current time period unless removal efforts are applied. Equation (6) requires that cell (i,j) become invaded at time t if it had an invaded neighbor in the previous time period, unless invasion is removed from cell (i,j) or control is applied to prevent invasion from the invaded neighbor; this condition must hold for cell (i,j) with each of its neighbors.

We solve for the infinite horizon optimal control solution using a finite horizon model. The infinite horizon steady state is reached by time T < ∞, and the scrap value at time T equals the flow of future steady state control costs and damages discounted over an infinite future. We choose T large enough (T = 100) so that the decision rules and equilibrium payoff are insensitive to changes in T (Additional details on solving for the infinite horizon solution can be found in the online appendix available at JEEM's supplementary repository, which can be accessed from http://aere.org/journals/).

3.4 Solution approach

Our specification employs two features that allow us to solve problems that have not been solved before. The first is judicious simplification and abstraction. The model developed here is arguably the simplest representation of a bioinvasion that still incorporates most critical features of the problem. The second feature of our specification that facilitates a solution is the transformation of the non-linear equality state equation system into an equivalent system of linear inequalities [equations (5) and (6)]. The state equation for each patch could have been specified as:

$$x_{i,j,t} = \left(1 - y_{i,j,t}\right)\left(1 - \left(1 - x_{i,j,t-1}\right)\prod_{(k,l)\in N}\left(1 - x_{k,l,t-1}\left(1 - z_{i,j,k,l,t}\right)\right)\right) \quad (8)$$

and the solution could then be attempted using Dynamic Programming or discrete numerical boundary value solution techniques. But the difficulty with this formulation is that the inherently large dimension of the problem precludes solving all but the simplest problems, since the number of states is approximately 2 raised to the power of the grid size.[3]

By specifying the state transition system as a system of linear inequalities, the model can be solved using integer programming rather than dynamic programming. The fact that the objective function and constraints are expressed linearly and the control and state variables are binary integers enables us to use a fast and efficient routine called SCIP. SCIP [27] is a framework for constraint integer programming problems that can solve certain large dimension problems with appropriate structure by using a linear relaxation method. Linear relaxation first ignores the binary control constraints and finds solutions to linear programs that typically involve fractional controls. It then performs branch and bound routines that resolve the parts of the original problem with fractional solutions. The branching is limited (bounded) by the fact that the subproblems' solutions are bounded by the integer constraint values. Branching thus

[3] The state of the art in computational algorithms for solving our kind of problem via conventional DP based algorithms is summarized in Farias et al. [26]. Their approximation technique solves a complex game theoretic equilibrium with 50 firms and 20 states per firm. In terms of state evaluations needed, this is equivalent to a problem like ours with a landscape grid of about 15 by 15.

divides the initial problem into smaller subproblems that are easier to solve, and the best of all solutions found in the subproblems yield the global optimum. Bounding avoids enumeration of all (exponentially many) solutions of the initial problem by eliminating subproblems whose lower (dual) bounds are greater than the global upper (primal) bound.

More general spatial-dynamic specifications would limit the size of the problem that can be solved. For example, adding non-linearity to the objective function or greater complexity to our state equations, making species density in each patch continuous rather than present/absent, or making controls non-integer would make solving large and complex landscape problems difficult. Our simplified formulation permits rapid solutions for problems with very large landscapes (greater than 25 by 25) and many time periods. We are able to exploit the speed of the algorithm by solving hundreds of optimizations for a large span of the parameter space and for many geometric depictions of the invasion and landscape. This helps identify specific parametric assumptions and topological conditions that influence control strategies. By solving numerous cases, we are able to synthesize the intuition behind results, even though we are unable to derive analytical solutions to this complex problem.

4. Results

Optimal control strategies for invasions vary dramatically across invasion, landscape, and economic characteristics, ranging from no control to complete eradication depending upon parameters. Between these two extremes, optimal policies include: eradication of part of the invasion and containment or abandonment of the rest, immediate complete containment, partial containment that allows some spread prior to complete containment, partial containment followed by abandonment of control efforts, and directed containment that shapes and redirects the wave front. For all scenarios examined, if clearing or eradication efforts are employed, they are optimally completed in the first time step. We report here selected results chosen from interesting problem configurations.

4.1 Some expected results

Although not presented here, we first examined how economic parameters and potential invasion and range size affected optimal control policies in simple settings. As expected, high control costs, low damages, and high discount rates reduce the amount of optimal control. All else equal, invasions that have a larger potential for spread warrant greater control. Holding landscape size constant, larger invasions are less likely to be optimally controlled, implying, importantly, that inadvertent delay of control (e.g., by late discovery) reduces the likelihood that eradication or containment will be optimal. The net present value of costs and damages also increases with delays, highlighting the importance of finding and controlling invasions early.

4.2 Landscape shape

Landscape shape impacts the optimal policy of an invasion because landscape boundaries (i.e., invasion range boundaries) affect the costs of invasion control and damages by constraining invasion spread. Eradication or containment is optimal across a larger range of economic parameters for invasions occuring in compact (e.g., square) landscapes than in same-sized narrow landscapes. The boundaries of narrow landscapes confine the spread of species more than compact landscapes, so damages accrue more slowly, resulting in lower potential total damages. The particular shape of the landscape, beyond length and width, also affects optimal control policies. For example, constrictions and expansions in the landscape influence optimal control strategies via their affects on the cost of controlling the invasion or on the spread rate (Several examples can be found in the online appendix available at JEEM's supplementary repository, which can be accessed from http://aere.org/journals/).

4.3 Invasion location

The effect of invasion location on optimal control policy is ambiguous because invasion location affects long-term damages and costs of control in opposing ways. An invasion beginning near an edge takes longer to fully invade the landscape than an invasion near the center because the furthest reaches of the landscape are more distant. Thus, while an uncontrolled invasion will eventually spread throughout the landscape regardless of its starting location, the net present value of potential damages from an invasion beginning near an edge are lower, reducing the range of control costs for which eradication or containment is optimal. On the other hand, invasions that occur along an edge of the landscape have lower containment costs because the landscape boundaries prevent spread along the bounded edge at no cost, mediating the effects of lower damages on optimal policy.

Invasion location can influence control costs even if the invasion does not begin immediately adjacent to a landscape boundary. Figure 1 shows the spread of an optimally controlled invasion across four time steps, demonstrating how landscape boundaries are strategically employed. The initial invasion (t=0) is a 4 by 4 block of cells located 2 cell widths from the corner of a 15 by 15 cell landscape. Optimal policy contains the invasion along its frontal edges, directing spread towards the corner of the landscape, after which the invasion is contained in perpetuity. This strategy reduces the number of exposed borders and periodic containment costs by 25% for the long-term by using landscape boundaries. For an identical invasion located centrally, immediate containment is optimal because landscape boundaries cannot be employed to reduce long-term containment costs and total potential damages are larger.

Figure 2 provides another example of the effect of invasion location on optimal policy for a two patch invasion, in which one patch occupies a corner cell (the upper left hand patch) and the other patch (lower right hand corner) is one cell width from the opposite corner. A large number of optimal policies are possible for this invasion depending on the economic parameters, including no control, eradication, or containment of both patches. However, because the two patches are differently located relative to the landscape boundaries, optimal policy applies dramatically different types of control to each patch for small variations in cost parameters. For example, the lower right hand patch is more costly to contain (4 exposed edges), so in some circumstances it is optimal to eradicate that patch and perpetually contain the other patch, which has only 2 exposed edges. But with slightly higher eradication costs, the optimal policy switches so that initial containment of the upper left hand patch is still optimal, but the patch with more exposed borders is neither contained nor cleared (Fig. 2). In this case the lower right invasion spreads, and this reduces the benefits of containing the upper left hand cell, so that eventually all control efforts are optimally abandoned.

Although the landscape boundaries cannot be used to reduce the amount of exposed edge on the lower right hand patch in Figure 2, the boundaries are still employed strategically to slow the invasion. Specifically, control is optimally applied to the lower right hand patch to slow its advance into the interior and direct growth towards the corner for the first two periods. This approach, which was also employed for the invasion in Figure 1, reduces the present value of damages from the invasion by delaying spread in the direction with the highest potential growth.

4.4 Invasion shape and contiguity

Geometric characteristics of the invasion, beyond size and location, affect optimal control policies in complex and interesting ways. In particular, the shape and contiguity of an invasion (holding size constant) affect optimal levels and spatial allocation of control effort. For example, containment is optimal across a wider range of border control costs for a compact invasion than for a similarly-located and sized patchy invasion which has a higher edge to area ratio. In addition, because containment is more costly for patchy invasions, eradication is optimal across a larger range of marginal (average) eradication costs than it is for compact invasions.

Clearly, a critical feature of invasion geometry is the effect of the length of invasion edge on containment costs. Figure 1 showed that the extent of exposed edge can be reduced by employing landscape boundaries. In other scenarios it is optimal to reduce the length of the invasion front by altering the shape of the invasion by clearing cells or by allowing spread prior to containment. For example, Figure 3 shows an initially "edgey" invasion for which optimal policy combines removal and spread prevention to reduce the number of exposed edges from 11 to 8 prior to complete containment.

With respect to non-contiguous (patchy) invasions, our scenarios also show that optimal control strategies can vary across patches of invasion within a landscape and that control strategies for individual patches depend on the entire landscape context (e.g., Fig. 2). Just as dynamic problems involve choosing an entire time path of decisions that are interdependent, optimal control of a spatial-dynamic system involves simultaneously choosing control efforts across spatially separated patches, because the benefits (avoided future damages) of controlling each patch depend on the control efforts and spread rates at other patches. Figure 4 shows an invasion for which optimal policy requires eradication of one patch and slowing, followed by abandonment, of the other. However, with slightly higher border control costs, the system-wide net benefits of slowing the spread of the large patch are reduced, and this reduces the gains from eradicating the small patch so that eradication of the small patch ceases to be optimal.

4.5 Landscape heterogeneity

Control costs and damages can vary across the landscape, and this heterogeneity can affect optimal policy. For example, optimal policy generally applies amplified controls to prevent or delay invasion of high-valued patches of land. Figure 5 shows a small initial invasion in a landscape that contains a distant but high value patch of land that would incur large damages from invasion. Optimal policy initially prevents spread in the direction of both high- and low-valued patches and directs spread toward the nearby landscape boundary. As the invasion grows, control is temporarily abandoned (periods 3-6) until the invasion eventually reaches the edge of the high value area. Then control is applied to prevent encroachment into that patch. In addition, the border controls that prevent invasion into the high-valued patch open up opportunities to initiate a new round of control (periods 7-10) that slows the rest of the wave front over the whole remaining area. The slowing policy eventually is abandoned, but in a manner that delays for as long as possible the relatively high costs of permanent protection of the high-valued patch.[4]

Figure 6 additionally illustrates some of the intricacies of spatial-dynamic optimal control policies in response to landscape complexities and heterogeneity. This example is identical to that in Figure 5, except for the presence of an additional high value patch in a protruding part of the landscape to the north. Optimal control initially directs the spread of the invasion toward the nearest landscape boundary to the west, slowing its progression toward the interior of the landscape. But then control is relaxed to the south so that the invasion is directed in periods 2-6 toward the southwest corner, away from the interior and the two high value patches. In periods 7-11, the barrier control on the frontal edge is gradually relaxed and then eliminated, allowing spread in the direction of both high value patches. In periods 13 and 14 control efforts are briefly resumed to further delay the invasion of the high value patch in the east by applying

[4] For the same initial invasion, but with higher costs of spread prevention, control again slows the spread of the invasion towards the high value patch, but eventually allows its invasion.

control along several edges, but eventually in periods 15-16 control is abandoned and the invasion encompasses the eastern high value patch and areas near the northern high value patch. In periods 17-20, as the invasion begins to spread into the region that protrudes to the north, spread prevention controls are sequentially implemented to prevent the invasion of both the high value patch located there and the low value area beyond it. The narrowness of the landscape in this region allows the high value patch to be protected for the long term, whereas the other high value patch (identical except for location) was too costly to protect in perpetuity. This is an interestingly complex optimization solution that would be difficult to predict in advance.

5. Synthesis and Discussion

The novel parts of our findings are those that explore the manner in which the topology of an invasion and the landscape determine the optimal policy, in addition to basic economic factors. Invasions that are identical in relative size can have dramatically different optimal control policies if they differ in shape and location. While this appears to militate against deriving simple rules of thumb, we are able to synthesize the intuition behind many results.

5.1 Landscape shape

Invasions in more compact landscapes generally warrant more control because spread is less constrained, resulting in higher potential damages. Landscape shape also affects the likelihood that an invasion will appear near enough to landscape borders that can be used to reduce long-term containment costs. Nonconvexities in the landscape, such as constrictions and expansions, influence optimal control policies by affecting the costs of containment and invasion spread rates in those regions. Interestingly, the presence of landscape nonconvexities is the only situation we found for which delaying the start of control efforts can be optimal.

5.2 Invasion location in the landscape

The initial location of an invasion affects both potential long-term damages and costs of control. Central invasions generate higher potential damages because the invasion can spread through the landscape more rapidly, while control costs may be lower for invasions that begin distally if landscape boundaries can help contain the invasion. For optimally controlled invasions with similar characteristics, the net present value of costs and damages is thus higher for central invasions than for invasions that begin distally. Location also influences the optimal spatial allocation of control by determining the direction of greatest potential invasion spread.

5.3 Invasion shape and contiguity

The shape of an invasion affects optimal control policies by affecting containment costs and spread rates. A greater amount of invasion edge, due to invasion shape, decreases the range of control costs for which containment is optimal, shifting policies toward eradication or abandonment. For non-compact (edgy) invasions, it is often optimal to remove or allow spread prior to containment to reduce the amount of exposed edge and hence long-term control costs. Optimal control of patchy invasions depends integrated actions over the entire landscape, and control efforts can vary across patches based on patch and total invasion characteristics.

5.4 Landscape aspects of control

Landscape features, such as bottlenecks, can be used strategically to reduce long-term containment costs, also highlighting the role of landscape geometry in invasion control. In addition, control is applied to delay or prevent spread in directions of high potential damage accrual due to either large areas of potential spread or presence of high valued patches of land.

Our examination of multi-patch invasions contributes some insight into an unanswered question about where to focus control effort: on large, core patches or on smaller, satellite patches (Moody and Mack [28]). Established invasions can contribute to invasion expansion both through growth of the main invasion and the creation of new satellite populations. While we do not consider long-distance dispersal processes or differential densities among invaded patches, our results support two points. First, greater control may be optimal for smaller, satellite invasions because eradication and containment costs are lower. Second, optimal control for each patch of an invasion depends on the entire invasion and landscape, so that patches cannot be considered independently. A blanket strategy or prioritization is thus unlikely to be optimal.

Many invasive plants are not regulated because they are classified as too widespread to justify eradication. Our results show, however, that under some circumstances it is optimal to eradicate one patch of an invasion even while allowing other patches to spread. Furthermore, it can be optimal to slow or contain widespread invasions, even when eradication is not justified, especially when large potential for further spread exists.

The main limitation of our deterministic model is its inability to allow for stochastic, rare, long-distance dispersal events. Unfortunately, this is a very difficult problem to address in the context of explicit space and is a problem that neither we, nor others, have yet solved. However, our results suggest some conjectures. For species that exhibit long distance dispersal, we expect eradication to be optimal across a greater range of economic parameters, because damages would accrue faster with long distance dispersal. Also, containment should be optimal across a smaller range of economic parameters, because the costs of preventing spread would be higher (due to the costs of removing satellite invasion patches) or the benefits would be lower (as the invasion established beyond the containment zone). In contrast, we expect a shift away from both eradication and containment in regions that incur continual propagule pressure (i.e., repeat invasions), because the benefits of both types of control are reduced. With respect to spatial strategies, even under stochastic spread we expect that greater control will be applied to direct spread away from the directions of highest potential long-term damages. Furthermore, optimal control will favor the maintenance or formation of compact and landscape-constrained invasions to minimize local containment costs and the potential for long-distance dispersal. However, we expect that controls may be applied earlier when employing landscape features for controlling stochastic invasions, so that long-distance dispersal also will be more constrained.

6. Conclusions

This paper has two purposes. The first is to provide understanding of economically optimal spatial control of bioinvasions. Optimal solutions for spatially explicit optimization problems generate a far richer set of solution characteristics than work that treats space only implicitly. In addition to the control principles we have derived, our approach could be applied to specific invasion problems to guide on the ground management. Data requirements include estimates of expected damages from invasion and of costs of species removal and spread prevention. In addition, knowledge of the current invasion extent and predictions of the potential geographic range of eventual spread and the existence of any existing natural barriers to spread are needed. The potential range of an invading species often can be predicted using ecological niche modeling (Peterson [29]).

The second purpose of this paper is to use the bioinvasion problem as a model case study for learning about a wider class of problems characterized by diffusion or spread processes that generate patterns over space and time. Other examples include groundwater contamination, epidemics, forest fires, migration and movement, technology adoption, etc.

Some of what emerges from accounting for both space and time is consistent with our intuition about the dynamic components of the problem, while other features are space-dependent. Most importantly, adding space necessitates concern about *geometric* characteristics of problems in addition to concern about more familiar metrics such as size or quantity. To highlight some of our new findings, we compare general principles that apply to dynamic problems with some new results that emerge from our consideration of spatial-dynamics:

- In dynamic problems, the index that differentiates decisions (time) runs only forward. In spatial-dynamic problems, the index that identifies decisions is both a time index and a directional spatial index. In general, the dynamic parts of the solution (concerned with *when* and at *what level* of intensity to initiate controls) are intertwined in complex ways with, and are not separable from, the spatial part of the solution of *where* to initiate controls.
- The solutions to interesting dynamic problems are forward-looking at each date, scanning the complete horizon, adding up the marginal impacts over that horizon (all evaluated along the optimal path), and comparing those anticipated impacts with current marginal costs. Spatial-dynamic problems also are forward-looking but over both time and space, accounting for the size and character of the potential space (and hence damages) that lies ahead in both time and space of the advancing invasion front. Directionally-differentiated damages influence the degree of control exerted at any point in time and space. Large prospective damages (either from a large amount of space or from high damages per unit of space) in the path of a spreading front will call forth higher levels of control early and at locations often roughly orthogonal to the path of the front.
- Dynamic optimization solutions depend critically upon the initial state of the system, generally measured by the *size* of capital or resource level at some starting date. For spatial-dynamic problems, the *geometry* of the initial state, as well as its size, matters. Small variations in shape and location in the landscape can lead to qualitatively different optimal solutions. For example, whether eradication or containment may be optimal depends not only

upon basic costs, damages, size of invaded area, and discount rate, but also upon how large the initial invasion is relative to the landscape, where it is located, the extent of exposed invasion edge, etc.

These are just a few of the characteristics that we conjecture may emerge as general properties of solutions of other spatial-dynamic optimization problems. In the end, economists will need to develop new intuition about spatial-dynamic problems by analyzing these and other cases before we can understand what features of the solutions to this class of problems appear to be general, and what features are specific to particular cases.

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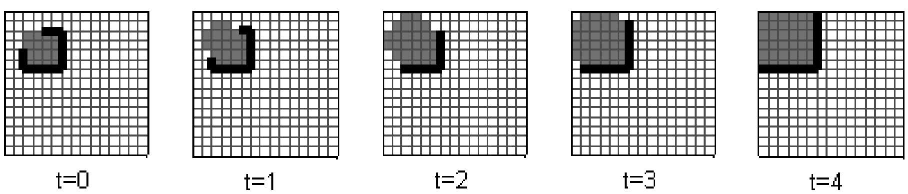

Figure 1. Optimal control of an invasion in a 15 by 15 cell landscape by a 4 by 4 patch of cells near a corner of the landscape. Invaded area shown in gray. Spread prevention shown by thick black lines. (r = 0.05, b = 10, e = 230, d = 1).

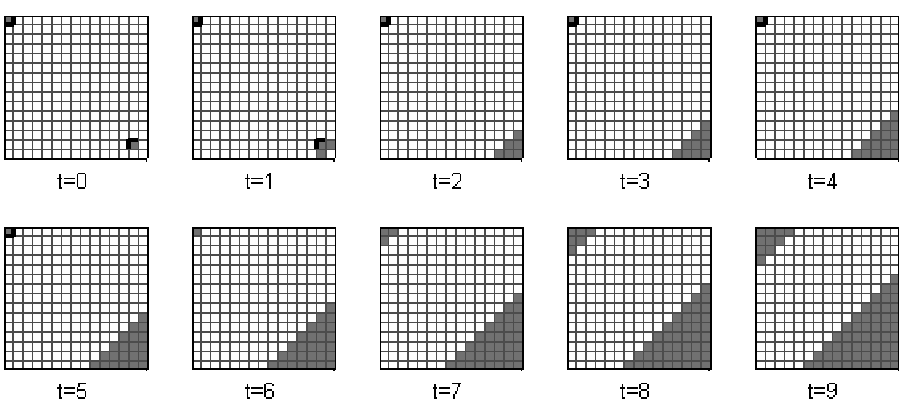

Figure 2. Optimal control of a 2 patch invasion in a 15 by 15 cell landscape. Invaded area shown

in gray. Spread prevention shown by thick black lines. (r = 0.05, b = 27, e = 2100, d = 1).

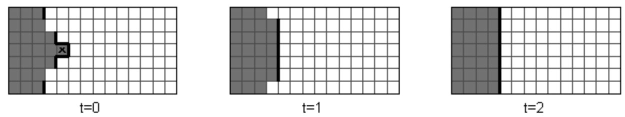

Figure 3. Optimal control of an invasion in a 7 by 14 cell landscape by a patch of cells with local concavities. Invaded area shown in gray. Spread prevention shown by thick black lines. Clearing efforts shown by 'x'. (r = 0.05, b = 7, e = 83, d = 1).

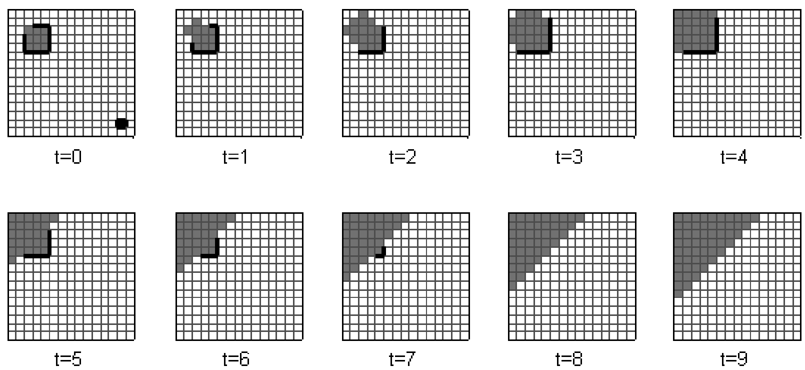

Figure 4. Optimal control of an invasion in a 15 by 15 cell landscape by a small (1 cell) and large (9 cells) patch. Invaded area shown in gray. Spread prevention shown by thick black lines. (r = 0.05, b = 14, e = 450, d = 1).

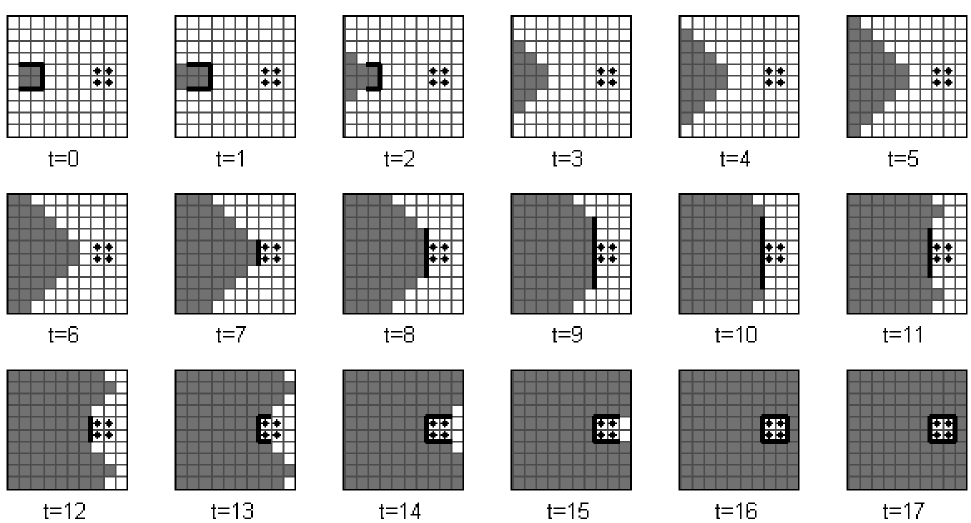

Figure 5. Optimal control of a 2 by 2 invasion in a 15 by 15 cell landscape with heterogeneous damages. A high value 2 by 2 patch (d =100), indicated by small black diamonds, lies in front of the initial invasion. Invaded area shown in gray. Spread prevention shown by thick black lines. (r = 0.05, b = 50, e =10000, d = 1).

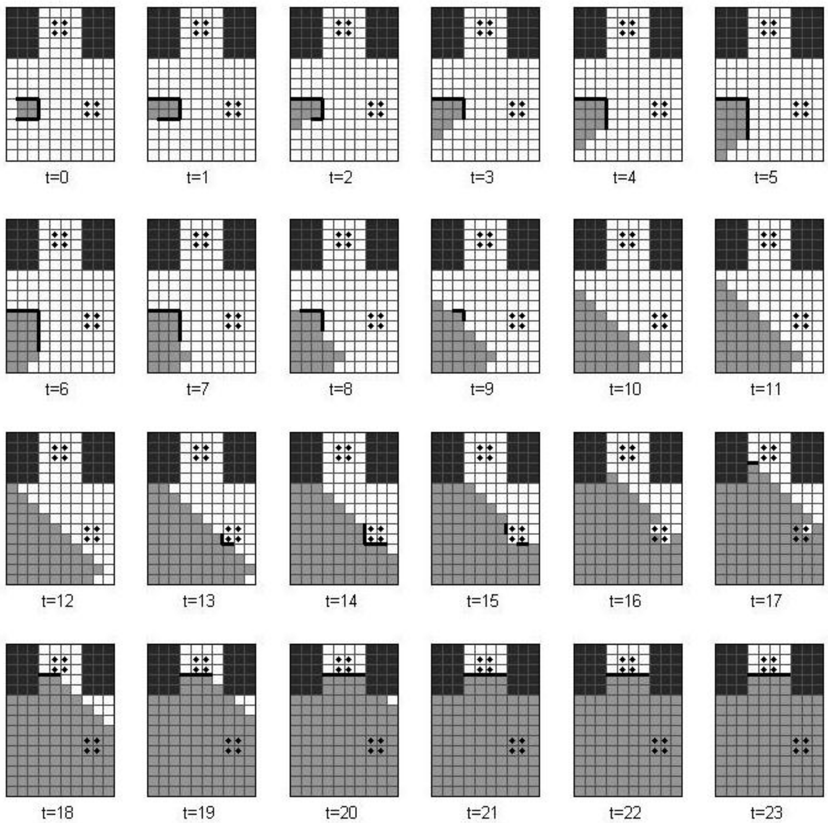

Figure 6. Optimal control of a 2 by 2 patch invasion in a constricted landscape with two high value patches (d = 100), indicated by black diamonds. The region is 15x10 with two 4x3 sections removed. The white area is invadable, gray area is invaded, and black area is not invadable. Spread prevention shown by thick black lines. (r = 0.05, b = 75, e =10000, d = 1).

Online appendix for "Optimal spatial control of biological invasions"

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A1. Solving an infinite time horizon problem using a finite horizon framework

We are interested in the optimal control solution over an infinite time horizon. However, solving for this directly as an integer-programming problem would require specifying an infinite number of constraints and variables, making the problem infeasible. Instead, we solve for the infinite time horizon solution in a finite time horizon framework by taking advantage of Bellman's principle of optimality and by specifying appropriate terminal values. With a sufficiently long time horizon, our system reaches the infinite time horizon steady-state equilibrium in which none to all of the landscape is invaded. With an infinite time horizon, time consistency requires that the system has reached this equilibrium if the invasion landscape remains unchanged between two time periods. In contrast, for a finite time horizon, the system can reach and maintain the steady-state equilibrium for many time periods but can depart from the steady state toward the end of the finite time horizon. To deal with this difficulty, the steady-state equilibrium solution can be locked in using constraints after the equilibrium has been reached, and appropriate transversality conditions can be added to account for control costs and damages accrued after the finite time horizon. We add the following constraints to the model defined in the main text [equations 1-7] to lock in the equilibrium solution:

$$y_{i,j,t} = y_{i,j,t\_mid} \qquad \forall (i,j) \in C, t \in T, t > t\_mid \text{ ;}$$

$$z_{i,j,k,l,t} = z_{i,j,k,l,t\_mid} \qquad \forall (i,j,k,l) \in N, t \in T, t > t\_mid \text{ ; and}$$

$$x_{i,j,t} = x_{i,j,t\_mid} \qquad \forall (i,j) \in C, t \in T, t > t\_mid \; ;$$

where $1 < t\_mid < T_{max}$. We choose t_mid and T_{max} large enough for equilibrium to have been reached by time $t < t\_mid$. We calculate the terminal value as the net present value of steady-state control costs and damages from time $T+1$ to infinity:

$$\sum_{t=T+1}^{\infty} \beta_t * \left(\sum_{(i,j)\in C} x_{i,j,T} d + \sum_{(i,j)\in C} y_{i,j,T} e + \sum_{(i,j,k,l)\in N} z_{i,j,k,l,T} b \right)$$

and include this value in the objective function (1).

A2. Additional bioinvasion examples

Figure A1 illustrates how landscape geometry can be employed strategically to optimally reduce long-term containment costs. In this scenario, complete containment in the first time period is not optimal because the extent of the exposed invasion edge (11 cell edges) is large. Instead, optimal policy slows the growth of the invasion along the center of the invasion front, delaying damages centrally, and then contains the invasion in perpetuity when it reaches the landscape constriction. This control policy slows the invasion along the region of the invasion front that has the greatest potential long-term growth of damages (because it is spreading toward the largest extent of uninvaded area) and delays complete containment until landscape features constrain long-term costs.

Landscape geometries that include areas with potentially large rates of damage accumulation, as illustrated in Figure A2, also can lead to interesting strategic containment of an invasion. In this scenario, the invasion is spreading along a narrow section of the landscape toward a region where the landscape becomes wider (and future damages from spread become larger). The narrow section of the landscape confines the invasion to spread at a rate of four cells per time period, and neither containment nor eradication is optimal because the costs of control are high relative to the avoided damages. However, if the invasion were to spread beyond the narrow region of the landscape, the rate of damage accumulation would increase rapidly because the invasion would spread in three directions rather than one. Consequently, optimal policy contains the invasion when it reaches the end of the constricted region, at which point the containment costs remain the same but the avoided damages increase.

Additional details and examples are available in Epanchin-Niell and Wilen [1].

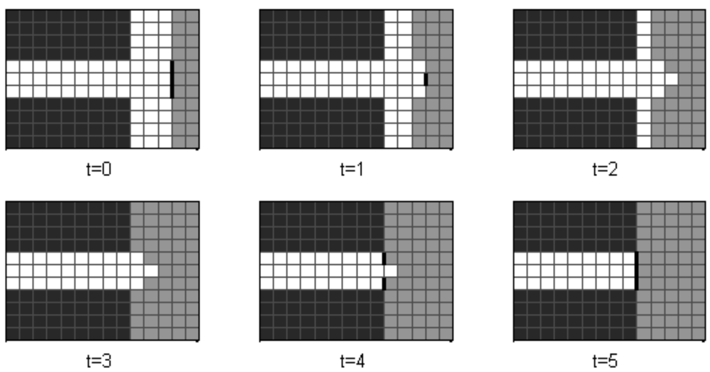

Figure A1. Optimal control in a landscape with a constriction. The region is 11x15 with two 4x9 sections removed. The white area is invadable, gray area is invaded, and black area is not invadable. Spread prevention shown by thick black lines. (*r* = 0.05, *b* = 7, *e* = 250, *d* = 1).

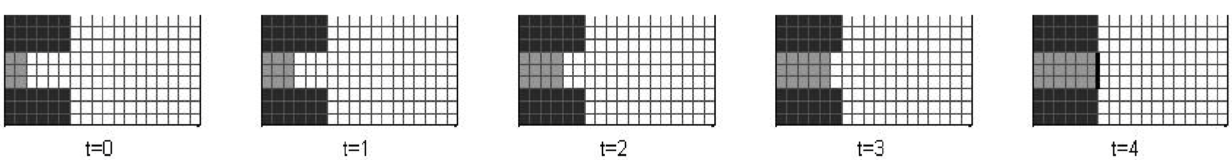

Figure A2. Optimal control in a landscape with an expansion. The region is 9x18 with two 3x6 sections removed. The white area is invadable, gray area is invaded, and black area is not invadable. Spread prevention shown by a thick black line. (*r* = 0.05, *b* = 22, *e* =250, *d* = 1).

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