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# Linking Carbon Markets with Different Initial Conditions

**Matt Woerman**

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# About the Author

**Matt Woerman** is an assistant professor in the Department of Resource Economics at the University of Massachusetts Amherst. He studies energy and environment economics and industrial organization with a focus on electricity, groundwater, and climate. Woerman's recent research examines the role of energy technologies on strategic firm behavior and market outcomes in wholesale electricity markets; how increasing groundwater pumping costs affect farm pumping and cropping decisions; and the design of carbon markets and other environmental policies. He has also developed applied econometric methods for use in empirical work, including tools for designing field experiments using panel data.

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# Linking Carbon Markets with Different Initial Conditions

Matt Woerman [*]

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**Abstract**

Despite the global nature of climate change, carbon pricing is driven by regional and sectoral carbon taxes or trading programs, each with unique features and disparate marginal costs. Linking these fragmented regional or sectoral programs could improve environmental and economic outcomes, but differing initial conditions pose a challenge to linking. We explore the use of an allowance exchange rate, which denominates the compliance value of an emissions allowance differently in each program. In a theoretical model, we find that linking with an exchange rate in the politically plausible range—between the benchmark regimes of autarky and traditional 1:1 trading—may reduce abatement in one program but achieves greater aggregate emissions abatement than the amount achieved at each bookend. Linking in this range also yields lower total abatement costs and greater economic surplus in each program, compared to autarky. Thus, a linked trading system with allowance exchange rates can be expected to yield benefits for the environment and each regional economy. When program caps achieve inefficiently low abatement, it is socially optimal to link at an exchange rate that increases total abatement in the linked system, which occurs within the politically plausible range of exchange rates, not at 1:1. We illustrate these results, and identify additional outcomes of interest to policymakers, using a simulation model of electricity markets.

**Key Words:** greenhouse gas, climate change, climate policy, carbon market, policy coordination

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[*] University of Massachusetts Amherst, Department of Resource Economics. Email: mwoerman@umass.edu. This paper is based on a previous manuscript coauthored with Dallas Burtraw and Karen Palmer of Resources for the Future and Clayton Munnings, who serves as a US Strategic Advisor for the International Emissions Trading Association. I greatly appreciate their ongoing support and contributions. I also thank Garth Heutel, John Stranlund, and seminar participants at the AERE Summer Conference, the ASSA Annual Meeting, the EAERE Annual Conference, the RFF-PIK climate policy workshop, and OSWEET for helpful comments and suggestions. Nicholas Roy provided excellent research assistance. This work was supported by Mistra Carbon Exit and the Resources for the Future Electric Power Program.

# Linking Carbon Markets with Different Initial Conditions

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## 1. Introduction

The environmental consequences of greenhouse gas emissions are felt around the globe, regardless of where those emissions originate. Correspondingly, in the 1990s, numerous economists heralded a single international carbon market as the cost-effective solution to climate change. Such a market would, in principle, lead to a single global carbon price through the trade of emissions allowances, which would serve to identify and realize emissions reductions at the lowest possible cost and yield the cost-effective geographic distribution of abatement. Despite the logic of this approach, international policymakers were unable to implement this vision and climate governance has taken a different path.

Today 68 international, national, regional, state, provincial, and municipal carbon pricing or trading programs are in operation, instead of the single international carbon market that was once imagined (World Bank 2022). Across these programs, the prevailing carbon prices range from \$0.00 to \$137.30 USD per ton of carbon dioxide ($CO_2$) equivalent,[1] representing an immense variety of environmental ambitions and abatement costs. Additionally, in some jurisdictions, different sectors of the economy are covered by separate existing or proposed pricing or trading programs (Perino, Ritz, and van Bentham 2020). For example, in Germany, power plants are covered by the European Union's trading program and face a price of \$87 per ton, while emissions from the buildings and transportation sectors are covered by Germany's trading program and face a price of only \$33 per ton.[2] This fragmentation leaves important opportunities for improved cost-effectiveness on the table and coordination could enable greater environmental stringency at lower total costs.

A central way to improve the cost-effectiveness of this patchwork is to aggregate through bilateral or multilateral linking, a process in which the regulatory authorities in each program

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[1] Carbon prices are for April 1, 2022, and are available from the World Bank's Carbon Pricing Dashboard: https://carbonpricingdashboard.worldbank.org/. Mexico's and Oregon's trading programs each have prices of \$0.00 per ton, indicating they are currently not binding. Excluding these programs, the lowest carbon price is Poland's carbon tax of \$0.08 per ton. The highest carbon price of \$137.30 per ton is Uruguay's carbon tax on gasoline.

[2] For more examples of overlapping sectoral programs, see https://climate-xchange.org/regional-cap-and-invest/ and https://ec.europa.eu/commission/presscorner/detail/en/qanda_21_3542.

mutually allow their regulated firms to use emissions allowances from any of the linked programs to meet compliance obligations (Jaffe, Ranson, and Stavins 2009).[3] The expected cost savings in linking carbon markets stem from differences in marginal abatement costs, but linking markets with different emissions reduction opportunities will create a revenue transfer that is viewed as politically challenging (Verde et al. 2020). Recent policy discussions regarding the linking of trading programs have considered an allowance exchange rate, which potentially denominates the compliance value of an emissions allowance (i.e., the quantity of emissions per allowance) differently in each program. An exchange rate provides policymakers with a mechanism to improve environmental and economic outcomes, to better balance the costs and benefits of linking programs, and to reconcile different program characteristics that otherwise might prohibit programs from linking. An exchange rate also provides policymakers with a way to link separate sectoral programs in the same jurisdiction without fully harmonizing the programs. We discuss these benefits in more detail below.

In this paper, we demonstrate that an allowance exchange rate can be used to improve both the environmental outcome of a linked system and the economic outcomes in each linked region or sector, as well as to harmonize trading programs with different initial program conditions including marginal costs, program stringency, and price collars. We develop an analytical model of a linked trading system to describe how an exchange rate affects overall efficiency and market outcomes, including distributional effects. We further explore these results by simulating the linking of two trading programs using a detailed model of regional electricity markets within the US that characterizes different specific design features in each program, accounting for how they interact with their respective regional electricity markets.

Carbon trading or pricing programs, as with many public policy choices, are the result of complex decision-making processes that include many competing stakeholder interests (Cohen, March, and Olsen 1972; Kingdon 1984), policy sequencing (Pahle et al. 2018), and political economy constraints (Rabe 2018). Once specific policy components—such as emissions cap levels or price containment mechanisms—are enshrined in law, they typically exhibit policy inertia, making them difficult for future policymakers to change, even during scheduled program

[3] We primarily focus on bilateral links, although a variety of other linking types exist, including incremental alignment of carbon policies, which Burtraw et al. (2013) refer to as "linking by degrees", unilateral linking, various forms of restricted links (Mehling and Haites 2009; Schneider et al. 2017; Quemin and de Perthuis 2019), and multilateral linking (Doda, Quemin, and Taschini 2019).

reviews. Additionally, many authors argue for long-run program stability because administrative changes to carbon markets undermine investor confidence (Salant and Henderson 1978; Aldy et al. 2017; Burtraw et al. 2022). To reflect these realities of real-world climate policy, in our analytical model of linking, we take the designs of the existing trading programs as pre-determined and fixed for the time horizon considered.[4] We then explore how linking at various allowance exchange rates alters market outcomes. In particular, we seek to determine what exchange rates, if any, yield both economic benefits in each region and environmental improvements globally, as well as what exchange rate is socially optimal.

Our analytical model yields novel and non-intuitive results about linking trading programs with an exchange rate. We first find that linking with an exchange rate between the benchmark regimes of autarky (no linking) and traditional 1:1 trading (linking without an exchange rate) yields greater abatement than the amount achieved at each of the bookends, and this additional abatement is achieved at strictly lower costs than autarky. We also show that exchange rate values in this range yield economic surplus gains in each region, as compared to autarky.[5] This range of exchange rates is likely to be the only set of exchange rates that would be considered by policymakers because it represents an intermediate step toward full integration, moving from the status quo toward traditional 1:1 linking. Thus, linking programs with any plausible exchange rate can be expected to yield benefits for the environment and each regional economy that dominate the outcomes under autarky, and yield environmental benefits when compared to the conventional approach to linking.

We further find that traditional linking (i.e., one-for-one trading between programs) with fully fungible allowances, which achieves an efficient distribution of abatement through a uniform allowance price (Mehling, Metcalf, and Stavins 2019), is rarely socially optimal. Trading programs typically fail to set emissions caps at optimal levels—as shown by the wide range of prices in existing programs—and linking with an exchange rate can alter the total amount of abatement, because one allowance no longer corresponds to one unit of emissions. As a result, it is socially optimal to use an allowance exchange rate that trades off the cost-

[4] Investment and emissions reduction efforts that are incentivized by linking may trigger an evolution in relevant market characteristics, including abatement opportunities and their associated costs. That dynamic process is beyond the scope of this initial model of linking with allowance exchange rates, but it likely would be a consequence of the outcomes we identify in this static analytical model.

[5] Our analytical model and these results generalize those of Schneider et al. (2017) and Quemin and de Perthuis (2019).

effectiveness of equating marginal costs in favor of moving the linked system closer to the efficient level of total abatement. When trading program emissions caps achieve an inefficiently low level of emissions abatement, as has been generally observed to date, this socially optimal exchange rate lies within the same likely range described above. This result contributes to the literature showing the optimality of non-uniform carbon prices in many contexts.[6]

Our analytical model also considers the use of an allowance exchange rate when linking programs with price collars, which is novel in the literature. When considering the linking of trading programs with price collars, a potential criticism is that one program's price collar might erode the environmental ambition of the other linked program (Vivid Economics 2020; Doda, Verde, and Borghesi 2022). We show that for a politically plausible range of allowance exchange rates, linked price collars are not expected to bind. Thus, even in the presence of price collars, environmental and economic benefits are expected when linking with an exchange rate.

Our simulation modeling explores the linking of two hypothetical trading programs that cover regions of the United States electricity sector. Using this more robust modeling framework, we confirm our analytical results about abatement, abatement costs, and the socially optimal allowance exchange rate. We find, however, that the efficiency gains may be distributed in ways not predicted by our simple analytical framework. Additionally, linking with an exchange rate can greatly affect the co-benefits of reducing other air pollutants. The consideration of co-benefits usually reinforces the climate benefits and efficiency gains of linking, but it may alter the choice of a specific allowance exchange rate.

A wealth of qualitative literature describes the potential advantages of linking in economic terms and the institutional arrangements that would be necessary under international agreements (Mehling, Metcalf, and Stavins 2018). For example, in principle, bilateral or multilateral linking achieves a unified price per unit of $CO_2$ emissions across the newly linked system that is expected to lower overall abatement costs. The potential gains from efficiently allocating abatement are greater the greater are differences in pre-linked allowance prices. Linking also can dampen allowance price volatility caused by regional variations in the demand or supply of allowances because typically the factors that influence emissions such as weather or economic activity are imperfectly correlated across jurisdictions (Flachsland, Marschinski, and

[6] Other examples include asymmetric information (Chávez and Stranlund 2009; Holland and Yates 2015), enforcement costs (Stranlund, Chávez, and Villena 2009), and intertemporal concerns (Leiby and Rubin 2001; Yates and Cronshaw 2001; Feng and Zhao 2006).

Edenhofer 2009; Burtraw et al. 2013; Doda, Quemin, and Taschini 2019). In some circumstances, linking can ameliorate concerns over competitiveness impacts by explicitly addressing the possibility for leakage of economic activity between jurisdictions that may result from differences in program stringency (Jaffe, Ranson, and Stavins 2009). Moreover, there are other potentially significant benefits to linking that are not economic in nature. From an environmental perspective, the reduction in abatement costs achieved by linking could make it easier to enhance ambition (Bodansky et al. 2016). From a political perspective, linking starts to dispel the free-rider narrative that can prevent individual jurisdictions from pricing carbon in the absence of an international carbon price (Flachsland, Marschinski, and Edenhofer 2009).

There is also a significant qualitative literature that outlines the potential costs of linking. First and foremost, established links between trading programs have required significant negotiations between jurisdictions to harmonize the design of the programs; the time and resources spent on this process of harmonization can be thought of as a fixed cost of linking. In addition, the efficiency gains achieved by linking may come with associated costs. For example, linking requires ceding some control over domestic allowance prices, which might be regarded as a political cost (Ranson and Stavins 2016), or a virtue when it insulates policymakers from narrow interest groups within their jurisdiction (Burtraw et al. 2013). While linking may reduce overall abatement costs, it may have negative economic impacts on particular actors in each jurisdiction (Newell, Pizer, and Raimi 2013).[7] Moreover, linking can exacerbate allowance price volatility in certain cases (Doda and Taschini 2017). From an environmental perspective, a broader market is likely to reduce leakage, but linking could increase emissions leakage if allowance prices increase in the program that is more susceptible to leakage (Jaffe, Ranson, and Stavins 2009) and may alter incentives for cap setting, encouraging programs to set higher emissions caps to achieve lower prices and therefore export more allowances, thereby resulting in higher emissions than would occur without linking (Bohm 1992; Helm 2003). Weitzman (2019) refers to the former issue as the primary free-rider problem and describes elements of program design including price floors and ceilings to affect distributional outcomes as a potential secondary free-rider problem. Strategic considerations may influence the decision about how to initially distribute emissions allowances, for example, through the use of output-based allocation

[7] In jurisdictions where allowance prices increase due to linking, compliance entities or consumers who purchase goods from these entities will experience greater costs. Conversely, in jurisdictions where allowance prices decrease due to linking, any agent holding excess permits will experience a reduction in the value of these assets and governments will receive less revenue from allowance auctions.

to provide a production incentive to mitigate leakage that would likely increase the allowance price (Palmer et al. 2017). Linking also might provide an incentive to introduce companion policies, such as technology support policies, that reduce local demand for allowances, to increase allowance exports and associated government revenues.

Weighing the advantages and disadvantages of a specific link requires an accounting of the unique designs of each of the involved trading programs and how they would interact under a particular linking architecture. Quantitative approaches are useful in this regard. One vein of the quantitative literature on linking utilizes models to provide estimates of the efficiency gains achieved by linking (Springer 2003) or the emissions outcomes of different coalitions of linked trading systems (Paltsev 2001). A second vein of the quantitative literature on linking takes an analytical approach to investigate the impact of different linking architectures (e.g., a link between mass and rate-based trading programs or a restricted one-way link that discounts incoming allowances) (Fischer 2003; Schneider et al. 2017), or the impacts of unique program design features (e.g., market size) on the economic implications of linking (Doda and Taschini 2017).

Jurisdictions considering a potential link have some control over the domestic costs and benefits of the link by using an allowance exchange rate, which denominates the compliance value of an emissions allowance (i.e., the quantity of emissions per allowance) differently in each program. That is, an exchange rate mandates that an allowance from one program is worth more or less, in terms of compliance (allowable tons per allowance), than is an allowance from another program. While economists typically discuss exchange rates in the context of pollutants that impose local damages that vary by the source of emissions (Hung and Shaw 2005), the scholarly interest in applying exchange rates in the context of greenhouse gas emissions has increased in recent years (Fischer 2003; Metcalf and Weisbach 2012; Holland and Yates 2015; Böhringer and Fischer 2020). Greenhouse gas allowance exchange rates have also been included in recent policy discussions, including efforts by the World Bank's Networked Carbon Market Initiative[8] (Marcu 2015; Macinante 2016) and China's stated intentions to discount allowances from regional emissions markets when its national trading program launches (Carbon Pulse

[8] The World Bank's Networked Carbon Market Initiative is focused on facilitating cross-border allowance trades based on a shared understanding of the relative value of different actions, instead of "harmonizing" climate actions so that units can be traded on a one-to-one basis.

2016).[9] In addition, a discount rate—which is effectively an "asymmetric" exchange rate—is being negotiated under Article 6 of the Paris Agreement to achieve overall mitigation in global emissions (Schneider et al. 2018; Piris-Cabezas and Lubowski 2019). A fluctuating exchange rate may emerge implicitly in programs linking jurisdictions with different monetary currencies if at least one has a binding price collar, such as the California-Quebec link. Quemin and de Perthuis (2019) and Schneider et al. (2017) compare exchange rates with other mechanisms, such as quantitative limits on trading and border adjustments as transitional mechanisms, to guide heterogeneous programs towards cost-effective outcomes. Linking with an explicit exchange rate would involve accounting for the jurisdiction of origin in allowance portfolios in allowance exchanges and portfolios, which incidentally may offer an administrative remedy to some of the challenges of potential de-linking (Pizer and Yates 2015).

Where carbon pricing already exists in segments of the economy, many jurisdictions are considering new and separate programs in other sectors. One motivation for separate programs is that the price elasticity of emissions reductions is relatively low, for example in the transportation and building sectors, and a uniform carbon price linking to existing programs would impose a cost burden in those new sectors without yielding substantial investment or emissions reductions.[10] Nonetheless, carbon pricing can accelerate diffusion of technology and provide revenue to fund infrastructure and other investments. Even within existing regional programs, the ambitions of individual jurisdictions often evolve differently over time. Many jurisdictions have mandated accelerated emissions reductions even while they are linked within a broader regional market, which could result in internal emissions leakage within the broader market, sometimes described as the waterbed effect (Perino 2018; Perino, Ritz, and van Bentham 2020). By using an allowance exchange rate to link sectors or jurisdictions that have different ambition and abatement opportunities, policymakers can sustain the environmental ambition of each program and improve cost effectiveness while attenuating distributional concerns and financial flows among sectors or programs.

Both the qualitative and quantitative veins of the literature are useful in characterizing the theoretical benefits and costs of linking but tend to assume that trading programs are nearly

[9] See also https://icapcarbonaction.com/en/ets-map?etsid=55.

[10] Different sectors will also yield different ancillary air quality benefits and are subject to different companion policies, trade exposure, and implicit tax interaction effects, any of which could motivate a jurisdiction to price carbon differently across sectors.

identical in design. We extend the existing literature by evaluating the linking of regional and sectoral trading programs with various and different designs (i.e., different levels of program stringency and price collars) and considering how different design parameters interact with alternative architectures for linking (e.g., different exchange rates for allowances). We make two primary contributions with this work. First, we develop an analytical model that formalizes the economic implications and emissions market outcomes of regional or sectoral linking with an exchange rate. With this model, we are the first to analytically describe the linking of sectoral trading programs with an allowance exchange rate. The model yields novel findings on the results of linking emissions markets, as well as the formalization of results that had previously been described only qualitatively. Second, we test several of our analytical results and illustrate other important market outcomes of linking by simulating a link between regional trading programs. We use a simulation model of electricity markets within the US to characterize the specific design features of two hypothetical regional trading programs, accounting for how they interact with their respective regional electricity markets. We simulate the trading programs in autarky and under various exchange rates. The electricity market model allows us to illustrate a range of efficiency and distributional implications and emissions outcomes that can arise from linking without losing the detailed designs of the two emissions markets as well as the nuanced and important interactions that might occur between them when linked.

## 2. Analytical Model of Linking

We model a regional economic sector with production supplied by a representative firm. We first show how this representative firm responds when faced with a policy that imposes a price on $CO_2$ emissions. We next describe the equilibrium outcomes of regional or sectoral emissions trading programs in two separate markets, which we describe as autarky. We then show how the outcomes change when two emissions markets link through the trade of allowances.[11] We describe a link as "regional" when it involves two programs that cover different geographic regions and are administered by different governmental bodies. We describe a link as "sectoral" when it involves two programs covering different sectors of the economy within the same geographic region.

[11] We initially only consider an emissions cap, but we later introduce a price collar on emissions allowances. This more general model also applies to the linking of a broader set of carbon pricing policies, such as a carbon tax, which can be interpreted as an emissions trading program with a price floor that is coincident with a price ceiling. Metcalf and Weisbach (2012) consider linking between a cap and trade program and a carbon tax.

This analytical model generalizes Quemin and de Perthuis (2019) by using more general cost functions and allowing for more program asymmetry.[12] We also consider additional market outcomes and focus on a specific range of outcomes that we believe are particularly policy-relevant. This model also bears similarities to Holland and Yates (2015), but we note two important differences. First, Holland and Yates (2015) use trading ratios that allow for more program flexibility than our allowance exchange rate;[13] we argue below that our more restrictive framework better replicates the real-world policy environment. Second, Holland and Yates (2015) include asymmetric information, which we abstract away from in order to focus on allowance exchange rates. These different assumptions yield important differences in our policy-relevant conclusions.

### *Production*

Production within a regional economic sector is characterized by a representative firm that uses a particular production technology and energy to produce a fixed level of output at lowest cost. The cost to the representative firm of producing output, $K(E)$, is a function of $CO_2$ emitted during production, $E$. We assume that $K(E)$ is a positive, convex function of emissions that is minimized at emissions level $\bar{E}$.[14] This production cost function is region- and sector-specific and depends on the quantity of output produced and the firm's production technology.

[12] Quemin and de Perthuis (2019) assume abatement costs are quadratic in abatement and, hence, marginal abatement costs are linear in abatement. They also assume each trading program has the same level of unregulated emissions and the same emissions caps. These assumptions improve tractability and allow them to make important comparisons between different linking schemes. As we focus on only one form of linking—an allowance exchange rate—we are able to make fewer assumptions. We assume abatement costs are increasing and convex in abatement; in Appendix B, we consider linear marginal abatement costs. Also, we allow initial conditions—including unregulated emissions and emissions caps—to vary by program.

[13] Holland and Yates (2015) use trading ratios that can be individually altered to affect both relative allowance prices and individual program stringency. For example, increasing a program's trading ratio not only increases its relative price but also increases the number of domestic allowances required for compliance, effectively making its cap more stringent. We argue below that increasing program stringency—that is, reducing the emissions cap—is politically challenging and often not a part of linking negotiations. Thus, we consider an allowance exchange rate that affects only relative allowance prices but not the number of domestic allowances required for compliance.

[14] We formally define these assumptions in Appendix A.

We further assume this function is fixed over the time horizon considered.[15] With no carbon emissions policy in place, this firm minimizes production cost by emitting $\bar{E}$.

If the firm is subject to an emissions policy that imposes an opportunity cost of $p$ on each unit emitted, the firm deviates from this baseline level of emissions. The additional cost of producing output with fewer emissions is a function of the level of abatement, $A = \bar{E} - E$, and is given by $C(A) = K(\bar{E} - A) - K(\bar{E})$. This abatement cost function, $C(A)$, is an increasing, convex function of emissions abatement with $C(0) = 0$. The marginal abatement cost function, which we denote as $c(A) = \frac{\partial C}{\partial A}(A)$, is also an increasing and convex function of emissions abatement with $c(0) = 0$.

Each unit of abatement reduces the firm's cost of policy compliance by $p$. When faced with this opportunity cost on emissions, the firm selects the level of abatement that minimizes its total cost:

$$\min_{A} C(A) - pA$$

This optimization problem yields the first-order condition:

$$c(A) = p$$

This is the familiar result that the representative firm's optimal level of abatement equates its marginal abatement cost, $c(A)$, to the marginal cost of emissions, $p$.

### *Emissions Trading in Autarky*

We now consider the specific design of the emissions trading program and the resulting outcomes—abatement, allowance prices, and abatement cost—that occur in this market in autarky. Although an emissions trading policy has many design parameters through which the program can be adjusted, this analytical model focuses initially on only one, and arguably the most important, of these policy parameters: the level of the cap; we later also introduce a price collar on emissions allowances.

[15] Holding this production cost function fixed implicitly assumes that productive capital and the level of production are fixed over the time horizon considered. This simplifying assumption corresponds well with the electricity sector, which features long lead times for new capacity and demand that is highly inelastic. Our simulation model also assumes electricity demand and aggregate production are fixed.

A government setting climate policy, or any major public policy, can be viewed as an "organized anarchy," which Cohen, March, and Olsen (1972) describe as a collection of "choices looking for problems, issues and feelings looking for decision situations in which they might be aired, solutions looking for issues to which they might be an answer, and decision makers looking for work."[16] Thus, because it is infeasible to accurately model the policymaker's complex and highly dimensional decision making as a mathematical optimization problem, we take the policy design parameters—the level of the emissions cap and, later, the price collar—that result from this process as given.[17]

The intended emissions cap yields $\bar{A}$ units of abatement by initially distributing a number of allowances equal to $\bar{E} - \bar{A}$, each of which authorizes the holder to emit one unit of $CO_2$. Combining this emissions cap with the firm's first-order condition yields the level of abatement, allowance price, and cost of abatement in autarky: $A^0 = \bar{A}$, $p^0 = c(\bar{A})$, and $C^0 = C(\bar{A})$.[18]

### *Linked Emissions Trading*

We now consider two independent emissions trading programs that link through the trade of emissions allowances.[19] Program $h$ has the higher allowance price and program $l$ has the lower allowance price in autarky: $p_h^0 > p_l^0$.[20] All characteristics of the representative firm and policy—such as the abatement cost function and number of allowances issued—can vary across the different programs. Emitters in each program can comply with the emissions policy by holding allowances issued by either program, but allowances are traded between the programs at

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[16] Cohen, March, and Olsen (1972) originally applied this description to universities in their garbage can model of organizational choice. Kingdon (1984) later adapted this model to a government setting public policy in his multiple streams framework. Rabe (2018) describes the complexities of setting climate policy in particular.

[17] Once a linked system has been established, these parameters might subsequently evolve endogenously. We leave this dynamic policy game as an important area for further research.

[18] The 0 superscript indicates outcomes in autarky.

[19] We assume the two representative firms only trade emissions allowances and do not interact in their input or output markets. This simplifying assumption allows us to focus our analysis on the emissions trading market. Additionally, the assumption corresponds well with the electricity sector, which is the context for our simulation model.

[20] We do not consider the linking of emissions trading programs with equal allowance prices in autarky and instead focus on programs with different market conditions in autarky.

a mutually agreed exchange rate. We assume each program continues to issue its own unique allowances, rather than jointly issuing a single compliance instrument.[21]

The exchange rate, $r$, is the number of allowances from program $l$ that are equivalent for compliance purposes to one allowance from program $h$. In other words, for each unit of $CO_2$ emitted by the firm in program $h$, it must have either one allowance from program $h$ or $r$ allowances from program $l$. Similarly, for each unit of $CO_2$ emitted by the firm in program $l$, it must have either one allowance from program $l$ or $\frac{1}{r}$ allowances from program $h$.

In this model, the exchange rate, $r$, is the only policy parameter that policymakers can alter. We observe that climate policy design parameters, such as the level of the cap or the price collar, exhibit strong policy inertia once they are codified in law and are often difficult for future policymakers to alter, and typically they have not been negotiated as a part of linkage negotiations.[22] Reflecting this reality, we assume these other policy parameters are pre-determined and fixed; they are not concurrently up for negotiation when policymakers choose to link and select an allowance exchange rate.[23] Thus, we consider how the value of the exchange rate affects market outcomes while holding these other policy parameters constant.

When linking emissions trading programs with an exchange rate, total abatement is not necessarily equal to abatement in autarky because one allowance no longer corresponds to one unit of emissions. Every $r$ allowances traded from program $l$ to program $h$ reduces emissions by $r$ units in program $l$ and increases emissions by one unit in program $h$. That is, at the linked market equilibrium, the following expression must hold:

$$r(A_h - \bar{A}_h) = \bar{A}_l - A_l$$

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[21] The same results can be achieved using a single compliance instrument, but the number of allowances issued must be adjusted to account for the exchange rate.

[22] This situation could arise for many reasons, including differences in the analytical and administrative resources available among jurisdictions. The “hub” model of carbon market development has been evident in the EU and Western Climate Initiative, where an existing thoroughly developed program provides a template for others to adopt (ICAP 2021). Additionally, linking of trading programs is likely to be less salient than changes to the number of emissions allowances and, hence, subject to less restrictive political constraints.

[23] Over the long run, policy preferences may evolve differently in a linked market context compared to autarky. This static model provides the initial conditions for future work that considers the longer-run dynamics of linking.

Additionally, we assume there are no arbitrage opportunities across the programs, [24] so the price of an allowance from program $h$ is $r$ times the price of an allowance from program $l$:

$$p_h = rp_l$$

We use this framework to analyze how important market outcomes of the linked system are affected by the selection of the allowance exchange rate. Due to the "organized anarchy" of climate policy, we do not attempt to rationalize a policymaker's objective function and solve for their optimal exchange rate. Instead, we examine outcomes that are likely to be important to policymakers and their constituents: abatement, abatement costs, and distributional effects. In doing so, we seek to determine what exchange rates, if any, yield both economic benefits in each region and environmental improvements globally, as well as what exchange rate is socially optimal.

**Abatement and Abatement Cost**

The choice of allowance exchange rate determines both the direction of net allowance flow between programs and the number of emissions embedded in every imported allowance. When linked, allowances flow from the program with the lower allowance cost in autarky to the program with the higher allowance cost, and those relative costs depend on the chosen exchange rate. For example, the firm in program $h$ will import allowances only if the cost of $r$ imported allowances in autarky is less than the cost of one domestic allowance: $rp_l^0 < p_h^0$. Thus, the direction of net allowance flow depends on whether the exchange rate is greater than or less than the ratio of allowance prices in autarky, $\frac{p_h^0}{p_l^0}$.[25] At the same time, whether each allowance imported

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[24] If this were not the case, then any emitter holding the higher-valued allowance could arbitrage the allowance price difference by selling the higher-valued allowance and buying the comparable number of lower-valued allowances.

[25] Linking at an exchange rate equal to the ratio of autarkic allowance prices, $\frac{p_h^0}{p_l^0}$, yields outcomes that are equivalent to autarky, so there is no net trade of allowances. Linking at a lower exchange rate, $r < \frac{p_h^0}{p_l^0}$, yields a net flow of allowances traded from the lower-priced program $l$ to the higher-priced program $h$ as the price gap shrinks compared to autarky. Linking at a higher exchange rate, $r > \frac{p_h^0}{p_l^0}$, yields a net flow of allowances traded from the higher-priced program $h$ to the lower-priced program $l$ as the price gap grows compared to autarky.

into the higher-cost program enables more or less than one unit of emissions depends on whether the exchange rate is greater than or less than 1.[26]

These two allowance exchange rate benchmarks, $\frac{p_h^0}{p_l^0}$ and 1, represent important linking regimes. An exchange rate of $\frac{p_h^0}{p_l^0}$, which yields outcomes equal to autarky, represents the status quo if the programs do not link. An exchange rate of 1, which achieves a uniform allowance price across the two programs, represents a traditional link with fully fungible allowances. If policymakers are interested in using an exchange rate as an intermediate step from the status quo to full integration, then an exchange rate between these two benchmark regimes is most likely: $1 < r < \frac{p_h^0}{p_l^0}$. Our first result is how the exchange rate—in particular, whether it is within or outside this likely range—affects total abatement and total abatement cost.[27]

**Result 1.** Linking at an allowance exchange rate of:

i. $1 < r < \frac{p_h^0}{p_l^0}$ yields greater total abatement and less total abatement cost than autarky.

ii. $r < 1$ or $r > \frac{p_h^0}{p_l^0}$ yields less total abatement and less total abatement cost than autarky.

Importantly, when the exchange rate is within the likely range of $1 < r < \frac{p_h^0}{p_l^0}$, the linked system achieves greater abatement at lower cost, compared to autarky.[28] In this range, net allowances flow from the lower-priced program $l$ to the higher-priced program $h$. Each additional unit of emissions in program $h$ requires more than one allowance from program $l$, so the overall cap is effectively tightened. This outcome reinforces the political preferences that

[26] At a lower allowance exchange rate of $r < 1$, every allowance traded from program $l$ to program $h$ permits more than one unit of emissions in $h$. At a higher exchange rate of $r > 1$, every allowance traded from program $l$ to program $h$ permits less than one unit of emissions in $h$, and every allowance traded from program $h$ to program $l$ permits more than one unit of emissions in $l$.

[27] Proofs of all results are in Appendix A.

[28] Schneider et al. (2017) and Quemin and de Perthuis (2019) find a similar range of exchange rates that increase abatement and improve cost-effectiveness—which they describe as "effective exchange rates" and the "reduction zone," respectively—under the assumption of linear marginal abatement costs. We confirm that these prior abatement results generalize with less restrictive cost assumptions. We further show that abatement costs are reduced at all exchange rates, which has not been described in the existing literature.

motivate the trading programs, strengthening the argument that an exchange rate in this range is most likely. Additionally, this trade of allowances shifts abatement from the program with higher marginal cost to the program with lower marginal cost, and this reallocation of abatement reduces the total abatement cost across the linked system.

This result is shown graphically in Figure 1, in which we plot the levels of abatement and the abatement costs for an illustrative example of trading programs with quadratic abatement costs.[29] In this example, the ratio of autarkic allowance prices is $\frac{p_h^0}{p_l^0} = 3$, which, along with an exchange rate of 1, are denoted by dashed vertical lines. At exchange rates in the likely range between 1 and 3, the linked system achieves greater total abatement at lower cost than in autarky. Total abatement costs are also lower than autarky outside this core range of exchange rates, but only because these more extreme exchange rates effectively loosen the cap and reduce the total level of abatement achieved by the linked system.[30]

**Optimal Exchange Rate**

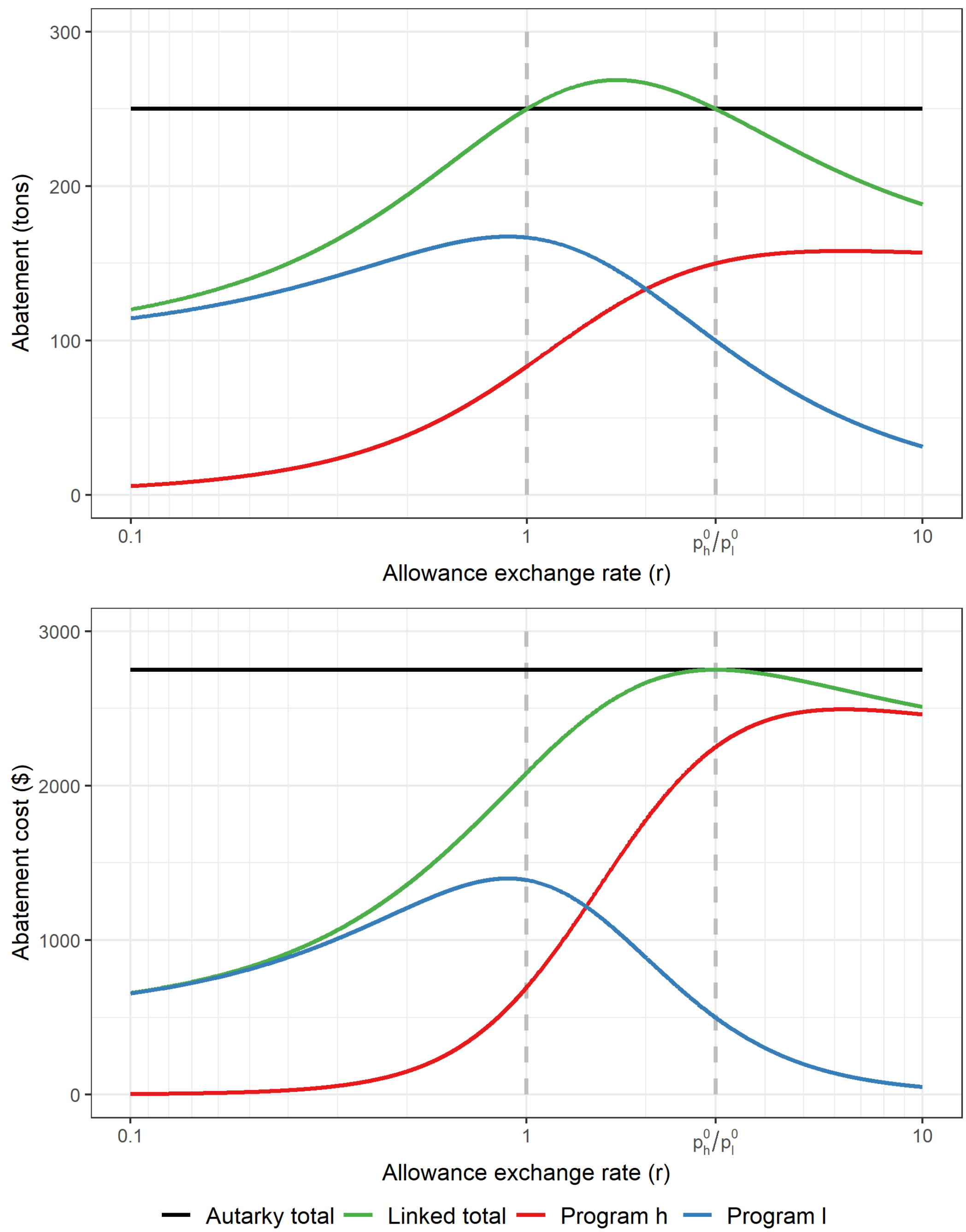
The social welfare gains or losses from linking are a function of abatement and abatement costs in the linked system, which we have shown depend on the allowance exchange rate. Although policymakers are not simply maximizing welfare—rather, they respond to a complex set of political incentives—it is still instructive to determine what exchange rate maximizes the welfare gains of linking. That socially optimal exchange rate can serve as a benchmark against which to compare the plausible exchange rates under consideration by policymakers, to understand if their political incentives may be aligned with the broader social incentives.

It is well known that abatement is efficiently allocated across programs when allowance prices are equal and all emitters face the same marginal incentive to abate, which occurs at an exchange rate of $r = 1$. If the total level of abatement is not optimal, however, the theory of the second best suggests it may be socially beneficial to trade off this efficient allocation of abatement in favor of approaching the optimal quantity of abatement.

[29] To generate this figure, we use: $\bar{A}_h = 150$, $C_h(A_h) = 0.1A_h^2$, $\bar{A}_l = 100$, and $C_l(A_l) = 0.05A_l^2$. Results are qualitatively similar for other abatement cost functions and abatement levels.

[30] Average abatement cost across the linked system—given by the ratio of total abatement cost to total abatement—increases monotonically with the exchange rate as abatement shifts from the lower-cost program $j$ to the higher-cost program $i$.The total level of abatement also changes with the exchange rate, however, so average abatement cost is not an appropriate metric to compare the economic efficiency of allowance exchange rates.

**Figure 1: Analytical results—Abatement and cost when linked**




*Notes*: This figure plots an illustrative numerical example of abatement (top panel) and abatement cost (bottom panel) in each program and in the linked system as a function of the allowance exchange rate. This example is parameterized with autarky abatement and abatement costs of $\bar{A}_h = 150$, $C_h(A_h) = 0.1A_h^2$, $\bar{A}_l = 100$, and $C_l(A_l) = 0.05A_l^2$. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates. The darkest horizontal line represents aggregate outcomes in autarky to highlight comparisons with that regime.

To determine the socially optimal exchange rate, consider a global pollutant with constant marginal damages $d$. As described previously, we assume each program's level of abatement and abatement cost curve are fixed, so the only choice variable is the exchange rate. The social planner seeks to maximize welfare, given by the benefits of abatement net of the costs:

$$\max_{r} d(A_h + A_l) - C_h - C_l$$

This optimization problem yields the first-order condition:

$$(d - p_h)\frac{\partial A_h}{\partial r} = -(d - p_l)\frac{\partial A_l}{\partial r}$$

where $d - p_h$ is the net benefit of the marginal unit of abatement in program $h$ and $\frac{\partial A_h}{\partial r}$ is the marginal quantity of abatement in program $h$ with respect to the exchange rate, so the product is the marginal welfare improvement with respect to the exchange rate. Note that both $p_h$ and $\frac{\partial A_h}{\partial r}$ are functions of $r$. The optimal exchange rate equalizes the magnitude of the marginal welfare improvement in each program.

From this first-order condition, we see that an exchange rate of $r = 1$ is optimal only when the linked allowance price—and, hence, the marginal abatement cost in each program—is equal to the marginal damage of emissions, $p_h = p_l = d$.[31] If an exchange rate of 1 yields a linked allowance price that is below $d$, however, then social welfare can be improved with an exchange rate that increases abatement. Conversely, if an exchange rate of 1 yields a linked allowance price that is above $d$, then social welfare can be improved with an exchange rate that decreases abatement. This intuition gives our next result.[32]

**Result 2.** The social optimal allowance exchange rate, $r^*$, lies in the interval:

i. If $r = 1$ would yield $p_h = p_l < d$, then $1 < r^* < \frac{p_h^0}{p_l^0}$.

ii. If $r = 1$ would yield $p_h = p_l > d$, then $r^* < 1$ or $r^* > \frac{p_h^0}{p_l^0}$.

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[31] Allowance prices when linked are equal if and only if the exchange rate is $r = 1$.

[32] Proofs of all results are in Appendix A.

This result adds to the literature documenting that non-uniform allowance prices are optimal—even for uniformly mixed pollutants—in many practical contexts. Prior work has shown this optimality due to asymmetric information (Chávez and Stranlund 2009; Holland and Yates 2015), enforcement costs (Stranlund, Chávez, and Villena 2009), and intertemporal concerns (Leiby and Rubin 2001; Yates and Cronshaw 2001; Feng and Zhao 2006). In contrast, we find non-uniform allowance prices are also optimal when emissions caps are set at inefficient levels.[33]

All existing carbon pricing programs have prices below the most recently estimated social cost of carbon, \$185 per metric ton of $CO_2$ (Rennert et al. 2022), as described in Section 1. Thus, for the linking of any real-world trading programs, social welfare is maximized when the programs are linked with an allowance exchange rate that is between 1 and $\frac{p_h^0}{p_l^0}$, which increases abatement in the linked system. Because this interval also represents the exchange rates most likely to be considered by policymakers, linking programs using exchange rates theoretically can be expected to achieve outcomes close to the social optimum, even though policymakers may not directly intend to maximize social welfare.

The optimal exchange rate result is shown graphically in Figure 2, in which we plot the climate benefits, abatement cost, and net social welfare gains of the linked system, as compared to autarky, for the same illustrative trading programs used in Figure 1.[34] The social welfare gains of linking are maximized when these programs link at an exchange rate of $r \approx 1.6$. Any exchange rate in the range $1 < r < \frac{p_h^0}{p_l^0}$ yields welfare gains compared to autarky, and nearly all of this range yields welfare gains compared to an exchange rate of $r = 1$ that efficiently allocates abatement between the two programs.

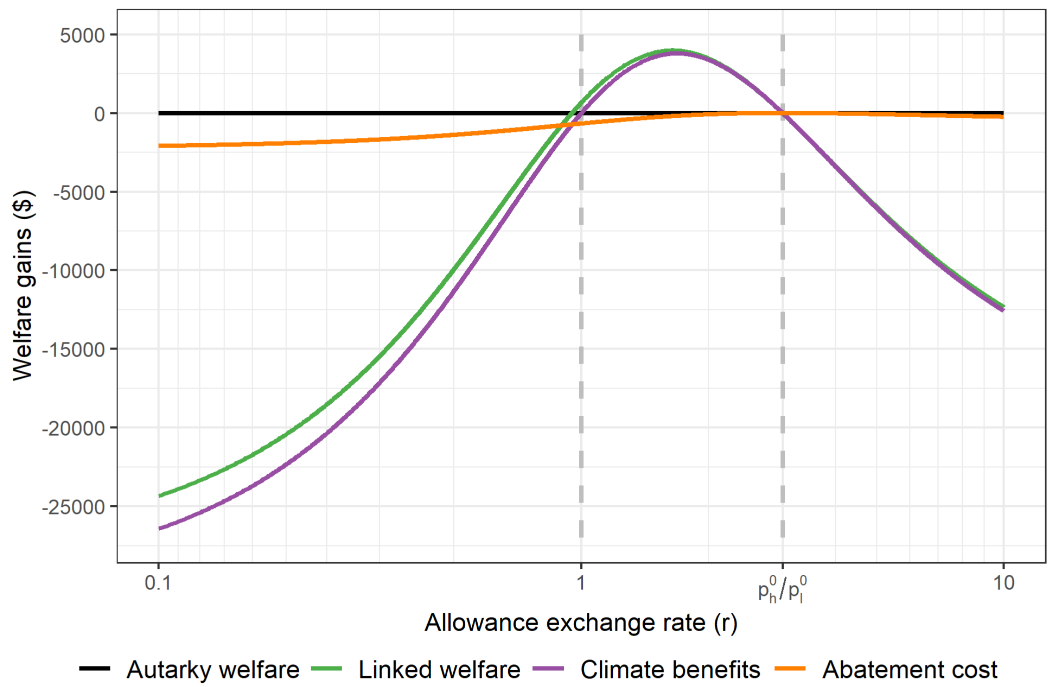
**Distributional Effects**

Policymakers may be interested not in setting the exchange rate to maximize social welfare gains or minimize systemwide abatement costs, but rather to benefit their own

33 Quemin and de Perthuis (2019) define the mathematical optimization problem for each program's optimal exchange rate but do not solve that complex optimization. We build on that work to consider the exchange rate that is socially optimal when jointly considering both programs, and we place that optimal exchange rate in relation to important benchmark regimes of autarky and 1:1 linking.

34 To generate this figure, we use: $\bar{A}_h = 150$, $C_h(A_h) = 0.1A_h^2$, $\bar{A}_l = 100$, and $C_l(A_l) = 0.05A_l^2$. Results are qualitatively similar for other abatement cost functions and abatement levels yielding too little abatement in autarky.

**Figure 2: Analytical results—Welfare gains compared to autarky when linked**




*Notes*: This figure plots an illustrative numerical example of welfare gains, climate benefits, and abatement costs in the linked system, compared to autarky, as a function of the allowance exchange rate. This example is parameterized with autarky abatement and abatement costs of $\bar{A}_h = 150$, $C_h(A_h) = 0.1A_h^2$, $\bar{A}_l = 100$, and $C_l(A_l) = 0.05A_l^2$. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates. The darkest horizontal line indicates zero to highlight these comparisons with autarky.

government or region. To this end, we next show how the choice of the exchange rate differently affects several distributional metrics in each region or sector. In doing so, we generalize a similar analysis of Quemin and de Perthuis (2019).

We first consider the revenue raised by the regional government through the sale of allowances.[35] The government revenue ($GR$) generated by each trading program is the product of the allowance price and the difference between emissions with no carbon emissions policy in place and the intended abatement:

$$GR_h = p_h(\bar{E}_h - \bar{A}_h) \quad \text{and} \quad GR_l = p_l(\bar{E}_l - \bar{A}_l)$$

[35] For this metric, we assume all emissions allowances are sold by the regional government at the market-clearing price; a multi-unit, uniform-price auction is an example of a mechanism that would achieve this outcome. If this is not the case and some allowances are freely allocated, $\bar{E}_h - \bar{A}_h$ and $\bar{E}_l - \bar{A}_l$ should be replaced by the quantity of allowances sold by the respective government.

Under regional linking, these expressions give the revenues raised by each regional government. Under sectoral linking, however, the two sectoral programs are overseen by the same government, so the expressions describe the two distinct revenue streams for that single government.

We also consider the net flow of allowance revenues into or out of a trading program due to the export or import of emissions allowances. The net revenue flow ($NRF$) into each trading program is:

$$NRF_h = p_h(A_h - \bar{A}_h) \quad \text{and} \quad NRF_l = p_l(A_l - \bar{A}_l)$$

This metric is more relevant for regional linking and represents revenue flow into or out of the region. Under sectoral linking, which occurs within the same region, there is no interregional exchange of allowances or revenues.

We finally consider total economic surplus—excluding the climate benefits—generated in each trading program,[36] which is a function of net revenue flows less abatement costs. The total economic surplus ($TS$) in each trading program is:

$$TS_h = p_h(A_h - \bar{A}_h) - C_h + Z_h \quad \text{and} \quad TS_l = p_l(A_l - \bar{A}_l) - C_l + Z_l$$

where $Z_h$ and $Z_l$ represent the economic surplus generated in the output market in each regional sector.[37] Under regional linking, these expressions represent the economic surplus generated for each regional economy. Under sectoral linking, which occurs within the same region, the economic surplus generated in that single regional economy is the sum of these two values.

The revenue metrics are functions of the direction of net allowance flow, which depends on whether the exchange rate is greater than or less than the ratio of allowance prices in autarky, $\frac{p_h^0}{p_l^0}$. Total economic surplus, however, is a function of a firm's full compliance cost: the sum of abatement cost and the cost of purchasing allowances. The firm in either program will only trade

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[36] Our measure of total economic surplus excluding climate benefits is conceptually the same as the "efficiency gains" analyzed by Quemin and de Perthuis (2019). Due to the trade of allowances, which transfers value between the two programs, it is not immediately obvious that the efficiency gains of trading would capture the full welfare gains—excluding climate benefits—that accrue to each program, so we use the different terminology to clarify this point.

[37] As described previously, the level of production and consumption are assumed to be fixed over the time horizon considered. This assumption corresponds well with demand for electricity, which is highly inelastic. Our simulation model also assumes electricity demand and aggregate production are fixed.

allowances if the trade reduces its compliance cost. Thus, any allowance trade—no matter the direction of net allowance flow—will reduce compliance cost and improve economic surplus in each program. Following this intuition, the effect of the exchange rate on each of the three distributional metrics is our third result, which we report separately for regional linking (Result 3a) and sectoral linking (Result 3b).[38]

**Result 3a.** Regional linking (at a rate other than $r = \frac{p_h^0}{p_l^0}$) always yields greater total economic surplus—excluding climate benefits—than autarky in each region, but it results in opposite revenue effects in each region:

i. Linking at $r < \frac{p_h^0}{p_l^0}$ yields less government revenue than autarky and negative net revenue flows in region $h$ but greater government revenue than autarky and positive net revenue flows in region $l$.

ii. Linking at $r > \frac{p_h^0}{p_l^0}$ yields greater government revenue than autarky and positive net revenue flows in region $h$ but less government revenue than autarky and negative net revenue flows in region $l$.

**Result 3b.** Sectoral linking (at a rate other than $r = \frac{p_h^0}{p_l^0}$) within the same region always yields greater total economic surplus—excluding climate benefits—than autarky in the region, but it may increase or decrease the regional government's total revenues.[39]

In particular, linking within the likely range of $1 < r < \frac{p_h^0}{p_l^0}$ will result in economic surplus gains—even when excluding the climate benefits—in each regional economy. Thus, because the exchange rate is expected to fall within this plausible range, linking with an exchange rate is likely to yield environmental benefits globally and economic benefits in each region—and the

[38] Proofs of all results are in Appendix A.

[39] The effect of sectoral linking on government revenue depends not only on how the exchange rate compares to the ratio of allowance prices in autarky, but also on the relative size of the two sectors and the relative stringency of the two trading programs.

linked system may approach the socially optimal outcome—even if none of these are the explicit goals of policymakers when linking.

Linking within this likely range of exchange rates will, however, also affect government revenues and the flow of allowance revenues, which may be of particular importance to policymakers. For example, under regional linking, an exchange rate in this likely range will always yield net revenue flows from region $h$ to region $l$. Under sectoral linking, an exchange rate in this likely range may increase or decrease government revenues, depending on characteristics of the sectors and trading programs. Thus, if a policymaker has a different objective function—such as maximizing government revenue or revenue flows into the region to fund investment—then this distributional outcome could instead create a barrier to linking. Nonetheless, linking in this likely range of exchange rates will yield distributional outcomes between the bookend cases of no linking ($r = \frac{p_h^0}{p_l^0}$) and traditional linking with fully fungible allowances ($r = 1$). As a result, if distributional concerns create a barrier to traditional linking at $r = 1$, an exchange rate in this likely range of $1 < r < \frac{p_h^0}{p_l^0}$ could moderate the distributional effects and engender links that otherwise would not occur, increasing total economic surplus in each region and increasing aggregate abatement.

**Price Containment Mechanisms**

Many emissions trading programs include a price containment mechanism to constrain the price of an emissions allowance from rising too high or falling too low or both. This price collar ensures that the allowance price and the resulting distributional outcomes of the trading program remain in politically acceptable ranges. When linking, however, policymakers may believe they are forced to cede some control over this important policy mechanism.[40]

We add to our model a price collar in each trading program. Each price collar is composed of a price floor, $\bar{p}^F$, and a price ceiling, $\bar{p}^C$.[41] We assume price collars do not bind in either program in autarky, as a binding price collar suggests the policy is constrained by political

[40] Vivid Economics (2020) and Doda, Verde, and Borghesi (2022) provide detailed discussions of linking emissions trading programs with market stability measures and call for price collars to be aligned before linking.

[41] There are several ways to achieve a price floor and price ceiling in practice. For example, the regional government could auction allowances with a reserve price of $\bar{p}^F$ and further offer an unlimited number of additional allowances for sale at a price of $\bar{p}^C$.

considerations that could also restrict linking.[42] When the two trading programs link with allowance exchange rate $r$, the effective price collar in each program may differ from the nominal price collars. For example, the firm in program $h$ faces a price ceiling of $\bar{p}_h^C$ in its own program but a price ceiling of $r\bar{p}_l^C$ when importing allowances, and the lesser of these two prices will bind. Thus, the effective price floors are:

$$p_h^F = rp_l^F = \max\{\bar{p}_h^F, r\bar{p}_l^F\} \quad \text{and} \quad p_l^F = \frac{1}{r}p_h^F = \max\left\{\frac{1}{r}\bar{p}_h^F, \bar{p}_l^F\right\}$$

The effective price ceilings are:

$$p_h^C = rp_l^C = \min\{\bar{p}_h^C, r\bar{p}_l^C\} \quad \text{and} \quad p_l^C = \frac{1}{r}p_h^C = \min\left\{\frac{1}{r}\bar{p}_h^C, \bar{p}_l^C\right\}$$

One important implication is that, when linking with price collars, an allowance exchange rate can enable links that would not be feasible with a traditional 1:1 link. For example, if the nominal price collars have no overlap—that is, the higher-priced floor is above the lower-priced ceiling ($\bar{p}_h^F > \bar{p}_l^C$)—then the programs could not link at the traditional exchange rate of $r = 1$. An allowance exchange rate provides an additional policy lever that can create overlap in the effective price collars and allow programs to link despite nominal price collars that are seemingly incompatible.

Another aspect of feasibility, though, is the political plausibility of the link. In a linked system, the effective price collar each program faces will always be weakly tighter than its nominal price collar in autarky,[43] and policymakers may face political constraints on the extent

[42] There are several reasons why the design of the emissions trading program might yield prices at the price floor or price ceiling, which we abstract away from in this analytical framework. Policymakers may not know *ex ante* what business-as-usual emissions or abatement costs will be, and this uncertainty may result in allowance prices at the floor or ceiling (Borenstein et al. 2019). Other companion policies could reduce the demand for allowances within the state and tend to suppress prices. Additionally, policymakers may face many political constraints when designing a trading program, and in trying to balance competing demands, an allowance price at the floor or ceiling may be the only politically feasible outcome. In practice, however, prices have only occasionally rested at a collar. One reason is that the price collar censors the distribution of potential payoffs and affects the market price even when it is not binding (Burtraw, Palmer, and Kahn 2010; Salant, Shobe, and Uler 2022).

[43] There is a practical limit to the extent of this tightening. Each program's effective price collar must have a ceiling that is weakly greater than the floor. This constraint restricts the feasible set of exchange rates to the range $\frac{\bar{p}_h^F}{\bar{p}_l^C} \leq r \leq \frac{\bar{p}_h^C}{\bar{p}_l^F}$. At the extremes of $r = \frac{\bar{p}_h^F}{\bar{p}_l^C}$ or $r = \frac{\bar{p}_h^C}{\bar{p}_l^F}$, each program's effective price ceiling equals its effective price floor, so the effective price collar in each program is tightened to the maximum extent possible and the linked trading system becomes equivalent to a carbon tax.

of this tightening. Because allowance prices in autarky reflect the political realities in each jurisdiction, they provide a reasonable constraint on the tightening of the price collar. Using this benchmark, we assume each program's allowance price in autarky must remain within its effective price collar when linked.[44]

This political plausibility may provide an additional constraint on the feasible range of allowance exchange rates when linking. For example, in order for allowance prices in program $h$ to not be overly restricted by the price collar in program $l$, the exchange rate must be such that $r\bar{p}_l^F < p_h^0 < r\bar{p}_l^C$. Similarly, program $l$ is not overly restricted when $\frac{1}{r}\bar{p}_h^F < p_l^0 < \frac{1}{r}\bar{p}_h^C$. We combine these constraints to define:

$$r_{min} = \max\left\{\frac{p_h^0}{\bar{p}_l^C}, \frac{\bar{p}_h^F}{p_l^0}\right\} \quad \text{and} \quad r_{max} = \min\left\{\frac{p_h^0}{\bar{p}_l^F}, \frac{\bar{p}_h^C}{p_l^0}\right\}$$

An allowance exchange rate within the range of $r_{min} \le r \le r_{max}$ ensures that each program's allowance price in autarky remains (weakly) within its effective price collar when linked.[45] Our final result describes market outcomes within this politically plausible range of exchange rates.[46]

**Result 4.** When linking programs with price collars at an exchange rate in the range $r_{min} \le r \le r_{max}$, effective price collars will not bind and all previous results will hold.

In other words, when linking occurs within the politically plausible range of exchange rates, the prior results from linking—environmental benefits globally and economic benefits in each region—remain.[47] Thus, an allowance exchange rate gives policymakers an additional policy lever to retain some sovereignty over allowance price collars when linking. This

[44] For example, if program $h$ has an allowance price of $30 in autarky, then it would be implausible politically for policymakers in that jurisdiction to agree to a link that yields an effective price ceiling of only $20.

[45] We previously describe that exchange rates between 1 and $\frac{p_h^0}{p_l^0}$ are most likely. The value of $r_{min}$ may be greater than 1 in some—but not all—cases and further restrict this likely range of exchange rates. The value of $r_{max}$ is always greater than $\frac{p_h^0}{p_l^0}$ and, hence, will not further restrict the likely range of exchange rates.

[46] Proofs of all results are in Appendix A.

[47] We derive this result from our deterministic analytical model. A stochastic model that accounts for uncertainty in emissions and abatement costs could further refine the likelihood of price collars binding as a function of the exchange rate. This result provides an important benchmark for future work on that topic.

additional control can help to ensure that each trading program's distributional outcomes remain within the politically acceptable range while maintaining the environmental and economic benefits of linking.

## 3. Simulation Model of Linking

In the previous section, we used a simple analytical model to demonstrate how the linked allowance exchange rate can affect abatement, welfare, and distributional outcomes. To do so, however, we abstracted from some of the complexities of real-world policies and markets. In this section, we turn to the Haiku electricity sector simulation model—which incorporates many of these additional characteristics—to provide an in-depth analysis of a link between two hypothetical emissions trading programs. We use the simulation model to confirm the main results and to investigate additional outcomes that are beyond the scope of the analytical model.

### Model Description

Haiku is a linear programming model of the United States electricity sector that minimizes costs in the 48 contiguous states across a 30-year time horizon with annual operations determined by 24 representative time blocks. The model represents generators in each state as model plants characterized by fuel and technology, including options for new investment in both fossil-fired generators and renewables technologies. The linear program uses a constrained optimization approach to represent capacity investment, emissions regulations, performance standards, and other important characteristics of the electricity sector. Output from the model is determined with perfect foresight allowing policies in the future to affect capacity decisions in earlier years. The Haiku model has been used on many occasions to inform the policy-making process.[48]

To align the simulation model structure more closely with the analytical model framework and to simulate a single-year emissions cap, we constrain elements of the full simulation model including transmission and investment functions.[49] The simulation modeling

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[48] See Shobe, Artuso, and Domeshek (2021) and Burtraw et al. (2022) for recent examples. A prior version of the model is used in Burtraw, Woerman, and Krupnick (2016), Palmer et al. (2017), and Palmer, Paul, and Keyes (2018), among many others.

[49] To negate for the possibility of emissions leakage, we shut down transmission to states outside of each linked region. To focus on contemporaneous rather than dynamic effects, we fix capacity and generation to a baseline scenario except in the year that the emissions trading scheme is in existence.

indicates that the results of our simple analytical model generally persist in this more complex framework that uses empirical data on an emissions-intensive industry, and it identifies additional outcomes that will be important for policymakers to evaluate and consider in real-world policy decisions.

**Model Scenarios**

The policy case we construct is a scenario of two regional emissions caps that exist for a single year, 2025.[50] The first regional cap encompasses all states in the Western Interconnection, and we model an emissions cap that reduces $CO_2$ emissions by 95 million tons from baseline in this region. The second regional cap includes the states in the Eastern United States that are members of the Regional Greenhouse Gas Initiative or the PJM market, and we model an emissions cap that reduces $CO_2$ emissions by 85 million tons from baseline in this region. We simulate electricity markets outcomes with these regional trading programs in autarky, and we then simulate outcomes when these programs link at various allowance exchange rates. To achieve these emissions reductions in autarky, the allowance price in the West is 3.24 times that in the East, so we consider allowance exchange rates that range from 0.8 to 4, spanning beyond the core range that would likely be considered by policymakers.[51] We also simulate two additional scenarios that include price containment mechanisms, which we describe later.

**Abatement and Abatement Cost**

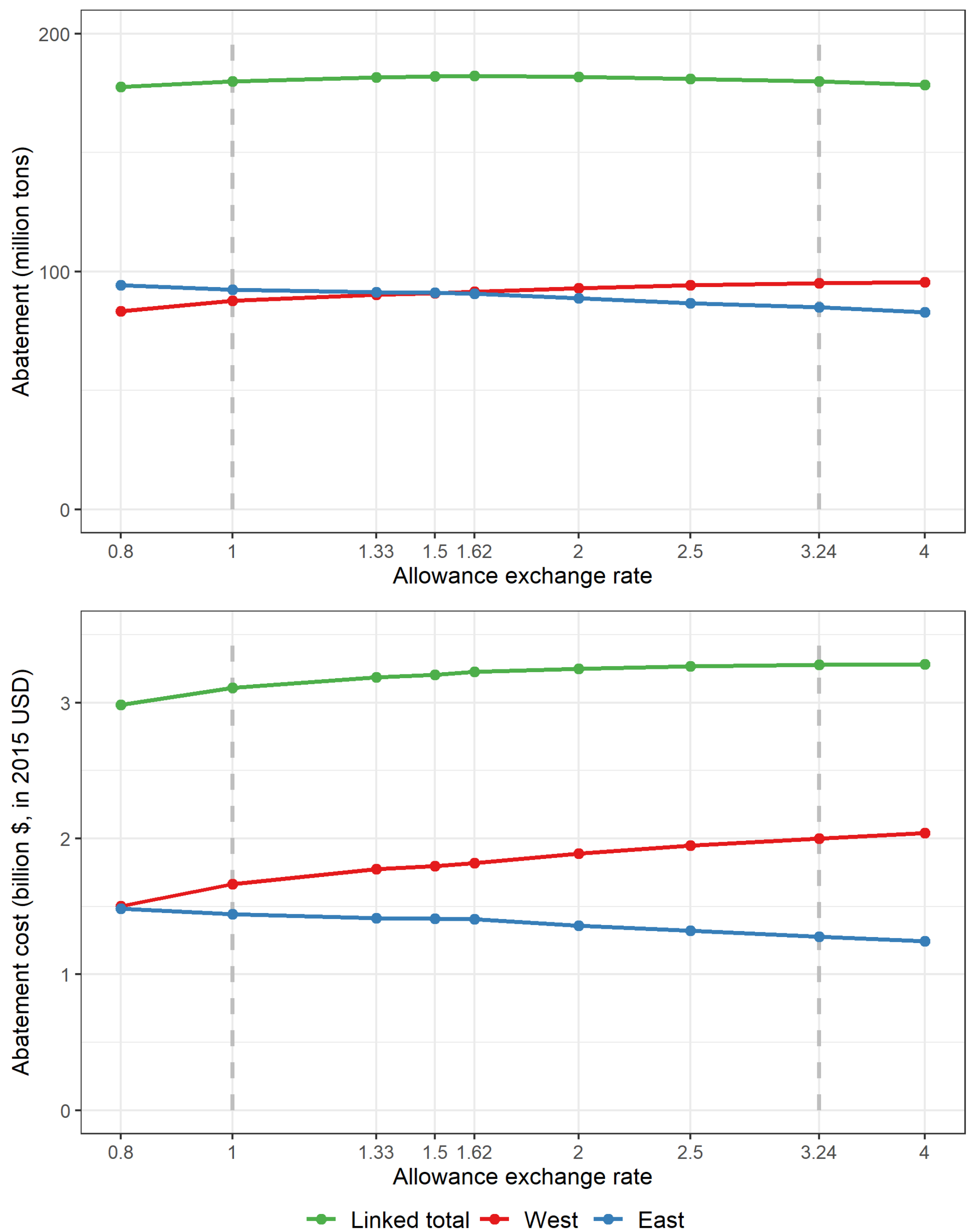
Figure 3 plots the levels of abatement and the abatement costs in each program and for the linked system in total under each of the simulated allowance exchange rates. These results are qualitatively similar to the results of the analytical model: at exchange rates in the core range between 1 and the autarky price ratio of 3.24, the linked system achieves greater total abatement at lower cost than in autarky. As we decrease the exchange rate from 3.24 down to 1, more abatement is achieved in the lower-cost East region and less in the higher-cost West, reducing the aggregate cost of abatement. This shift in abatement is achieved because emitters in the West import allowances from the East, and in this range of exchange rates, more than one allowance must be imported for every ton emitted, so this flow of allowances also reduces emissions. We

[50] A single-year emissions cap allows us to focus on the contemporaneous effects of the policies in that single year, rather than their longer-run dynamics over many years.

[51] We simulate links at allowance exchange rates of 0.8, 1, 1.33, 1.5, 1.62, 2, 2.5, 3.24, and 4.

find that total abatement is maximized at an exchange rate of 1.62, with a modest increase in abatement as compared to either autarky or linking at an exchange rate of 1.

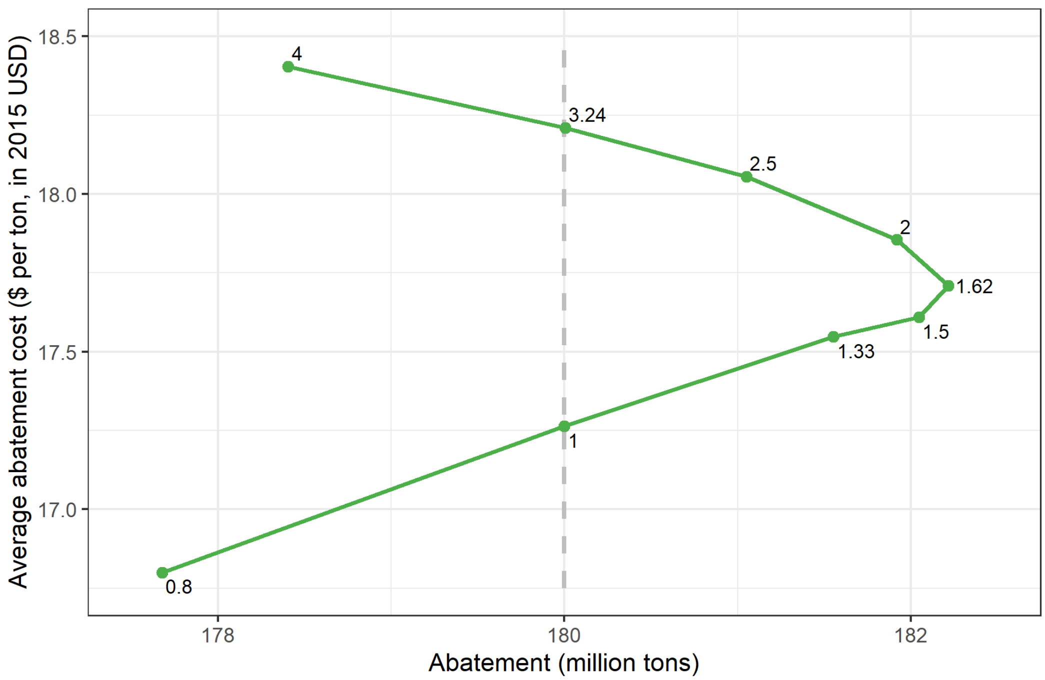
**Figure 3: Simulation results—Abatement and cost when linked**




*Notes*: This figure plots simulation modeling results of abatement (top panel) and abatement cost (bottom panel) in each program and in the linked system under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

We further summarize these results in Figure 4, which plots the relationship between abatement and average abatement cost; the number next to each point indicates the allowance exchange rate that achieves that outcome.[52] We see that lowering the allowance exchange rate from the autarky price ratio of 3.24 down to 1.62 increases total abatement in the system and reduces the average abatement cost, generating both economic gains and climate benefits. Lowering the allowance exchange rate below 1.62 continues to reduce the average abatement cost, but it also reduces total abatement, creating a tradeoff between lower costs and improved environmental ambition.

**Figure 4: Simulation results—Average abatement cost when linked**



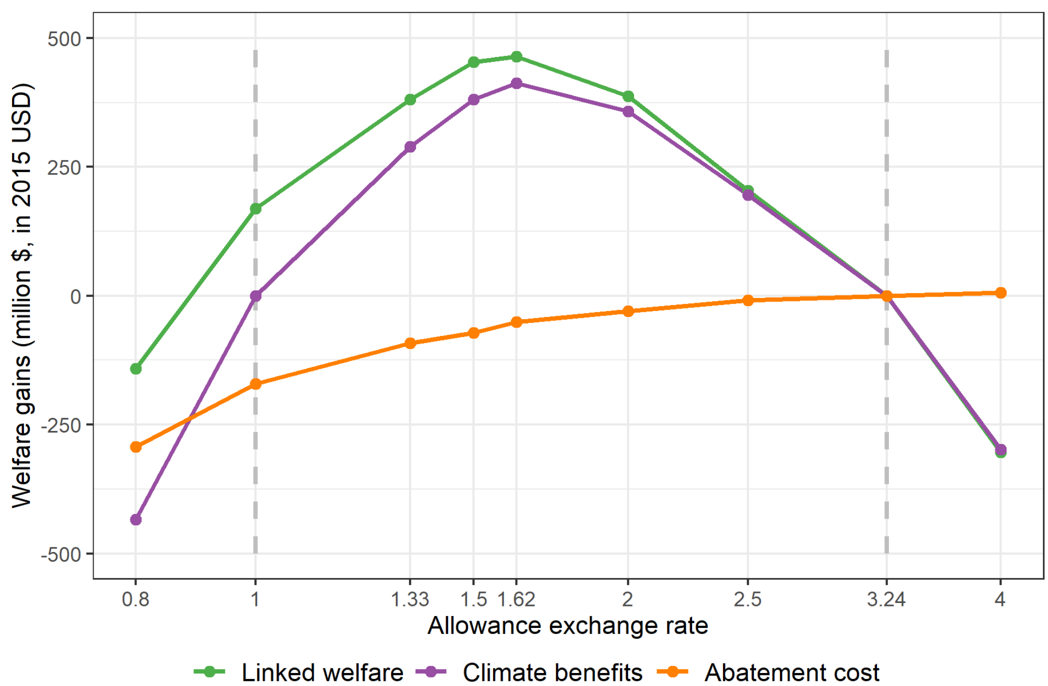

*Notes*: This figure plots simulation modeling results of abatement (x-axis) and average abatement cost (y-axis) in the linked system under different allowance exchange rate scenarios. Numerical labels display the exchange rate in each scenario. The dashed vertical line indicates aggregate abatement in each of the benchmark regimes of full integration and autarky; outcomes to the right of this line correspond to the politically plausible range of exchange rates.

[52] Quemin and de Perthuis (2019) plot a similar relationship, showing abatement and cost-efficiency at different exchange rates, based on their theoretical model. Here we show that a similar result holds in our numerical simulations.

**Welfare**

We also see this tradeoff between abatement costs and climate benefits in Figure 5, in which we plot the welfare gains from linking compared to autarky. We again see that abatement costs monotonically decrease as the allowance exchange rate declines, but climate benefits are a nonmonotonic function of the exchange rate.[53] As a result, social welfare is maximized when these regional trading programs link at an allowance exchange rate of 1.62. Importantly—and in line with the results of our analytical model—social welfare is not maximized at an exchange rate of 1, which equates marginal abatement costs across the two programs. Instead, even though an exchange rate of 1.62 yields a less efficient allocation of abatement across the two programs, it achieves greater abatement and climate benefits that more than offset that inefficiency.

**Figure 5: Simulation results—Welfare gains compared to autarky when linked**



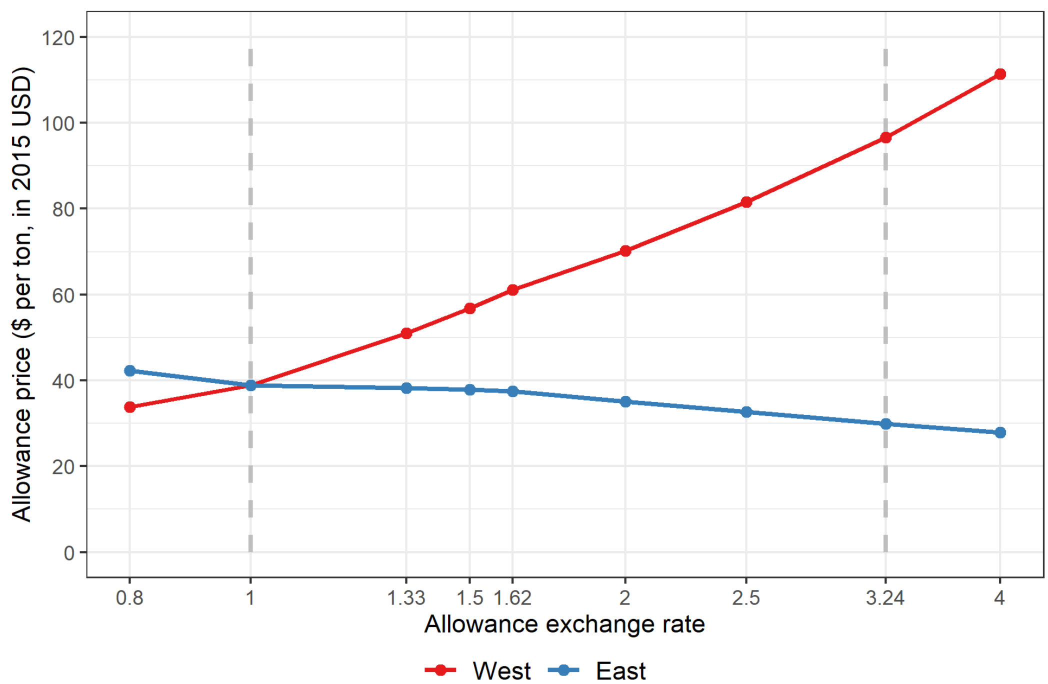

*Notes*: This figure plots simulation modeling results of welfare gains, climate benefits, and abatement costs in the linked system, compared to autarky, under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

[53] We value $CO_2$ abatement at the social cost of carbon of $185 per metric ton (Rennert et al. 2022). We convert this value to $186.70 per short ton in 2015 USD.

## Distributional Effects

The climate benefits of linking accrue globally, rather than within the regions where the abatement occurs, so the local economic consequences of linking come from changes in abatement costs and flows of allowance values. These economic benefits or costs of linking may not be distributed evenly across the two trading programs or among different participants within each program. For example, higher allowance prices increase government revenues from allowances sales but also increase compliance costs for emitters. Figure 6 shows how the choice of allowance exchange rate affects the allowance price, or marginal cost of abatement, in each trading program. We see that lowering the exchange rate greatly reduces the allowance price in the West and yields a relatively small price increase in the East.

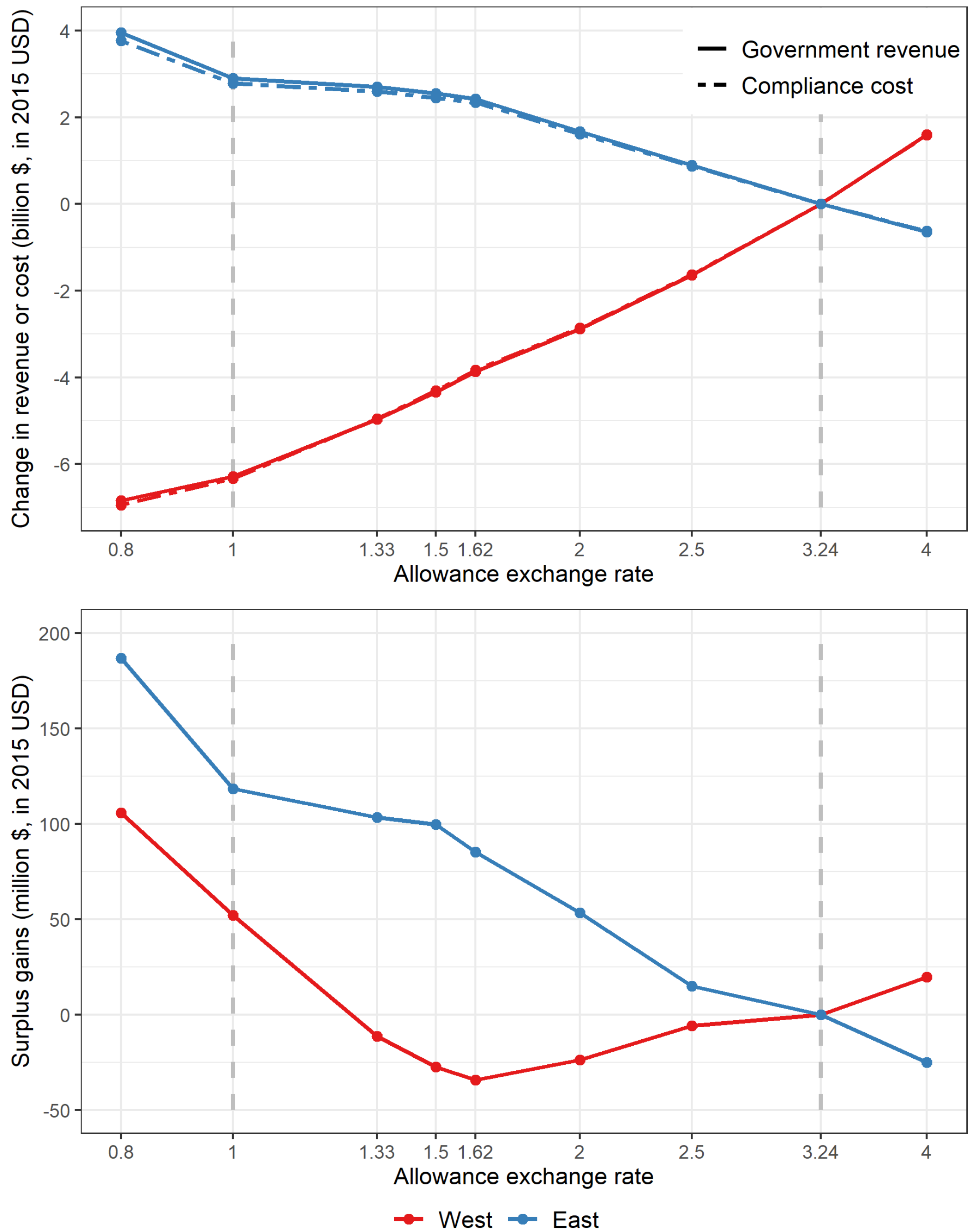
**Figure 6: Simulation results—Allowance prices when linked**




*Notes*: This figure plots simulation modeling results allowance prices in each program under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

The top panel of Figure 7 plots how these allowance prices translate into changes in revenue for the governments that issue the allowances and changes in compliance costs for emitters covered by each trading program. Lowering the allowance exchange rate, which decreases the allowance price in the West, greatly reduces both the revenue collected by the government and the compliance cost incurred by emitters in the West; at an exchange rate of 1, these values fall by more than $6 billion compared to autarky. Conversely, linking at an allowance exchange rate of 1 increases both government revenue and compliance cost in the East by roughly $3 billion compared to autarky. In fact, at any exchange rate below the autarky price

**Figure 7: Simulation results—Regional distributional effects and surplus compared to autarky when linked**




*Notes*: This figure plots simulation modeling results of changes in government revenues and compliance costs (top panel) and surplus gains (bottom panel) in each program, compared to autarky, under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

ratio of 3.24, emitters in the West and governments in the East experience benefits, while governments in the West and emitters in the East experience large losses.

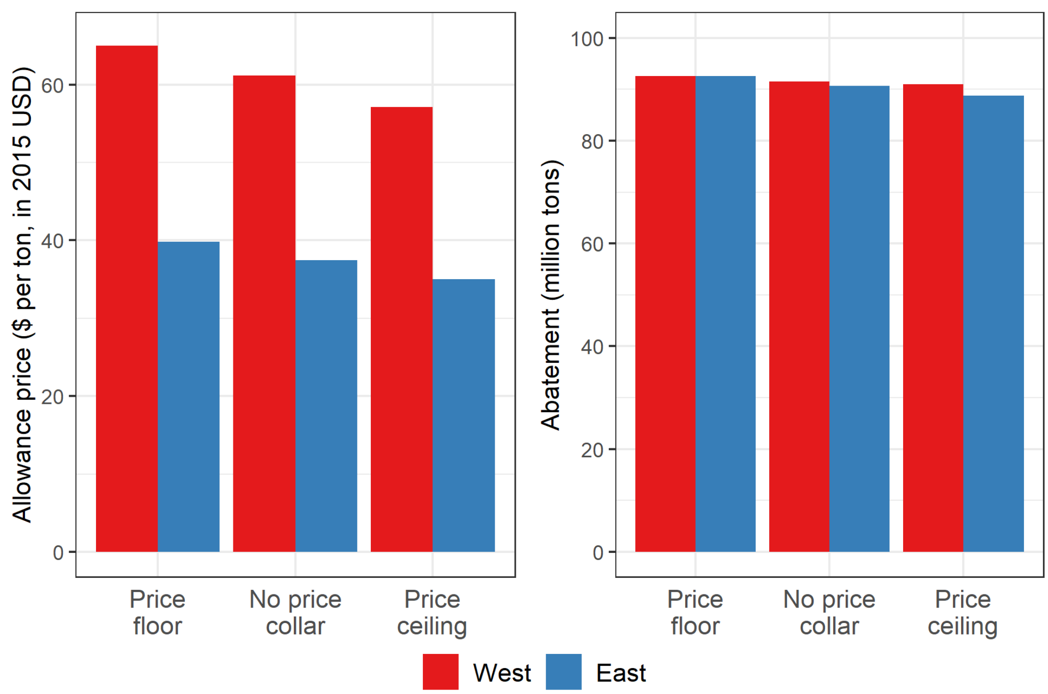
These distributional effects of linking—particularly the losses for governments in the West and emitters in the East—may create opposition to the link among important stakeholders. Using an allowance exchange rate other than the traditional rate of 1 can moderate these distributional effects and may reduce opposition to linking; for example, at an exchange rate of 2, these distributional effects are only half to two-thirds of what they are at a rate of 1. Additionally, within-region transfers could be used to offset revenue reductions or compliance cost increases. In that case, a more relevant metric to evaluate linking would be the total economic surplus gain or loss in each region from linking, which is the difference between changes in government revenue and compliance cost, and which we plot in the bottom panel of Figure 7. Although our analytical result states that both regions should gain economic surplus from linking at any exchange rate, here we see that the West loses surplus for many of the exchange rates in the core range, which could impede the linking of these trading programs. These surplus losses occur because—as shown by the allowance prices and abatement levels that result from simulation modeling—the marginal abatement costs in the West are convex over this range of exchange rates, rather than linear as in our analytical model, which yields less cost reductions from linking. Thus, we find that the use of an exchange rate can moderate the distributional effects of linking, but it may not be sufficient to overcome all political economy constraints in all cases.

**Price Containment Mechanisms**

Another aspect of the political economy of negotiating a linking regime is the integrity of the price control mechanism in each program and the ability of one program's price collar to affect allowance prices and abatement in both programs. To explore this issue, we consider two price control mechanisms: a price floor of $65 per ton in the higher-priced West and a price ceiling of $35 per ton in the lower-priced East. We simulate a link at an allowance exchange rate of 1.62—the rate that maximizes linked abatement in the absence of price containment mechanisms—with each of these price containment mechanisms in effect. With neither in place, a rate of 1.62 yields allowance prices of $61.15 per ton in the West and $37.44 per ton in the East. Thus, with both price containment mechanisms in place, it would be impossible for the programs to link at an exchange rate of 1.62, or any rate below 1.86,[54] because it would be infeasible to satisfy the price floor and the price ceiling simultaneously.

Figure 8 plots the resulting allowance prices and abatement when these programs link at an allowance exchange rate of 1.62 under three price collar scenarios: the West's $65 per ton price floor, no price containment mechanism, and the East's $35 per ton price ceiling. As expected, compared to the no price collar scenario, we find that the price floor in the West bolsters allowance prices in both programs, yielding greater abatement than with no price containment mechanism; although not shown, these higher allowance prices also imply greater government revenue and greater compliance costs in each program. Conversely, the price ceiling in the East suppresses allowance prices in both programs and yields less abatement, lower

**Figure 8: Simulation results—Outcomes when linked with a price containment mechanism**




*Notes*: This figure plots simulation modeling results of allowance prices (left panel) and abatement (right panel) in each program under different price containment mechanism scenarios: a $65 price floor in the West, no price collar, and a $35 price ceiling in the East. All three scenarios have an allowance exchange rate of 1.62.

54 The ratio of the West's $65 per ton price floor to the East's $35 per ton price ceiling is 1.86. At an allowance exchange rate of 1.86, both price containment mechanisms would bind simultaneously, and the linked system would effectively have regionally differentiated carbon taxes of $65 per ton in the West and $35 per ton in the East. At any rate below 1.86, including a traditional link at a rate of 1, the effective price floor in the West would exceed the effective price ceiling in the East, so it would be impossible to maintain a functioning market at these exchange rates.

government revenues, and lower compliance costs in both programs. If some of these outcomes are not in line with stakeholder preferences, then the presence of price containment mechanisms may further complicate the political economy of linking trading programs.

**Generation and Capacity Mix**

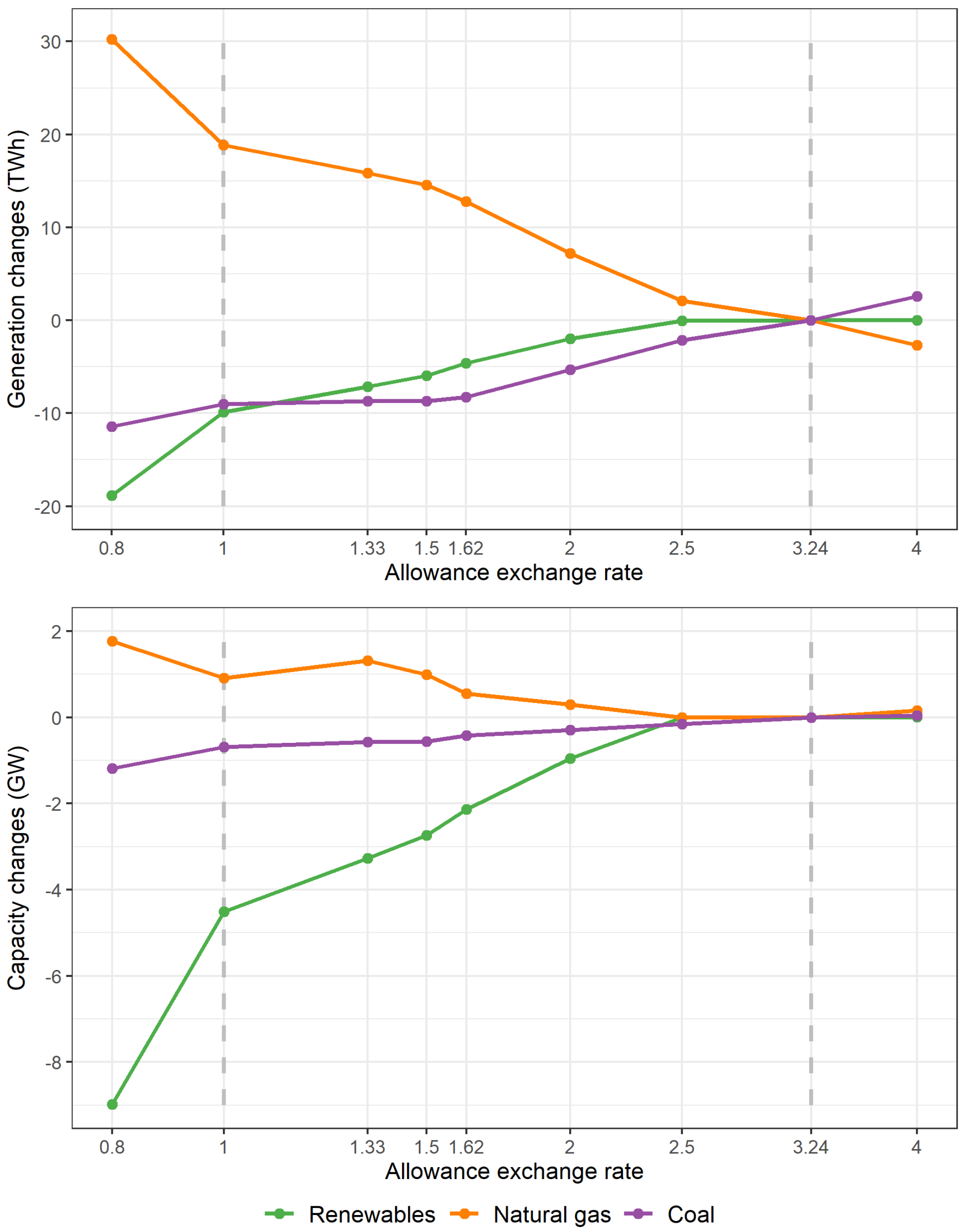
The Haiku electricity sector simulation model includes a detailed representation of both annual operations and capacity investments, so we can investigate how abatement is achieved in autarky and when the trading programs link. Figure 9 shows how the allowance exchange rate affects the generation and capacity mix across the linked system. In the higher-marginal-cost West, abatement is achieved by substituting to renewables; in the lower-marginal-cost East abatement is achieved by fuel switching between coal-fired and natural gas-fired units. Thus, lowering the allowance exchange rate, which shifts abatement from the West to the East, yields less generation from both renewables and coal units with a corresponding increase in generation from natural gas units. The reduction in generation from renewables occurs because less renewables capacity is built in the West at lower exchange rates, whereas fuel switching between fossil fuels results in only small changes in coal and natural gas capacity.

**Other Air Pollutants**

These shifts in generation sources when linked, compared to autarky, yield changes in emissions of not only $CO_2$ but also other air pollutants, such as sulfur dioxide ($SO_2$) and nitrogen oxides ($NO_x$). The top panel of Figure 10 plots how abatement of $SO_2$ and $NO_x$ vary with the allowance exchange rate. Lowering the exchange rate increases abatement of both $SO_2$ and $NO_x$ in the East through fuel switching from coal to natural gas, and it slightly reduces abatement of both pollutants in the West due to lower renewables penetration. Reducing emissions of $SO_2$ and $NO_x$ yields health benefits within the region where the pollutants are emitted. These benefits are regionally heterogenous and are estimated by the U.S. Environmental Protection Agency (2011) to be 3.5 times greater in the East than in the West. Additionally, there is uncertainty about the health benefits of abating $SO_2$ and $NO_x$, so we calculate a range of benefits in each trading program.[55]

[55] We use estimates from Krewski et al. (2009) as a lower bound and estimates from Lepeule et al. (2012) as an upper bound for the health benefits of abating $SO_2$ and $NO_x$. Our calculations of health benefits are relatively simple and could be improved upon with a higher resolution consideration of the geographic location of emissions changes and the simulation of local air pollutants and health benefits due to fuel switching. Additional details on the incidence of these health benefits could also help further refine the calculation.

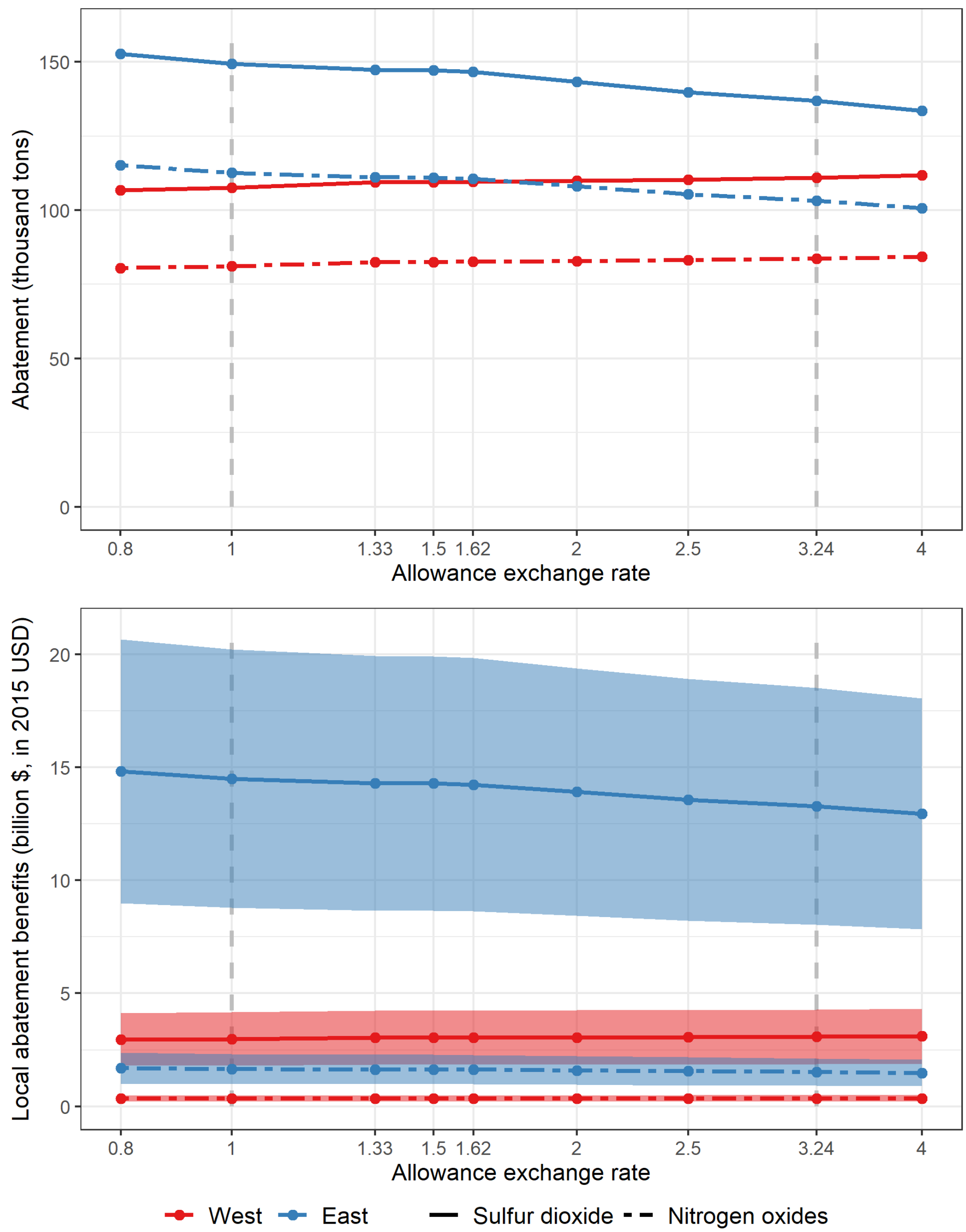
**Figure 9: Simulation results—Generation and capacity when linked**




*Notes*: This figure plots simulation modeling results of changes in generation (top panel) and capacity (bottom panel) by technology in the linked system, compared to autarky, under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

The bottom panel of Figure 10 plots the local health benefits from abatement of $SO_2$ and $NO_x$; the shaded areas give the full range of health benefits, and each point gives the midpoint of

**Figure 10: Simulation results—Abatement and abatement benefits of other air pollutants when linked**

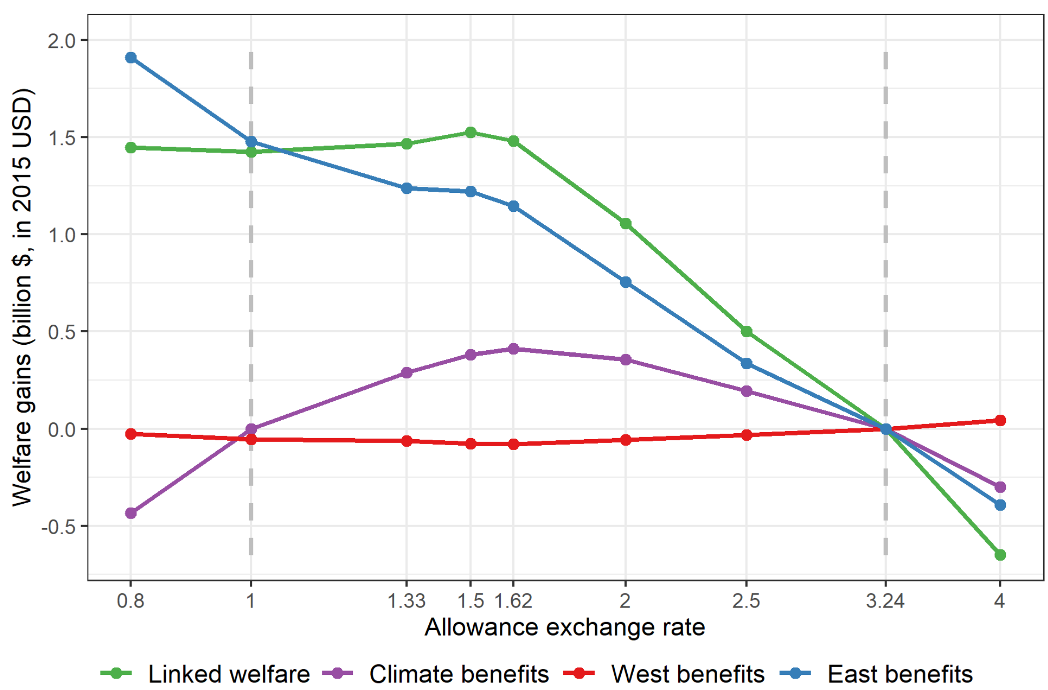



*Notes*: This figure plots simulation modeling results of abatement (top panel) and local abatement benefits (bottom panel) of other air pollutants in each program under different allowance exchange rate scenarios. In the bottom panel, the shaded areas give the ranges of benefits and each point gives the midpoint of the range. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

the range. We see that $SO_2$ reductions in the East generate \$8 billion to \$18 billion of health benefits at the autarky price ratio of 3.24, and lowering the allowance exchange rate further increases these benefits. The West also experiences health benefits from abatement, but they are smaller and tend to decrease as the allowance exchange rate declines.

To fully account for the welfare effects of linking these trading programs with an exchange rate, we must also account for these local health benefits from $SO_2$ and $NO_x$ abatement. Figure 11 plots the welfare gains across the linked system, including the health benefits from $SO_2$ and $NO_x$ abatement, due to linking with an allowance exchange rate; the figure also indicates the gains that accrue globally to the climate and locally in in each trading program.[56] The vast majority of welfare gains from linking at an allowance exchange rate in the core range, or even below 1, is due to the health benefits in the East from reducing $SO_2$ and $NO_x$ emissions. Additionally, the welfare loss in the West is minor by comparison. Thus, depending on the priorities of the governments in each region, the consideration of co-benefits could dramatically alter the political economy of negotiating an allowance exchange rate. In fact, from

**Figure 11: Simulation results—Welfare gains including health benefits compared to autarky when linked**




*Notes*: This figure plots simulation modeling results of welfare gains (including health benefits) and climate benefits in the linked system and local benefits in each program, compared to autarky, under different allowance exchange rate scenarios. Dashed vertical lines indicate the benchmark regimes of full integration and autarky, which bookend the politically plausible range of exchange rates.

[56] In calculating the welfare gains from linking in Figure 11, we use the midpoint of the health benefits depicted in Figure 10.

a social welfare perspective, these results indicate that an exchange rate of 0.8, which maximizes reductions of $SO_2$ emissions in the East—or an even lower allowance exchange rate—would be the optimal policy choice.

**Summary**

In summary, our simulation modeling replicates several of the main results from our analytical model. In particular, linking at an allowance exchange rate in the core range—that is, between 1 and the autarky price ratio—achieves greater $CO_2$ abatement than either bookend exchange rate, and it does so at lower abatement cost than autarky. As a result, the socially optimally exchange rate is not a rate of 1, which equates marginal costs across the two programs, but rather an exchange rate in this core range that yields greater $CO_2$ abatement and climate benefits. In contrast to the analytical model, however, we find that linking in this core range causes one region to lose economic surplus, which may pose political economy challenges for a potential link.

This simulation modeling also explores several aspects of linking that are beyond the scope of our analytical model and reveals outcomes that deserve careful evaluations when policymakers consider a real-world link. In particular, we find that the choice of allowance exchange rate can greatly affect the generation mix and other air pollutant emissions in each program. In some settings, these co-benefits from $SO_2$ and $NO_x$ abatement could be large enough to drive the decision to link $CO_2$ trading programs.

## 4. Conclusion

This paper provides a framework to analyze the linking of regional or sectoral emissions trading programs with different abatement opportunities and different features, including stringency as measured by allowance prices, and different price containment mechanisms. We develop an analytical framework for linking of emissions trading programs using an allowance exchange rate, which has the potential to align program stringency or price collars and can substitute for more direct efforts to harmonize program details before linking. We then apply that framework to simulate the potential linking of two carbon markets covering electricity sector emissions in the Western and Eastern United States.

Our analytical and simulation models demonstrate that formal linking of emissions trading systems with an allowance exchange rate may lead to aggregate emissions that differ from the sum of the caps of the two programs when they operate independently. We find two-way potential differences to the emissions outcome of linking; that is, emissions can theoretically be either lower or higher under a linked market. The use of an exchange rate to reconcile differences in stringency between the programs can have the effect of changing aggregate emissions compared to autarky, and an exchange rate in the policy-relevant range will reduce emissions. An emissions outcome that differs from the sum of the emissions caps in autarky also might become apparent if relative marginal abatement costs change over time, for example, due to changes in fuel prices or electricity demand. Another reason this difference in emissions outcomes could theoretically result is the presence of price containment measures, either price floors or ceilings, that adjust the number of emissions allowances introduced in one program in response to allowance prices, but which have effects that propagate across both programs when they are linked. An exchange rate in the policy-relevant range, however, is not expected to affect emissions in this way. In addition, other aspects of program design that could lead to emissions that differ from the sum of the emissions caps in autarky include the treatment of offsets or efforts to contain leakage, some of which have been anticipated previously in the literature.

Linking with an allowance exchange rate also has important implications for the economic costs of the trading systems. Our analytical model finds that traditional 1:1 linking improves the cost-effectiveness of emissions reductions, although the use of an exchange rate undoes some of these cost reductions. Because the exchange rate affects both emissions and abatement costs, its choice has implications for the overall efficiency of the linked system, and in the policy-relevant range of exchange rates between autarky and 1:1, linking yields welfare gains.

These welfare gains from linking accrue unequally throughout the linked system, and some constituencies in each program may benefit from linking, while others may incur losses. Increasing attention is being given to the distribution of emissions reductions that result from carbon trading programs, as well as the distribution of costs that are incurred to meet the goals of these programs, despite the global nature of climate change. This attention is focused on the concern that not all communities see reductions in criteria air pollutants or receive other environmental benefits in equal measure, and some may be made worse due to the flexible implementation of emissions trading and other carbon pricing schemes. Additionally, some constituencies may bear disproportionate costs of emissions reductions, while other constituencies are poised to benefit. Economic approaches to environmental policy typically separate these effects from the central goal of carbon pricing, which is to achieve greenhouse gas reductions at the least cost. In general, we show that linking programs with a plausible exchange rate is expected to contribute to the central goal of reducing costs in each program while increasing the aggregate greenhouse gas abatement that is achieved. However, our research highlights other issues that should be anticipated, including potential uneven distributional outcomes among the affected constituencies, and more generally changes in conventional air pollutants. Policymakers may need to consider and compensate for these distributional effects if linking occurs.

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## Appendix A: Proofs of results

### *Assumptions*

We assume the representative firm's production cost is a function of emissions and is given by $K(E)$. This function is positive at all levels of emissions. It is also convex in emissions, and it reaches a minimum at some value of emissions denoted by $\bar{E} > 0$. This production cost function can vary by representative firm, and we assume each cost function is fixed over the time horizon we consider.

The representative firm may abate emissions below the uncontrolled level, with abatement given by $A = \bar{E} - E$, but at a cost. This abatement cost, which is a function of the level of abatement, is given by $C(A) = K(\bar{E} - A) - K(\bar{E})$. Due to the properties of $K(E)$, $C(0) = 0$ and is positive at all positive levels of abatement. The abatement cost function is also increasing and convex in the level of abatement. The marginal abatement cost function, which we denote $c(A)$ is the derivative of the abatement cost function, $c(A) = \frac{\partial C}{\partial A}(A)$. Due to the properties of $C(A)$, $c(0) = 0$ and is positive at all positive levels of abatement. The marginal abatement cost function is also increasing and convex in the level of abatement. These cost functions can also vary by representative firm, and we assume each cost function is fixed over the time horizon we consider.

### *Abatement and allowance prices*

Before providing proofs of our main results, we first consider the effect of linking with an allowance exchange rate on the abatement and allowance prices in each program. We use the notation $\Delta$ to denote a comparison between a linked outcome and its respective outcome in autarky—that is, $\Delta A_h = A_h - \bar{A}_h$. Then we can restate the linking condition for abatement as: $r\Delta A_h = -\Delta A_l$. Then the signs on $\Delta A_h$ and $\Delta A_l$ must be opposite, or both must equal zero. Hence, the signs on $\Delta p_h$ and $\Delta p_l$ must also be opposite, or both must equal zero. There are three cases to consider:

Case 1: $\Delta A_h > 0$, $\Delta p_h > 0$, $\Delta A_l < 0$, and $\Delta p_l < 0$. This case occurs if and only if $p_h^0 < p_h = rp_l < rp_l^0$, where the equality is due to the linking condition for prices, which gives $r > \frac{p_h^0}{p_l^0}$. Thus, $\Delta A_h > 0$, $\Delta p_h > 0$, $\Delta A_l < 0$, and $\Delta p_l < 0$ if and only if $r > \frac{p_h^0}{p_l^0}$.

Case 2: $\Delta A_h < 0$, $\Delta p_h < 0$, $\Delta A_l > 0$, and $\Delta p_l > 0$. This case occurs if and only if $p_h^0 > p_h = rp_l > rp_l^0$, where the equality is due to the linking condition for prices, which gives $r < \frac{p_h^0}{p_l^0}$. Thus, $\Delta A_h < 0$, $\Delta p_h < 0$, $\Delta A_l > 0$, and $\Delta p_l > 0$ if and only if $r < \frac{p_h^0}{p_l^0}$.

Case 3: $\Delta A_h = 0$, $\Delta p_h = 0$, $\Delta A_l = 0$, and $\Delta p_l = 0$. This case occurs if and only if $p_h^0 = p_h = rp_l = rp_l^0$, where the equality is due to the linking condition for prices, which gives $r = \frac{p_h^0}{p_l^0}$. Thus, $\Delta A_h = 0$, $\Delta p_h = 0$, $\Delta A_l = 0$, and $\Delta p_l = 0$ if and only if $r = \frac{p_h^0}{p_l^0}$.

### *Proof of Result 1*

We first consider the effect of the allowance exchange rate on total abatement in the linked system as compared to total abatement in autarky. This comparison of total abatement is given by $\Delta A \equiv \Delta A_h + \Delta A_l = \Delta A_h - r\Delta A_h = (1-r)\Delta A_h$, where the first equality is due to the linking condition for abatement. Thus, the sign of $\Delta A$ depends on the signs of $1 - r$ and $\Delta A_h$. There are five cases to consider:

Case 1: $r > \frac{p_h^0}{p_l^0}$. Then $1 - r < 0$ and $\Delta A_h > 0$, so $\Delta A < 0$.

Case 2: $r = \frac{p_h^0}{p_l^0}$. Then $1 - r < 0$ and $\Delta A_h = 0$, so $\Delta A = 0$.

Case 3: $1 < r < \frac{p_h^0}{p_l^0}$. Then $1 - r < 0$ and $\Delta A_h < 0$, so $\Delta A > 0$.

Case 4: $r = 1$. Then $1 - r = 0$ and $\Delta A_h < 0$, so $\Delta A = 0$.

Case 5: $r < 1$. Then $1 - r > 0$ and $\Delta A_h < 0$, so $\Delta A < 0$.

In summary, if $1 < r < \frac{p_h^0}{p_l^0}$, then $\Delta A > 0$; if $r < 1$ or $r > \frac{p_h^0}{p_l^0}$, then $\Delta A < 0$; and if $r = 1$ or $r = \frac{p_h^0}{p_l^0}$, then $\Delta A = 0$.

We now consider the effect of the allowance exchange rate on total abatement cost in the linked system as compared to total abatement cost in autarky, given by $\Delta C \equiv \Delta C_h + \Delta C_l$. Ignoring $r = \frac{p_h^0}{p_l^0}$, which yields outcomes equal to autarky, there are two cases to consider:

Case 1: $r > \frac{p_h^0}{p_l^0}$. Then $\Delta A_h > 0$ and $\Delta C_h > 0$. The additional abatement in program $h$ incurs marginal costs below the linked price in program $h$, or $c_h(A) < p_h$ for all $A < A_h$, so $\Delta C_h < p_h \Delta A_h$. Conversely, $\Delta A_l < 0$ and $\Delta C_l < 0$. The foregone abatement in program $l$ corresponds to marginal costs above the linked price in program $l$, or $c_l(A) > p_l$ for all $A > A_l$, so $\Delta C_l < p_l \Delta A_l$. Combining the linking conditions for abatement and prices, $p_l \Delta A_l = \left(\frac{p_h}{r}\right)(-r\Delta A_l) = -p_h \Delta A_h$, so $\Delta C_l < -p_h \Delta A_h$. Thus, $\Delta C = \Delta C_h + \Delta C_l < p_h \Delta A_h - p_h \Delta A_h = 0$.

Case 2: $r < \frac{p_h^0}{p_l^0}$. Then $\Delta A_h < 0$ and $\Delta C_h < 0$. The foregone abatement in program $h$ corresponds to marginal costs above the linked price in program $h$, or $c_h(A) > p_h$ for all $A > A_h$, so $\Delta C_h < p_h \Delta A_h$.Conversely, $\Delta A_l > 0$ and $\Delta C_l > 0$. The additional abatement in program $l$ incurs marginal costs below the linked price in program $l$, or $c_l(A) < p_l$ for all $A < A_l$, so $\Delta C_l < p_l \Delta A_l$. Combining the linking conditions for abatement and prices, $p_l \Delta A_l = \left(\frac{p_h}{r}\right)(-r\Delta A_l) = -p_h \Delta A_h$, so $\Delta C_l < -p_h \Delta A_h$. Thus, $\Delta C = \Delta C_h + \Delta C_l < p_h \Delta A_h - p_h \Delta A_h = 0$.

In summary, if $r \neq \frac{p_h^0}{p_l^0}$, then $\Delta C < 0$. ■

### *Proof of Result 2*

The welfare generated by the linked system is $W \equiv d(A_h + A_l) - C_h - C_l$. Consider allowance the exchange rate $r = 1$. From Result 1, $\Delta A(r = 1) = 0$ and $\Delta C(r = 1) < 0$. Thus, welfare is greater at an exchange rate $r = 1$ than in autarky, or $\Delta W(r = 1) > 0$. The derivative of welfare with respect to the allowance exchange rate is $\frac{dW}{dr} = (d - p_h)\frac{\partial A_h}{\partial r} + (d - p_l)\frac{\partial A_l}{\partial r}$. Again, consider the exchange rate $r = 1$. At $r = 1$, $p_h = p_l$, so the derivative simplifies to $\frac{dW}{dr}(r = 1) = (d - p_h)\frac{\partial A}{\partial r}$. Result 1 implies that, when evaluated at $r = 1$, $\frac{\partial A}{\partial r}(r = 1) > 0$. Hence, the sign of $\frac{dW}{dr}(r = 1)$ depends on the value of $p_h(r = 1)$ compared to $d$. There are three cases to consider:

Case 1: At $r = 1$, $p_h = p_l < d$. Then $\frac{dW}{dr}(r = 1) > 0$, so welfare improves if the exchange rate is increased to a value just above $r = 1$. Hence, there exists some exchange rate $r'$ that is in the interval $1 < r' < \frac{p_h^0}{p_l^0}$ and that yields greater welfare than both $r = 1$ and autarky, or $W(r') > W(r = 1) > W(r = \frac{p_h^0}{p_l^0})$. Between these two endpoints, welfare must reach a maximum. Thus, the welfare-maximizing exchange rate, $r^*$, lies in the interval $1 < r^* < \frac{p_h^0}{p_l^0}$.

Case 2: At $r = 1$, $p_h = p_l = d$. Then $\frac{dW}{dr}(r = 1) = 0$, so welfare is maximized at $r^* = 1$.

Case 3: At $r = 1$, $p_h = p_l > d$. Then $\frac{dW}{dr}(r = 1) < 0$, so welfare decreases if the exchange rate is increased to a value just above $r = 1$, and it improves if the exchange rate is reduced to a value just below $r = 1$. Hence, $W(r') < W(r = 1)$ for all $r'$ in the interval $1 < r' < \frac{p_h^0}{p_l^0}$. But there exists some exchange rate $r'' < 1$ that yields greater welfare than both $r = 1$ and autarky, or $W(r'') > W(r = 1) > W(r = \frac{p_h^0}{p_l^0})$. Similar exchange rates may also exist greater than $\frac{p_h^0}{p_l^0}$. Thus, the welfare-maximizing exchange rate, $r^*$, lies in the interval $r^* < 1$ or $r^* > \frac{p_h^0}{p_l^0}$.

In summary, if $r = 1$ yields $p_h = p_l < d$, then $1 < r^* < \frac{p_h^0}{p_l^0}$; if $r = 1$ yields $p_h = p_l = d$, then $r^* = 1$; and if $r = 1$ yields $p_h = p_l > d$, then $r^* < 1$ or $r^* > \frac{p_h^0}{p_l^0}$. ■

### *Proof of Result 3*

Each program's government revenue when linked, as compared to autarky, is given by $\Delta GR_h = \Delta p_h(\bar{E}_h - \bar{A}_h)$ and $\Delta GR_l = \Delta p_l(\bar{E}_l - \bar{A}_l)$. Then the signs of $\Delta GR_h$ and $\Delta GR_l$ depend on the signs of $\Delta p_h$ and $\Delta p_l$, respectively. Thus, if $r > \frac{p_h^0}{p_l^0}$, then $\Delta GR_h > 0$ and $\Delta GR_l < 0$; if $r < \frac{p_h^0}{p_l^0}$, then $\Delta GR_h < 0$ and $\Delta GR_l > 0$. Under sectoral linking, the sum of government revenues may be more relevant, but the sign is indeterminant and depends on the relative magnitude of each program.

The net revenue flows into each program—there are no revenue flows in autarky—are given by $NRF_h = p_h \Delta A_h$ and $NRF_l = p_l \Delta A_l$. Thus, if $r > \frac{p_h^0}{p_l^0}$, then $NRF_h > 0$ and $NRF_l < 0$; if $r < \frac{p_h^0}{p_l^0}$, then $NRF_h < 0$ and $NRF_l > 0$.

Each program's total surplus—excluding climate benefits—when linked, as compared to autarky, is given by $\Delta TS_h = p_h \Delta A_h - \Delta C_h$ and $\Delta TS_l = p_l \Delta A_l - \Delta C_l$. From an earlier result, when linked at an exchange rate $r \neq \frac{p_h^0}{p_l^0}$, $\Delta C_h < p_h \Delta A_h$ and $\Delta C_l < p_l \Delta A_l$. Thus, if $r \neq \frac{p_h^0}{p_l^0}$, then $\Delta TS_h > 0$ and $\Delta TS_l > 0$. Under sectoral linking, the sum of these surpluses may be more relevant, and $\Delta TS \equiv \Delta TS_h + \Delta TS_l > 0$ if $r \neq \frac{p_h^0}{p_l^0}$. ■

### *Proof of Result 4*

A price floor binds if the unconstrained allowance price would be at or below the floor, meaning $p_h \leq \bar{p}_h^F$ or $p_l \leq \bar{p}_l^F$. Similarly, a price ceiling binds if the unconstrained allowance price would be at or above the ceiling, meaning $p_h \geq \bar{p}_h^C$ or $p_l \geq \bar{p}_l^C$. We assume that price collars do not bind in autarky, so $\bar{p}_h^F < p_h^0 < \bar{p}_h^C$ and $\bar{p}_l^F < p_l^0 < \bar{p}_l^C$. We further assume the allowance exchange rate is within the politically plausible range of $\max\left\{\frac{p_h^0}{\bar{p}_l^C}, \frac{\bar{p}_h^F}{p_l^0}\right\} \leq r \leq \min\left\{\frac{p_h^0}{\bar{p}_l^F}, \frac{\bar{p}_h^C}{p_l^0}\right\}$. We will show that a binding price collar when linked violates one of these assumptions. Ignoring $r = \frac{p_h^0}{p_l^0}$, which yields outcomes equal to autarky, there are two cases to consider:

Case 1: $r > \frac{p_h^0}{p_l^0}$. Then $\Delta p_h > 0$ and $\Delta p_l < 0$. There are four sub-cases to consider, one for the price floor and one for the price ceiling in each of the two programs.

Case 1a: Suppose $p_h \leq \bar{p}_h^F$, which is equivalent to $p_l \leq \frac{1}{r}\bar{p}_h^F$. But $p_h^0 < p_h \leq \bar{p}_h^F$, so the floor in program $h$ would have been binding in autarky, which violates an assumption.

Case 1b: Suppose $p_l \leq \bar{p}_l^F$, which is equivalent to $p_h \leq r\bar{p}_l^F$. But $p_h^0 < p_h \leq r\bar{p}_l^F$, or $r > \frac{p_h^0}{\bar{p}_l^F}$, so the exchange rate is not politically plausible, which violates an assumption.

Case 1c: Suppose $p_h \geq \bar{p}_h^C$, which is equivalent to $p_l \geq \frac{1}{r}\bar{p}_h^C$. But $p_l^0 > p_l \geq \frac{1}{r}\bar{p}_h^C$, or $r > \frac{\bar{p}_h^C}{p_l^0}$, so the exchange rate is not politically plausible, which violates an assumption.

Case 1d: Suppose $p_l \geq \bar{p}_l^C$, which is equivalent to $p_h \geq r\bar{p}_h^C$. But $p_l^0 > p_l \geq \bar{p}_l^C$, so the ceiling in program $l$ would have been binding in autarky, which violates an assumption.

Case 2: $r < \frac{p_h^0}{p_l^0}$. Then $\Delta p_h < 0$ and $\Delta p_l > 0$. There are four sub-cases to consider, one for the price floor and one for the price ceiling in each of the two programs.

Case 2a: Suppose $p_h \leq \bar{p}_h^F$, which is equivalent to $p_l \leq \frac{1}{r}\bar{p}_h^F$. But $p_l^0 < p_l \leq \frac{1}{r}\bar{p}_h^F$, or $r < \frac{\bar{p}_h^F}{p_l^0}$, so the exchange rate is not politically plausible, which violates an assumption.

Case 2b: Suppose $p_l \leq \bar{p}_l^F$, which is equivalent to $p_h \leq r\bar{p}_l^F$. But $p_l^0 < p_l \leq \bar{p}_l^F$, so the floor in program $l$ would have been binding in autarky, which violates an assumption.

Case 2c: Suppose $p_h \geq \bar{p}_h^C$, which is equivalent to $p_l \geq \frac{1}{r}\bar{p}_h^C$. But $p_h^0 > p_h \geq \bar{p}_h^C$, so the ceiling in program h would have been binding in autarky, which violates an assumption.

Case 2d: Suppose $p_l \geq \bar{p}_l^C$, which is equivalent to $p_h \geq r\bar{p}_l^C$. But $p_h^0 > p_h \geq r\bar{p}_l^C$, or $r < \frac{p_h^0}{\bar{p}_l^C}$, so the exchange rate is not politically plausible, which violates an assumption.

In summary, in no case can a price collar bind in the linked system without violating an assumption. Thus, if price collars do not bind in autarky, then an exchange rate in the range $r_{min} \leq r \leq r_{max}$ means price collars will not bind. ■

## Appendix B: Results with quadratic abatement costs

In this appendix we derive closed-form results for the special case with abatement costs that are quadratic in abatement and, hence, marginal abatement costs that are linear in abatement. The assumption of quadratic abatement costs and linear marginal abatement costs is common in the literature (Quemin and de Perthuis 2019).

### *Production*

We now assume the cost to the representative firm of producing output is a quadratic function of $CO_2$ emitted during production, $E$:

$$K(E) = \alpha - \beta E + \frac{\gamma}{2}E^2$$

We assume the parameters are positive: $\alpha, \beta, \gamma > 0$. With no carbon emissions policy in place, this firm minimizes production cost by emitting $\bar{E} = \frac{\beta}{\gamma}$. The cost of abating emissions from this baseline level is:

$$C(A) = \frac{\gamma}{2}A^2$$

When faced with a carbon price of $p$, the firm selects the level of abatement that minimizes its total cost:

$$\min_A C(A) - pA$$

This optimization problem yields the first-order condition:

$$\frac{\partial C}{\partial A} = \gamma A = p$$

### *Emissions Trading in Autarky*

An emissions cap with an intended level of abatement $\bar{A}$ yields the resulting level of abatement, allowance price, and cost of abatement in autarky:

$$A^0 = \bar{A}$$
$$p^0 = \gamma \bar{A}$$
$$C^0 = \frac{\gamma}{2}\bar{A}^2$$

### *Linked Emissions Trading*

When two independent emissions trading programs—denoted by $h$ and $l$—link with an allowance exchange rate of $r$, the same conditions must hold as for the general case. Abatement in each program must satisfy:

$$r(A_h - \bar{A}_h) = \bar{A}_l - A_l$$

Allowance prices in each program must satisfy:

$$p_h = rp_l$$

**Abatement and Abatement Cost**

Combining each firm's first-order condition with the linking conditions, we first solve for the allowance prices in the linked system:

$$p_h = \frac{\bar{A}_h + \frac{1}{r}\bar{A}_l}{\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}} \quad \text{and} \quad p_l = \frac{r\bar{A}_h + \bar{A}_l}{\frac{r^2}{\gamma_h} + \frac{1}{\gamma_l}}$$

We similarly solve for the level of abatement in each program when linked:

$$A_h = \frac{\bar{A}_h + \frac{1}{r}\bar{A}_l}{\gamma_h\left(\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}\right)} \quad \text{and} \quad A_l = \frac{r\bar{A}_h + \bar{A}_l}{\gamma_l\left(\frac{r^2}{\gamma_h} + \frac{1}{\gamma_l}\right)}$$

We finally solve for the cost to achieve this level of abatement in each of the programs:

$$C_h = \frac{\left(\bar{A}_h + \frac{1}{r}\bar{A}_l\right)^2}{2\gamma_h\left(\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}\right)^2} \quad \text{and} \quad C_l = \frac{(r\bar{A}_h + \bar{A}_l)^2}{2\gamma_l\left(\frac{r^2}{\gamma_h} + \frac{1}{\gamma_l}\right)^2}$$

Then total abatement and abatement cost in the linked system are given by:

$$A_h + A_l = \left(\frac{\bar{A}_h + \frac{1}{r}\bar{A}_l}{\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}}\right)\left(\frac{1}{\gamma_h} + \frac{1}{r\gamma_l}\right)$$

$$C_h + C_l = \frac{\left(\bar{A}_h + \frac{1}{r}\bar{A}_l\right)^2}{2\left(\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}\right)}$$

We compare these expressions for the linked system to total abatement and abatement cost in autarky:

$$A_h^0 + A_l^0 = \bar{A}_h + \bar{A}_l$$
$$C_h^0 + C_l^0 = \frac{\gamma_h}{2}\bar{A}_h^2 + \frac{\gamma_l}{2}\bar{A}_l^2$$

For our first result of this special case, we determine which allowance exchange rates yield greater or less abatement and abatement cost when linked. This result is the same as in the general case.

**Result 1.** Linking at an allowance exchange rate of:

i. $1 < r < \frac{p_h^0}{p_l^0}$ yields greater total abatement and less total abatement cost than autarky.

ii. $r < 1$ or $r > \frac{p_h^0}{p_l^0}$ yields less total abatement and less total abatement cost than autarky.

**Optimal Exchange Rate**

The optimal exchange rate serves as a benchmark to understand if the political incentives of policymakers are aligned with the broader social incentives. As in the general case, the optimization problem is:

$$\max_r d(A_h + A_l) - C_h - C_l$$

This optimization problem again yields the first-order condition:

$$(d - p_h)\frac{\partial A_h}{\partial r} = -(d - p_l)\frac{\partial A_l}{\partial r}$$

In this special case, we have expressions for the prices and derivatives, which we substitute into this first-order condition. Then we have a closed-form expression for the optimal exchange rate, which is our second result of this special case.

**Result 2.** The socially optimal exchange rate is[57]

[57] This expression is correct so long as the programs in autarky are not over-abating to an extreme level. If they are, the second term in the numerator is subtracted from the first, rather than added.

$$r^* = \frac{\gamma_h\left(\bar{A}_h(2d - p_h^0) - \bar{A}_l(2d - p_l^0)\right) + \gamma_h\sqrt{(\bar{A}_h p_h^0 + \bar{A}_l p_l^0)\left(\frac{1}{\gamma_h}(2d - p_h^0)^2 + \frac{1}{\gamma_l}(2d - p_l^0)^2\right)}}{p_h^0(2d - p_l^0) + p_l^0(2d - p_h^0)}$$

Using this closed-form expression, it can be shown that, if an exchange rate of 1 yields a linked system with too little abatement—that is, the allowance prices when linked are below marginal damages—then this socially optimal exchange rate will always lie within the open interval bounded by 1 and $\frac{p_h^0}{p_l^0}$, as in the general result.

**Distributional Effects**

In this special case, we can express the distributional metrics as functions of the underlying program parameters and the allowance exchange rate by substituting in the earlier closed-form expressions for allowance prices, abatement, and abatement costs. For government revenue and net revenue flows, this special case does not yield new insights—as in the general case, these revenue metrics depend on how the exchange rate compares to $\frac{p_h^0}{p_l^0}$. For total economic surplus, however, this special case does yield a closed-form expression that can be more easily interpreted. Each program's total surplus—excluding climate benefits—when linked, as compared to autarky, is:

$$\Delta TS_h = \frac{\gamma_h (p_l^0)^2}{2(\gamma_h + r^2\gamma_l)^2}\left(r - \frac{p_h^0}{p_l^0}\right)^2 \quad \text{and} \quad \Delta TS_l = \frac{r^2\gamma_l (p_l^0)^2}{2(\gamma_h + r^2\gamma_l)^2}\left(r - \frac{p_h^0}{p_l^0}\right)^2$$

At exchange rates other than $r = \frac{p_h^0}{p_l^0}$, each of these expressions is positive. Thus, the third set of results is the same as in the general case.

**Result 3a.** Regional linking (at a rate other than $r = \frac{p_h^0}{p_l^0}$) always yields greater total economic surplus—excluding climate benefits—than autarky in each region, but it results in opposite revenue effects in each region:

i. Linking at $r < \frac{p_h^0}{p_l^0}$ yields less government revenue than autarky and negative net revenue flows in region $h$ but greater government revenue than autarky and positive net revenue flows in region $l$.

ii. Linking at $r > \frac{p_h^0}{p_l^0}$ yields greater government revenue than autarky and positive net revenue flows in region $h$ but less government revenue than autarky and negative net revenue flows in region $l$.

**Result 3b.** Sectoral linking (at a rate other than $r = \frac{p_h^0}{p_l^0}$) within the same region always yields greater total economic surplus—excluding climate benefits—than autarky in the region, but it may increase or decrease the regional government's total revenues.

**Price Containment Mechanisms**

In this special case, we use the expressions for allowance prices in the linked system to show how the choice of exchange rate determines if the linked system is at a price floor, price ceiling, or within the price collar. For example, the price floor of program $h$ binds when the allowance price in program $h$ would otherwise have been below the price floor

$$p_h = \frac{\bar{A}_h + \frac{1}{r}\bar{A}_l}{\frac{1}{\gamma_h} + \frac{1}{r^2\gamma_l}} \leq \bar{p}_h^F$$

We then solve for the values of $r$ that yield this inequality. We similarly solve for the exchange rates that yield the other price collar outcomes, which gives our final result for this special case.

**Result 4.** The exchange rate, $r$, determines the linked system outcome:[58]

i. If $r \leq \frac{-\frac{\gamma_h}{\gamma_l}p_l^0 + \sqrt{\left(\frac{\gamma_h}{\gamma_l}p_l^0\right)^2 + 4\frac{\gamma_h}{\gamma_l}\bar{p}_h^F(p_h^0 - \bar{p}_h^F)}}{2(p_h^0 - \bar{p}_h^F)}$, then the price floor of program $h$ binds with $p_h = \bar{p}_h^F$ and $p_l = \frac{1}{r}\bar{p}_h^F$.

[58] It is possible, depending on the parameters of each program, for any of these exchange rate intervals 4 to fall outside the feasible set of exchange rates—that is, $\frac{\bar{p}_i^F}{\bar{p}_j^C} \leq r \leq \frac{\bar{p}_i^C}{\bar{p}_j^F}$—and, thus, not be a feasible outcome for the linked market. It is additionally possible, in theory, for overlap to occur between the intervals in (i) and (ii) or the intervals in (iii) and (iv). This overlap only occurs outside the feasible set of exchange rates, however, so each exchange rate within the feasible set yields only one of the outcomes given in this result.

ii. If $\frac{p_h^0-\sqrt{\left(p_h^0\right)^2-4\frac{\gamma_h}{\gamma_l}\bar{p}_l^C\left(\bar{p}_l^C-p_l^0\right)}}{2\bar{p}_l^C} \leq r \leq \frac{p_h^0+\sqrt{\left(p_h^0\right)^2-4\frac{\gamma_h}{\gamma_l}\bar{p}_l^C\left(\bar{p}_l^C-p_l^0\right)}}{2\bar{p}_l^C}$, then the price ceiling of program $l$ binds with $p_h = r\bar{p}_l^C$ and $p_l = \bar{p}_l^C$.

iii. If $\frac{\frac{\gamma_h}{\gamma_l}p_l^0-\sqrt{\left(\frac{\gamma_h}{\gamma_l}p_l^0\right)^2-4\frac{\gamma_h}{\gamma_l}\bar{p}_h^C\left(\bar{p}_h^C-p_h^0\right)}}{2\left(\bar{p}_h^C-p_h^0\right)} \leq r \leq \frac{\frac{\gamma_h}{\gamma_l}p_l^0+\sqrt{\left(\frac{\gamma_h}{\gamma_l}p_l^0\right)^2-4\frac{\gamma_h}{\gamma_l}\bar{p}_h^C\left(\bar{p}_h^C-p_h^0\right)}}{2\left(\bar{p}_h^C-p_h^0\right)}$, then the price ceiling of program $h$ binds with $p_h = \bar{p}_h^C$ and $p_l = \frac{1}{r}\bar{p}_h^C$.

iv. If $r \geq \frac{p_h^0+\sqrt{\left(p_h^0\right)^2+4\frac{\gamma_h}{\gamma_l}\bar{p}_l^F\left(p_l^0-\bar{p}_l^F\right)}}{2\bar{p}_l^F}$, then the price floor of program $l$ binds with $p_h = r\bar{p}_l^F$ and $p_l = \bar{p}_l^F$.

v. Otherwise, the linked system clears inside the price collar.

It can be shown that, if the price collars do not bind in autarky, then the first four cases—which yield a binding price collar in the linked system—only occur if $r < r_{min}$ or $r > r_{max}$. Conversely, if the exchange rate is within the range $r_{min} \leq r \leq r_{max}$, then price collars will not bind in the linked system, as in the general case.