# Read This Paper Even Later: Procrastination with Time-Inconsistent Preferences

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Discussion Paper 99-20

April 1999

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# Procrastination with Time-Inconsistent Preferences[1]

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*Resources for the Future*

**Abstract**

Salience costs, along with imperfect foresight, have been used in previous studies to explain procrastination of a one-time task. A companion to this paper, "Read This Paper Later: Procrastination with Time-Consistent Preferences" analyzes the extent to which procrastination of a divisible task is compatible with rational behavior. While the fully rational model explains key qualitative observations, it requires an extremely high rate of time preference or elasticity of intertemporal substitution to generate serious procrastination and cannot explain undesired procrastination at all. This paper investigates the extent to which dynamically inconsistent preferences can better explain such impatience and address the issue of self-control failures. Two types of discount functions are presented, motivated by previous salience cost explanations. Hyperbolic discounting corresponds to a salient present; short-term discount rates are higher than long-term ones. A new form, differential discounting, arises from salient costs; utility from leisure is discounted at a higher rate than rewards from work. The model of a divisible task with delayed rewards generates clear predictions that can be used to distinguish between types. When workers have rational expectations about future behavior, both regimes induce self-control problems and sharper procrastination than standard exponential discounting. However, they have different implications for policies to induce work, reduce procrastination, and improve welfare.

**Keywords.** procrastination, natural resource economics, hyperbolic discounting, differential discounting

**JEL Classification No(s).:** Q3, D9, J22, J31

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# 1 Introduction

Procrastination, particularly in academics, has been the source of several psychological studies, but few economic ones. Presumably, this dichotomy exists because psychologists love to explore irrational behavior and economists usually restrict themselves to rational conduct, and procrastination is perceived to be irrational, that is, not utility maximizing. However, if not rational, the behavior is certainly normal: Ellis and Knaus (1977) estimated that 95% of college students procrastinate. What seems truly irrational is that economists should ignore such pervasive work-related behavior.

Fischer (1999) approaches leisure time as an exhaustible resource and finds that the standard models from natural resource economics explain many facets of academic procrastination. However, in this fully rational model, the implicit rate of time preference (and/or elasticity of intertemporal substitution) needed to generate serious procrastination is much larger than is typically assigned to people in standard economic models. Such seemingly high rates of time preference may be explained within the context of the rational model, but the perception that procrastination is problematic cannot be explained with time-consistent preferences.

Of course, one always wishes one had more time or less work, so regret about not having worked hard enough earlier is not in itself indicative of irrational or problem procrastination. However, if one looks ahead at the expected path of behavior and wishes it would be different, a self-control problem does exist. Solomon and Rothblum (1984) found that about half of American university students surveyed reported that procrastination was a personal problem of "moderate" or more serious proportions. Furthermore, nearly two-thirds of respondents wanted to decrease their tendency to procrastinate on tasks like writing term papers and studying for exams. The prospective nature of these questions is truly revealing of self-control failures and undesired procrastination, issues which cannot be captured in the rational model.[3]

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[3]That people are willing to engage in costly commitment devices (such as Christmas clubs or fat farms)

This paper explores the impact of dynamically inconsistent preferences for time discounting on the performance and procrastination of a divisible task when expectations are rational.[4] Two types of time-inconsistent discounting are explored: hyperbolic discounting, which applies a higher discount rate to the short term than the long term, and differential discounting, which applies a higher discount rate to the costs of work (utility from leisure) than to rewards. Each type generates more procrastination than standard time-consistent discounting, but they have different implications for policies to induce work, reduce procrastination, and improve welfare. Section 2 discusses the existing literature, its limitations, and how the "salience costs" it uses to explain procrastination can motivate these types of discounting preferences. Section 3 presents the basic framework of procrastination as an exhaustible resource problem. Sections 4 and 5 then apply it to model time allocation with hyperbolic and differential discounting, respectively. Section 6 compares the effects of different reward schemes under exponential, hyperbolic, and differential discounting. Section 7 concludes.

## 2   Salience and Time Discounting

Salience costs, as opposed to pure impatience, are frequently advanced as an explanation for procrastination. For example, opportunity costs of time today may be more salient than those tomorrow; that is, today's opportunities are clear while tomorrow's are vague, making the former seem more pressing. Akerlof (1991) offered a salience-cost model to explain procrastination of a task requiring action at a single point in time. With a salience cost to acting today, and none attributed to tomorrow, one always wants to postpone action, even though the stream of benefits is maximized with immediate action.

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to overcome apparent self-control failures is frequently cited as evidence for dynamically inconsistent preferences. See Ainslie (1992) and Loewenstein and Elster (1992) for overviews, as well as Laibson (1995), Schelling (1978, 1984), Thaler (1980), and Thaler and Shefrim (1981).

[4]Several economists have advanced models of time-inconsistent or changing preferences to examine savings behavior: Strotz (1956), Phelps and Pollak (1968), Pollak (1968), Peleg and Yaari (1973), Laitner (1980), Laibson (1995).

The Akerlof model combines several important features, each of which can contribute to procrastination, but not all of which are necessary to explain the behavior in general:

1. a salient present, creating a temporary preference for the short term;

2. salient costs, implying a heightened sensitivity to the work compared to the rewards;

3. imperfect foresight about future behavior; and

4. an indivisible task.

With today's costs more salient than tomorrow's, a one-time, short-term discount factor is effectively applied to all future action; after today, future costs are evaluated with equal weight (a long-run discount factor of 1). At the same time, only the costs are more salient; benefits today are weighted equally with benefits tomorrow. Effectively, different discount factors are being used for the costs and benefits of the action.

Each of these types of discounting alone generate dynamically inconsistent preferences: in each case, current costs tomorrow are weighted more heavily than tomorrow's costs are today. However, such preferences by themselves are not sufficient to produce indefinite procrastination of a fixed task: one must have irrational expectations of the future. If a person realizes she will want to postpone again every day, the rational expectations strategy is to perform the task at once. O'Donoghue and Rabin (1999) examine the decision to procrastinate a one-time task with Akerlof-style salience costs, allowing for rewards or costs to be salient and expectations to be sophisticated (rational) or naïve.[5]

Psychologists describing patterns of relative preferences for rewards and delays have found they indicate highly bowed discount curves, "predict[ing] that there will be some pairs of alternative rewards such that a larger, later reward is preferred when the choice is seen from a distance, but the smaller, earlier reward is preferred as it becomes imminent."[6] Such studies of temporary preferences typically focus on a heightened sensitivity to the

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[5]They show that rational expectations can still allow for some procrastination. In a four-period model, if the task is worth performing in period 4, and knowing this the person in period 3 will procrastinate, the person in period 2 may find waiting two more periods for the reward too costly and want to perform it. Consequently, the person in the first period will procrastinate, confident the task will be performed in the next period.

[6]Lowenstein and Elster, p. 68.

present.[7] However, these observations, cited as evidence for hyperbolic discounting, may also be generated by differential discounting.

The Akerlof and O'Donogue-Rabin models effectively combine these two types of discounting in the form of a salient present with salient costs. By concentrating on a different type of task, one which can be divided and performed over time, this paper can separate out the different impacts of a preference for the short term and differential evaluation of costs and rewards.

# 3 Model Framework: Time as an Exhaustible Resource

Consider a type of task which can be divided into many (if not an infinite number of) small actions to be completed over time, such as writing a term paper. Procrastination of this kind of work is not necessarily characterized by missing deadlines or leaving tasks incomplete; it can exhibit itself in an increasing workload, as more of the task is performed the closer the deadline. Academic procrastination is a familiar example: assignments may be handed in on time, yet procrastination is deemed to have occurred since much of the work was accomplished at the eleventh hour. In psychology as well as in economics, definitions of procrastination differ widely; perhaps the one most closely corresponding to mine is presented by Solomon and Rothblum (1984): "the act of needlessly delaying tasks to the point of experiencing subjective discomfort."[8] In general, I will consider procrastination to be a steep, rather than constant, work path; however, an additional problem of insufficient total work may also coincide with this behavior in some situations.

The basic problem is that posed in Fischer (1999): when the work requirement demands many units of effort over a finite amount of time, when will that effort take place? The question is similar to a resource extraction problem, as leisure time in the interim is an

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[7]See Ainslie (1992), Loewenstein and Elster (1992), and Thaler (1991) for overviews.

[8]I interpret "needless" to be in terms of feasibility and "subjective discomfort" to be significant disutility of work near the deadline.

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exhaustible resource.

Suppose a person faces a deadline $T$ days from now, at which time she gets reward $F(\cdot)$, a function of total work completed by the deadline. She gets utility $u(l_t)$ for $l$ hours of leisure on any day $t$; assume $u(\cdot)$ is strictly increasing and concave. A maximum of 24 hours per day are available to divide between work and leisure;[9] work thus equals the excess of 24 minus hours of leisure: $w_t = 24 - l_t$. In addition, she has a maximum daily leisure "extraction" rate — a "capacity constraint" — of 24 hours.[10]

Since preferences may change over time, the dynamic problem is best explained by dividing the representative worker into "selves" according to time period. Each self at each point in time chooses how much work to do that day, taking into account the behavior of her future selves. Thinking of total leisure time as a resource stock, the problem for each self can be characterized as a question of how much of the resource stock to leave for the next self.

Let $S_t$ represent the total stock of available time not yet used in leisure inherited by self $t$. Let $R$ denote the minimum amount of work that must be completed by the deadline. The worker starts out with total potential leisure time of $S_0 = 24T - R$, and each self inherits the stock of the preceding self, less the leisure she extracted: $S_{t+1} = S_t - l_t$. This construction will facilitate backing out the optimal path and allow for different types of reward functions. If the reward is a variable function of cumulative work, $R = 0$ and $S_T$ will equal the amount of work completed by the deadline. If a fixed reward is offered, $R$ is the cumulative work requirement and $S_T = 0$.

This basic framework is the same for all types of discounting. Each self maximizes her stream of utility from leisure and the reward, with respect to the stock of potential leisure time she leaves her successor, subject to the aforementioned constraints. Although preferences may not be dynamically consistent, procrastination is still rational in this sense:

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[9]Those of less hardy stock who need a minimum of sleep may feel free to pick a smaller number.

[10]The capacity constraint occurs when marginal utility at leisure all day is positive; in a sense, one would like to take more than 24 hours of leisure in a day but is technologically constrained by the rate of the earth's rotation.

if the worker had it to do all over again, knowing how she behaved all along the way, she would act the same.

# 4 Hyperbolic Discounting

The first salience story is that opportunities today are more salient than those tomorrow or any day following. Akerlof's salience costs were effectively a high one-term discount rate for leisure, a simple form similar to hyperbolic discounting. Hyperbolic discount functions have high discount rates for small delays and low discount rates for long delays, a dichotomy which generates dynamically inconsistent preferences. From a distance, a person may prefer a larger, later reward, but when the time draws near, the smaller, more imminent reward will be chosen.

Hyperbolic discount functions have been suggested by some studies of human and animal behavior which show that preferences for time discounting are more deeply bowed than exponential discount functions.[11] For example, facing hypothetical tradeoffs of certified checks, most people would take $100 today over $200 in 2 years, but not $100 in 6 years over $200 in 8 years.[12] Exponential discounters should find these tradeoffs identical in relative terms.

Consider a simplified, "quasi-hyperbolic" discount function put forth by Laibson (1995) to mimic hyperbolic discounting: from the perspective of time $t$, utility of current consumption is weighted at unity, while utility of consumption at future time $t + i$ is weighted by the discount factor $\beta \delta^i$. This function has the advantages of tractability and the intuitive component of combining a short-run ($\beta \delta$) and long-run ($\delta$) discount factor.

The dynamic inconsistencies created by hyperbolic discounting are best explained by comparing two "selves."[13] Today's self discounts tomorrow's utility by $\beta \delta$, yet he wants

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[11]Ainslie (1992).

[12]Lowenstein and Elster (1992), p. 69.

[13]In honor of David Laibson, I will make the hyperbolic discounter a "he."

tomorrow's self to discount the following period's utility by $\delta$, which tomorrow's self will not do when tomorrow becomes today. Hence, the dynamic inconsistency (or intertemporal schizophrenia).

In deciding how much work to do and when to do it, the self at time $t$ would desire the current and future work path that maximizes total discounted utility, $U_H(\cdot)$:

$$U_H(l_t, l_{t+1}, ..., l_{T-1}) = u(l_t) + \sum_{i=1}^{T-t-1} \beta \delta^i u(l_{t+i}) + \beta \delta^{T-t} F(S_T), \qquad (1)$$

where $u(l_t)$ is utility of leisure at time $t$, $F(\cdot)$ is the reward function, and $S_t$ is the total stock of available time not yet used in leisure inherited by self $t$, as presented in the previous section. Both $u(\cdot)$ and $F(\cdot)$ are strictly increasing, concave functions. Furthermore, let us assume that $u'(0) = \infty$, which will ensure that $l_t > 0$ for $t \in [0, T-1]$.

The current self wants future selves to behave as if they have standard exponential discounting. If self $t$ could control not only $l_t$ but also $l_{t+1}$, $l_t + 2$, etc., he would have $u'(l_t) = \beta \delta^{T-t} F'(\cdot)$ and $u'(l_{t+i}) = \delta^{T-t-i} F'(\cdot)$ for all $i \geq 1$; i.e., after today marginal utility would rise at the rate of time preference.[14]

However, the sophisticated self realizes that he cannot achieve this optimum, since future selves will not behave as he desires, and he reacts accordingly. He maximizes utility of current and future consumption, subject to the constraints that his successors will maximize their utility. The equilibrium path can thus be found by backwards induction.

Consider simple log utility functions for both leisure and the reward:[15] $u(l) = ln(l)$ and $F(S_T) = \alpha ln(S_T)$.

The last working self $(T-1)$ maximizes current utility from leisure plus the discounted

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[14]These equations must hold with equality only for $l_{t+i} \in (0, 24)$. If the capacity constraint binds (he would prefer to consume more than 24 hours of leisure in a day but cannot), then $u'(24) \geq \beta \delta^{T-t} F'(\cdot)$ and $u'(24) \geq \delta^{T-t-i} F'(\cdot)$ for $i \geq 1$. If the "choke price" is reached (he would prefer to work more than 24 hours that day but cannot), then $u'(0) \leq \beta \delta^{T-t-i} F'(\cdot)$ and $u(0) \leq \delta^{T-t-i} F'(\cdot)$ for $i \geq 1$. However, the assumption of $u'(0) = \infty$ eliminates this latter possibility.

[15]The assumption that the utility from leisure and the reward have the same, constant relative risk-aversion coefficient (in this case 1) is needed for the time path of the marginal propensity to consume leisure out of the inherited stock of time to be independent of the final amount of work. This independence ensures the equilibrium time path is unique.

reward next period from passing on the remaining stock of potential leisure time:

$$U_H(S_{T-1}, 1) = \max_{l_{T-1}\in[0,24]} \{ln[l_{T-1}] + \beta\delta\alpha ln[S_{T-1} - l_{T-1}]\}. \qquad (2)$$

The resulting $l_{T-1}$ is a linear function of $S_{T-1}$, with a kink where the capacity constraint binds:[16]

$$l_{T-1} = \min\left[\frac{1}{1 + \beta\delta\alpha}S_{T-1}, 24\right]. \qquad (3)$$

Let $\mu_{T-1}^H$ denote $T - 1$'s (constant) marginal propensity to consume leisure out of the stock of time he inherits, in this case $1/(1 + \beta\delta\alpha)$. Suppose for now that $T - 1$ is not capacity constrained and thus follows this proportional consumption rule. Self $T - 2$ maximizes current utility from leisure, plus the discounted utility from $T - 1$'s leisure, plus the discounted reward, knowing how $T - 1$ will behave given how much of the current stock of time not spent in leisure is left him:

$$U_H(S_{T-2}, 2) = \max_{l_{T-2}\in[0,24]} \Big\{ ln[l_{T-2}] + \beta\delta ln[\mu_{T-1}(S_{T-2} - l_{T-2})]$$
$$+ \beta\delta^2\alpha ln[(1 - \mu_{T-1})(S_{T-2} - l_{T-2})]\Big\}. \qquad (4)$$

The interior solution for $l_{T-1}$ is also a linear function of inherited stock:

$$l_{T-2} = \min\left[\frac{1}{1 + \beta\delta\alpha + \beta\delta^2\alpha}S_{T-2}, 24\right]. \qquad (5)$$

This simple rule actually suffices even if self $T - 1$ does not face an interior solution, because if $T - 1$ is capacity constrained, so is self $T - 2$. I.e., if one self would like to consume 24 hours of leisure, the preceding self would prefer to consume more, but is by definition capacity constrained at 24 hours. Therefore, either the interior solution will hold, or the self will be capacity constrained. (See Proposition 1 in the Appendix.)

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[16]The result that the next period's stock is a linear of this period's stock holds as long as utility from leisure and the reward have the same, constant relative risk-aversion coefficient (see previous footnote).

Self $T - 3$ maximizes discounted utility knowing the marginal propensities to consume leisure of his successors, and the process continues through backwards induction. Let $\mu_t^H$ denote $t$'s marginal propensity to consume leisure out of the stock of time he inherits, which is constant in this case. On the equilibrium path, the change in leisure consumption due to the change in inherited stock is given by the backwards recursion,[17]

$$\mu_{T-i}^H = \frac{\mu_{T-i+1}^H}{\mu_{T-i+1}^H + \delta - (1 - \beta)\delta\mu_{T-i+1}^H},$$ (6)

for $i = 1, ..., T$, beginning at the end with $\mu_{T-1}^H = 1/(1 + \beta\delta\alpha)$.

This recursion implies

$$\mu_t^H = \frac{1}{1 - \beta + \sum_{i=0}^{T-t-1} \beta\delta^i + (\beta\delta)^{T-t}\alpha},$$ (7)

for $t = 0, ..., T - 1$.

For a fixed reward, $\alpha = 0$ and $\mu_{T-1}^H = 1$. Taking into account the capacity constraint, the stock of potential leisure time remaining in any period $t + 1$ is then $S_{t+1}^H = \max[(1 - \mu_t^H)S_t^H, \ S_t^H - 24]$.

The general problem can be written as a modified Bellman equation. To understand this problem better, consider selves as occupying a day in the week. Monday's and Tuesday's selves agree on the relative valuation of utility flows received on Wednesday and all following days: those discount factors decline by $\delta$ each day. However, Monday thinks Tuesday overvalues his own utility. Therefore, Monday views his subsequent stream of utility as $\delta$ times Tuesday's total discounted utility, minus a bit of Tuesday's leisure utility.

Thus, for any self $t$, today's discounted utility equals utility of current leisure, plus tomorrow's self's total utility discounted by long-run factor $\delta$, minus the discounted amount

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[17]See Proposition 2 in the Appendix for the full derivation.

by which the next self overvalues his current leisure consumption:

$$U_H(S_t, T-t) = \max_{l_t \in [0,24]} \{u(l_t) + \delta \left[ U_H(S_{t+1}, T-t-1) - (1-\beta)u(l_{t+1}) \right] \}, \quad (8)$$

where $S_{t+1} = S_t - l_t$ and leisure in $t+1$ is implicitly a function of the inherited stock: $l_{t+1}(S_{t+1}, T-t-1)$.

Self $t$ maximizes with respect to his leisure and thereby the stock that he leaves for the next self. For $l_t \in (0, 24)$ the following first-order condition must hold:

$$u'(l_t) - \delta U_H'(S_{t+1}, T-t-1) + \delta(1-\beta)u'(l_{t+1})\frac{\partial l_{t+1}}{\partial S_{t+1}} = 0. \quad (9)$$

By the Envelope Theorem, we see the change in discounted utility when any self $t+1$ inherits a bit more stock:

$$U_H'(S_{t+1}, T-t-1) = \delta U_H'(S_{t+2}, T-t-2) - \delta(1-\beta)u'(l_{t+2})\frac{\partial l_{t+2}}{\partial S_{t+2}}. \quad (10)$$

From the first-order condition of self $t+1$, we can rewrite this equation as

$$U_H'(S_{t+1}, T-t-1) = u'(l_{t+1}). \quad (11)$$

Equations (9) and (11) lead to the following relationship between marginal utilities (for any $\{l_t, l_{t+1}\} \in (0, 24)$):[18]

$$u'(l_t) = u'(l_{t+1})\delta \left[ 1 - (1-\beta)\frac{\partial l_{t+1}}{\partial S_{t+1}} \right]. \quad (12)$$

Note from Equation (12) that if $\beta = 1$, standard exponential discounting applies.

Since the marginal propensity to consume leisure from the inherited stock of time is positive, for $\beta < 1$ marginal utility grows faster than the long-run discount rate. In other

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[18]For another explicit derivation, see Laibson (1995).

words, the short-term discount factor raises the effective discount rate (lowers the effective discount factor) in every period, although to different extents.

Let $R_{t+1}^H = \delta(1 - (1 - \beta)\mu_{t+1}^H(S_{t+1}, T - t - 1))$ denote the effective discount factor between $t$ and $t + 1$, where $\mu_t^H(S_t, T - t) = \partial l_t/\partial S_t$, self $t$'s marginal propensity to consume leisure out of the stock of time he inherits. Note that the effective discount factor falls as $\mu^H$ rises, as the impact on the next self's leisure consumption of leaving a bit more stock grows stronger. Therefore, if $\mu^H$ increases as the ending date approaches, the rate of change in marginal utility (and thereby leisure) increases as well.

Using a utility function with constant relative risk aversion (CRRA) results in $\mu$ being simply a function of time to the deadline (and the parameters of discount factors, reward and risk-aversion coefficients), independent of the current stock of potential leisure time. Thus, a unique equilibrium path will exist. The example of log utility represents a particular case of this type, and the corresponding equilibrium path was presented in Equation (6).

Figure 1: Hyperbolic Discounters and Fixed Reward
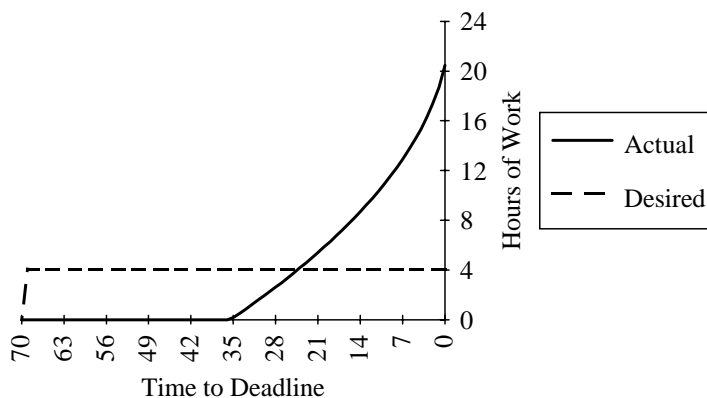


To illustrate the results, consider an example with a long-run discount factor of one (the effective daily factor corresponding to traditional annual rates of discount) and $\beta = 2/3$ (today has a 50% premium over any other day). Utility from work is a log function, and the subject has 10 weeks to complete a task that requires an average of 4 hours of work a day. Figure 1 compares the equilibrium work path to the desired path of self 0 for a fixed

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reward.Essentially, the current self would like to do little or no work today, but after today he would like himself to work at a steady pace. However, knowing that his successors would squander extra leisure time, he ends up procrastinating more, forcing future selves to work more.

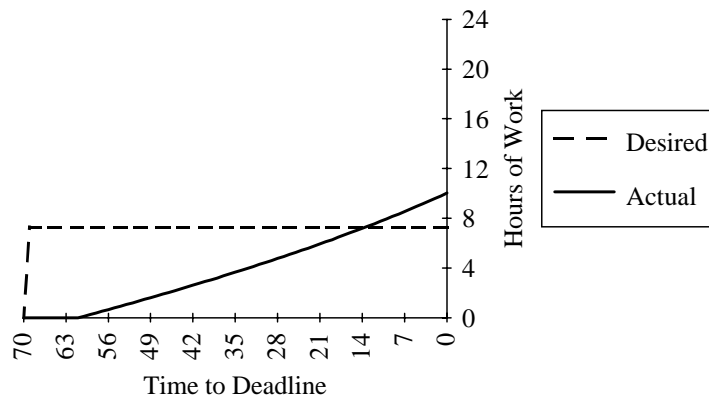Figure 2: Hyperbolic Discounters and Variable Reward



Figure 2 compares the equilibrium work path to the desired path of self 0 when a hyperbolic discounter faces a variable reward.[19] Hyperbolic discounting with rational expectations causes less work to be performed overall than the initial self would desire. Furthermore, it induces procrastination in the form of a rising, rather than level, workload.

The procrastination induced by these dynamically inconsistent preferences, though "rational," is no longer welfare maximizing as it was with exponential discounting. Proposition 3 in the Appendix proves that, with a variable reward, the rational-expectations equilibrium can be pareto dominated for all selves if they all collectively worked a bit more.[20]

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[19]The reward takes the form of $\alpha ln[S_T]$ where $\alpha$ is set such that the rational self just completes the work requirement.

[20]Laibson (1995) demonstrates how hyperbolic discounting can generate undersaving and lower welfare for all selves. While in his infinite-horizon model the result is a lower steady-state level of consumption, in this finite model, the problem is that the ultimate reward is too small, due to too little work.

# 5 Differential Discounting

Self-control problems occur when different events are discounted at different rates. The story of the salient present said that events may be discounted at different rates according to their timing. The second salience story says that the effects of certain actions may be more salient than those of other activities. In other words, events with different characteristics may be discounted at different rates.

Examining several anomalies in discounting preferences, Loewenstein and Prelec (1992) noted that not only do discount rates appear to change over a time horizon, but they also seem to vary with different types of rewards or costs. Lowenstein (1987) asserted that discount rates may not be independent of the types of goods consumed; rather, they may vary with goods of differing characteristics. Luckert and Adamowicz (1993) found empirical evidence of different time preferences for environmental versus financial goods.

Akerlof's salience costs operated like different discount rates for leisure and rewards: today's costs of performing the task were more salient than tomorrow's but today's benefits were not. I will refer to this different treatment over time of costs and benefits (in this case foregone utility from leisure and the rewards for work, respectively) as "differential discounting."

With salient costs, the present value of postponing a one-time task declines faster than does the flow of benefits. From any point in time, there will be another point in the future where the marginal gain from postponement equals the marginal benefits foregone, but once arrived there, postponing is again optimal. Thus, differential discounting, like hyperbolic discounting, creates dynamically inconsistent preferences.

Furthermore, differential discounting can create preferences that seem hyperbolic. Consider again the hypothetical tradeoffs of certified checks; now suppose people get utility from receiving a prize, in addition to liking more money. If people discount the joy of winning more heavily than cash, they may prefer taking $100 now over $200 in 2 years. However, looking ahead 6 years, the thrill of victory is so heavily discounted that relatively

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more of the overall prize value is in the cash, so rather than \$100 in 6 years they choose \$200 in 2 more years.[21]

Suppose that people do not discount leisure (or disutility from researching a paper) in the same way they discount the reward or penalty; specifically, let us assume that utility from leisure is more heavily discounted than the reward for work. Benefits obtained in $t$ periods are valued at $\delta^t$, while the utility of leisure in $t$ periods is valued at $(\gamma\delta)^t$, where $\gamma < 1$.

In deciding how much work to do and when to do it, the self at time $t$ would desire the current and future work path that maximizes total discounted utility:

$$U_t^D(l_t, l_{t+1}, ..., l_{T-1}) = u(l_t) + \sum_{i=1}^{T-t-1} (\gamma\delta)^i u(l_{t+i}) + \delta^{T-t}F(S_T), \qquad (13)$$

where $u(l_t)$ is again utility of leisure at time $t$, $F(\cdot)$ the reward function, and $S_t$ the total stock of available time not yet used in leisure inherited by self $t$, as presented in Section 3.

On the desired path of self $t$, the first-order conditions with respect to work in any period imply that marginal utility should rise according to the discount factor for costs; however, they depend on the initial perspective:

$$u'(l_{t+i}) = \gamma^{-i}\delta^{T-t-i}F'(S_T), \qquad (14)$$

where $S_T$ equals the cumulative amount of work completed by the deadline.

Thus, preferences change over time. Self $t$ wants marginal utility at $T-1$ to be $\gamma^{t-T+1}\delta F'(S_T)$, while the self at $T-1$ equates marginal utility to the lower $\delta F'(S_T)$ (holding total work constant). In other words, self $t$ wants later selves to work harder than they will.

However, if selves have rational expectations, they will choose their current workload

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[21]Although this story requires believing people derive intrinsic utility from receiving cash, so does the hyperbolic story: a hyperbolic discounter maximizing a stream of consumption should still want to maximize his present value (in market terms) of income.

knowing how subsequent selves will react. Consider the problem for the last self $(T-1)$ with log utility and reward functions:

$$U_D(S_{T-1}, 1) = \max_{l_{T-1} \in [0,24]} \{ln[l_{T-1}] + \delta \alpha ln[S_{T-1} - l_{t-1}]\}. \qquad (15)$$

The resulting $l_{T-1}$ is a linear function of $S_{T-1}$ up to the capacity constraint:[22]

$$l_{T-1} = \min\left[\frac{1}{1+\delta\alpha} S_{T-1}, 24\right]. \qquad (16)$$

Let $\mu_{T-1}^H$ denote $T-1$'s (constant) marginal propensity to consume leisure out of the stock of time she inherits, in this case $1/(1+\delta\alpha)$. Self $T-2$ maximizes current utility from leisure plus the discounted utility from $T-1$'s leisure plus the discounted reward, deciding how much of the current stock of time not spent in leisure to leave $T-1$, knowing how she will behave:

$$U_D(S_{T-2}, 2) = \max_{l_{T-1} \in [0,24]} \{ln[l_{T-1}] + \delta \alpha ln[S_{T-1} - l_{t-1}]\}. \qquad (17)$$

The unconstrained $l_{T-2}$ is also a linear function of inherited stock:

$$l_{T-2} = \min\left[\frac{1}{1+\gamma\delta+\delta^2\alpha} S_{T-2}, 24\right]. \qquad (18)$$

As in the hyperbolic case, if self $T-1$ is capacity constrained, so is self $T-2$. Self $T-3$ maximizes discounted utility knowing the marginal propensities to consume leisure of her successors, and the process continues through backwards induction. Let $\mu_t^D$ denote the differential discounter's (constant) marginal propensity to consume leisure out of the stock inherited at time $t$. Proposition 4 in the Appendix derives explicitly the recursive

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[22]Again, the result that the next period's stock is a linear function of this period's stock holds as long as utility from leisure and the reward have the same, constant relative risk-aversion coefficient.

relationship determining $\mu_{T-i}^D$, showing that

$$\mu_{T-i}^D = \frac{\mu_{T-i+1}^D}{\mu_{T-i+1}^D + \gamma\delta + (1-\gamma)\delta^i \alpha \mu_{T-i+1}^D}, \tag{19}$$

for $i = 1, ..., T$ and starting at the end with $\mu_{T-1}^D = 1/(1 + \delta\alpha)$.

This recursion implies

$$\mu_t^D = \frac{1}{\sum_{i=0}^{T-t-1}(\gamma\delta)^i + \delta^{T-t}\alpha}, \tag{20}$$

for $t = 0, ..., T - 1$.

The general problem of the rational differential discounter can be rewritten as a modified Bellman equation. Consider each self as a day in the week. Monday and Tuesday agree on the relative valuation of leisure utility received on all the days, discounting by an additional $\gamma\delta$ each day. However, Monday thinks Tuesday undervalues the reward received at the end. (Recall Equation (14).) Therefore, Monday views her subsequent stream of utility as $\gamma\delta$ times Tuesday's total discounted utility, plus an extra bit of the reward utility.

Thus, self $t$ maximizes her current utility of leisure plus her successor's utility stream, discounted by factor $\gamma\delta$, plus a term to make up for her successor's relative undervaluation of the delayed reward:

$$U_D(S_t, T - t) = \max_{l_t \in [0,24]} \left\{ u(l_t) + \gamma\delta U_D(S_{t+1}, T - t - 1) + (1 - \gamma)\delta^{T-t}F(S_T) \right\}. \tag{21}$$

Under both types of discounting, self $t$ knows that self $t + 1$ will overvalue her own consumption of leisure relative to the total stream of utility, in the hyperbolic case because she overvalues current leisure relative to tomorrow and in the differential case because she overvalues it relative to the marginal discounted reward, from $t$'s perspective.

The differential discounting self maximizes total utility with respect to the stock of potential leisure time she leaves her successor, taking into account how much of that stock

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gets passed through to the deadline. The following first-order condition results for $l_t \in (0, 24)$:

$$u'(l_t) - \gamma \delta U'_D(S_{t+1}, T - t - 1) - (1 - \gamma)\delta^{T-t} F'(S_T)\frac{\partial S_T}{\partial S_{t+1}} = 0. \tag{22}$$

By the Envelope Theorem, we see the change in discounted utility when any self $t + 1$ inherits a bit more stock:

$$U'_D(S_{t+1}, T - t - 1) = \gamma \delta U'_D(S_{t+2}, T - t - 2) + (1 - \gamma)\delta^{T-t-1} F'(S_T)\frac{\partial S_T}{\partial S_{t+2}}. \tag{23}$$

From t+1's first-order condition, we can rewrite this equation as

$$U'_D(S_{t+1}, T - t - 1) = u'(l_{t+1}). \tag{24}$$

Combining (22) and (24) gives the relationship between marginal utilities on the optimal path:

$$u'(l_t) = \gamma \delta u'(l_{t+1}) + (1 - \gamma)\delta^{T-t} F'(S_T)\frac{\partial S_T}{\partial S_{t+1}}. \tag{25}$$

Logically, $0 \leq \partial S_T/\partial S_{t+1} \leq 1$: at least part of another hour not spent in leisure left for tomorrow will be added to the cumulative stock of work at the end; some will likely be taken by successors as more leisure, but not more than the initial additional amount. For $\gamma < 1$, marginal utility will grow more slowly than under exponential discounting with factor $\gamma\delta$ but faster than with $\delta$. With a fixed reward and work requirement, $\partial S_T/\partial S_{t+1} = 0$, and time allocation proceeds as with high exponential discounting.

Let $\mu_t^D(S_t, T - t)$ denote the differential discounter's marginal propensity to consume leisure out of the stock inherited at time $t$. As in the hyperbolic case, using a CRRA utility function makes $\mu$ a function of time to the deadline and independent of the current stock of potential leisure time, resulting in a unique equilibrium path. The marginal change in the final stock of time not spent in leisure (cumulative work) is what remains after each self

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consumes its portion of leisure: $S_{t+1} = S_t(1 - \mu_t^D)$, $S_T = S_t(1 - \mu_t^D) \prod_{i=t+1}^{T-1}(1 - \mu_i^D)$, and $\partial S_T/\partial S_{t+1} = \prod_{i=t+1}^{T-1}(1 - \mu_i^D)$. For the log reward function, many of the terms in the impact on the marginal reward cancel. Specifically, $F'(S_T)\partial S_T/\partial S_{t+1} = \alpha/S_{t+1}$. Equation (19) presented the corresponding equilibrium path of $\mu^D$.

If a differential discounter faces a fixed reward, her preferences will be time-consistent: although each self evaluates the reward differently, the behavior of any one self does not affect the marginal reward, conditional on all selves wanting to satisfy the work requirement. Therefore, she behaves like an exponential discounter with a high rate of time preference (corresponding to the discount factor $\gamma\delta$).

Figure 3: Differential Discounters and Variable Reward
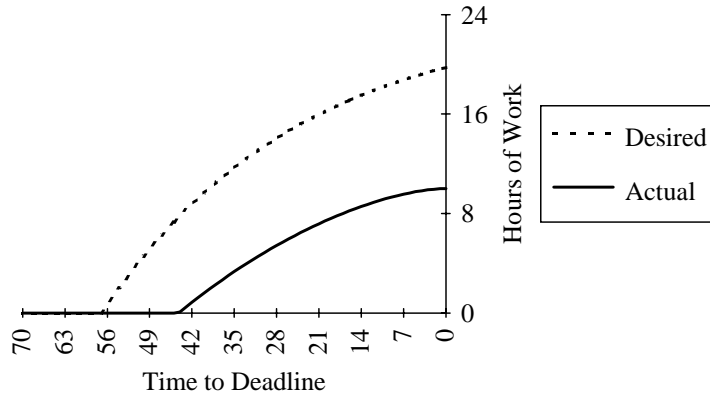


With a variable reward, however, the marginal reward is affected and evaluated differently by each self. Figure 3 compares the equilibrium work path to the desired path of self 0 with a variable reward.[23] With rational expectations, differential discounting causes the start of work to be postponed until nearer the deadline and, with a concave reward (or convex penalty) function, less work to be performed overall than desired.

However, the differential discounter actually does not procrastinate as much as she would like. Her marginal utility rises more slowly than on the desired path, and she does not build up her workload as rapidly as she prefers. Essentially, she sees the cost of work de-

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[23]The reward in the illustration takes the form of $\alpha ln[S_T]$ where $\alpha$ is set such that the rational self just completes the work requirement. As before, $\delta = 1$ and $\gamma = .97$.

clining much faster over time than the benefits and therefore would like to postpone work; however, knowing her successors will value their leisure time more relative to the reward and not work enough, she compensates by doing more work early (actually procrastinating less).[24] Still, less work is performed overall than she would like.

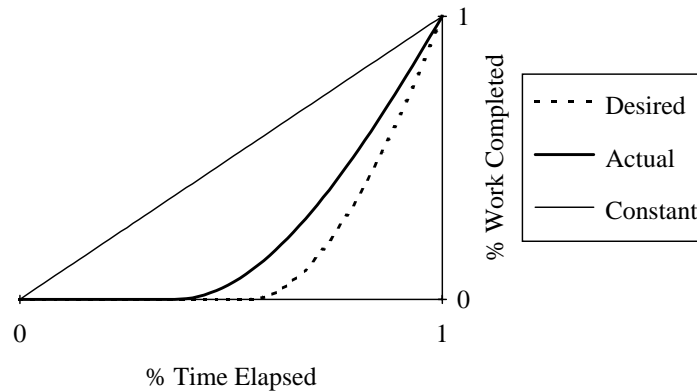Figure 4: Procrastination Rates of Differential Discounters



Figure 4 shows this result from another angle. A Lorenz curve compares the percentage of work completed to the percentage of time elapsed. For the same work requirement (with $\alpha$ adjusted accordingly), the path of the rational differential discounter lies inside the desired path of work accumulation, closer to the constant-work path of the 45-degree line. This result implies that the differential discounter ends up procrastinating less than the initial self would like to behave, given a work requirement.

Thus, the self-control problem actually has two facets:

1. the total amount of work performed, and

2. the steepness of the path of work.

While both hyperbolic and differential discounters perform too little work in equilibrium, they have different perspectives on the path of work and the amount of procrastination. If the desired paths were to be followed, and the reward parameter lowered to achieve the same work requirement (shifting down the desired path of marginal utility), the "ideal"

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[24]For example, a student would like to write an "A" paper the last week of the semester. However, if she actually waited until then to start, at that point she would settle for a passing grade.

(from the point of view of self 0) hyperbolic work path would be less steep than the actual one, while the ideal differential work path would be steeper. Thus, the initial hyperbolic self thinks his successors will procrastinate too much, while the differential discounter seems to think they don't procrastinate enough.

This result has important implications for welfare and the setting of deadlines. Procrastination can be reduced by dividing the task into segments, each with its own deadline, forcing the student to accomplish more work earlier than she would normally.[25] While this technique could help hyperbolic selves improve welfare by flattening the work path, it would certainly lower welfare for differential discounters (holding total work constant).

# 6   Comparing Regimes and Rewards

For employers desiring to get work out of their employees, methods of inducement are important. This section considers the relative effectiveness of delayed fixed and variable rewards (such as bonuses) and contemporaneous rewards (such as wages) on exponential, hyperbolic, and differential discounters.

## 6.1   Fixed Rewards

For the worker to achieve a fixed work requirement, a sufficient fixed delayed reward or penalty must exist. The amount of the minimum required reward, of course, will depend on the discounting regime. The differential discounter behaves like an exponential discounter with a high rate of time preference ($\gamma\delta$), but she requires a lesser reward to induce work since it is discounted at a lower rate. She also requires less inducement than an exponential discounter using $\delta$, since the higher rate of time preference for leisure reduces the present values of the alternative paths of utility and also their difference.[26]

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[25]Psychological studies of college students have documented the effectiveness of closer supervision and more frequent deadlines in reducing procrastination: Lamwers and Jazwinski (1989), Reiser (1984), and Wesp (1986).

[26]See Proposition 5 in the Appendix for proof.

Hyperbolic discounters, on the other hand, may require more inducement than an exponential discounter using $\delta$. First of all, the fixed reward is evaluated using a higher effective discount rate: $\beta\delta^T < \delta^T$; second of all, since subsequent selves will not behave ideally, the utility differential may be greater. However, that utility differential is also evaluated at a higher effective discount rate, so the net effect is ambiguous.

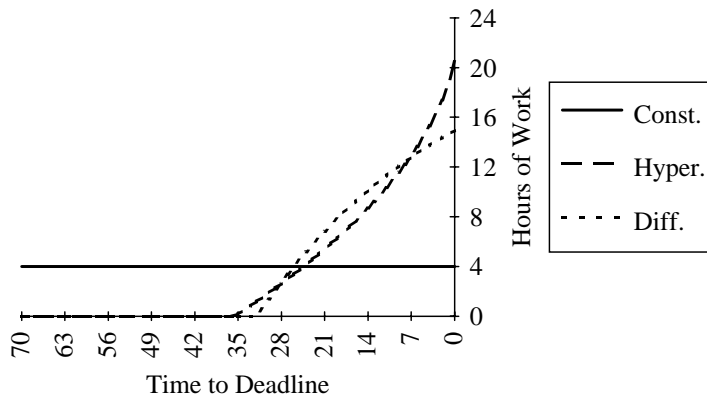Figure 5: Comparison with Fixed Rewards



Figure 5 compares the optimal work paths of hyperbolic and differential discounters for identical work requirements when rewards are fixed. The parameters are calibrated to generate similar, stylized amounts of procrastination, although other characteristics will differ.[27] The constant work path thus represents an exponential discounter at traditional rates (which are effectively zero on a daily basis). The differential discounter procrastinates like a high exponential discounter. The hyperbolic discounter, on the other hand, has lower but increasing effective discount rates, leading to a work path which builds slowly initially and accelerates toward the deadline.

## 6.2 Variable Delayed Rewards

Incentives change, however, if the reward varies with the amount of work completed. At any point in time, the hyperbolic discounter evaluates a given marginal reward with a lower

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[27]Specifically, $\delta = 1$, $\beta = 2/3$, and $\gamma = .97$.

discount factor than the differential discounter ($\beta\delta^s$ versus $\delta^s$ at any point $s$ periods from the deadline). Given the same reward schedule, the hyperbolic discounter performs less work than the differential discounter (who performs less work than her exponential counterpart with the long-run discount factor). Correspondingly, for a given amount of work, a hyperbolic discounter would require a greater reward parameter than a differential discounter.
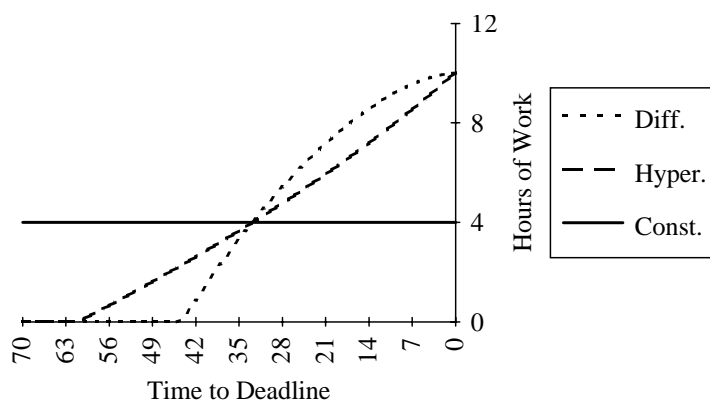
Figure 6: Comparison with Variable Rewards



Figure 6 shows the optimal paths for workers with ("low"=$\delta$), constant ($\delta = 1$), hyperbolic, and differential discount functions when the variable reward parameter ($\alpha$) for each type is set to achieve the same work requirement.[28] The differential discounter procrastinates the most, starting last and building up quickly. The hyperbolic discounter requires the highest reward parameter to cope with the self-control problem as well as the heavier discounting of the reward by all selves, compared to the other discounters. Meanwhile the exponential discounter (here a non-discounter) requires the lowest marginal reward, since she has no self-control problem to overcome.

Although not readily apparent from the figure, the paths of the daily effective discount rate also differ according to discounting regime: With hyperbolic discounting, the effective

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[28]In this log utilities case with $\beta = 2/3$, $\delta = 1$, $\gamma = .97$ and a work target of 284 hours, the hyperbolic discounter requires a marginal reward about 50% higher than that of the differential discounter and more than twice as high as the low exponential discounter.

discount rate rises over time; with exponential discounting it is constant; and with differential discounting, it declines as the deadline approaches.[29] Thus, the time path of the effective discount rate presents a distinguishing characteristic for each regime which may be directly observable.

## 6.3 Contemporaneous Rewards

On the other hand, the employer may not be restricted to delayed compensation; another option is to offer a pay-as-you-go reward. Consider a contemporaneous reward that is a linear function of that period's work, like a wage; the worker's net utility in the current period is then $u(l) + y(24 - l)$. The wage acts like a constant variable cost of extraction in the natural resource analog: extracting another unit of the resource, leisure time, costs the worker $y$ in foregone wages.

First suppose only a wage is offered. Without a delayed reward depending on cumulative work or task completion, behavior today has no impact on behavior tomorrow. Thus, regardless of type of discounting, each self works until the marginal utility of leisure equals the wage, and no procrastination occurs.

However, if the differential discounter could control successive selves, she could be made better off. Since future utility from leisure is discounted more heavily than future wages, she would prefer future selves to work until marginal utility of leisure is higher than the wage (self $t$ would want $u(l_{t+i}) = \gamma^{-i} y$). Essentially, the wages earned in the future cost less in present value terms of utility from leisure, so the differential discounter would rather do less work now and earn more later. If the preferences of the initial self were realized, procrastination would occur (i.e., the workload would build up over time) and for the same work requirement the necessary wage would be much smaller. Figure 7 compares

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[29]Propositions 6 and 7 in the Appendix show these results for the log utility example. However, there may be some combinations of $\alpha$ and the risk-aversion coefficient for which the effective discount rates decline near the deadline for both hyperbolic and differential discounters; still, the implied paths look quite different. In fact, for inelastic elasticities of intertemporal substitution, the effective discount factor for the differential discounter can follow a U-shaped path, while that of the hyperbolic discounter remains monotonic.

Figure 7: Differential Discounters and Contemporaneous Rewards
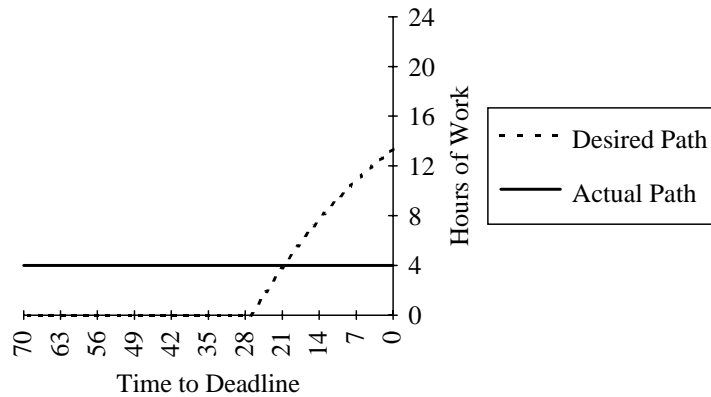


the actual constant path to what would occur if the desired path were followed and wages were adjusted accordingly.

Combining wages with delayed rewards flattens work paths for all types of discounters.[30] Thus, offering wages reduces procrastination. (Combination paths are not shown since adding wages makes the marginal propensity to take leisure a function of the inherited stock of time, rather than an independent and unique function of time and the reward and discounting parameters).

## 6.4 Choosing Rewards

For a supervisor or employer, the optimal reward schedule will likely be a mixture of the options. He will have to trade off the relative effectiveness of each type of compensation in attaining a certain work goal, his need to reduce procrastination in the face of uncertainty,[31] and his desire possibly to *encourage* procrastination if he faces positive interest rates. The present value of his wage payments will be lower if more of the work is performed later.

The tradeoffs for different rewards also depend on the worker's preference for time discounting. A fixed reward will be cheapest for the employer of the differential discounter,

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[30]In the resource extraction analog, since net marginal surplus must rise by the discount rate, higher constant marginal extraction costs cause marginal utility to rise more slowly.

[31]Fischer (1999) shows that with a fixed penalty and uncertainty about the true size of the work requirement, high procrastinators are more likely to fail to meet a deadline.

but it will induce the most procrastination. Delayed variable rewards are more costly for hyperbolic discounters than for differential or (low-rate) exponential ones. Contemporaneous wages reduce procrastination for all types, relative to either delayed reward. Given only wages, no one will procrastinate, but differential discounters would be willing to take lower wages if they could get future selves to work more. Variable rewards do not offer flexibility if the ultimate work requirement is uncertain (unless the reward schedules can be changed); fixed rewards allow for changes in the work requirement, if they are high enough to prevent abandonment in enough cases. Multiple deadlines are effective in reducing procrastination and allow more delay of payment than wages, but they tend to be more costly to administrate. Furthermore, differential discounters would require a much higher fixed reward compared to a single deadline; hyperbolic discounters, on the other hand, could conceivably require less, if the earlier deadlines help them cope with their self-control problem.

An additional problem occurs if an employer faces a distribution of workers with different tastes: selection. What type of workers will he attract with a given compensation package?

## 6.5  Experimental and Empirical Extensions

While the existence of changes in temporary preferences has been substantiated in several cases, their influence on more general economic behavior still has not been well documented, theoretically or empirically. Much of the evidence for temporary preference theory revolves around experiments involving one-time rewards or punishments, rather than repeated choices which impact future choice sets and reward flows.[32]

Laibson shows that over the long horizon, hyperbolic discounting mimics a constant discount rate, higher than the long-run rate (the same logic should hold for differential discounting). Since the rate is effectively constant, distinguishing between the exponen-

[32]See Loewenstein and Prelec or Ainslie for good overviews.

tial and time-inconsistent regimes empirically is difficult. In the finite horizon, however, the effective discount rate changes over time. For example, in the log utilities case, the effective discount rate rises over time with hyperbolic discounting; with a cumulative extraction constraint (as in the fixed-reward case), it rises quite sharply as the resource nears exhaustion. The effective discount rate of the differential discounter, on the other hand, declines as the deadline approaches (and is constant for a fixed reward).[33] The model of procrastination as a finite-horizon resource extraction problem may provide an opportunity to test explicitly for time-inconsistent discounting. Varying reward schemes could also help distinguish between hyperbolic and differential discounting.

# 7    Conclusion

To date, much of the procrastination literature has approached it as being generated by dynamically inconsistent, and thereby irrational, choices. Yet psychological studies reveal an awareness of a self-control problem, suggesting that people are rational about their future behavior but have difficulty influencing it to conform to current preferences. For a divisible task, imperfect foresight is not necessary to generate procrastination; in fact, mere impatience can explain many aspects of the behavior. However, the time-consistent model cannot account for the inherent self-control problem. Both types of time-inconsistent preferences motivated by salience stories can explain these puzzles, as well as the seemingly high rates of time preference implicit in procrastinating behavior. When compared to exponential discounting with a low, long-run rate, discount factors motivated by salience costs produce more procrastination, whether it is today that is more salient than tomorrow or the disutility of work that is more salient than the reward. Furthermore, the time path of work, though "rational," is not welfare maximizing in either case.

Both hyperbolic and differential discounters wish they would do more work in the fu-

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[33]See Footnote 23.

ture, but the actual self-control problems are quite different. While the differential discounter wishes she could get herself to do lot more work near the deadline, the hyperbolic discounter wishes that, after today, he would work at a fairly steady pace. Thus, in the actual equilibrium, the hyperbolic discounter procrastinates "too much" while the differential discounter does not procrastinate "enough" (build up work fast enough).

The different reactions to different reward types, as well as the different time paths of effective discount rates, may allow an observer to distinguish which type of salience induces common forms of procrastination. Making such a distinction is important, since the prescription for influencing behavior depends on what aspect is salient. Hyperbolic discounters can be made better off if compelled to reduce procrastination; differential discounters, on the other hand, would have to be compensated to perform more work earlier rather than later.

The results here are important for models of short-term labor-supply decisions when compensation packages include delayed rewards, (such as bonuses at the completion of a project) or penalties (such as being fired if the deadline is not met). Although designed for procrastination, the model can also represent the consumption of an exhaustible good over a fixed period.A natural application would involve the path of lifetime savings behavior in the presence of a bequest motive (or charitable contributions and the Great Reward), when the evaluation of the utility of the bequest changes over time.[34] In conclusion, if salience matters, making preferences dynamically inconsistent, the implications for many economic models are profound.

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[34]With a positive interest rate for savings, this would in a sense be a renewable resource problem.

# Appendix

**Proposition 1** *If self $t$ is capacity constrained, so is self $t - 1$.*

This result holds for any concave utility and reward functions. Start by considering the last two selves. A capacity constraint binding on self $T - 1$ implies

$$\beta\delta F'(S_{T-1} - 24) < u'(24). \tag{26}$$

The problem of self $T - 2$ is then

$$U_H(S_{T-2}, 2) = \max_{l_{T-2}\in[0,24]} \left\{u(l_{T-2}) + \beta\delta u(24) + \beta\delta^2 F(S_{T-1} - 24)\right\}. \tag{27}$$

where $S_{T-1} = S_{T-2} - l_{T-2}$.

For an interior solution to hold, then

$$u'(l_{T-2}) = \beta\delta^2 F'(S_{T-1} - 24). \tag{28}$$

But from (26) we know that the right-hand side is less than $\delta u'(24)$. Thus, (5) cannot hold with equality since $u'(l_{T-2}) \geq u'(24)$ by definition of the capacity constraint. Therefore, if self $T - 1$ *is* capacity constrained, then $T - 2$ must be as well.

This result holds for any capacity-constrained self and his predecessors. Suppose self $t$ finds that, given the impact he has on subsequent utilities, he would prefer to take more than 24 hours of leisure but cannot. His predecessor has no impact on the behavior of $t$ and the same impact on his successors, but discounted more heavily, implying he would prefer to take even more leisure than $t$ would. Therefore, self $t - 1$ (and by the same logic, all preceding selves) must be capacity constrained at 24 hours of leisure as well. The same logic holds for the differential discounter (as long as $\gamma \leq 1$).

Furthermore, the interior solution rule for leisure consumption will always exceed the

capacity constraint when the successor is capacity constrained. If the next self is capacity constrained, the marginal impact of the current self passing on more stock is greater, since the next self cannot raise his consumption. If the capacity constraint were lifted for subsequent selves (the interior solution rule held), the marginal impact of leaving more stock would be even less, implying the current self would want to consume even more leisure now if he could.

**Proposition 2** *The hyperbolic discounter's marginal propensity to consume leisure at* t *is given by* Equation (6)*:*

$$\mu_{T-i}^{H} = \frac{\mu_{T-i+1}^{H}}{\mu_{T-i+1}^{H} + \delta - (1 - \beta)\delta\mu_{T-i+1}^{H}},$$

*for* $i = 1, ..., T$.

Using the log utility example, we can rewrite Equation (12), the equilibrium relationship between marginal utilities of selves $t$ and $t + 1$ when the capacity constraint is not binding:

$$\frac{1}{l_t} = \frac{\delta(1 - (1 - \beta)\partial l_{t+1}/\partial S_{t+1})}{l_{t+1}}. \tag{29}$$

As seen earlier, with log (or other CRRA) utility functions, any self's leisure consumption is a linear function of the inherited stock. Thus, in (29) we can substitute $\mu_t S_t$ for $l_t$ and $\mu_{t+1} S_{t+1}$ (or $\mu_{t+1}(1 - \mu_t)S_t$) for $l_{t+1}$, and $\mu_{t+1}$ for $\partial l_{t+1}/\partial S_{t+1}$:

$$\frac{1}{\mu_t^H S_t} = \frac{\delta(1 - (1 - \beta)\mu_{t+1}^H)}{\mu_{t+1}^H(1 - \mu_t^H)S_t}. \tag{30}$$

Note that $S_t$ cancels out; the marginal propensity to consume leisure is independent of the inherited stock.

Solving for $\mu_t$, we get Equation (6), the backwards recursion giving the change in leisure consumption due to the change in inherited stock on the equilibrium path.

**Proposition 3** *The rational-expectations equilibrium of a hyperbolic discounter can be pareto-dominated for all selves.*

Consider what would happen to the welfare of any self $t$ if he and all his successors reduced their number of leisure hours along the equilibrium path by $\epsilon$:

$$U_H(S_t, T - t) = u(l_t - \epsilon) + \sum_{i=1}^{T-t-1} \beta\delta^i u(l_{t+i} - \epsilon) + \beta\delta^{T-t} F(S_t - \sum_{i=0}^{T-t-1} (l_{t+i} - \epsilon)). \quad (31)$$

Recall that along the equilibrium path, the effective discount factors are less than $\delta$, which implies that $u'(l_{t+i})\delta^i < u'(l_{T-1})\delta^{T-1-t} = \beta\delta^{T-t}F'(\cdot)$, for $0 \leq i < T - t - 1$. Using this fact, differentiating (31) with respect to $\epsilon$ and evaluating at $\epsilon = 0$, we see that welfare would rise for each self if all selves worked a little bit more:

$$\begin{aligned}
\frac{\partial U_H(S_t, T - t)}{\partial \epsilon}\Big|_{\epsilon=0} &= -u'(l_t) - \sum_{i=1}^{T-t-1} \beta\delta^i u'(l_{t+i}) + (T - t)\beta\delta^{T-t}F'(S_T) \\
&\geq -\beta\delta^{T-t}F'(S_T) - (T - t - 1)\beta^2\delta^{T-t}F'(S_T) \\
&\quad + (T - t)\beta\delta^{T-t}F'(S_T) \\
&= (T - t - 1)(1 - \beta)\beta\delta^{T-t}F'(S_T) \geq 0. \quad (32)
\end{aligned}$$

Self $T - 1$ is just indifferent to working a bit more, since that is his first-order condition; all preceding selves would be strictly better off. Intuitively, since each self's successors don't behave "properly," the gains from the bigger reward at the deadline due to the cumulative additional work effort outweigh the discounted stream of lost utility from leisure. If *all* selves worked a bit more (including earlier ones), even $T - 1$ would be strictly better off since his reward would be much larger.

The same exercise can be performed for the differential discounter.

**Proposition 4** *The differential discounter's marginal propensity to consume leisure at* t *is*

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*given by* Equation (19)*:*

$$\mu_{T-i}^D = \frac{\mu_{T-i+1}^D}{\mu_{T-i+1}^D + \gamma\delta + (1-\gamma)\delta^i \alpha \mu_{T-i+1}^D},$$

*for* $i = 1, ..., T$.

Using the log utility example, we can rewrite Equation (25), the equilibrium relationship between marginal utilities of selves $t$ and $t+1$ when the capacity constraint is not binding:

$$\frac{1}{l_t} = \frac{\gamma\delta}{l_{t+1}} + (1-\gamma)\delta^{T-t}\frac{\alpha}{S_T}\frac{\partial S_T}{\partial S_{t+1}}. \tag{33}$$

As in the hyperbolic case, we can substitute $\mu_t S_t$ for $l_t$ and $\mu_{t+1} S_{t+1}$ (or $\mu_{t+1}(1-\mu_t)S_t$) for $l_{t+1}$. The following identities also hold:

$$S_{t+1} = S_t(1 - \mu_t^D), \tag{34}$$

$$S_T = S_t(1 - \mu_t^D) \prod_{i=t+1}^{T-1} (1 - \mu_i^D), \tag{35}$$

$$\frac{\partial S_T}{\partial S_{t+1}} = \prod_{i=t+1}^{T-1} (1 - \mu_i^D). \tag{36}$$

The marginal impact on the reward can then be reexpressed,

$$F'(S_T)\frac{\partial S_T}{\partial S_{t+1}} = \frac{\alpha}{S_{t+1}}. \tag{37}$$

Thus, (33) simplifies to

$$\frac{1}{\mu_t^D S_t} = \frac{\gamma\delta}{\mu_{t+1}^D(1 - \mu_t^D)S_t} + (1-\gamma)\delta^{T-t}\frac{\alpha}{(1 - \mu_t^D)S_t}. \tag{38}$$

Again, $S_t$ cancels out; the marginal propensity to consume leisure is independent of the inherited stock.

Solving for $\mu_t^D$, we get Equation (19), the backwards recursion giving the change in leisure consumption due to the change in inherited stock on the equilibrium path.

**Proposition 5** *The differential discounter requires less of a fixed reward than an exponential discounter.*

Given any work requirement and corresponding path of leisure, to induce work, the discounted value of the reward must be greater than or equal to the difference between discounted utility of the no-leisure path and that of the path with work for every self. The reward will equal the greatest difference:

$$F = \max_{t \in [0, T-1]} \left\{ \delta^{t-T} \sum_{i=t}^{T-1} (\gamma \delta)^{i-t} (u(24) - u(l_i)) \right\}. \tag{39}$$

Differentiating the contents of the brackets with respect to $\gamma$, and recognizing that discounted marginal utility remains constant and that the total amount of leisure cannot change, one can see the remaining discounted difference between utility is increasing with $\gamma$ for all $t$; thus, $dF/d\gamma > 0$. In other words, the more heavily leisure is discounted, the less high the reward must be to induce work. The same method can be used to see that $dF/d\delta < 0$. Thus, the differential discounter requires less of an inducement than either the low- or high-rate exponential discounter.

**Proposition 6** *For log utilities, the hyperbolic discounter's marginal propensity to consume leisure (MPC) — and thereby the effective discount rate — increases as the deadline approaches for any $\alpha < 1/(1 - \delta)$.*

Equation (7) presents $\mu_t^H$. The MPC rises over time if $\mu_t^H > \mu_{t-1}^H$, or if

$$1 - \beta \sum_{i=0}^{T-t-1} \beta \delta^i + (\beta \delta)^{T-t} \alpha > 1 - \beta \sum_{i=0}^{T-t} \beta \delta^i + (\beta \delta)^{T-t+1} \alpha. \tag{40}$$

This expression can be reduced to

$$\beta \delta^{T-t} (1 - (1 - \delta)\alpha) > 0, \tag{41}$$

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which holds for any $\alpha < 1/(1-\delta)$.

**Proposition 7** *For log utilities, the differential discounter's effective discount factor increases as the deadline approaches.*

Let $R_{t+1}^D$ represent the effective discount factor between $t$ and $t+1$ (the ratio of the marginal utilities):

$$R_{t+1}^D = \gamma\delta + (1-\gamma)\delta^{T-t}\alpha\mu_{t+1}^D. \tag{42}$$

$R_{t+1}^D > R_t^D$ if $\mu_{t+1}^D > \delta\mu_t^D$. Using the expression for $\mu^D$ from Equation (20), we can rewrite this inequality as

$$\sum_{i=0}^{T-t-1}(\gamma\delta)^i + \delta^{T-t}\alpha > \delta\left(\sum_{i=0}^{T-t-2}(\gamma\delta)^i + \delta^{T-t-1}\alpha\right), \tag{43}$$

which reduces to

$$(1-\delta)\sum_{i=0}^{T-t-2}(\gamma\delta)^i + (\gamma\delta)^{T-t-1} > 0, \tag{44}$$

which always holds.

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