DISCUSSION PAPER

APRIL 2006 ■ RFF DP 06-25

India's Firewood Crisis Re-examined

Klaas Van 't Veld, Urvashi Narain, Shreekant Gupta, Neetu Chopra, and Supriya Singh

1616 P St. NW
Washington, DC 20036
202-328-5000 www.rff.org

RESOURCES FOR THE FUTURE

India's Firewood Crisis Re-examined*

Klaas van 't Veld

Department of Economics & Finance, University of Wyoming

Urvashi Narain

Resources for the Future, Washington DC

Shreekant Gupta, Neetu Chopra, and Supriya Singh

Delhi University, Delhi

Abstract

Households in rural India are highly dependent on firewood as their main source of energy, partly because non-biofuels tend to be expensive. The prevailing view is therefore that, when faced with shortages of firewood in the village commons, such households, and especially the women in them, have to spend more and more time searching for firewood and eventually settle for poorer-quality biomass such as twigs, branches and dry leaves.

Using data from a random sample of rural households in the Indian state of Madhya Pradesh, we come to very different conclusions, however. We find that households in villages with degraded forests do not spend longer hours searching for firewood, but instead switch to either using firewood from private trees or to using agricultural waste for fuel. In the long run, moreover, households respond to the firewood shortage by altering the mix of private trees on their land in favor of firewood, as opposed to fruit, trees. We find also that, Joint Forest Management, a government program initiated in the 1990s, is having a positive impact on the firewood economy.

JEL: O13, O18, Q23, Q42

Keywords: Firewood crisis, time allocation, fuel switching, JFM, India

*We are very grateful to our eight post-graduate students, Sambit Mishra, Indrani Sen, Sushil Adhikari, Manojit Saha, Hassan Imam, Supriya Singh, Piyali Mitra, and Shilpa Jain, and our five local enumerators, Karishma Gupta, Archana Kamlawat, Manish Maheswari, Om Prakash Rai, and Mukesh Upadhyay for helping us collect the data analyzed in this paper. We also gratefully acknowledge financial support from Resources for the Future, the South Asia Region Environment Sector of the World Bank, and the South Asia Network of Development and Environmental Economists. Corresponding author: Urvashi Narain, Resources for the Future, 1616 P Street NW, Washington, DC 20036. Tel: (202) 328-5098, Fax: (202) 939-3460, E-mail: narain@rff.org.

1. Introduction

Starting in the early 1970s, it was widely held that India would soon face a severe firewood shortage. Demand for firewood, which together with animal dung and agricultural residue is one of the main cooking fuels used by rural households, was thought to be leading to widespread forest degradation. The degradation, it was believed, would soon be so severe that households would face a firewood "famine" (Agarwal 1998). Natarajan (1990) reinforced this sense of an impending firewood crisis by arguing that compared to relatively richer urban households, it was harder for poor rural households to switch to costly non-biofuels such as kerosene and liquefied petroleum gas. Meanwhile, the rural livestock population remained stagnant, thereby offering little prospect of dung for fuel providing a cheap alternative, and a largely ineffective government program to introduce more energy-efficient cooking stoves was doing little to reduce the demand for firewood (Natarajan 1990). These dire predictions conjured up images of households—and in particular women—having to spend endless hours searching for firewood, and eventually settling for poorer-quality biomass such as twigs, branches, and dry leaves.

Drawing on data collected in the early 1990s by the National Council for Applied Economic Research in New Delhi (for more details see (Natarajan 1995)), Agarwal (1998) questioned the validity of some of these previous predictions. Twenty years after the first warnings of a crisis, rural households were still found to use firewood as their main cooking fuel. Moreover, households had *not* been forced to switch to poorer-quality biomass. In fact, the percentage of rural households using logs, a superior type of biomass, was found to have increased over time. Also, fewer households were found to be collecting firewood from the village commons, and more were collecting firewood from trees grown on their own lands. In short, the firewood crisis had not come to be by the early 1990s.

Two possible reasons were put forward by Agarwal to explain the apparent increase in the supply of logs. One was the invasion of exotic woody plant specie, such as *Prosopis juliflora*, which are non-browsable and can be heavily lopped for firewood. Agarwal noted, however, that this invasion had largely been on public lands and could therefore not explain the observed increase in the supply of logs from private lands. The second possible reason was that farmers had reacted to the firewood crisis by starting to grow more trees on their own lands, a practice promoted by a state-sponsored farm forestry project in the 1980s (Saxena and Ballabh 1995). Agarwal noted, however, that no study to date had investigated this response, and that the farm forestry project had declined in importance in the 1990s.

What, then, has since happened to the firewood crisis, especially given the demise of the farm forestry project? If firewood shortages have developed, how are households—and especially women—coping? Are they spending more time collecting firewood from the commons in areas where forest lands are degraded? Are they substituting towards firewood collected from their own land, or towards alternative biofuels such as dung and crop residues? What, if any, has been the role of Joint Forest Management (JFM), a government program initiated in the mid-1990s under which the state agrees to share forest produce—firewood, timber, as well as fodder—from state-owned forest lands with villagers in return for their participation in the management of these lands (Khare, Sarin, Saxena, Palit, Bathla, Vania and Satyanarayana 2000)?

In this paper we examine these questions using data collected from 539 households in 60 villages in the Jhabua district of the state of Madhya Pradesh, covering the period from June 2000 to May 2001. The data include gender-specific information on time spent collecting firewood from the commons, the quantity of firewood collected, the quantity of firewood produced from own land, as well as the quantities of animal dung and agricultural residue used for fuel. Moreover, 21 of the villages have JFM projects.

As we discuss in the next section, the more recent literature—which is very limited in scope, and based largely on anecdotal evidence and case studies—does in fact point to a developing firewood crisis in India. Women are reported to be spending more time collecting firewood in villages with degraded forests, households are found to be unable to cope with the crisis by switching to alternative fuels, and JFM is said to be restricting women's access to forests and thereby aggravating the firewood crisis. Our data, however, paints a very diffrent picture. We find that fewer women choose to collect firewood from the village commons in villages with degraded forests. These households are able to cope with the shortage of firewood by using crop residues as cooking fuel and by planting more firewood trees, as opposed to fruit trees, on their lands and thereby producing more firewood privately. Finally, JFM does appear to be easing the firewood crisis in Jhabua. In villages with JFM projects, women are more likely to collect firewood from the village commons, and spend less time collecting essentially the same quantity of firewood.

The rest of the paper is organized as follows. Section 2 reviews the more recent, post-1990 literature on the firewood crisis in India. Section 3 describes our study site and provides some background information about Joint Forest Management projects. Section 4 outlines a theoretical model that forms the basis of our empirical model. Section 5 outlines the empirical model, the estimation techniques, and then describes the data. Section 6 discusses the results and section 7 concludes.

2. Literature Review

Studies of household fuel use, based on data from the 1990s, have continued to find that firewood and animal dung are the dominant fuels used by households in rural India (ESMAP 2003), and that there is little evidence of an energy transition occurring. According to a 1993-1994 survey by the National Sample Survey Organization (NSSO), an entity of the central Indian government, over 80% of rural households use firewood as their primary energy source for cooking and lighting, while another 10% use dung as such. A 1999-2000 NSO survey found that the first percentage had fallen to 75%, and that the difference was made up by an uptake in liquid petroleum gas. The share of dung, meanwhile, had remained stable at 10%.[1]

The only other study of fuel switching in rural India that we are aware of is that by Heltberg, Arndt and Sekhar (2000), which considers the extent to which households in rural Rajasthan cope with the firewood crisis by switching from firewood collected from common forests to private fuels. The latter are defined as a combination of firewood from private lands, agricultural residue, and animal dung. The authors find that households do not increase their use of private fuels in areas with degraded forests. Households with larger landholdings and a larger number of trees on their lands do collect less firewood from common forest lands and use more private fuels, but this is found to be true irrespective of the state of the local forests. Studies using data from Nepal do find that households in some areas are able to cope with firewood scarcity in common forest lands by switching to agricultural residue (Amacher, Hyde and Joshee 1993) or to firewood from private trees (Cooke 2000).

As for the relationship between time spent in firewood collection and firewood availability, evidence on this relationship in India is largely anecdotal. A number of studies point to the fact that women devote a significant amount of time per day to the collection of firewood (Laxmi, Parikh, Karmakar and Dabrase 2003), but fail to link time spent to the state of the local forests. An exception is the study by Heltberg et al. (2000), using data collected in the Indian state of Rajasthan

[1]The NSSO survey did not ask about the amount of agricultural residue used as fuel.

in the late 1990s, which finds that households as a whole spend more time in collection in villages with more degraded forests.

Additional evidence comes from three studies in neighboring Nepal. Using data from the Nepal Energy and Nutrition Survey conducted in the early 1980s, Kumar and Hotchkiss (1988) and Cooke (1998) both find that women devote more time to the collection of forest resources—firewood and fodder—in villages with degraded forests. Using data from the late 1980s, Amacher, Hyde and Kanel (1996) find, however, that time spent in collection increases in the marginal productivity of such time, where the latter serves as a proxy for firewood availability in the commons. Contrary to Kumar and Hotchkiss (1988) and Cooke (1998), therefore, households in the Amacher et al. (1996) study spend *less* time collecting resources in villages where forests are degraded.

A shortcoming of all four studies discussed above is that in the regressions used to estimate the relationship between time spent collecting and forest degradation, the measures of firewood availability are indirect and likely endogenous. Kumar and Hotchkiss (1988) and Heltberg et al. (2000) use the time needed to collect a fixed quantity (20 kilograms and 1 kilogram, respectively) of firewood as their measure of timber scarcity, while Cooke (1998) uses the same measure multiplied by the imputed female wage rate. Since unobservable, household-level characteristics that affect the total time spent in collection are likely to affect the time spent collecting a fixed quantity of firewood in the same direction, these measures of scarcity are likely endogenous, and possibly biased towards finding the result that these studies report (i.e., that time spent collecting increases with scarcity).[2] Similarly, unobservable, household-level characteristics that affect the total time spent in collection are also likely to affect the marginal productivity of such time, and again in the same direction. The measure employed by Amacher et al. (1993) is therefore also likely endogenous, and possibly biased towards finding the result that their study reports (i.e., that time spent collecting *decreases* with scarcity). In contrast to these studies, our paper uses a direct physical measure of firewood availability, namely, the volume of biomass available per household in the village.

As for JFM, very few studies have documented the impact of this government program on either the firewood crisis or household incomes. This is surprising given how widespread the program has become: according to Sarin, Singh, Sundar and Bhogal (2003), as of October 2001, 27 out of

[2]Brouwer, Hoorweg and van Liere (1997), in studying the responses of households in rural Malawi to firewood shortages, note that "...distance to collection place and collection time are not reliable indicators of firewood shortages as so often postulated in the literature." In their study, households with relatively more adult females are found to travel further and more frequently than households in the same village with fewer adult females. Collection time for these households is therefore not correlated with physical scarcity of the resource.

28 Indian states had issued JFM orders, and 18% of India's total forest area had been brought under the purview of JFM. The studies that do exist again rely mostly on anecdotal evidence, and offer conflicting views. Some contend that JFM has lead to an increase in the amount of firewood collected by the household ((Pathan, Arul and Poffenberger 1900), (Banyopadhyay and Shyamsunder 2006)); others argue that, by placing restrictions on the amount of firewood that can be harvested and by emphasizing timber benefits over firewood benefits, JFM has placed additional hardships on women because they have to walk further to collect firewood (Khare et al. 2000).[3]

3. Study Site

Household- and village-level data analyzed in this paper was collected from a random sample of 539 households living in 60 different villages in the district of Jhabua in the Indian state of Madhya Pradesh (for more information on the data collection process, see Narain, Gupta and van 't Veld (2005)).

Jhabua is a semi-arid hilly region lying in the southwest corner of the state of Madhya Pradesh. Compared to other Indian districts, Jhabua is sparsely populated, with 1.3 million people living on $6,793 km^2$. The district's population is largely rural, with only 9% living in urban areas and the rest in 1,313 villages, and largely tribal or indigenous, with only 14% of the population being classified as non-tribal. Jhabua is also a poor district, with 47% of the population living below the poverty line, and with low rates of literacy (information from the Madhya Pradesh government's website on Jhabua—www.jhabua.nic.in/default.htm).

Agriculture is the main occupation of households in Jhabua, with over 90% of the workforce employed in this sector (HDR 1998). The *kharif*, or main agricultural season, runs from June through October and thereby spans the monsoon months July and August. The *rabi*, or winter agricultural season, runs from November through March. Whether or not households in fact engage in any agriculture during the rabi season depends in large part on the strength of the monsoons, because agriculture in Jhabua is mainly rain-fed. Finally, the summer season is made up of the months of April and May, two months where there is little economic activity in the village. The household- and village-level data used in this paper was collected for the period June 2000 to May 2001 and therefore spanned all three seasons—kharif, rabi, and summer.

[3]Kohlin and Parks (2001) consider the impact of a different state-initiated program, which promotes the establishment of plantations on village common lands (so called village woodlots). The authors find that these woodlots have helped increase the supply of firewood. This program, however, is not as prevalent as JFM.

As for land use, 54% of the total land area in Jhabua is classified as agricultural land, 19% as forest land, and the rest as "degraded" land (HDR 1998). These forest lands are considered to be state property and have traditionally been managed by the state forest department to maximize timber revenue. Only about 50% of the total forest area currently has any vegetative cover, and the majority of that is low-density forests (for the history of forest degradation in Jhabua, see TERI (2000)). Nevertheless, households all over Jhabua depend on these forests as well as on other village common lands for firewood, construction wood, fodder, and other minor forest products. In fact, villagers have been given rights, called *nistar*, that permit households to collect forest products from state forest lands for non-commercial household use (PRNRM 2002a).

3.1. **Joint Forest Management.** Forest degradation is and has been a problem in large parts of India, not just in Jhabua. The 1988 Forest Policy, laid down by the Government of India, recognized this problem and called for the development of a new forest management system that would involve local people and would also be geared towards meeting their needs. These changes, it was hoped, would reverse years of alienation of local villagers from the management of forest lands, one of the factors said to be responsible for years of degradation (Sarin et al. 2003). The new policy was implemented through a circular issued by the Ministry of Environment and Forests on June 1, 1990 (GOI 1990), and came to be known as Joint Forest Management (JFM). The Madhya Pradesh government passed its first JFM order in 1991 and the program was formally initiated in Jhabua the following year.

Though some details vary from state to state, under JFM the state forest department agrees to share forest produce from state-owned forest lands with local villagers in return for their participation in the management of these forest lands. Villagers are allowed to collect dry and fallen branches for firewood, are given access to wood removed during thinning operations, are permitted to gather fodder and minor forest products, and are given a share of the final timber harvest. Collection is, however, meant for domestic needs and not for commercial sale. In return, villagers participate in the development of forest working plans, agree to protect the forests against encroachment and timber smuggling, and agree to restrict their use of certain forest products (Khare et al. 2000).

In Madhya Pradesh, as in most states, JFM was initiated on forest lands that were degraded. Unlike most other states, however, Madhya Pradesh amended its policy in 1995 to allow well-stocked forest areas to be eligible for JFM.[4] Also in 1995, the World Bank initiated a large forestry project in Madhya Pradesh that gave a considerable boost to the state's JFM program. The project channeled large sums of money to JFM committees for so-called "village development activities," meaning activities that aimed at reducing the dependence of households on local forests. The project's first phase ended in 1999, and by mid-2000 about 38% of the state's total forest area was being managed under JFM ((MPFD Undated) cited by (Sarin et al. 2003)).

3.2. **Evaluation of JFM.** A number of studies have evaluated the strengths and weaknesses of the institutions established under JFM. By and large, these studies argue that people's "participation" under JFM has been limited. State forest departments have made little effort to engage villagers in forest management and have done little to incorporate the priorities of forest-dependent households in forest management goals. These criticisms have also been directed at JFM in Madhya Pradesh. A case study by (Sarin et al. 2003) of 13 villages in the Bastar district and Harda forest divisio of Madhya Pradesh, revealed limited participation by villagers in JFM committees, especially by women and other disempowered groups. Villagers were found to have little knowledge of the decisionmaking process, and therefore little say in the management of the local forests, or for that matter in the management of funds targeted for village development activities. Other studies that have found institutional deficiencies with JFM in Madhya Pradesh include a a two-year study in the district of Dewas, by the University of Edinburgh (PRNRM 2002b), and a study providing a mid-term review of the World Bank's forestry project. The latter review pointed especially to the corruption surrounding the use of village development funds ((PRNRM 2002b) (Sundar, Roger and Thin 2001)).

Aside from these institutional studies, few studies have documented the impact that JFM has had on the firewood crisis, or for that matter on household incomes. Based on their case study, Sarin et al. (2003) argue that restrictions placed under JFM, on the extraction of firewood and animal grazing, especially in the early years of the program, have placed undue burdens on poor households.[5] The authors also claim, on the other hand, that in villages in the Harda forest division,

[4]Village institutions established to manage forest in well-stocked areas are called *van suraksha samities* (forest protection committees) while those in degraded areas are called *gram van samities* (village forest committees) (Sarin et al. 2003).

[5]Similar criticisms have been raised by local tribal organizations ((PRNRM 2002b)).

funds made available for village development, and specifically for irrigation development, did lead to improvements in agricultural productivity. None of these impacts were quantified, however. Similarly, the mid-term review of the World Bank project noted that JFM was leading to forest regeneration but did not document what benefits this regeneration was conferring on villagers (PRNRM 2002b).

4. Theoretical Model

Our empirical analysis of the relationship between time spent in firewood collection and forest degradation is informed by the following theoretical model.

Consider a representative farm household that derives utility from the consumption of a cooked staple and from the consumption of fruit. The cooked staple is produced by combining the raw staple with firewood, and by substituting the production function into the household's utility function, we obtain the derived utility function

$$U = U(c^a, c^f, c^r),$$

where c^a denotes consumption of the raw staple, c^f consumption of firewood, and c^r the consumption of fruit. Note that by not explicitly including the time input required for cooking, we implicitly treat this time input as fixed.

The household allocates some of its time and some of its land to production of the raw staple. The agricultural production function is

$$q^a = q^a(t^a, L^a),$$

where t^a is the time spent on the farm, L^a is the amount of own land used for the production of the staple. The household also allocates some of its time to collecting firewood from the commons. The production function for firewood collection is

$$q_c^f = q_c^f(t^f, F_c^f),$$

where t^f is the time spent collecting, and F_c^f denotes the stock of firewood in the commons. The household may also produce firewood privately, from firewood trees on its own land:

$$q_p^f = q_p^f(F_p^f),$$

where F_p^f is the stock of private firewood trees. Lastly, the household may produce fruit privately as well, from fruit trees on its own land:

$$q_p^r = q_p^r(F_p^r),$$

where F_p^r is the stock of private fruit trees. Given that households are likely to live close to their agricultural fields, we do not explicitly consider the time spent collecting either firewood or fruit from private trees.

The household's budget constraint is

$$q^a + p^f q_c^f + p^f q_p^f + p^r q_p^r + wt^w + v = c^a + p^f c^f + p^r c^r, \tag{1}$$

where p^f is the market price of firewood, p^r the market price of fruit, w the wage rate, t^w the time spent in wage labor, and v the household's exogenous income. Note that we are treating the staple as the numeraire, so $p^a = 1$. Also, we are assuming that markets exist for the staple, firewood, and fruit, and therefore that consumption of these commodities is not constrained to equal their production by the household. In particular, a household can meet its desired consumption of firewood by collecting it from the commons, collecting it from its private trees, or purchasing it on the market.

To simplify the analysis, we treat leisure time as fixed, so that the household's time constraint is

$$T = t^a + t^f + t^w. \tag{2}$$

By expressing the stock of private trees in units of land area taken up by those trees, the household's land constraint becomes

$$L^a + F_p^f + F_p^r = L. \tag{3}$$

Lastly, although households may purchase firewood, we assume that they never sell firewood on the market. That is,

$$c^f \geq q_c^f + q_p^f. \tag{4}$$

This assumed asymmetry is consistent with our data set: out of a total of 539 households in our final sample, over a hundred purchase firewood but only one household—a single male owning very little land—sells any. This may be explained by the existence of large fixed costs associated with selling firewood, promoting specialization.

The household's optimization problem is to choose the utility-maximizing combination of variables $c^a, c^f, c^r, t^a, t^f t^w, F_p^f, F_p^r$, and L^a, subject to constraints (1) through (3), as well as non-negativity constraints on t^f, t^w, F_p^f, and F_p^r. Note that by assuming $t^w \geq 0$, we abstract from the possibility of hiring labor to either work on the land or collect firewood. We also assume that c^a, c^f, c^r, t^a and L^a are always strictly positive.

The Lagrangian associated with this optimization problem is

$$
\begin{aligned}
L = {} & U(c^a, c^f, c^r) \\
& + \lambda^B[q^a(t^a, L^a) + p^f q_c^f(t^f, F_c^f) + p^f q_p^f(F_p^f) + p^r q_p^r(F_p^r) + wt^w + v - c^a - p^f c^f - p^r c^r] \\
& + \lambda^T[T - t^a - t^f - t^w] \\
& + \lambda^F[L - L^a - F_p^f - F_p^r] \\
& + \mu^c[c^f - q_c^f(t^f, F_c^f) - q_p^f(F_p^f)] \\
& + \mu^t t^f + \mu^w t^w + \mu^f F_p^f + \mu^r F_p^r,
\end{aligned}
$$

4.1. **Interpreting the First-Order Conditions.** As we show in appendix A, the first-order conditions associated with the Lagrangian can be pared down to the following six equations:

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f + \mu^c = 0, \tag{5}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{6}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^t = 0, \tag{7}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{8}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^f = 0, \tag{9}$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^r = 0, \tag{10}$$

The first two conditions help determine the household's optimal consumption of the staple, firewood, and fruit. If the household buys any firewood from the market (so $\mu^c = 0$), then equation 5 implies that the household equates the marginal utility per rupee of firewood and staple consumption. If, on the other hand, it does not buy any firewood ($\mu^w > 0$), then at the prevailing prices its marginal utility per rupee of firewood must fall short of that of staple consumption. Similarly, equation 6 implies that the household chooses quantities of the staple and fruit so as to equate the marginal utility per rupee of fruit and staple consumption.

The next two conditions help determine how the household allocates its labor between agricultural production, firewood collection, and wage employment. If the household collects any firewood (so $\mu^t = 0$), then equation 7 implies that it equates the marginal utility per hour of time spent collecting to that of time spent working on the farm. Similarly, equation 8 implies that if the household engages in wage labor (so $\mu^w = 0$), then it equates the marginal utility of time spent working on the farm to that of working for wages.

The final two conditions help determine how the household allocates land to either agriculture or private firewood and fruit trees. Specifically, if the household plants any private firewood trees (so $\mu^f = 0$), it equates the marginal utility of consumption of firewood produced from those trees to that of consumption of the staple produced from its land, and similarly for fruit trees.

4.2. **Model Results.** The question that is the main focus of this paper is how forest degradation, i.e., a reduction in the stock of firewood in the commons, F_c^f, affects the amount of time that the household devotes to firewood collection from the commons, t^f, and the number of firewood trees that it plants on its own land, F_p^f.

The theoretical model allows us to answer this question for a wide range of cases, distinguished by what combination of the 5 multipliers μ^c, μ^t, μ^w, μ^f and μ^r is zero. In appendix B, we restrict attention to the subset of cases where $\mu^c = \mu^t = 0$, i.e., the household purchases at least some firewood from the market, but also collects some firewood from the commons. We show that if in addition we assume that the household engages in wage labor, so that $\mu^w = 0$ (cases I and II in the appendix), then a decrease in firewood biomass in the commons unambiguously induces a *reduction* in the time spent collecting, with an equal increase in the time spent working for wages. The change in biomass does not affect the time spent working on the farm, however, nor the number of either firewood or fruit trees planted privately, nor thereby the amount of land allocated to trees.

If, on the other hand, the household does *not* engage in wage labor, so that $\mu^w > 0$ (cases III through VI), then a decrease in firewood biomass in the commons again unambiguously induces a *reduction* in the time spent collecting, but now induces an equal increase in the time spent working on the farm. Moreover, it also induces an increase in the amount of land allocated to agriculture, with a corresponding reduction in the number of firewood and fruit trees planted privately. Intuitively, the increase in time spent working on the farm in favor of time spent collecting firewood results in an increased productivity of agricultural land relative to that of land planted with private trees. To restore equilibrium, the household reallocates land from trees to agriculture.

While these comparative-statics results do not exhaust all possible theoretical cases, they indicate, at least, that there is no *a priori* reason to expect decreased firewood availability in the commons to necessarily induce an increase in time spent collecting. Under plausible conditions, the opposite may well be true. In the next section, we turn to our data to examine this issue empirically.

5. Empirical Analysis

In order to examine the different responses of households to variations in forest biomass across the villages in our sample, we estimate equations for the amount of time spent by households in firewood collection, for the quantities of firewood, agricultural residue, and dung used by households, and for the number of firewood and fruit trees that households plant on their own lands. The first of these equations, which we are able to estimate separately for men, women, and children, allows us to examine whether, as forests degrade, households—and particularly women within those households—spends more time collecting firewood. The fuel use equations allow us to analyze the extent to which, as forests degrade, households switch either to alternative sources of firewood, such as private supplies or the market, or to alternative fuels, such as dung and agricultural residue. Finally, the firewood- and fruit-tree equations allow us to examine whether, as forests degrade, households grow more firewood trees on their private lands.

5.1. **Empirical Strategy.** The two main data issues that we have to confront in estimating these equations are those of censoring and seasonality.

The dependent variable in almost all the equations we estimate is censored. Not all households choose to collect any firewood from the commons, for example, and not all households have any trees, especially firewood trees, on their land. Three common regression models used to account for such censoring are the Tobit model, the Heckman two-step selection model, and the two-part model. Of these, the Tobit model is the most restrictive, as it imposes the assumption that the participation equation (i.e., the process that determines whether or not the dependent variable is censored) is identical to the outcome equation (the process that determines the value of the dependent variable if it is not censored). The Heckman two-step model relaxes this assumption, but has the well-known drawback of relying on essentially arbitrary functional form assumptions for identification if the same set of regressors is used in both equations. In general, it is difficult to come up with plausible arguments for *not* including the same set of regressors, however, and this is true in the setting of our study as well. Below, we report results for the two-part model alone; our results are generally robust across all three models, however.

A further issue is the existence of important seasonal differences in the choices that the household makes with regards to time allocation and fuel consumption. Fortunately, we have data on several of these choices at the seasonal level, with three observations for each household: one for the *kharif* (main harvest) season, one for the *rabi* (secondary harvest) season, and one for the summer (dry) season. We are therefore able to take seasonal differences into account by including seasonal dummies in panel regressions. This has the additional advantage that we can control, to some extent, for unobserved household-specific effects. Because many of our regressors are *not* available at the seasonal level—including, in particular, our estimate of forest biomass—we adopt a random-effects specification for these regressions, however, which assumes that the unobserved household-specific effects are not correlated with the regressors. Hausman tests comparing fixed and random-effects specifications fail to reject this assumption.

5.2. **Description of Variables.** Household time-allocation decisions, fuel-consumption choices, and decisions to plant private firewood and fruit trees depend on a number of household- and village-specific variables. Table 1 lists the household-specific endogenous variables that we examine below, while Table 2 lists household- and village-specific exogenous variables.

6. Empirical Results

6.1. **Seasonal Differences.** The regression estimates for time spent collecting firewood from the commons (see Table 3) and for fuel use (see Table 6) reveal strong seasonal differences. The probit random-effects regressions reported in Table 3 estimate whether or not the household in fact spends any time collecting firewood. We estimate these equations separately for the household as a whole (P.COL.H) and for the men, women, and children within each household (P.COL.M, P.COL.F, and P.COL.C). The ordinary random-effects regressions reported in the same table estimate the determinants of the log of the time spent collecting (net of time taken to travel to and from the collection site), conditional on this time being positive, again separately for the household as a whole (L.COL.H) and for the men, women, and children within each household (L.COL.M, L.COL.F, and L.COL.C).

Similarly, the probit random-effects regressions reported in Table 7 estimate whether or not the household in fact collects any firewood (P.FWD.COL), purchases any firewood (P.FWD.PUR) or consumes any dung for fuel (P.DFU.CON), while the ordinary random-effects regressions estimate the determinants of the log of the quantities of firewood collected (L.FWD.COL), firewood purchased (L.FWU.PUR), or dung for fuel consumed (L.DFU.CON), conditional on these quantities being positive.[6]

The coefficients on the KHARIF and SUMMER seasonal dummies in the probit equations of Table 3 as well as the first probit equation of Table 6 indicate that households are less likely to collect firewood in the kharif season, which has high rainfall relative to the rabi season (the excluded category), and more likely to collect in the dry summer season. This is also indicated by the coefficient on the actual level of rainfall in each of the seasons, which is negative and significant in the probit equations for the household as a whole (as well as in the probit equation for time spent collecting by women). A likely explanation for these findings is that higher rainfall increases the marginal product of labor in the household's main occupation, namely agriculture, thereby increasing the opportunity cost of time spent collecting firewood. Another possible explanation is that households prefer to collect dry rather than wet firewood.

The positive and significant coefficient on the summer dummy in the probit equation for firewood purchases in Table 6 indicates that households are also more likely to buy firewood in the summer relative to the rabi season.

Interviews from the field reveal that households in fact store firewood in the summer for the succeeding kharif season, which is typically the main agricultural season. Apart from the higher opportunity cost of time in the kharif, a further reason for such storage is that biofuel substitutes are less available in the kharif. Agricultural residue for fuel becomes available only in the rabi season following the main kharif harvest, or in the summer following the secondary rabi harvest. Moreover, any animal dung collected during the kharif season is more likely to be applied to agricultural fields as manure at the beginning of the rabi season, just as dung collected in the summer season is applied to the fields in the following kharif season. It is only in the rabi season, preceding the summer, that there is no demand for manure, so that dung collected during the rabi can be used

[6] The minor discrepancy between the coefficients in the participation equations for time spent collecting and quantity collected arises because two households reported collecting a positive amount of firewood, but requiring essentially no time to do so. Accordingly, the number of observations in the conditional outcome equations differs slightly as well.

as cooking fuel. The negative and significant coefficient on both the kharif and summer dummies in the probit equation for consumption of dung for fuel in Table 6 is consistent with this.

The same trends show up also in simple tabulations by season of the number of households that collect firewood from the commons, purchase firewood, or use dung for fuel (see Table 5), pointing to the importance of analyzing household firewood collection and fuel use at the seasonal level.

6.2. **Time Spent Collecting Firewood from the Commons.** Two general features of the equations reported in Table 3 for time spent collecting firewood is that very few of the regressors enter significantly into the estimated equations for men and children, and that the results for the household as a whole appear to be driven by those for women. Both features are explained by the relatively small numbers of men and children that collect: the job of collecting firewood clearly falls mainly on women. As the coefficients on the proportion of females (PROP.F) and children (PROP.C) in each household indicate, men are more likely to collect in households where there are relatively few women and children, while children are more likely to collect in households in which there are relatively many children.

A key result in these regressions is that the coefficient on the log of per-capita biomass availability (L.BIO) is positive and significant in the participation equations for both the household as a whole and for women in particular, but is not statistically significant at even the 10% level in any of the conditional outcome equations. Higher biomass availability in a village makes households, and especially the women in those households, more likely to collect firewood from the commons, but does not significantly affect the time spent collecting (conditional on collecting at all).

One possible interpretation of this result is that, contrary to some of the claims discussed in the introduction about a looming rural ‘firewood crisis’, households in villages with relatively degraded forests are able to switch to substitutes for firewood from the commons; that is, they are not forced to significantly increase the time spent collecting, and in fact are more likely to simply abandon collecting altogether. Some evidence in support of this interpretation (additional evidence is reported below) comes from the negative and significant coefficient on the log of private firewood trees owned (L.FWD.TR) in the participation equations for households as a whole and for women, and in the conditional outcome equation for women.

Two likely proxies for the opportunity cost of time spent in firewood collection—the average distance to agricultural markets (MKTDIS.AGR) and the level of education of the head of the household (HEAD.EDU)—enter significantly in the participation equations, with the expected sign.

Households in villages that are remote from markets and households with less-educated household heads are likely to have fewer outside labor opportunities, and are therefore more likely to spend time collecting. Conversely, households with more land and more farm capital (as measured by the product of the log of land and the log of the value of capital, L.LAND×L.CAP) are less likely to collect, as the opportunity cost of time spent collecting rather than working the farm is likely higher for such households.

Apart from the average depth of the water table (AV.DEP), the effect of is difficult to interpret, the only other regressors that enter significantly into the conditional outcome equations are the distance to the nearest market for firewood, and an interaction term between biomass per household and the presence in the village of a JFM project. As one would expect, households spend less time collecting in villages where firewood is available from nearby markets. For reasons to be discussed below, households also spend less time collecting, but are more likely to collect, in villages with JFM projects *and* high levels of per-household biomass (L.BIO.H).

6.3. **Fuel Use.** Tables 6 and 7 present the results on household fuel use.

Consider first the effect of biomass availability in the commons (L.BIO) on fuel use. As noted before, the positive and highly significant coefficient on biomass in the first column of Table 6 shows that fewer households collect firewood from the commons as forests degrade. In the last column of the table, the positive and weakly significant coefficient on biomass in the equation for the log of dung consumption for fuel (L.DFU.CON) indicates that as forests degrade, households also use less dung. This then raises the question how do households meet their demand for fuel.

Table 7 provides some suggestion that they do so by collecting firewood from private trees and by using agricultural residues for fuel. The coefficient on biomass is negative and significant in the probit regressions for both private firewood consumption and consumption of agricultural residue for fuel. As is to be expected, both the likelihood that households collect firewood from private trees and the amount of private firewood they collect increase in the (log of the) number of private trees owned by the household (L.FWD.TR and L.FRU.TR).[7] Not surprising either is that both the likelihood that households use agricultural residue for fuel and the amount that they use increase in the amount of land cultivated by the household (as captured by the interaction effect L.LAND×L.CAP), or that, as shown in Table 6, both the likelihood of using dung for fuel and the amount used increase the (log of the) number of animals owned by the household (L.ANIMAL).

[7]Although no fruit is collected from firewood trees, some firewood is commonly collected from fruit trees.

As for purchases of firewood, the third and fourth column of Table 6 indicate that households that own private trees are less likely to buy firewood. Reassuringly, for households that do buy, the quantity bought declines in the price.

Other determinants of household fuel-use choices are the level of education of the household head and the level of water scarcity in the village. As already noted above, households with more educated household heads are less likely to collect firewood from the commons. The probit equation in the fifth column of Table 6 indicates that they are also less likely to consume dung for fuel, but the remaining equations in the fuel-use tables suggest that they are no more likely to collect private firewood, purchase firewood on the market, or use agricultural residue for fuel. One possible explanation for these results is that these households meet more of their fuel demand from non-biofuels such as kerosene or LPG. Unfortunately, our data set does not contain information on non-biofuel use, so we cannot confirm if this is in fact the case.

A final result concerns the effect on fuel use choices of long-run water scarcity in the village (WAT.SCRC), as measured by the drop over time in the village groundwater level. Households in villages where groundwater has on average been falling over the years prior to our study year appear less likely to use agricultural residues for fuel, and appear to use more dung for fuel. A possible explanation is that agriculture is less productive in those villages, making agricultural residue scarcer.

6.4. **Planting of Firewood and Fruit Trees.** Thus far, we have considered household fuel-use choices conditional on the number of private firewood and fruit trees that households have on their land. The resulting estimates therefore capture the short-run response of households to changes in the regressors, over a period of time where the household cannot adjust its private tree holdings. In the long run, however, the household may respond to these changes by planting or uprooting trees on its land, or altering the proportion of these trees that are firewood or fruit trees. The two regressions reported in Table 8 estimate this long-run response.

The first equation estimates the likelihood that a household owns any private trees at all, conditional on having some land to plant them on (PR.PRI.TR).[8] Not surprisingly, this likelihood increases significantly in the amount of land that households have, as well as in the distance to the nearest firewood market. The likelihood does not vary significantly with per-capita biomass availability, however.

[8] 49 of the 539 households in our sample are landless.

The second equation estimates the fraction of private trees that are grown exclusively for firewood (FR.FWD.TR). Here, biomass availability clearly does matter: households in villages with degraded forests are significantly more likely to grow firewood rather than fruit trees on their land. Both in the short run and the long run, then, households appear to respond to firewood scarcity in the commons by increasing their use of privately grown firewood.

As one might expect, households that own fewer animals (L.ANIMAL), and therefore presumably have less access to dung for fuel, also grow relatively more private firewood trees.

6.5. **JFM.** Finally, we consider how household firewood collection and consumption patterns differ between villages with and without JFM projects. The data suggests two possible differences. First, as shown in Table 3, the presence of a JFM project in a village appears to reinforce the effect of higher biomass availability in the commons on the likelihood that households, and in particular women, collect firewood. Furthermore, although biomass availability does not appear to affect the time spent collecting in villages without JFM, it does appear to reduce the time spent collecting in village with JFM.

Since we separately control for biomass availability, these differences cannot be explained by a tendency for villages with JFM projects to have more biomass. It should be noted, however, that we are only controlling for the *volume* and not the type of biomass, and therefore one possible explanation for our finding is that JFM villages have a more favorable mix of trees in their commons, with perhaps a larger fraction of firewood trees. Another possible explanation is that the presence of JFM projects makes collection of firewood from the commons a more "legitimate" activity. Numerous case studies of pre-JFM forest management have documented stories of women being harassed by state forest guards as they attempted to collect firewood from the commons; such collection was viewed suspiciously by forest managers and considered to be responsible for the rampant degradation of state forest lands. Because JFM projects more clearly define villagers' rights to state forest products, women may feel less inhibition about collecting firewood from the commons.

The increase in firewood collection from the commons in JFM villages appears to reduce households' reliance on dung for fuel. As shown in the last two columns of Table 6, households in villages with JFM *and* high biomass availability are less likely to use dung for fuel, and those that do use it, use less.

7. Conclusion

The main objective of this paper is to understand how households in rural India, which are highly dependent on firewood as their main source of energy, respond to shortages of firewood in village commons. As discussed in the introduction, the prevailing view is that when faced with such shortages, because non-biofuels are expensive, such households, and especially the women in those households, have to spend more and more time searching for firewood. Not only does this place an ever increasing burden on women, but because it is likely that the increased time devoted to firewood collection comes at the expense of time that would be otherwise be spent on income-generating activities, shortages of firewood will have additional negative impacts on household welfare.

Using data from a random sample of rural households in the Indian state of Madhya Pradesh, we come to very different conclusions, however. First, we find that faced with firewood shortages in village forests, households do not spend longer hours searching for firewood. Instead, fewer households choose to collect firewood from degraded forests at all. These households instead switch to either using firewood from private trees or to agricultural waste. In the long run, moreover, households respond to the firewood shortage by altering the mix of private trees on their land in favor of firewood, as opposed to fruit, trees. The latter response offers a more robust, long-term solution to forest degradation than the more short-term solutions often provided by government-initiated programs such as the farm forestry program of the 1990s. That said, we nonetheless find that a different government program initiated since the 1990s, namely JFM, is having an impact on the firewood economy. In villages with JFM projects *and* relatively high levels of biomass, women appear to collect more firewood from the commons, and to spend less time doing so. Only time, obviously, will tell whether this impact of JFM will be long-lived.

References

Agarwal, A. (1998). False predictions, *Down to Earth* **7**(1): 29–39.

Amacher, G. S., Hyde, W. F. and Joshee, B. R. (1993). Joint production and consumption in traditional households: Fuelwood and crop residues in two districts in Nepal, *Journal of Development Studies* **30**(1): 206–225.

Amacher, G. S., Hyde, W. F. and Kanel, K. R. (1996). Household fuelwood demand and supply in Nepal's Tarai and mid-hills: Choice between cash outlays and labor opportunity, *World Development* **24**(11): 1725–1736.

Banyopadhyay, S. and Shyamsunder, P. (2006). Fuelwood consumption and participation in community forestry in India, Environment Department, World Bank.

Brouwer, I. D., Hoorweg, J. C. and van Liere, M. J. (1997). When households run out of fuel: Responses of rural households to decreasing fuelwood availability, Ntcheu district, Malawi, *World Development* **25**(2): 255–266.

Cooke, P. A. (1998). Intrahousehold labor allocation responses to environmental good scarcity: A case study from the hills of Nepal, *Economic Development and Cultural Change* **46**(4): 807–830.

Cooke, P. A. (2000). Changes in intrahousehold labor allocation to environmental goods collection: A case study from rural Nepal, 1982 and 1997, Food Consumption and Nutrition Division Discussion Paper 87, International Food Policy Research Institute, Washington DC.

ESMAP (2003). India: Access of the poor to clean household fuels, *Technical Report, Energy Sector Management Assistance Programme 263/03*, World Bank, Washington, D.C.

GOI (1990). Involvement of village communities and voluntary agencies in regeneration of degraded forests, *Circular No.6.21/89-F.P.*, Ministry of Environment and Forests, Government of India‘, New Delhi.

HDR (1998). *The Madhya Pradesh Human Development Report*, Government of Madhya Pradesh.

Heltberg, R., Arndt, T. C. and Sekhar, N. U. (2000). Fuelwood consumption and forest degradation: A household model for domestic energy substitution in rural India, *Land Economics* **76**(2): 213–232.

Jodha, N. S. (1986). Common property resources and rural poor in dry regions of India, *Economic and Political Weekly* **21**(27): 1169–1181.

Khare, A., Sarin, M., Saxena, N., Palit, S., Bathla, S., Vania, F. and Satyanarayana, M. (2000). Joint forest management: Policy, practice, and prospects, *Policy that works for forests and people series 3*, World Wide Fund for Nature-India, New Delhi and International Institute for Environment and Development, London.

Kohlin, G. and Parks, P. J. (2001). Spatial variability and disincentives to harvest: Deforestation and fuelwood collection in south Asia, *Land Economics* **77**(2): 206–218.

Kumar, S. K. and Hotchkiss, D. (1988). Consequences of deforestation for women's time allocation, agricultural production, and nutrition in hill areas of Nepal, Research Report 69, International Food Policy Research Institute, Washington DC.

Laxmi, V., Parikh, J., Karmakar, S. and Dabrase, P. (2003). Household energy, women's hardship and health impacts in rural Rajsthan, India: need for sustainable energy solutions, *Energy for Sustainable Development* **7**(1): 50–68.

MPFD (Undated). Status of jfm in madhya pradesh, *Technical report*, Madhya Pradesh Forest Department, Bhopal.

Narain, U., Gupta, S. and van 't Veld, K. T. (2005). Poverty and the environment: Exploring the relationship between household incomes, private assets and natural assets, Discussion Paper No. 05 , Resources for the Future, Washington DC.

Natarajan, I. (1990). Firewood balance in the nineties, *Margin* **22**(4): 38–49.

Natarajan, I. (1995). Trends in firewood consumption in rural India, *Margin* **28**(1): 41–47.

Pathan, R., Arul, N. and Poffenberger, M. (1900). Forest protection committees in Gujarat: Joint management initiative, *Working Paper 7*, Ford Foundation, New Delhi.

PRNRM (2002a). Panchayati raj and natural resource management: How to decentralize management of natural resources, Madhya Pradesh: State Report.

PRNRM (2002b). Panchayati raj and natural resource management: How to decentralize management of natural resources, Madhya Pradesh: Situation Analysis and Literature Review.

Sarin, M., Singh, N. M., Sundar, N. and Bhogal, R. K. (2003). Devolution as a threat to democratic decision-making in forestry? Findings from three states in India, Working Paper 197, Overseas Development Institute, London UK.

Saxena, N. and Ballabh, V. (eds) (1995). *Farm Forestry in South Asia*, Sage Publications, Delhi.

Sundar, N., Roger, J. and Thin, N. (2001). *Branching Out: Joint Forest Management in India*, Oxford University Press, New Delhi.

TERI (2000). *Green Beginnings: Joint Forest Management in Jhabua*, Tata Energy Research Institute, New Delhi.

	Variable Definitions	Mean	Standard Deviation
Time Spent by Household	Minutes spent per day in area by household members collecting firewood	7.2	1.4
Time Spent by Women	Minutes spent per day in area by women members collecting firewood divided by the number of working-age women	11.1	1.8
Firewood from Commons	Kilograms of firewood collected per day from local forests and village common lands	58.7	14.7
Firewood Bought	Kilograms of firewood bought per day from local markets	7.1	1.3
Firewood from Own Trees	Kilograms of firewood collected from private trees per day	0.1	0
Dung as fuel	Kilograms of animal manure from own and common lands used per day as cooking fuel	0.4	0
Ag. Residue as fuel	Kilograms of agricultural residue from various crops used per day as cooking fuel	0.1	0
Fuel Trees	Number of babul and low-quality-timber trees owned in June 2000	1.2	0.3
Fruit Trees	Number of trees, other than fuel trees, owned in June 2000	5.9	0.2

Table 1. Definitions, means and standard deviations of endogenous variables

	Variable Definitions	Mean	Standard Deviation
Age of Head	Age of the head of the household	43.9	0.6
Education of Head	Highest class completed by the head of the household	2.9	0.3
Proportion of Women	Number of working-age women divided by the total number of working-age members	0.37	0.01
Proportion of Children	Number of working-age children divided by the total number of working-age members	0.28	0.01
Land and Farm Capital	Number of hectares of land cultivated times the value of farm capital owned in June 2000 per working-age member	533832	303798
Animal Holdings	Number of bullocks, cows, buffaloes, goats, sheep and donkeys owned in June 2000 per working-age member. As per Jodha (1986), goats and sheep are given a weight of 0.1 while other animals a weight of 1	0.5	0.02
Biomass Availability	Thousands of tonnes of timber and fodder biomass available within a 5km radius from the center of the village divided by the number of households living in the village	3.2	0.49
JFM	Presence of Joint Forest Management project in village		
Average Depth	Average depth in meters of village well monitored by Madhya Pradesh Groundwater Department between 1973 and 2000	5.6	0.3
Long-run Water Scarcity	Difference in meters between depth of the monitored village well in 1999 and 1973	-0.9	0.4
Rainfall	Millimeters of rainfall measured at the Block within which the village lies	374	18
Distance to Markets	Average distance in kilometers between the village and markets for agricultural inputs and outputs	10	1
Distance to Firewood Markets	Distance in kilometers between the village and the closest firewood market	1.9	0.6
Firewood Price	In-village rupee price of firewood per kilogram	1.4	0.1

Table 2. Definitions, means and standard deviations of exogenous variables

Estimation method	PROBITRE	RE	PROBITRE	RE	PROBITRE	RE	PROBITRE	RE
Dependent variable	P.COL.H	LT.COL.H	P.COL.M	LT.COL.M	P.COL.F	LT.COL.F	P.COL.C	LT.COL.C
HD.AGE	–0.009	–0.009	–0.005	–0.010	–0.010	–0.007	0.010	–0.037***
HD.EDU	–0.079***	0.013	–0.047	0.041	–0.081***	0.009	–0.126**	0.035
PROP.F	0.777	–0.488	–2.056*	–0.707	2.342***	–0.991	0.969	–2.659
PROP.C	1.068**	–0.930**	–1.481*	–0.211	1.581***	–0.107	5.618***	–3.958***
L.LAND×L.CAP	–0.024***	0.011	–0.022	0.017	–0.013	0.009	–0.013	–0.023
L.ANIMAL	–0.077	–0.053	–0.053	–0.123	–0.073	0.008	–0.117	0.104
L.FWD.TR	–0.066*	–0.016	0.103	–0.083	–0.091**	–0.077*	0.015	0.126
L.FRU.TR	0.084**	–0.046	0.042	–0.029	0.018	–0.051	0.112	–0.020
L.BIO	0.179**	0.046	0.222	0.086	0.145*	0.034	–0.108	0.215
L.BIO.H×JFM	0.368***	–0.198**	0.291	–0.148	0.414***	–0.310***	0.312	–0.316*
AV.DEP	–0.069**	0.072**	–0.012	0.070	–0.065*	0.072**	0.003	0.030
WTR.SCARCE	0.047	0.038	0.078	0.056	0.025	0.047	0.005	0.057
RAIN	–0.262**	–0.077	–0.098	–0.035	–0.406**	–0.023	–0.371	0.041
MKTDIS.FWD	–0.008	0.048***	0.015	0.023	–0.011	0.038*	0.002	0.044
MKTDIS.AGR	0.031***	0.001	0.040***	–0.004	0.027***	0.006	0.027*	0.002
P.FWD	0.025	–0.002	–0.198	–0.062	0.093	0.141	–0.698***	–0.332
KHARIF	–0.959***	0.127	–0.838*	–0.109	–0.555	0.026	–0.787	0.086
SUMMER	0.247**	0.096*	0.148	0.089	0.282***	0.070	0.231	0.092
CONSTANT	0.032	2.644***	–1.095	3.383***	–1.111*	3.062***	–4.700***	6.867***
No. of obs.	1617	423	1617	114	1617	335	1617	113
R^2		0.14		0.15		0.14		0.30
χ^2	187.44***	37.23***	44.84***	16.96	140.22***	27.58*	56.77***	31.12**

TABLE 3. Time spent collecting firewood from the commons, for the household as a whole and by gender.

Estimation method	PROBITRE	RE	PROBITRE	RE	PROBITRE	RE
Dependent variable	P.FWD.COL	L.FWD.COL	P.FWD.PUR	L.FWD.PUR	P.DFU.CON	L.DFU.CON
HD.AGE	–0.011*	–0.003	–0.004	–0.003	0.000	–0.001
HD.EDU	–0.076***	0.030**	0.019	–0.011	–0.027**	0.007
PROP.F	0.689	–0.321	–0.203	–0.083	0.124	0.023
PROP.C	0.986**	–0.741***	–0.606*	0.243	–0.034	–0.222*
L.LAND×L.CAP	–0.025***	0.001	–0.007	0.009*	–0.012**	–0.002
L.ANIMAL	–0.090	–0.010	0.000	–0.017	0.135***	0.062***
L.FWD.TR	–0.073*	–0.000	–0.071**	0.028	0.048**	–0.005
L.FRU.TR	0.090**	–0.019	–0.086***	–0.021	–0.024	–0.017
L.BIO	0.180**	0.040	0.056	0.071	0.015	0.059***
L.BIO.H×JFM	0.376***	–0.048	–0.077	0.052	–0.223***	–0.078***
AV.DEP	–0.066**	0.000	–0.009	–0.010	0.040**	0.010
WTR.SCARCE	0.047	0.013	–0.011	–0.023	0.019	0.017**
RAIN	–0.278**	–0.025	–0.191	0.039	–0.527**	–0.242
MKTDIS.FWD	–0.008	0.024**	0.002	0.006	–0.008	–0.000
MKTDIS.AGR	0.030***	0.008**	0.001	–0.007	–0.009*	–0.001
P.FWD	0.031	–0.035	0.150	–0.305***	–0.175**	0.028
KHARIF	–0.982***	–0.038	0.378	–0.116	–1.924***	0.396
SUMMER	0.235**	0.040	0.363***	0.262***	–1.003***	0.003
CONSTANT	0.228	1.236***	–0.802*	1.114***	0.949***	0.503***
No. of obs.	1617	429	1617	134	1617	697
R^2		0.12		0.35		0.12
χ^2	185.11***	40.51***	50.06***	62.20***	291.79***	56.70***

TABLE 4. Collection and purchases of firewood and consumption of dung for fuel.

	Seasons		
Number of Households	Kharif	Rabi	Summer
Collecting Firewood	40	181	208
Purchasing Firewood	33	38	63
Using Dung for Fuel	12	431	254

TABLE 5. Number of Households that Collect Firewood from Commons, Purchase Firewood, and Use Dung for Fuel by Season

Estimation method	PROBITRE	RE	PROBITRE	RE	PROBITRE	RE
Dependent variable	P.FWD.COL	L.FWD.COL	P.FWD.PUR	L.FWD.PUR	P.DFU.CON	L.DFU.CON
HD.AGE	–0.011*	–0.003	–0.004	–0.003	0.000	–0.001
HD.EDU	–0.076***	0.030**	0.019	–0.011	–0.027**	0.007
PROP.F	0.689	–0.321	–0.203	–0.083	0.124	0.023
PROP.C	0.986**	–0.741***	–0.606*	0.243	–0.034	–0.222*
L.LAND×L.CAP	–0.025***	0.001	–0.007	0.009*	–0.012**	–0.002
L.ANIMAL	–0.090	–0.010	0.000	–0.017	0.135***	0.062***
L.FWD.TR	–0.073*	–0.000	–0.071**	0.028	0.048**	–0.005
L.FRU.TR	0.090**	–0.019	–0.086***	–0.021	–0.024	–0.017
L.BIO	0.180**	0.040	0.056	0.071	0.015	0.059***
L.BIO.H×JFM	0.376***	–0.048	–0.077	0.052	–0.223***	–0.078***
AV.DEP	–0.066**	0.000	–0.009	–0.010	0.040**	0.010
WTR.SCARCE	0.047	0.013	–0.011	–0.023	0.019	0.017**
RAIN	–0.278**	–0.025	–0.191	0.039	–0.527**	–0.242
MKTDIS.FWD	–0.008	0.024**	0.002	0.006	–0.008	–0.000
MKTDIS.AGR	0.030***	0.008**	0.001	–0.007	–0.009*	–0.001
P.FWD	0.031	–0.035	0.150	–0.305***	–0.175**	0.028
KHARIF	–0.982***	–0.038	0.378	–0.116	–1.924***	0.396
SUMMER	0.235**	0.040	0.363***	0.262***	–1.003***	0.003
CONSTANT	0.228	1.236***	–0.802*	1.114***	0.949***	0.503***
No. of obs.	1617	429	1617	134	1617	697
R^2		0.12		0.35		0.12
χ^2	185.11***	40.51***	50.06***	62.20***	291.79***	56.70***

Table 6. Collection and purchases of firewood and consumption of dung for fuel.

Estimation method	PROBIT	OLS	PROBIT	OLS
Dependent variable	P.FWD.PRI	L.FWD.PRI	P.CFU.PRI	L.CFU.PRI
HD.AGE	0.004	0.002*	0.003	0.000
HD.EDU	–0.000	0.015***	–0.016	0.008***
PROP.F	–0.456	–0.122	0.234	–0.112
PROP.C	0.170	–0.052	0.425	–0.099
L.LAND×L.CAP	–0.004	0.009**	0.026**	0.006***
L.ANIMAL	0.122*	–0.050***	0.095	–0.003
L.FWD.TR	0.231***	0.033***	0.068**	0.001
L.FRU.TR	0.268***	0.030***	0.034	–0.003
L.BIO	–0.152**	–0.003	–0.211**	0.006
L.BIO.H×JFM	0.090	0.013	0.115	–0.003
AV.DEP	–0.050	–0.000	–0.058	–0.003
WTR.SCARCE	0.005	0.004	–0.064**	–0.010*
RAIN	0.000	0.000	–0.000	–0.000
MKTDIS.FWD	–0.004	0.004	–0.042	0.002
MKTDIS.AGR	–0.014*	–0.001	–0.003	–0.001
P.FWD	0.101	–0.043*	–0.047	–0.029
CONSTANT	–2.215***	–0.181	–0.581	0.156**
No. of obs.	539	253	539	371
R^2		0.40		0.22

Table 7. Private production of firewood and of agricultural residue used for fuel.

Estimation method	PROBIT	OLS
Dependent variable	P.PRI.TR	FR.FWD.TR
HD.AGE	0.008	–0.002*
HD.EDU	0.004	0.002
PROP.F	0.809	–0.090
PROP.C	0.387	–0.161*
L.LAND×L.CAP	0.056***	0.003*
L.ANIMAL	–0.143	–0.044***
L.BIO	–0.083	–0.067***
L.BIO.H×JFM	0.290	–0.053*
AV.DEP	–0.061	–0.006
WTR.SCARCE	0.012	–0.006
MKTDIS.FWD	0.121**	–0.001
MKTDIS.AGR	0.020	0.003
CONSTANT	0.357	0.488***
No. of obs.	490	472
R^2		0.17

Table 8. Ownership of private firewood and fruit trees.

Appendix A. First-order Conditions

As described in section 4, the Lagrangian associated with the household's optimization problem is

$$
\begin{aligned}
L = {} & U(c^a, c^f, c^r) \\
& + \lambda^B[q^a(t^a, L^a) + p^f q_c^f(t^f; F_c^f) + p^f q_p^f(F_p^f) + p^r q_p^r(F_p^r) + wt^w + v - c^a - p^f c^f - p^r c^r] \\
& + \lambda^T[T - t^a - t^f - t^w] \\
& + \lambda^F[L - L^a - F_p^f - F_p^r] \\
& + \mu^c[c^f - q_c^f(t^f; F_c^f) - q_p^f(F_p^f)] \\
& + \mu^t t^f + \mu^w t^w + \mu^f F_p^f + \mu^r F_p^r,
\end{aligned}
$$

with the following first-order conditions

$$\frac{\partial L}{\partial c^a} = \frac{\partial U}{\partial c^a} - \lambda^B = 0, \tag{11}$$

$$\frac{\partial L}{\partial c^f} = \frac{\partial U}{\partial c^f} - \lambda^B p^f + \mu^c = 0, \tag{12}$$

$$\frac{\partial L}{\partial c^r} = \frac{\partial U}{\partial c^r} - \lambda^B p^r = 0, \tag{13}$$

$$\frac{\partial L}{\partial t^a} = \lambda^B \frac{\partial q^a}{\partial t^a} - \lambda^T = 0, \tag{14}$$

$$\frac{\partial L}{\partial t^f} = \lambda^B p^f \frac{\partial q_c^f}{\partial t^f} - \lambda^T - \mu^c \frac{\partial q_c^f}{\partial t^f} + \mu^t = 0, \tag{15}$$

$$\frac{\partial L}{\partial t^w} = \lambda^B w - \lambda^T + \mu^w = 0, \tag{16}$$

$$\frac{\partial L}{\partial F_p^f} = \lambda^B p^f \frac{\partial q_p^f}{\partial F_p^f} - \lambda^F - \mu^c \frac{\partial q_p^f}{\partial F_p^f} + \mu^f = 0, \tag{17}$$

$$\frac{\partial L}{\partial F_r^f} = \lambda^B p^r \frac{\partial q_p^r}{\partial F_p^r} - \lambda^F + \mu^r = 0, \tag{18}$$

$$\frac{\partial L}{\partial L^a} = \lambda^B \frac{\partial q^a}{\partial L^a} - \lambda^F = 0. \tag{19}$$

Substitute λ^B using (11):

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f + \mu^c = 0, \tag{20}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{21}$$

$$\frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} - \lambda^T = 0, \tag{22}$$

$$\frac{\partial U}{\partial c^a} p^f \frac{\partial q_c^f}{\partial t^f} - \lambda^T - \mu^c \frac{\partial q_c^f}{\partial t^f} + \mu^t = 0, \tag{23}$$

$$\frac{\partial U}{\partial c^a} w - \lambda^T + \mu^w = 0, \tag{24}$$

$$\frac{\partial U}{\partial c^a} p^f \frac{\partial q_p^f}{\partial F_p^f} - \lambda^F - \mu^c \frac{\partial q_p^f}{\partial F_p^f} + \mu^f = 0, \tag{25}$$

$$\frac{\partial U}{\partial c^a} p^r \frac{\partial q_p^r}{\partial F_p^r} - \lambda^F + \mu^r = 0, \tag{26}$$

$$\frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} - \lambda^F = 0. \tag{27}$$

And then substitute λ^T and λ^F using (22) and (27):

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f + \mu^c = 0, \tag{28}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{29}$$

$$\frac{\partial U}{\partial c^a} p^f \frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} - \mu^c \frac{\partial q_c^f}{\partial t^f} + \mu^t = 0, \tag{30}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{31}$$

$$\frac{\partial U}{\partial c^a} p^f \frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} - \mu^c \frac{\partial q_p^f}{\partial F_p^f} + \mu^f = 0, \tag{32}$$

$$\frac{\partial U}{\partial c^a} p^r \frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} + \mu^r = 0, \tag{33}$$

Finally, substitute (28) into (30) and (32), and (29) into (33):

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a}p^f + \mu^c = 0$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a}p^r = 0$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^t = 0$$

$$\frac{\partial U}{\partial c^a}w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^w = 0$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^f = 0$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^r = 0$$

Appendix B. Analysis

Case I: none of the inequality constraints bind.

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a}p^f = 0, \tag{34}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a}p^r = 0, \tag{35}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{36}$$

$$\frac{\partial U}{\partial c^a}w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{37}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} = 0, \tag{38}$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} = 0, \tag{39}$$

Substitute (34) into (36) and (38), substitute (35) into (39), and then cancel $\partial U/\partial c^a$ terms:

$$p^f\frac{\partial q_c^f(t^f;F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a,L^a)}{\partial t^a} = 0, \tag{40}$$

$$w - \frac{\partial q^a(t^a,L^a)}{\partial t^a} = 0, \tag{41}$$

$$p^f\frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(t^a,L^a)}{\partial L^a} = 0, \tag{42}$$

$$p^r\frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(t^a,L^a)}{\partial L^a} = 0. \tag{43}$$

Use (2) to substitute away t^a and use (3) to substitute away L^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial t^a} = 0, \tag{44}$$

$$w - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial t^a} = 0, \tag{45}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0, \tag{46}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0. \tag{47}$$

Substitute (45) into (44) to simplify the latter:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - w = 0, \tag{48}$$

$$w - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial t^a} = 0, \tag{49}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0, \tag{50}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(T - t^f - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0. \tag{51}$$

Treat (48) as implicitly defining a function $t^f(F_c^f, p^f, w)$, and note that

$$\frac{\partial t^f}{\partial F_c^f} = -\frac{\frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}}{\frac{\partial^2 q_c^f}{\partial {t^f}^2}} > 0. \tag{52}$$

Substitute this function into the other three equations:

$$w - \frac{\partial q^a(T - t^f(F_c^f, p^f, w) - t^w, L - F_p^f - F_p^r)}{\partial t^a} = 0, \tag{53}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(T - t^f(F_c^f, p^f, w) - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0, \tag{54}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(T - t^f(F_c^f, p^f, w) - t^w, L - F_p^f - F_p^r)}{\partial L^a} = 0. \tag{55}$$

These three equations are just the first-order conditions of the following unconstrained maximization problem:

$$\max_{t^w, F_p^f, F_p^r} \pi = wt^w + p^f q_p^f(F_p^f) + p^r q_p^r(F_p^r) + q^a(T - t^f(F_c^f, p^f, w) - t^w, L - F_p^f - F_p^r).$$

Totally differentiate with respect to the three endogenous variables and the variable t^f, which we can treat as exogenous to the system of equations:

$$\begin{bmatrix} \frac{\partial^2 q^a}{\partial t^{a2}} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} + \frac{\partial^2 q^a}{\partial L^{a2}} & \frac{\partial^2 q^a}{\partial L^{a2}} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial L^{a2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} + \frac{\partial^2 q^a}{\partial L^{a2}} \end{bmatrix} \begin{bmatrix} dt^w \\ dF_p^f \\ dF_p^r \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 q^a}{\partial t^{a2}} \\ -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \end{bmatrix} dt^f$$

A simpler, but equivalent system (with the same solution) is obtained if we subtract row 2 from row 3:

$$\begin{bmatrix} \frac{\partial^2 q^a}{\partial t^{a2}} & 0 & -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & -\frac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & -p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{bmatrix} \begin{bmatrix} dt^w \\ dF_p^f \\ dF_p^r \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 q^a}{\partial t^{a2}} \\ -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ 0 \end{bmatrix} dt^f$$

Assuming the second-order condition is satisfied, which requires the determinant D of the first matrix to have a negative sign, the comparative statics are

$$\frac{\partial t^w}{\partial t^f} = \frac{1}{D} \begin{vmatrix} -\frac{\partial^2 q^a}{\partial t^{a2}} & 0 & -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ -\frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & -\frac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & -p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{vmatrix} = -1,$$

since the determinant in the numerator is identical to D, except that the first column is multiplied by -1, which changes the determinant's sign. Also,

$$\frac{\partial F_p^f}{\partial t^f} = \frac{1}{D} \begin{vmatrix} \frac{\partial^2 q^a}{\partial t^{a2}} & -\frac{\partial^2 q^a}{\partial t^{a2}} & -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & -\frac{\partial^2 q^a}{\partial t^a \partial L^a} & -\frac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & 0 & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{vmatrix} = 0,$$

since the first and second columns of the determinant are linearly dependent, and

$$\frac{\partial F_p^r}{\partial F_c^f} = \frac{1}{D} \begin{vmatrix} \frac{\partial^2 q^a}{\partial t^{a2}} & 0 & -\frac{\partial^2 q^a}{\partial t^{a2}} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & -\frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ 0 & -p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & 0 \end{vmatrix} = 0,$$

for the same reason.

Using (4) and (52)

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2),

$$\begin{aligned}\frac{dt^a}{dF_c^f} &= -\frac{dt^w}{dF_c^f} - \frac{dt^f}{dF_c^f} \\ &= -\frac{dt^w}{dt^f}\frac{dt^f}{dF_c^f} - \frac{dt^f}{dF_c^f} \\ &= \frac{dt^f}{dF_c^f} - \frac{dt^f}{dF_c^f} = 0.\end{aligned}$$

Using (3),

$$\frac{dL^a}{dF_c^f} = -\underbrace{\frac{\partial F_p^f}{\partial F_c^f}}_{(0)} - \underbrace{\frac{\partial F_p^r}{\partial F_c^f}}_{(0)} = 0.$$

Case II: only the non-negativity constraints on F_p^f, and F_p^r bind (so the household has no private trees, although it does have private land).

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a}p^f = 0, \tag{56}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a}p^r = 0, \tag{57}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{58}$$

$$\frac{\partial U}{\partial c^a}w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{59}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^f = 0, \tag{60}$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^r = 0, \tag{61}$$

Since μ^f and μ^r each enter only one equation, their value can be determined after solving the remaining four equations together with the two constraints (1) and (2) for the remaining six unknowns (c^a, c^f, c^r, t^a, t^f, and t^w).

Substitute (56) into (58), and then cancel $\partial U/\partial c^a$ terms:

$$p^f\frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0, \tag{62}$$

$$w - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0. \tag{63}$$

$$\tag{64}$$

Use (2) to substitute away t^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f - t^w, L^a)}{\partial t^a} = 0, \tag{65}$$

$$w - \frac{\partial q^a(T - t^f - t^w, L^a)}{\partial t^a} = 0. \tag{66}$$

$$\tag{67}$$

Substitute (66) into (65) to simplify the latter:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - w = 0, \tag{68}$$

$$w - \frac{\partial q^a(T - t^f - t^w, L^a)}{\partial t^a} = 0. \tag{69}$$

We now have a recursive system where equation (68) implicitly defines a function $t^f(F_c^f, p^f, w)$ and, after substituting this function into (69), the latter implicitly defines a function $t^w(F_c^f, p^f, w)$.

Totally differentiating (65) yields

$$\frac{\partial t^f}{\partial F_c^f} = -\frac{\frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}}{\frac{\partial^2 q_c^f}{\partial {t^f}^2}} > 0. \tag{70}$$

Totally differentiating (66) after substituting $t^f(F_c^f, p^f, w)$ yields

$$\frac{\partial t^w}{\partial F_c^f} = -\frac{\frac{\partial^2 q^a}{\partial t^{a2}} \frac{\partial t^f}{\partial F_c^f}}{\frac{\partial^2 q^a}{\partial t^{a2}}} = -\frac{\partial t^f}{\partial F_c^f} < 0. \tag{71}$$

Using (4),

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2),

$$\frac{dt^a}{dF_c^f} = -\frac{dt^w}{dF_c^f} - \frac{dt^f}{dF_c^f} = 0.$$

Case III: only the non-negativity constraint on t^w binds (so the household does not engage in wage labor).

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f = 0, \tag{72}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{73}$$

$$\frac{\partial U}{\partial c^f} \frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} = 0, \tag{74}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{75}$$

$$\frac{\partial U}{\partial c^f} \frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} = 0, \tag{76}$$

$$\frac{\partial U}{\partial c^r} \frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} = 0, \tag{77}$$

Since μ^w enters only equation (75), its value can be determined after solving the remaining five equations together with the three constraints (1), (2), and (3) for the remaining eight unknowns (c^a, c^f, c^r, t^f, t^a, F_p^f, F_p^r, and L^a).

Substitute (72) into (74) and (76), substitute (73) into (77), and then cancel $\partial U/\partial c^a$ terms:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0, \tag{78}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(t^a, L^a)}{\partial L^a} = 0, \tag{79}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(t^a, L^a)}{\partial L^a} = 0. \tag{80}$$

Use (2) and our assumption that $t^w = 0$ to substitute away t^a, and use (3) to substitute away L^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f, L - F_p^f - F_p^r)}{\partial t^a} = 0, \tag{81}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(T - t^f, L - F_p^f - F_p^r)}{\partial L^a} = 0, \tag{82}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(T - t^f, L - F_p^f - F_p^r)}{\partial L^a} = 0. \tag{83}$$

These three equations are just the first-order conditions of the following unconstrained maximization problem:

$$\max_{t^f, F_p^f, F_p^r} \pi = p^f q_c^f(t^f; F_c^f) + p^f q_p^f(F_p^f) + p^r q_p^r(F_p^r) + q^a(T - t^f, L - F_p^f - F_p^r).$$

Totally differentiate with respect to the three endogenous variables and the exogenous variable F_c^f:

$$\begin{bmatrix} p^f \frac{\partial^2 q_c^f}{\partial t^{f2}} + \frac{\partial^2 q^a}{\partial t^{a2}} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} + \frac{\partial^2 q^a}{\partial L^{a2}} & \frac{\partial^2 q^a}{\partial L^{a2}} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial L^{a2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} + \frac{\partial^2 q^a}{\partial L^{a2}} \end{bmatrix} \begin{bmatrix} dt^f \\ dF_p^f \\ dF_p^r \end{bmatrix} = \begin{bmatrix} -p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ 0 \\ 0 \end{bmatrix} dF_c^f$$

A simpler, but equivalent system (with the same solution) is obtained if we subtract row 2 from row 3:

$$\begin{bmatrix} p^f \frac{\partial^2 q_c^f}{\partial t^{f2}} + \frac{\partial^2 q^a}{\partial t^{a2}} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \frac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} + \frac{\partial^2 q^a}{\partial L^{a2}} & \frac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & -p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{bmatrix} \begin{bmatrix} dt^f \\ dF_p^f \\ dL^a \end{bmatrix} = \begin{bmatrix} -p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ 0 \\ 0 \end{bmatrix} dF_c^f$$

Assuming the second-order condition is satisfied, which requires the determinant D of the first matrix to have a negative sign, the comparative statics are

$$\frac{\partial t^f}{\partial F_c^f} = \frac{1}{D} \begin{vmatrix} -p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} & \frac{\partial^2 q^a}{\partial t^a \partial L^a} \\ 0 & p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} + \frac{\partial^2 q^a}{\partial L^{a2}} & \frac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & -p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} & p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{vmatrix}$$

$$\stackrel{s}{=} -\underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}} + \frac{\partial^2 q^a}{\partial L^{a2}}\right)}_{(-)} \underbrace{\left(p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}}\right)}_{(-)}$$

$$+ \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(-p^f \frac{\partial^2 q_p^f}{\partial F_p^{f2}}\right)}_{(+)} \underbrace{\left(\frac{\partial^2 q^a}{\partial L^{a2}}\right)}_{(-)} > 0,$$

where $\stackrel{s}{=}$ denotes equality of sign.

$$\frac{\partial F_p^f}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f\,2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & 0 & \dfrac{\partial^2 q^a}{\partial L^{a2}} \\ 0 & 0 & p^r \dfrac{\partial^2 q_p^r}{\partial F_p^{r2}} \end{vmatrix}$$

$$\stackrel{s}{=} \underbrace{\left(\frac{\partial^2 q^a}{\partial t^a \partial L^a}\right)}_{(+)} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(p^r \frac{\partial^2 q_p^r}{\partial F_p^{r2}}\right)}_{(-)} > 0,$$

and

$$\frac{\partial F_p^r}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f\,2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \dfrac{\partial^2 q_p^f}{\partial F_p^{f\,2}} + \dfrac{\partial^2 q^a}{\partial L^{a2}} & 0 \\ 0 & -p^f \dfrac{\partial^2 q_p^f}{\partial F_p^{f\,2}} & 0 \end{vmatrix}$$

$$\stackrel{s}{=} -\underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(\frac{\partial^2 q^a}{\partial t^a \partial L^a}\right)}_{(+)} \underbrace{\left(-p^f \frac{\partial^2 q_p^f}{\partial F_p^{f\,2}}\right)}_{(+)}$$

$$= \underbrace{\left(\frac{\partial^2 q^a}{\partial t^a \partial L^a}\right)}_{(+)} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(p^f \frac{\partial^2 q_p^f}{\partial F_p^{f\,2}}\right)}_{(-)} > 0.$$

Using (4),

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2) and our assumption that $t^w = 0$,

$$\frac{dt^a}{dF_c^f} = -\underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} < 0.$$

Using (3),

$$\frac{dL^a}{dF_c^f} = -\underbrace{\frac{\partial F_p^f}{\partial F_c^f}}_{(+)} - \underbrace{\frac{\partial F_p^r}{\partial F_c^f}}_{(+)} < 0.$$

Case IV: only the non-negativity constraints on t^w and F_p^r bind (so the household does not engage in wage labor and has no fruit trees).

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f = 0, \tag{84}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{85}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{86}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{87}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} = 0, \tag{88}$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^r = 0, \tag{89}$$

Since μ^w and μ^r each enter only one equation, their value can be determined after solving the remaining four equations together with the two constraints (1) and (2) for the remaining 6 unknowns (c^a, c^f, c^r, t^a, t^f, and F_p^f).

Substitute (84) into (86) and (88), and then cancel $\partial U/\partial c^a$ terms:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0, \tag{90}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(t^a, L^a)}{\partial L^a} = 0. \tag{91}$$

Use (2) and our assumption that $t^w = 0$ to substitute away t^a, and use (3) and our assumption that $F_p^r = 0$ to substitute away L^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f, L - F_p^f)}{\partial t^a} = 0, \tag{92}$$

$$p^f \frac{\partial q_p^f(F_p^f)}{\partial F_p^f} - \frac{\partial q^a(T - t^f, L - F_p^f)}{\partial L^a} = 0. \tag{93}$$

These two equations are just the first-order conditions of the following unconstrained maximization problem:

$$\max_{t^f, F_p^f} \pi = p^f q_c^f(t^f; F_c^f) + p^f q_p^f(F_p^f) + q^a(T - t^f, L - F_p^f - F_p^r).$$

Totally differentiate with respect to the two endogenous variables and the exogenous variable F_c^f:

$$\begin{bmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & p^f \dfrac{\partial^2 q_p^f}{\partial F_p^{f2}} + \dfrac{\partial^2 q^a}{\partial L^{a2}} \end{bmatrix} \begin{bmatrix} dt^f \\ dF_p^f \end{bmatrix} = \begin{bmatrix} -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ 0 \end{bmatrix} dF_c^f$$

Assuming the second-order condition is satisfied, which requires the determinant D of the first matrix to have a positive sign, the comparative statics are

$$\frac{\partial t^f}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} \\ 0 & p^f \dfrac{\partial^2 q_p^f}{\partial F_p^{f\,2}} + \dfrac{\partial^2 q^a}{\partial L^{a2}} \end{vmatrix}$$

$$\stackrel{s}{=} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(p^f \frac{\partial^2 q_p^f}{\partial F_p^{f\,2}} + \frac{\partial^2 q^a}{\partial L^{a2}}\right)}_{(-)} > 0,$$

and

$$\frac{\partial F_p^f}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f\,2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & 0 \end{vmatrix}$$

$$\stackrel{s}{=} -\underbrace{\left(\frac{\partial^2 q^a}{\partial t^a \partial L^a}\right)}_{(+)} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} > 0.$$

Using (4),

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2) and our assumption that $t^w = 0$,

$$\frac{dt^a}{dF_c^f} = -\underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} < 0.$$

Using (3) and our assumption that $F_p^r = 0$,

$$\frac{dL^a}{dF_c^f} = -\underbrace{\frac{\partial F_p^f}{\partial F_c^f}}_{(+)} < 0.$$

Case V: the non-negativity constraints on t^w and F_p^f bind (so the household does not engage in wage labor and has no private firewood trees, although it does have private

fruit trees on its land).

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f = 0, \tag{94}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{95}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} = 0, \tag{96}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{97}$$

$$\frac{\partial U}{\partial c^f}\frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} + \mu^f = 0, \tag{98}$$

$$\frac{\partial U}{\partial c^r}\frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a}\frac{\partial q^a}{\partial L^a} = 0, \tag{99}$$

Since μ^w and μ^f each enter only one equation, their value can be determined after solving the remaining four equations together with the three constraints (1), (2), and (3) for the remaining seven unknowns (c^a, c^f, c^r, t^a, t^f, F_p^r, and L^a).

Substitute (94) into (96), substitute (95) into (99), and then cancel $\partial U/\partial c^a$ terms:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0, \tag{100}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(t^a, L^a)}{\partial L^a} = 0. \tag{101}$$

Use (2) and our assumption that $t^w = 0$ to substitute away t^a, and use (3) and our assumption that $F_p^f = 0$ to substitute away L^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f, L - F_p^r)}{\partial t^a} = 0, \tag{102}$$

$$p^r \frac{\partial q_p^r(F_p^r)}{\partial F_p^r} - \frac{\partial q^a(T - t^f, L - F_p^r)}{\partial L^a} = 0. \tag{103}$$

These two equations are just the first-order conditions of the following unconstrained maximization problem:

$$\max_{t^f, F_p^r} \pi = p^f q_c^f(t^f; F_c^f) + p^f q_p^r(F_p^r) + q^a(T - t^f, L - F_p^r).$$

Totally differentiate with respect to the two endogenous variables and the exogenous variable F_c^f:

$$\begin{bmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & p^r \dfrac{\partial^2 q_p^r}{\partial F_p^{r2}} + \dfrac{\partial^2 q^a}{\partial L^{a2}} \end{bmatrix} \begin{bmatrix} dt^f \\ dF_p^r \end{bmatrix} = \begin{bmatrix} -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ 0 \end{bmatrix} dF_c^f$$

Assuming the second-order condition is satisfied, which requires the determinant D of the first matrix to have a positive sign, the comparative statics are

$$\frac{\partial t^f}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} & \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} \\ 0 & p^r \dfrac{\partial^2 q_p^r}{\partial F_p^{r2}} + \dfrac{\partial^2 q^a}{\partial L^{a2}} \end{vmatrix}$$

$$\stackrel{s}{=} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} \underbrace{\left(p^f \frac{\partial^2 q_p^r}{\partial F_p^{r2}} + \frac{\partial^2 q^a}{\partial L^{a2}}\right)}_{(-)} > 0,$$

and

$$\frac{\partial F_p^r}{\partial F_c^f} = \frac{1}{D}\begin{vmatrix} p^f \dfrac{\partial^2 q_c^f}{\partial t^{f2}} + \dfrac{\partial^2 q^a}{\partial t^{a2}} & -p^f \dfrac{\partial^2 q_c^f}{\partial t^f \partial F_c^f} \\ \dfrac{\partial^2 q^a}{\partial t^a \partial L^a} & 0 \end{vmatrix}$$

$$\stackrel{s}{=} -\underbrace{\left(\frac{\partial^2 q^a}{\partial t^a \partial L^a}\right)}_{(+)} \underbrace{\left(-p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}\right)}_{(-)} > 0.$$

Using (4),

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2) and our assumption that $t^w = 0$,

$$\frac{dt^a}{dF_c^f} = -\underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} < 0.$$

Using (3) and our assumption that $F_p^f = 0$,

$$\frac{dL^a}{dF_c^f} = -\underbrace{\frac{\partial F_p^r}{\partial F_c^f}}_{(+)} < 0.$$

Case VI: the non-negativity constraints on t^w, F_p^f, and F_p^r bind (so the household does not engage in wage labor and has no private firewood or fruit trees, although it

does have private land).

$$\frac{\partial U}{\partial c^f} - \frac{\partial U}{\partial c^a} p^f = 0, \tag{104}$$

$$\frac{\partial U}{\partial c^r} - \frac{\partial U}{\partial c^a} p^r = 0, \tag{105}$$

$$\frac{\partial U}{\partial c^f} \frac{\partial q_c^f}{\partial t^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} = 0, \tag{106}$$

$$\frac{\partial U}{\partial c^a} w - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial t^a} + \mu^w = 0, \tag{107}$$

$$\frac{\partial U}{\partial c^f} \frac{\partial q_p^f}{\partial F_p^f} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} + \mu^f = 0, \tag{108}$$

$$\frac{\partial U}{\partial c^r} \frac{\partial q_p^r}{\partial F_p^r} - \frac{\partial U}{\partial c^a} \frac{\partial q^a}{\partial L^a} + \mu^r = 0, \tag{109}$$

Since μ^w, μ^f, and μ^r each enter only one equation, their value can be determined after solving the remaining three equations together with the two constraints (1) and (2) for the remaining five unknowns (c^a, c^f, c^r, t^a, and t^f).

Substitute (104) into (106) and then cancel $\partial U/\partial c^a$ terms:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(t^a, L^a)}{\partial t^a} = 0. \tag{110}$$

Use (2) and our assumption that $t^w = 0$ to substitute away t^a:

$$p^f \frac{\partial q_c^f(t^f; F_c^f)}{\partial t^f} - \frac{\partial q^a(T - t^f, L^a)}{\partial t^a} = 0. \tag{111}$$

$$\tag{112}$$

We now have a single equation in the single unknown t^f. Totally differentiating yields

$$\frac{dt^f}{dF_c^f} = -\frac{\overbrace{p^f \frac{\partial^2 q_c^f}{\partial t^f \partial F_c^f}}^{(+)}}{\underbrace{p^f \frac{\partial^2 q_c^f}{\partial {t^f}^2}}_{(-)} + \underbrace{\frac{\partial^2 q^a}{\partial {t^a}^2}}_{(-)}} > 0.$$

Using (4),

$$\frac{dq_c^f}{dF_c^f} = \underbrace{\frac{\partial q_c^f}{\partial t^f}}_{(+)} \underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} + \underbrace{\frac{\partial q_c^f}{\partial F_c^f}}_{(+)} > 0.$$

Using (2) and our assumption that $t^w = 0$,

$$\frac{dt^a}{dF_c^f} = -\underbrace{\frac{\partial t^f}{\partial F_c^f}}_{(+)} < 0.$$