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Putting free-riding to work: A Partnership Solution to the common-property problem

Martin D. Heintzelman [a], Stephen W. Salant [b,*], Stephan Schott [c]

[a] Clarkson University, School of Business, Potsdam, NY, USA
[b] Department of Economics, University of Michigan, Ann Arbor, MI, USA
[c] School of Public Policy and Administration, Carleton University, Ottawa, Ontario, Canada

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ABSTRACT

The common-property problem results in excessive mining, hunting, and extraction of oil and water. The same phenomenon is also responsible for excessive investment in R&D and excessive outlays in rent-seeking contests. We propose a "Partnership Solution" to eliminate or at least mitigate these excesses. Each of N players joins a partnership in the first stage and chooses his effort in the second stage. Under the rules of a partnership, each member must pay his own cost of effort but receives an equal share of the partnership's revenue. The incentive to free-ride created by such partnerships turns out to be beneficial since it naturally offsets the excessive effort inherent in such problems. In our two-stage game, this institutional arrangement can, under specified circumstances, induce the social optimum in a subgame-perfect equilibrium: no one has a unilateral incentive (1) to switch to another partnership (or create a new partnership) in the first stage or (2) to deviate from socially optimal actions in the second stage. The game may have other subgame-perfect equilibria, but the one associated with the "Partnership Solution" is strictly preferred by every player. We also propose a modification of the first stage which generates a unique subgame-perfect equilibrium. Antitrust authorities should recognize that partnerships can have a less benign use. By organizing as competing partnerships, an industry can reduce the "excessive" output of Cournot oligopoly to the monopoly level. Since no partner has any incentive to overproduce in the current period, there is no need to deter cheating with threats of future punishments.

1. Introduction

In some fisheries in Japan, fishermen from several vessels share their catch. Their pooled output is sold through a common outlet and members of each partnership divide equally the resulting gross revenue, no matter how little someone has contributed. Such egalitarian catch-sharing among vessels is a prescription for free-riding. Multiple partnerships compete for the catch, or the induced free-riding would be even greater.

Received economic theory cannot account for such partnerships. They do not appear to be a response to uncertainty or asymmetric information. Catch-sharing partnerships must have *some* advantage, however, since as of the census of 1988, 147 different fishing groups in Japan were engaging in such income pooling. To understand why such partnerships arise,

* Corresponding author. Fax: +1 734 764 2769.
E-mail address: ssalant@umich.edu (S.W. Salant).

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Platteau and Seki [24] interviewed skippers in the glass-shrimp industry in Japan and, when feasible, used more objective measures to validate their responses. They concluded that "The most prominent result emerging from this exercise is certainly the fact that stabilization of incomes was not mentioned a single time." Instead, the main motive appeared to be the reduction of congestion: "The desire to avoid the various costs of crowding while operating in attractive fishing spots appears as the main reason stated by Japanese fishermen for adopting pooling arrangements."[1]

These Japanese fishermen appear to have rediscovered an ancient solution to the common-property problem. According to anthropologists, hunter–gatherer cultures that have survived to the modern era typically share their kill and work short hours; moreover, they consume enough quantity and variety to be characterized by one distinguished anthropologist as the "Original Affluent Society" [25]. These phenomena, which have been studied extensively but separately, may be connected: those hunter–gatherer cultures surviving to modern times may owe their success to their practice of sharing the fish and game caught by groups of hunters since extensive sharing dulls hunting effort sufficiently to protect common property from overexploitation.[2]

At the opposite end of the technological spectrum, individuals who form research joint ventures to share revenue from their discoveries may have hit upon the same solution. Without joint ventures, individuals vying for a patent awarded to the best innovation will invest too much even taking account of the fact that the expected value of the winning patent grows with aggregate investment [4]. Such innovation tournaments are strategically equivalent to rent-seeking contests where the value of the prize increases with the total outlay. So, in the absence of prize-sharing within interest groups, rent-seeking outlays are also excessive [12].[3]

Common property extraction provides a particularly relevant illustration of the same strategic considerations. In the absence of sharing, fishing effort is excessive [16] even when account is taken of the fact that aggregate catch grows with aggregate effort. Sharing arrangements promote free-riding and, when effort would otherwise be excessive, this constitutes a social improvement. We investigate the circumstances when sharing agreements can be used to restore the social optimum in extraction from common properties. As we make clear, however, our conclusions apply equally to (1) innovation tournaments and (2) rent-seeking contests.

Besides these beneficial uses, partnerships can also be used for a more sinister purpose. In the absence of partnership agreements, service providers in an industry can be expected to reap at most oligopoly profits. But if they can organize themselves into a collection of competing revenue-sharing partnerships (a common organizational form in some service industries), they can potentially reap monopoly profits.[4] There is no need for interactions to be ongoing so that the prospect of a price war in the future deters the current temptation to expand output. The revenue-sharing inherent in a partnership structure *eliminates* the current temptation to expand output. Antitrust authorities should be aware that in industries where partnerships predominate, prices may approach monopoly levels even though competition among these partnerships is vigorous.

We consider a two-stage game where individuals choose their partnerships at the first stage and their effort levels at the second stage. If every individual chooses in the first stage to work in a different partnership as its sole "partner," aggregate effort in the second-stage equilibrium will exceed the social optimum. At the other extreme, if every player joins the same grand partnership, aggregate effort in the second stage will be inadequate due to free-riding. As we show, aggregate effort is a strictly increasing function of the number of partnerships formed at the first stage.[5] Socially optimal effort can, therefore, be induced (or approximated if there are integer problems) if the N players partition themselves into an intermediate number of partnerships in such a way that each agent's tendency to work too hard is exactly offset by his tendency to free ride. We refer to this as the "Partnership Solution."

In reality, of course, the Partnership Solution is viable if and only if each person in a given partnership has no incentive to switch to some other partnership (pre-existing or new). We refer to such partnerships as "stable." We refer to each

[1] A similar arrangement seems to have been adopted by some groups of New Jersey fishermen as well [15]. A more traditional society in Tonga also has features of the partnership arrangement we explore here [5].

[2] "The literature on traditional hunter–gatherers provides ample evidence that work effort is extremely low in traditional societies and that natural resources are not overexploited but rather under-exploited" [20, p. 45]. "We do not know whether traditional societies have introduced sharing consciously…Once introduced (or chosen by accident), however, it appears to be a stable means to regulate resource use." [20, p. 67].

[3] In a contest for a fixed prize among rent-seeking individuals exogenously allocated to partnerships, Nitzan showed that aggregate rent-seeking outlays decline because of free-riding [21]. Each partnership is assumed to use the same exogenous sharing rule (of which our egalitarian rule is a special case). In other cases, the rule partially rewards an individual for making larger effort relative to other members of his group, which implicitly requires that group members costlessly monitor each other's efforts. We extend Nitzan's original insight and show its implications for the common property and cartel problems. The case we examine is isomorphic to a rent-seeking contest where the prize is a strictly increasing function of aggregate outlays and where groups form endogenously (see footnote 7). Since the prize in Nitzan is fixed, the partition generating the highest social welfare occurs when every player is in a single group [21]. In our variable-prize case, putting every player in one group induces too little extraction effort and the social optimum instead occurs at an "interior" solution with the correct number groups. In Section 5, we show how our Partnership Solution easily generalizes when groups share according to Nitzan's rule provided the weight on relative effort is not excessive. Baik and Lee extend the group rent-seeking model with a fixed prize by endogenizing group formation and the choice of the sharing rule [3]. However, our analyses are quite different because there is no variable prize (the counterpart to our production) in their application. Consequently, it is efficient for everyone to join a single group, make no rent-seeking outlay, and share the fixed prize; in our context, the "prize" (production) would completely vanish in the absence of outlays (effort). Another key difference arises because we consider the role of team production.

[4] Law firms, medical firms, and consultancies are some examples.

[5] For experimental confirmation of this prediction, see [27].

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partition of players into partnerships in the first stage of our game as a "partnership structure." Whether a partnership structure is stable (i.e., part of a subgame-perfect Nash equilibrium) turns out to depend on the disadvantages of solo production relative to team production. This follows since the principal source of first-stage instability is going into business for oneself (deviating unilaterally to a *new* partnership).

There are typically multiple subgame-perfect equilibria in the game. In the equilibrium associated with the Partnership Solution, however, every individual receives the largest expected payoff. This suggests two ways the game can be modified to implement this "payoff-dominant" equilibrium. Before the agents select the partnerships, an outside mediator, with the power neither to monitor nor enforce compliance with his suggestion, can recommend this number of partnerships. His only function is to focus the expectations of the players on the payoff-dominant equilibrium. Alternatively, each agent can specify the number of partnerships he prefers and the proposal of one agent, selected randomly, would be implemented. Either of these modified game forms should result in the implementation of the Partnership Solution.

It is natural to ask what advantages the Partnership Solution has over Pigouvian taxes or auctioned quotas. While taxes or quotas could induce the same extraction effort as the Partnership Solution, resource users would receive larger aggregate surplus under the Partnership Solution as long as *any* of the revenue collected under these two alternative policies was diverted to general coffers rather than being redistributed to the N players.

2. Decentralization in a two-stage partnership game

Suppose N individuals expend effort to extract a common resource and consider two extremes.[6] At one extreme, suppose everyone pays his own effort cost and acts independently; then, as is well known, aggregate effort will be excessive because of congestion externalities. At the other, suppose everyone must share the fruits of his labor equally with the other $N-1$ individuals while paying his own cost of effort; then aggregate effort will be insufficient because of free-riding. In each extreme, players partitioned into competing partnerships simultaneously choose effort levels, with each partnership's share of aggregate revenue equal to its share of aggregate effort and every member of each partnership required to pay his own effort cost but to share equally with his colleagues the gross revenue he brings in. In the first extreme above, there are N "solo" partnerships, while in the second there is 1 "grand" partnership to which all N individuals belong. We show below that aggregate effort is a strictly increasing function of the number of partnerships and hence socially optimal effort can be achieved as a Nash equilibrium if a particular number of partnerships forms at the first stage. To begin, we define the following notation that will be used throughout the paper.

m_i number of members of group i
x_{ik} effort level of agent k in group i
Y_i^{-k} aggregate effort level of members of group i other than agent k
X_{-i} aggregate effort of other groups
X total effort level (sum of all agents' efforts)
$F(X)$ aggregate production function
c constant marginal cost of effort
n number of groups
N total number of agents
$A(\cdot)=F(X)/X$ average product
$\bar{x}_i=(x_{ik}+Y_i^{-k})/m_i$ mean effort level in group i
β hours of team effort needed to replace 1 hour of solo effort

Until Section 4, we make the assumption standard in the common-property literature that the price of output, $\bar{p}$, is a constant (normalized to unity). In addition, we assume $A(X)$ is bounded, strictly positive, strictly decreasing, and twice continuously differentiable; $A(0)-c>0$; and $A'(X)+XA''(X)<0$, holds for all $X\geqslant 0$ (the Novshek condition [22]).

Socially optimal effort, X^*, maximizes total net benefit $\bar{p}X(A(X)-c)$. As $\bar{p}$ is normalized to one, X^* satisfies

$$A(X^*)+X^*A'(X^*)-c=0. \tag{1}$$

Since the Novshek condition holds, X^* is unique. This aggregate effort level is the goal we seek to achieve by decentralization through our Partnership Solution.

Let $n\leqslant N$ denote the number of distinct partnerships formed at the first stage and index these groups $i=1,\dots,n$. Then, in the second stage, agents simultaneously choose their effort after observing each agent's choice of group. To verify that the Partnership Solution is subgame perfect, we must show that it forms a Nash equilibrium in every subgame. We demonstrate this through backwards induction, considering the problem of effort choice first.

[6] In assuming that N is exogenous, we are abstracting from the entry of outsiders. In reality, partnerships have also been effective in protecting their own territories from outsiders. For an interesting analysis of how this has been accomplished see Acheson and Gardner's [1] discussion of the Maine lobster fishery.

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2.1. Equilibrium effort choice in second-stage subgames

Consider second-period subgames in which individuals grouped into partnerships simultaneously choose their effort levels. An individual in group i would choose his own effort level (x_{ik}) taking as given the aggregate effort level of his colleagues in partnership i ($Y_i^{-k} = \sum_{l \neq k} x_{il}$) as well as the aggregate effort levels of the other partnerships (X_{-i}). Hence, he would maximize

$$\pi_{ik} = \underset{x_{ik}}{\mathrm{Max}} \left\{ \frac{1}{m_i} \left[\frac{x_{ik} + Y_i^{-k}}{X} \right] F(X) - cx_{ik} \right\},$$

where $X = x_{ik} + Y_i^{-k} + X_{-i}$ and m_i is the number of partners in his group.[7] This is equivalent to maximizing:

$$m_i \pi_{ik} = (x_{ik} + Y_i^{-k}) A(x_{ik} + Y_i^{-k} + X_{-i}) - m_i c x_{ik}. \tag{2}$$

To find the best response of member k in partnership i, we differentiate the objective function (2) with respect to x_{ik} and substitute $X = x_{ik} + Y_i^{-k} + X_{-i}$ to arrive at the following N first-order conditions:

$$A(X) + (x_{ik} + Y_i^{-k}) A'(X) = m_i c \quad \text{for } i = 1, \ldots, n \text{ and } k = 1, \ldots, m_i. \tag{3}$$

The N first-order conditions in (3) contain two effects, each of which leads each player i to reduce his effort: the "internalization effect" and the "diversion-of-benefits effect." First, since in a multiperson partnership, player i *receives* a share of the receipts generated by his partners, he would refrain from imposing as large a negative externality on them as he would if he operated solo. That is, the first factor in the second term is larger by Y_i^{-k} than it would be if he operated solo. This "internalization effect" would induce him to reduce his effort in a multiperson partnership even if $c = 0$ but the effect would disappear if under the rules of the partnership he received nothing from his partners. Second, since in a multiperson partnership, player i *relinquishes* a share of the benefits of his effort but pays the full cost of generating them, he would reduce his effort. That is, the right-hand side of the equation is $m_i > 1$ times as large as it would be if he were operating solo. This "diversion-of-benefits effect" would persist even if he received nothing from his partners but nonetheless had to give them a portion of his catch. This effect would disappear if $c = 0$.

Consider any solution to the N equations in (3) and the two equations defining X and Y_i^{-k}. Notice that if the efforts within any partnership are rearranged without altering their sum than each of these $N+2$ equations still holds. Formally, therefore, there are multiple Nash equilibria in the final stage of our game, with the payoff to player k lower in those equilibria in which he undertakes a larger effort. This multiplicity of equilibria in the second stage of our game is an artifact of our assumptions since introducing even the slightest convexity in the effort cost functions or the slightest weight on relative effort in the sharing rule would eliminate all of the asymmetric equilibria, leaving only the symmetric one. We therefore focus on the symmetric equilibrium by assuming as a "refinement" that agents anticipate at the first stage that efforts will be shared equally in any partnership they join.[8]

2.2. Partnership effects on effort choice

Since the first-order conditions in Eq. (3) depend only on the aggregate effort of partnership i and not on the efforts of its individual members, we can re-write the conditions in terms of $\bar{x}_i$, the mean effort in partnership i:

$$A(X) + m_i \bar{x}_i \cdot A'(X) - cm_i = 0 \quad \text{for } i = 1, \ldots, n. \tag{4}$$

These n equations plus the identity $X = \sum_{i=1}^{n} m_i \bar{x}_i$ uniquely determine the n mean effort levels $\{\bar{x}_i\}_{i=1}^{n}$ and X. We can solve Eq. (4) for $\bar{x}_i$, the mean effort level in group i:

$$\bar{x}_i = \left(\frac{1}{-A'(X)} \right) \left(\frac{A(X)}{m_i} - c \right). \tag{5}$$

[7] We can now formally show the connection between the common-property problem and the two other applications discussed in the Introduction. Let x_{ik} denote the rent-seeking outlay of player k in group i, Y_i^{-k} denote the outlays of his partners, X denote the total outlay of all contestants, and $F(X)$ denote the prize. Then, assuming the probability that a group wins the prize is proportional to its rent-seeking outlays and the prize is divided evenly among members of the winning group, the foregoing objective function of player k is simply his expected payoff from an outlay of x_{ik}. Similarly, consider an innovation tournament in which player k in group i makes research effort x_{ik}. Following Baye and Hoppe [4], conceive of each researcher's discovery effort as his drawing ideas worth y if awarded a patent ($y \in [0, U]$, where U is an exogenous highest conceivable value) from a differentiable probability distribution $G(y)$. Let x_{ik} denote the integer number of times ideas are drawn with replacement from the distribution. If each draw costs the researcher c, then researcher k's cost would be cx_{ik}. Moreover, if the prize goes to the partnership with the highest realized draw, then the probability that partnership i wins equals its aggregate number of draws as a percentage of the total number of draws of all players in the tournament (the factor in square brackets). Finally, the expected value of the prize is a strictly increasing, strictly concave function of the total number of draws [4, Theorem 1], as our analysis assumes.

[8] In exactly these circumstances, Benoit and Krishna [6] have shown that many first-stage configurations can be supported as subgame-perfect equilibria. To clarify what we are ignoring, suppose anyone deviating to a different partnership at the first stage anticipated that, as "hazing" reserved for new members, his new colleagues would undertake no effort and would make him carry the entire effort burden of the partnership by himself. The anticipated situation would deter his deviation and is technically "credible" since it is a Nash equilibrium. While exploiting this aspect of our problem would allow us to show that the Partnership Solution is stable in a wider set of circumstances, we refrain from doing so.

If partnerships of different sizes form at the first stage, then their mean effort levels will differ at the second stage. In particular, since the second of the positive factors in Eq. (5) decreases in m_i, we conclude:

Proposition 1. *In any equilibrium, strictly larger groups have strictly smaller mean effort levels.*

Intuitively, the larger the group, the more free-riding occurs within it.

Next we verify that *aggregate* effort in the second-stage depends only on the *number* (n) of groups formed at the first stage and not on the distribution of agents among the different groups:

Proposition 2. *Aggregate effort (X) in the second stage depends only on the number of groups formed in the first stage and not on the size of those groups.*

Proof. Summing the n first-order conditions in Eq. (4) yields[9]

$$nA(X) + XA'(X) - cN = 0. \tag{6}$$

Thus aggregate effort (X) induced in the Nash equilibria of second-stage subgames depends only on the number of groups formed at the first stage and not on the specific partition. □

A monotonic relationship exists between the number of partnerships formed at the first stage and the aggregate effort expended at the second stage.

Proposition 3. *If the number of groups formed at the first stage is strictly larger, the aggregate effort level at the second stage is strictly larger.*

Proof. Differentiating Eq. (6) implicitly, we obtain

$$\frac{dX}{dn} = \frac{A(X)}{-[(n+1)A'(X) + XA''(X)]} > 0,$$

where the inequality follows from $A(X) > 0, A'(X) < 0$, and the Novshek condition. □

Since aggregate effort in our game is a continuous, strictly increasing function of the number of groups formed at the first stage and since $n = 1$ induces too little aggregate effort and $n = N$ generates too much, some unique intermediate number of groups will induce the socially optimal level of effort at the second stage. We can find this number by plugging X^* into Eq. (6) and then solving for n^*.

Proposition 4. *If $n^* = 1 + [c(N-1)/A(X^*)]$ groups form at the first stage, then the aggregate effort chosen in the Nash equilibrium of the second stage will be socially optimal.*

Proof. Substitute $n^* = 1 + [c(N-1)/A(X^*)]$ and X^* into Eq. (6). This gives us

$$\left(\frac{c(N-1)}{A(X^*)} + 1\right)A(X^*) + X^*A'(X^*) - cN = 0.$$

Simplifying, we obtain

$$A(X^*) + X^*A'(X^*) - c = 0,$$

which is the same as Eq. (1), the condition defining X^*. □

Proposition 4 implies that whenever $c = 0$, the social optimum is achieved by putting everyone into a single partnership ($n = 1$), while for $c > 0$, the optimum is achieved by dividing them among several partnerships. To understand these results, suppose each of the N players joins a single partnership and everyone's effort equals $1/N$th of the aggregate profit-maximizing level. If costs are zero, aggregate revenue would also be maximized so that if someone marginally reduced his effort unilaterally, total revenue would not change nor would his share of it. Since deviating is unprofitable, the social optimum is achieved as a Nash equilibrium. On the other hand, if everyone is assembled into a single partnership and asked to undertake $1/N$th of the socially optimal effort when $c > 0$, there is an incentive to reduce effort. In this case, such a reduction does not affect aggregate profit because aggregate gross revenue and aggregate costs fall by equal amounts. However, since all of the cost savings accrue to the deviator while only $1/N$th of the aggregate revenue loss is borne by him, he has a strict incentive to reduce his effort and it is no longer possible to achieve the social optimum with a single partnership.

When $c > 0$, therefore, the Partnership Solution requires dividing the N players among *several* partnerships. Suppose again, each player undertakes $1/N$th of the socially optimal effort. If someone marginally reduced his effort unilaterally, he would save the same costs as before but now would incur a larger loss in revenue both because his partnership's losses are larger and because he bears a larger share of those losses.

[9] Our proposition reinterprets the result that, in an interior equilibrium of a Cournot oligopoly model with constant marginal costs, aggregate output depends only on the sum of the marginal costs [7].

In the Partnership Solution ($n = n^*$), the social optimum is achieved as a Nash equilibrium because each person fully anticipates the *social* consequences of his deviation. If anyone increased his effort, he would incur the entire cost increase. Moreover, since the gross revenue of his colleagues would expand by exactly as much as the gross revenue of non-colleagues would contract, the deviator's share of his partnership's increased revenue would just equal the aggregate gain in revenue from his deviation.[10]

2.3. Partnerships and Pigouvian taxes

We conclude this section by clarifying the relationship between partnerships and Pigouvian taxes. Partition the N players in any way into n partnerships and compute the Nash equilibrium in the simultaneous-move effort game. We could dissolve one or more of these partnerships and instead tax the effort of each of its members at the group-specific, constant rate $\tau_i = (1-1/m_i)A(X)$. In response to the taxes, each member of a group would choose $\hat{x}_i$ to maximize $\hat{x}_i A(\hat{x}_i + X_{-i}) - (c+\tau_i)\hat{x}_i$ by solving $A(X) + \hat{x}_i A'(X) = c + (1-1/m_i)A(X)$, or $A(X) + m_i \hat{x}_i A'(X) = m_i c$ (as $\sum_{i=1}^{n} m_i \hat{x}_i = X$). But this is just Eq. (4) and so $\hat{x}_i = \bar{x}_i$; taxing the groups in this way induces the same effort as when every group was a partnership. Notice that whereas before effort declined with partnership size because of free-riding, now it declines with tax-group size because larger groups are subjected to higher tax rates.

While dissolving a partnership and subjecting each of its former members to such taxes would not alter equilibrium effort levels, it would alter equilibrium payoffs. In particular, every member of group i would earn $\tau_i \bar{x}_i$ less when taxed than as a member of a partnership. Hence, unless all of the tax revenues are rebated, participants would strictly prefer the Partnership Solution to the Pigouvian tax scheme.

3. Partnership choice in the first stage: equilibria and Implementation

In the first stage, N players simultaneously choose partnerships, resulting in the formation of between 1 and N groups. We now investigate whether any agent would have an incentive to deviate from the chosen partnership structure. When no such incentive exists, the partnership structure will be part of a subgame-perfect Nash equilibrium.

3.1. Equilibrium in the first stage

There are two possible deviations at the first stage: an agent can abandon the colleagues in his prescribed group for the members of some other group or he can abandon his prescribed group to go into business for himself. The following proposition allows us to eliminate as potential subgame-perfect equilibria most partnership structures which could form at the first stage.

Proposition 5. *A necessary condition for a partnership structure to be part of a subgame-perfect Nash equilibrium is that no partnerships differ in size by more than one member.*

Proof. From Proposition 2, a deviation which maintains the number of groups formed at the first stage will not alter second stage aggregate effort, X^*. If each player in group i exerts mean effort $\bar{x}_i$ when aggregate effort is X, then, from (2) his payoff is

$$\pi_i = \bar{x}_i (A(X^*) - c). \tag{7}$$

This is strictly increasing in $\bar{x}_i$ since $A(X^*) - c > 0$. Each member's payoff is larger in groups with a larger mean level of effort. From Proposition 1, a group with a smaller number of members will have a larger mean effort. Hence, any player can strictly improve his payoff if he can switch to a partnership that will be smaller after he joins it than his existing partnership. Then profitable defections are only possible when two partnerships differ in size by two or more members. □

There remain N partnerships structures to consider as potential subgame-perfect Nash equilibria; for any n $(=1,\ldots,N)$ there will be a *unique* partnership structure where sizes differ by at most one member. To visualize this, imagine that the N players are being dealt sequentially like cards in a card game to the n partnerships arranged around a table. When all of the players have been dealt out (assigned to partnerships) some of the partnerships may have one fewer player than rest. The Partnership Solution with its n^* groups is among these potential equilibria.

[10] In the Partnership Solution, each player "internalizes" the aggregate effects of his actions. If any player deviates by increasing his effort marginally above its equilibrium level in the Partnership Solution, he imposes losses on those outside his partnership ($X_{-i}A'$) as well as gains on the others within his partnership ($((m-1)/m)[A + (X - X_{-i})A']$). However, when $n = n^*$ these externalities net to zero: $((m-1)/m)[A + (X - X_{-i})A' + X_{-i}A'] = 0$. As a result, each player imposes, in aggregate, neither a positive nor a negative externality on other players in the game: all net social benefits of a deviation are internalized. Rearranging the previous expression, we obtain $(1/m_i)[A + (X - X_{-i})A'] = A + XA'$. So when any player sets his own marginal private benefit (the left-hand side of this last equation) equal to the marginal social cost, he is also equating the marginal social benefit (the right-hand side of this equation) to the marginal social cost.

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Any of the remaining partnership structures will be subgame perfect if no agent can strictly improve his expected payoff by forming a new, singleton, group. Whether this is profitable or not depends upon the disadvantage of solo production compared to team production. The literature on the theory of the firm emphasizes that multiagent firms are rife with incentive problems to which single-agent firms are immune. But, since multi-agent firms abound, there must be a countervailing advantage to such arrangements—individuals working in teams must be able to produce more output per man-hour than those working alone.[11] Thus, we consider the possibility that a team can produce more than an individual working by himself the same number of man-hours; in extreme cases, a team may be necessary in order to produce at all.[12] Suppose that to duplicate the efforts of 1 man-hour of team effort, a single individual must work $1/\beta$ hours, for $\beta \in [0,1]$. The marginal cost of effort for an individual working alone is then $(1/\beta)c$.

Partition the N players into n groups in such a way that no two groups differ in size by more than 1 member (recall that for any n there is a unique partition that satisfies this restriction). From Proposition 1, if some partnerships are one member larger than others, these larger partnerships will generate more free-riding in the equilibrium of the second stage. Anticipating lower payoffs in the second stage, every member of a larger partnership would have a stronger incentive to deviate to a solo partnership at the first stage. Let $g(n,\beta)$ denote the gain a member of a larger partnership would achieve by setting up his own partnership. If $g(n,\beta)\leqslant 0$ then he has no incentive to deviate and *a fortiori* neither does any member of a smaller partnership; hence the partition under consideration is stable. If, however, $g(n,\beta)>0$ then he has an incentive to deviate and the partition under consideration is unstable. By analyzing properties of the $g(\cdot,\cdot)$ function, we show below that for any n, including n^*, there is a unique $\beta(n) \in (0,1]$ such that the Partnership Solution is stable for all $\beta \leqslant \beta(n)$.

3.1.1. *Team production is essential* ($\beta = 0$)

In many applications, "it takes two workers to perform a given task" (Holmstrom and Tirole [17, p. 67]); solo production is infeasible. For example, no matter how hard a person works he/she cannot catch a whale by himself; nor can he/she stay awake every day and night of his medical career to help patients with their medical emergencies. In other applications deviating to solo groups may be illegal since many partnership agreements contain "non-compete" clauses which prevent an individual, when leaving a partnership, from competing in the same market as the group he is leaving.[13] Whenever solo production is infeasible, $g(n,0)<0$ and we have:

Proposition 6. *When solo production is infeasible, the Partnership Solution is subgame perfect and solves the common-property problem.*

Proof. Since $g(n,0)<0$, no deviation to a solo partnership is profitable for any n, including n^*. □

If solo production is infeasible there will be other subgame-perfect equilibria besides the Partnership Solution. At the end of the section, we discuss two mechanisms for implementing the equilibrium inducing socially optimal effort.

3.1.2. *Solo production is feasible* ($\beta \in (0,1]$)

If solo partnerships are legal and feasible and $\beta<1$, then social welfare can never be maximized as long as any solo partnership is involved. Even if a partition with solo partnerships resulted in equilibrium optimal effort, X^*, the cost of achieving it will strictly exceed cX^*, which a planner could achieve by assembling a team of all N players and commanding that level of effort. Letting $\lfloor Z \rfloor$ denote the greatest integer less than or equal to Z, we assume that $n = 1,2,\ldots,\lfloor N/2 \rfloor$ partnerships.

Eq. (6) implicitly defines the aggregate effort which would result from n partnerships, each of which has two or more members. Denote the aggregate effort implicitly defined by this equation as $X(n)$. If $X(\lfloor N/2 \rfloor)\geqslant X^*$, then the Partnership Solution can potentially achieve the first best by generating more free riding and thereby bringing effort down toward X^*.

Denote the payoff of a potential deviator, prior to his deviation, as π^C and his payoff if he goes solo as π^D. A partner who deviates gains $g(n,\beta) = \pi^D - \pi^C$. While π^C is independent of β, his gain from going solo, his effort, everyone else's effort, and aggregate effort, will depend on the parameter β. Define $\underline{\beta}$ such that for any $\beta > \underline{\beta}$, the deviator going solo would make strictly positive effort while for any smaller β he would make zero effort. When $\beta \in [0,\underline{\beta}]$, the deviator would receive a zero payoff ($\pi^D = 0$) following his deviation; thus, $g(n,\beta) = g(n,0) = -\pi^C < 0$ for any $\beta \in [0,\underline{\beta}]$. When $\beta \in (\underline{\beta},1]$ the consequences of one agent's going solo are described by the four variables π^D, X, X_{-1}, and $\bar{x}_1$ which are defined by Eqs. (8)–(11) below, where for simplicity we assign the index "1" to the deviator's solo partnership (and therefore denote his effort as $\bar{x}_1$ and the

[11] The importance of team production in the theory of the multiperson firm was first emphasized by [2] and their insights have now percolated down to undergraduate treatments of that theory. For extensive discussion, consult [14] or [10].

[12] Alternatively, there could be subjective "social benefits" to remaining in a team [23].

[13] Various courts have upheld such clauses, including the Georgia Supreme Court in Rash v. Toccoa Clinic Med. Assoc., 253 Ga. 322, 320 S.E.2d 170 (1984).

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aggregate effort of all others after observing his first-stage deviation as X_{-1}):

$$\pi^{D} = \bar{x}_1\left(A(X) - \frac{c}{\beta}\right), \tag{8}$$

$$A(X) + \bar{x}_1 A'(X) - \frac{c}{\beta} = 0, \tag{9}$$

$$nA(X) + X_{-1}A'(X) - (N-1)c = 0, \tag{10}$$

$$\bar{x}_1 + X_{-1} = X. \tag{11}$$

Eq. (10) is obtained by adding up the first-order conditions of the n original partnerships (that is, excluding from the sum the solo partnership) after effort levels in the following stage have adjusted in response to the deviation.

Proposition 7. *$g(n,\beta)$ is a differentiable function of β in the neighborhood of any β in $(\underline{\beta},1]$ and $g_\beta(n,\beta) > 0$ for any β in $(\underline{\beta},1]$.*

Proof. See Appendix A.

The following lemma establishes that there is no discontinuity in g at the boundary $\beta = \underline{\beta}$.

Lemma 1. *The function $g(n,\beta)$ is continuous in β at the point $\underline{\beta}$.*

Proof. Since $g(n,\beta) = -\pi^{C}$ for $\beta \in [0,\underline{\beta}]$, it suffices to verify that $\lim_{\beta\to\underline{\beta}} g(n,\beta) = \lim_{\beta\to\underline{\beta}}(\pi^{D} - \pi^{C}) = -\pi^{C}$. But this follows from (8) since $\lim_{\beta\to\underline{\beta}}\bar{x}_1 = 0$ and A is bounded. □

We can therefore, conclude:

Proposition 8. *If the partition indexed by n is stable for some β, then it is stable for all smaller β.*

Proof. This follows from Proposition 7. □

We now use the results above to prove the existence and uniqueness of a "threshold" $\beta(n)$ which separates stable from unstable partitions.

Proposition 9. *For any $n \leqslant \lfloor N/2 \rfloor$, there exists a unique $\beta(n) \in (\underline{\beta},1]$ such that for any $\beta < \beta(n)$, the partition indexed by n can be supported as a subgame-perfect equilibrium, while for $\beta > \beta(n)$ the partition can never be supported.*

Proof. For any given n, suppose that at $\beta = 1, g \leqslant 0$. Then that partition can be supported as a subgame-perfect equilibrium for any $\beta \in (0,1]$ and we can define $\beta(n) = 1$. Now suppose that at $\beta = 1, g > 0$. Since $g(n,\beta)$ is continuous there will exist one or more roots, $\beta \in (0,1)$, such that $g(n,\beta) = 0$. Since g is strictly increasing (Proposition 7), there can be only one root, which we denote $\beta(n)$. □

This proposition makes precise the intuitive notion that the socially optimal partnership partition is stable if solo production is "sufficiently disadvantageous": solo production need not be infeasible ($\beta = 0$) but β cannot be strictly larger than $\beta(n^*)$. Alternatively, for any given $\beta, \beta(n)$ also defines partnership partitions which are stable: $\{n : \beta(n) > \beta\}$.

Is the Partnership Solution *always* stable even when team production confers no advantage whatsoever ($\beta = 1$)? No. Suppose $N = 12$ producers in an industry, each with constant marginal cost of $c = 3$, face an inverse demand curve of $P = 19 - X$ and attempt to achieve monopoly profits by dividing into $n = 4$ partnerships of equal size. It is easily verified that for any $\beta \leqslant .39$, full monopoly profits (\$64) can be achieved, but for $\beta > .39$ the configuration of four partnerships is unstable.

3.2. Implementation of the Partnership Solution

We consider two methods of implementing the Partnership Solution. Both methods designate the formation of a specific number of partnerships and assume that players will be assigned to these partnerships in such a way that sizes differ by at most one member.

The first method utilizes a coordination device first proposed by Schelling [26, p. 63, 302]. An outside "mediator" could recommend publicly to all the players the establishment of n^* partnerships. By assumption, this mediator could neither monitor nor enforce compliance with his suggestion. His only function would be to focus the expectations of the players on the payoff-dominant equilibrium.[14] While subjects in two-player experiments disregard such recommendations if they are not Nash or are Nash but not payoff dominant [18], they follow the recommendations which are Nash and payoff dominant 98% of the time. Moreover, in two-stage experimental games, the mediator is equally influential in the selection of subgame-perfect equilibria [8].[15]

[14] In certain settings, one could imagine a regulatory authority having the further ability to enforce the Partnership Solution, say in a fishery. In this case, there is no need for the Partnership Solution to be "stable" in the sense described above.

[15] See Camerer [9, pp. 362–365] for discussion. Further evidence that pre-play communication can secure the payoff-dominant profile is provided by "cheap talk" experiments where the players themselves communicate prior to play without benefit of a mediator [13].

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The second method involves a modification of the first stage of the game. Each agent could simply specify the number of partnerships he prefers and the proposal of one agent, selected randomly, would be implemented. Since the socially optimal number of partnerships is also privately optimal for each agent, and this selection mechanism makes each agent pivotal with positive probability, each agent should specify n^*.[16]

4. Generalizations using the same sharing rule

Until now, we have assumed that no costs were shared within a partnership and no individual or partnership had the power to change the price of output. According to Platteau and Seki [24], however, even in the case of the Japanese fishermen, neither of these simplifications is entirely realistic [24]. Some costs are shared among the members of each partnership. Moreover, although Japanese fisherman primarily partition themselves into partnerships to reduce congestion, a secondary reason is to raise prices: "Fishermen believe that by limiting effort they can cause fish prices to rise." Statistical analysis of price data confirmed this effect [24].

To relax these two maintained assumptions, we partition N homogeneous agents into n payoff-sharing groups indexed by i, each playing a simultaneous-move game, where agent k in group i chooses x_{ik} to maximize $(1/m_i)[x_{ik}+Y_i^{-k}]G(x_{ik}+Y_i^{-k}+X_{-i})-cx_{ik}$. If we make the same assumptions about $G(X)$ that we made about $A(X)$ then we will get the corresponding results. So assume that $G(X)$ is strictly positive, strictly decreasing, and twice continuously differentiable; $G(0)-c>0$; and $G'(X)+XG''(X)<0$, holds for all $X\geqslant 0$. These assumptions are sufficient to insure the existence of a pure-strategy Nash equilibrium in the simultaneous-move game. Because $G(\cdot)$ is downward-sloping, there is a negative externality: agent k is adversely affected by increases in X_{-i}. We have derived conditions sufficient for the aggregate payoff, $X(G(X)-c)$, to be maximized: provided $n^*\leqslant\lfloor N/2\rfloor$ and $\beta<\beta(n^*)$, the optimum can be achieved by setting up n^* partnerships differing in size by at most one member.

Suppose $G(X)=A(X)-K$, where K denotes cost per unit effort for those costs shared within the partnership. Then the Partnership Solution maximizes producer surplus. Since price is constant, this maximizes social welfare as well.

Next suppose $G(X)=P(F(X))A(X)-K$, where $P(\cdot)$ is the industry price when aggregate output $F(X)$ is put on the market. This generalization fits the case of the Japanese fishermen, who share some but not all costs and who use their partnerships not merely to curb congestion but to raise price. Again, the Partnership Solution maximizes producer surplus.

Finally, suppose $G(X)=P(X)-K$, where X is now interpreted as *output* and K (respectively, c) as the cost per unit *output* rather than effort, which is shared (respectively, not shared) within the partnership. In this case, there is no congestion externality and hence no common property problem. The Partnership Solution can be used to curb excessive output and permits a cartel to maximize profits without any need for supergame strategies. An outside observer would simply see an industry with a collection of firms organized as partnerships in competition with one another.

In cases where $\beta>\beta(n^*)$, the advantages of team production are insufficient to achieve the first-best using the Partnership Solution. In such cases, a generalization of the Partnership Solution can nonetheless lead to a *second-best* equilibrium with a large increase in the aggregate payoff. To illustrate, recall the example where $N=12, c=3$, and $G(X)=19-X$. In that case $n^*=4$ and $\underline{\beta}=.39$. Suppose that $\beta=.56>.39$. While dividing the agents into four partnerships of equal size is not feasible (since each member would have an incentive to go solo) if the 12 agents are divided into six partnerships of equal size industry profit is then \$54.12—not the first-best level of 64 but approximately *triple* the result in the common property (or oligopoly) solution.

5. Generalizing the sharing rule

In any game, one must specify the feasible strategies of each player and the payoff each receives for any strategy profile. In each second-stage subgame, we assume that each member of a partnership anticipated that he would receive an equal share of the gross revenue accruing to the partnership, and that there is no opportunity for players to deviate from this sharing rule *ex post*. This can, most easily, be justified by assuming that the sharing rule is contracted at the time of group formation, in which case agents are legally obligated to comply.[17] We now generalize our analysis to other sharing rules.

In the rent-seeking game of Nitzan [21], the group winning the fixed prize divides it using a sharing function S_{ik} which is an exogenous weighted average of our egalitarian rule and a rule rewarding relative effort:

$$S_{ik}=(1-a)\frac{1}{m_i}+a\frac{x_{ik}}{(x_{ik}+Y_i^{-k})}, \tag{12}$$

where $a\in[0,1]$.[18] We have so far restricted attention to the egalitarian case ($a=0$) because it is simpler and does not require that members perfectly and costlessly monitor each other's efforts. When such monitoring is possible, however, we can show that our results generalize. For any $a\in(0,1]$, it is straightforward to verify that any partition of players into

[16] Unlike the first method, this method achieves the Partnership Solution even in cases where, in the original game, going solo would be profitable.

[17] Another possible justification is that even if one of the partners were able to seize the entire allocation of the group, each partner would be equally likely to be the thief, and each would, in expectation, get an equal share of the total.

[18] Sen [28] previously used this weighted-average rule when discussing sharing within cooperatives.

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groups generates a Nash equilibrium in the second stage in which every member of a larger group makes smaller effort and receives a smaller payoff. Therefore, as in our model, the groups formed at the first stage will differ in size by at most one member.[19]

If member k of group i makes effort x_{ik}, his payoff is $S_{ik}(x_{ik}+Y_i^{-k})A(X)-cx_{ik}$. Summing each player's best reply, we obtain an equation linking effort to the number of groups and the weight on the relative effort component of the sharing rule[20]

$$KA(X)+XA'(X)-Nc=0, \tag{13}$$

where $K=aN+(1-a)n$. If sharing is purely egalitarian ($a=0$), this reduces to Eq. (6). As before, aggregate effort is strictly increasing in the number of groups and independent of the distribution of players among these groups. However, with this more general sharing rule, decreasing the weight (a) on relative effort provides a second channel through which to stimulate free riding and reduce aggregate effort.

Using our egalitarian sharing rule ($a=0$), we previously found that aggregate effort could be reduced to the socially optimal level (X^*) by reducing the number of groups from N to n^* defined in Proposition 4. Eq. (13) can be used to generalize the Partnership Solution when the sharing rule puts more weight on relative effort. Since n^* groups would generate socially optimal effort (X^*) when $a=0$, the number of groups ($n(a)$) which would achieve the social optimum when more weight is put on relative effort is

$$n(a)=\frac{n^*-aN}{1-a}. \tag{14}$$

As a increases, $n(a)$ decreases until, at $a^*=(n^*-1)/(N-1)$, a single group is required to achieve X^*. Partnerships can solve the common property problem only if $a\in[0,a^*]$.

6. Conclusion

In this paper, we showed how the free-riding induced in partnerships can be harnessed to increase payoffs when aggregate effort or output would otherwise be excessive. We showed how this idea can be applied to curb excessive extraction from common properties or excessive production from cartels. The same mechanism can achieve the social optimum in innovation tournaments and rent-seeking contests with variable prizes. Inducing limited free-riding may be beneficial in other contexts as well. For example, tips in some restaurants are *separately* solicited by various team members whose combined effort makes a dining experience pleasurable: the maître d', the sommelier, the waiter, the musician, the coat-room provider, the parking valet, etc. In such situations, aggregate effort per customer may be excessive and pooling tips among subsets of these service providers (an increasingly common practice in restaurants) can be used to raise their net payoffs. Japanese fishermen who have formed partnerships report that pooling revenues reduces congestion and raises price. As we have seen, these are consequences to be expected from such partnerships.

While there is a temptation in the Partnership Solution to flee one's free-riding partners by going solo, other forces often act as a counterbalance. Going solo is not attractive when there are sufficient benefits from team production, fixed costs of setting up a solo practice, or social benefits to remaining in existing groups. In such circumstances, the Partnership Solution can be used to maximize or at least to raise significantly the aggregate payoff.

Throughout, we assumed that a partnership had to admit every applicant. It might have been more realistic to assume that members of an existing partnership could deny admission to anyone if opposition to him within the partnership was "sufficiently widespread." This change in assumption would in fact have *increased* the scope of the Partnership Solution. For, every solution we identified as stable would continue to be stable since no one in such solutions has any incentive to join an existing partnership even when assured of admission. But partitions we identified as unstable under our old assumption would become stable under this new assumption. To illustrate, suppose going solo was infeasible and we set up n^* non-solo partnerships some of which differed by two or more members. Such an arrangement could not achieve the first-best under our old assumption because every member of the largest partnership would deviate unilaterally to a smaller partnership with less free-riding. But this same arrangement *would* achieve the first-best under the new assumption since admitting him would be blocked unanimously by existing members who anticipated that expanding the number of partners would stimulate free-riding and would lower each of their payoffs.[21] In assuming that no applicant could be rejected by existing members, therefore, we *understated* the usefulness of partnerships in solving the common-property and cartel problems.

It is natural to ask if partnerships can ever solve or ameliorate the common-property problem when agents differ in their constant marginal costs. The answer, surprisingly, is yes. Suppose N agents have marginal cost c and are grouped into

[19] As with the egalitarian rule, there will also be an incentive to deviate by going solo unless solo production is either impossible or sufficiently costly compared to production with teams of two or more members.

[20] The best reply (x_{ik}) solves $[(1-a)(1/m_i)+a]A(X)+[(1-a)(1/m_i)+ax_{ik}/x_i]x_iA'(X)-c=0$, where x_i denotes aggregate effort in partnership i. For any $a>0$, this equation implies that in equilibrium every individual in the same group makes the same effort. This explains why we *assumed* equal efforts when $a=0$ even though in that polar case individual effort levels in a group are indeterminate.

[21] Either the anticipation of rejection by one's new colleagues or of having to carry a disproportionate share of the new partnership's workload (see footnote 8) could explain why the pools of Japanese fishermen persist even with sizes differing by more than one member.

the socially optimal number of partnerships (n^*) of equal size. Suppose we now add σn^* more agents and assume they have a very high common marginal cost ($c_h \gg c$). Put σ of them in each of the n^* partnerships so that each group has the same composition. The social optimum is to have none of the new workers active and to generate the unchanged aggregate effort, X^*, from the more efficient workers. Suppose, for simplicity, that the common marginal cost of the additional agents is so high that they never have an incentive to work in the decentralized solution. Even so, their presence still has an effect. Because each more efficient worker now receives a smaller share of the gross revenue of his partnership, he has an incentive to work less hard. To enlarge aggregate effort, however, we could group individuals into a *larger* number of partnerships, each with fewer people. In this way, it may be possible to *approximate* the social optimum and, upon occasion, to duplicate it.[22] Since every partnership has the same composition, no agent would have any incentive to switch unilaterally to another partnership.

As in the homogeneous case, there may be more than one subgame-perfect equilibrium and the same issue of selecting the best one arises. For simplicity, suppose one of these equilibria replicates the social optimum. In the heterogeneous case, the two types of agents will disagree. The free-loaders will strictly prefer to have more than the socially optimal number of partnerships because that would generate more output; after all, someone else is expending the effort to produce that output. But since social welfare weakly declines when output expands, the low-cost workers would be made worse off with more than the socially optimal number of partnerships; after all, it is they who must work harder to expand the output. Hence, low-cost workers want fewer partnerships than high-cost workers. This conflict between heterogeneous members of the partnerships is reminiscent of conflicts identified in the literature on heterogeneous cartels and common properties although in contexts without sharing it is the higher cost agents which prefer reduced output.[23] A more complete analysis of partnerships with heterogeneous agents must be left to future research.

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Appendix A. Proof of Proposition 7

For the first part (differentiability), since π^C is independent of β, it is sufficient to show that π^D is differentiable in β. Use Eq. (11) to eliminate X from Eqs. (8)–(10). Eq. (10) does not involve β. Given the Novshek condition, $(n+1)A' + X_{-1}A'' \neq 0$; therefore, the implicit function theorem insures that, in a neighborhood of any solution $(\bar{x}_1, X_{-1}, X)$ induced by $\beta \in (\underline{\beta}, 1]$ we can write Eq. (10) as $X_{-1} = f(\bar{x}_1)$ where $f(\cdot)$ is a continuous function with derivative $f' = -(nA' + X_{-1}A'')/\ ((n+1)A' + X_{-1}A'') \in (-1, 0)$. Eq. (9) does involve β. Replace X_{-1} in this equation by $f(\bar{x}_1)$. Given the Novshek condition and $A'<0$, $(1+f')(A' + \bar{x}_1 A'') + A' \neq 0$; therefore, the implicit function theorem insures that we can write Eq. (9) locally as $\bar{x}_1 = h(\beta)$ for some continuous function $h(\cdot)$ with derivative $h' = -c/\beta^2/(A' + (1+f')(A' + x_1 A''))>0$. Substituting both of these differentiable functions into Eq. (8), we obtain

$$\pi^D(\beta) = h(\beta)[A(h(\beta) + f(h(\beta))) - c/\beta].$$

Since $A(\cdot)$ is differentiable and since sums, products, and compositions of differentiable functions are differentiable, π^D is a differentiable function of β in a neighborhood of any solution $(\pi_D, \bar{x}_1, X_{-1}, X, \beta)$. Given this conclusion, there can be no $\beta \in (\underline{\beta}, 1]$ where π^D is non-differentiable. It follows that $g(n, \beta)$ is differentiable in the neighborhood of any β in the interval $(\underline{\beta}, 1]$.

For the second part (monotonicity), since $\bar{x}_1 > 0$ for any β in $\underline{\beta}, [1]$, $h(\beta) > 0$. Differentiating $g(n, \beta)$, using Eq. (9) to simplify, and recalling that $A' > 0$, we conclude that

$$g_\beta(n, \beta) = h'[A + hA' - c/\beta] + h[A'f'h' + c/\beta^2] = h[A'f'h' + c/\beta^2] > 0$$

for any β in $(\underline{\beta}, 1]$.

[22] It is straightforward to produce an example of a partnership equilibrium where agents are heterogeneous, *every agent* is active, and the social optimum is approximated. Simply reduce the higher marginal cost in our example until each freeloader makes a slight effort. Admittedly, it is impossible to *achieve* the social optimum when the high cost agents are active.

[23] In their classic paper, Johnson and Libecap [19] argue that because of heterogeneity "limits on individual effort are extremely costly to agree to and enforce" in common property environments, such as fisheries. Cave and Salant [11, Section III,B], discuss how such cost differentials create similar tensions within cartels which vote on their quotas. Finally, Tarui [30] finds, in an overlapping-generations framework relying upon punishment strategies, that heterogeneity in agent productivity reduces the ability of agents to achieve a sustainable harvest when harvests are shared equally amongst agents. Sherstyuk [29], in a special case of partnerships with heterogeneous agents and no effort choice, and thus no shirking, finds that partnerships can be stable and efficient, but only when they are segregated by "ability," in contrast to the results suggested here.

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