Market power, private information, and the optimal scale of pollution permit markets for North Carolina's Neuse River*

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Abstract

We extend the analysis of optimal scale in pollution permit markets by allowing for both market power and private information. The effect of these considerations on optimal scale is determined by analyzing pollution of nitrogen from Waste Water Treatment Plants (WWTP) into North Carolina's Neuse River System. An economic model of damages and abatement costs is integrated with a hydro-ecological model of nitrogen flow through the Neuse. We determine the optimal allocation number of trading zones and allocate the WWTP into these zones. For many combinations of parameters, both the market power and private information cases lead to lower total costs than competition. Under the most likely regulatory scenario, we find cost savings of 1.5 million dollars per year under the optimal market design relative to the typical 303 (d) regulation in which the WWTP are not allowed to trade.

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1 Introduction

Pollution permit markets are a firmly established policy tool for the control of pollution. These markets have been extensively studied in the academic literature and frequently applied to real pollution problems. There is, however, a critical gap in the analysis of these markets. When pollution is non-uniformly mixed and abatement costs are uncertain to the regulator, then the proper scale of the market is a fundamental market design issue. Should polluting firms be clustered into a few large trading zones, or is it better to have many smaller zones, each with a few firms?

Questions of this type have received surprisingly little attention.[1] Williams (2003) considers the problem of dividing a pollution permit market into a set of zones. Firms are allowed to trade within a zone, but not between the zones. Williams identifies a fundamental trade-off between abatement costs and damages. As the number of zones increases, total abatement costs increase because, with more zones, firms have fewer trading partners. But total damages decrease because more zones allow a tighter control on the spatial distribution of pollution which in turn reduces the severity of "hot spots". Krysiak and Schweitzer (2010) consider a framework similar to Williams, but also allow uncertainty about the location of the polluting firms and uncertainty about the spatial distribution of pollution. They determine the optimal number of zones and identify additional trade-offs related to the additional sources of uncertainty in their model. In both of these existing papers, it is assumed that the permit markets are competitive.

In this paper, we extend the literature on optimal scale in pollution permit markets in several ways. First, we consider the possibility that firms may exert market power. As the number of firms in a zone decreases, the competitive assumption becomes increasingly less tenable. In fact, a permit market with a small number of firms has an interesting market structure, as both buyers and sellers may exert market power. Recently several authors have

[1] Mendolsohn (1986) analyzes the choice between uniform and differentiated regulation, but does not explicitly model the mixing of pollution.

analyzed such markets (Lange 2008, Malueg and Yates 2009, Wirl 2009) and we incorporate their insights into the study of optimal scale. Once we move away from the competitive model, then other industrial organization issues such as private information may arise. In addition to the regulator, firms themselves may be uncertain about the abatement costs of the other firms, so that a given firm's abatement costs may be private information to that firm. Our second extension, then, is to allow for this private information and to study its effect on the optimal scale of the market. Of the papers that consider market power in permit markets, only Malueg and Yates (2009) allow for both market power and private information, so we utilize their model in our analysis.

Including market power and private information modifies the trade-offs in determining the optimal scale. For example, Malueg and Yates (2009) show that, relative to competition, market power leads to a decrease in the quantity of trading, which in turn leads to an increase in total abatement costs. Although they do not consider the effect on damages, the reduction in trade is likely to lead to a reduction in the severity of hot spots. This suggests that the net effect of market power may not necessarily be a decrease in welfare. It is possible that total expected costs of a market in which firms exert market power may in fact be lower than than the total expected costs of a market in which the firms are competitive, provided that both markets are optimized with respect to scale. In conjunction with the results from Williams (2002) and Kyrsiak and Schweitzer (2010), this suggests that the characteristics of a market with optimal scale will depend critically on the parameters of the specific pollution problem being analyzed.

This brings us to the last, and perhaps most important extension to the literature. We analyze the optimal scale for pollution markets in an actual ecological system. We consider emissions of nitrogen from waste water treatment plants (WWTP) into North Carolina's Neuse River System. To do this, we integrate a state-of-the-art hydro-ecological model of the spatial distribution of nitrogen through the Neuse, and make a determination of actual abatement cost functions for the WWTP. Using the resulting model, we determine the optimal market design. This includes specifying the optimal number of zones as well as the optimal assignment of specific plants into these zones. We show how this optimal structure depends on various assumptions about market power and private information.

The results of our analysis have important implications for water quality regulation. The Neuse River is classified as section 303(d) impaired water under the Clean Water Act. The typical regulation of 303(d) impaired waters is fairly restrictive. The EPA issues a permit to each treatment plant which specifies a maximum emission level from that plant. There is no trading of permits among the treatment plants (although some trading with non-point sources is allowed in specific cases). In the specific case of the Neuse, the state of NC and the EPA, in conjunction with the waste treatment plants, have crafted a more flexible regulation that does allow some trading between a group of twenty two water treatment plants collectively called the Neuse River Compliance Association (NRCA). Our analysis of the Neuse takes this positive development as a starting point. Given that trading is allowed, what is the optimal scale for this trading? The answer to this question is our market design for the Neuse. We show that this optimal design can lead to significant decreases in total costs to society relative to the typical section 303(d) regulation.

2 Overview

Although our main goal in this paper is determining the optimal market design for the Neuse River System, we develop the model in a general manner so that it can be easily adapted to other pollution problems. Accordingly, let there be m spatially distributed firms that generate emissions of pollution and n spatially distributed sites at which pollution is measured. The optimal market design minimizes the expected sum of abatement costs and damages. Abatement costs are incurred by the firms as they decrease their emissions. We assume that these costs are uncertain to the regulator who designs the market. In particular, we consider abatement functions that depend on random variables with specified expected values and variances. The emission of pollution causes damages which are quantified at the measurement sites. In the case of the Neuse, damages are due to reduced water quality and other ecological effects of nitrogen. We do not explicitly model uncertainty about damages. This is not as restrictive as it may first appear. As is typical in models of pollution permit markets, if the uncertainty about damages is independent from the uncertainty about abatement costs, then the variance of the damage uncertainty will not influence the optimal market design. Under these conditions, the magnitude of the damage parameters can be implicitly interpreted as expected values of random variables.

Let the set of firms be given by $\{1, 2, 3, \ldots, m\}$, where each number refers to a specific firm. A particular market design is a partition of this set. For example, the partition

$$\{1, 3\}, \{2, 4 \ldots, m\}$$

defines two trading zones. The first zone contains firms 1 and 3 and the second zone contains the rest. Firms 1 and 3 may trade with each other, but they may not trade with the other firms. In one extreme, corresponding to the trivial partition $\{1, 2, 3, \ldots, m\}$, there is a single zone. All firms may trade with each other. In the other extreme, corresponding to the partition $\{1\}, \{2\}, \ldots, \{m\}$, there are m zones, which effectively precludes any trading (we call this the no-trading partition). The set of feasible market designs is equal to the set of all partitions of the set $\{1, 2, 3, \ldots, m\}$. The number of elements in this set increases rapidly in m (see Table 1).

To determine the optimal market design, we search over the feasible set. For a given partition, we determine the market equilibria in each zone (which will depend on the assumptions about market power and private information) and the resulting emissions of pollution. This determines expected abatement costs and damages and hence expected total costs for the partition. We repeat this process over all partitions and select the one with the lowest total costs. The final output will be the optimal market design, which not only specifies the

Table 1: Number of partitions as a function of number of elements in the set bell

m	partitions
4	15
8	4140
10	115,975
12	4,213,597
15	1.3×10^{9}
20	5.1×10^{13}

number of zones, but also assigns specific firms into these zones.

Our approach contains a number of assumptions which we now delineate and justify. The first set of assumptions reflect an implicit minimization of transactions costs. We assume permits trade on a one for one basis within a zone, and trading is not allowed across zones. Alternatively, one might consider defining trading ratios within a zone or even across zones. But this greatly increases the complexity of the problem from the point of view of the polluting firm. As it stands, they need only consider a single market and a simple trading rule in that market. Following Malueg and Yates (2009), we use a "net-trade function" to analyze the market equilibria in each zone.[2] Firms submit a net-trade function to the market maker. The net-trade function specifies how many permits the firms are willing to buy or sell at various prices. The market maker selects the equilibrium price such that overall net trades are zero. The net-trade function approach is just one particular way to model a market mechanism in which both buyers and sellers may exert market power. But net-trade functions are likely to have lower transactions costs than plausible alternatives such as bilateral trading.

In light of table 1, our second set of assumptions reflects an implicit minimization of analysis costs. We make these assumptions so that all partitions in the feasible set can be evaluated in a reasonable time on a personal computer. It is conceptually easy to relax most

[2]Net-trade function equilibria are an extension of Klemperer and Meyer (1989)'s supply function equilibria to the case in which both buyers and sellers exert market power See Hendricks and McAfee (2010) and Malueg and Yates (2009).

of these assumptions, but actually performing the computations would require additional computing power and/or more sophisticated search techniques. First, we restrict our attention to the ten largest WWTP in the NRCA. From table 1, this implies we must evaluate 115,975 partitions. These ten WWTP currently generate about 80 percent of the permitted emissions of Nitrogen from the entire set of 22 WWTP in the NRCA. Next, we assume that the spatial distribution of pollution is known with certainty.[3] We also assume that the abatement cost functions are quadratic and the slope of the marginal abatement cost functions is the same across firms.[4] Under these assumptions about the abatement cost functions, we can obtain an analytical solution for market equilibria. Relaxing them would require numerical solutions. The damage function is assumed to be quadratic as well. Turning now to the initial permit endowment, each firm is given the endowment that is optimal for the no-trading partition.[5] Finally, we do not consider uncertainty about the location of firms. In our specific problem, the location of the WWTP is relatively stable.

The details of the Neuse River including pollution sources and measurement sites is given in Figure 1. The map shows the actual geographic location for these elements and the schematic diagram shows their spatial relationship in a network system. In the schematic diagram, the lines represent the flow of water through the river system. The boxes represent the location of the WWTP and/or measurement sites. There are 10 WWTP. For example, the box labelled 1 is the first, and most upstream on it's branch, WWTP. There are 15 measurement sites. There is a measurement site corresponding to each WWTP. Measurement site 11 corresponds to Falls Lake reservoir. Measurement sites 12, 13, and 14 correspond to the confluence of two rivers. Site 15 is the estuary.

[3]We point out at the appropriate point in the development of the model how one might relax this assumption.

[4]Although many models of permit markets, including Williams (2002) and Krysiak and Schweitzer (2010), use quadratic functions, it may be more realistic to allow more flexible functional forms.

[5]Because the market equilibria may involve market power, such an endowment is not necessarily optimal for the other partitions, and so one might also consider optimizing over this dimension as well.

3 Model Components

The three main components of the model are abatement costs, damages, and the method of determining market equilibria.

3.1 Abatement Costs

Let the abatement costs for firm i be written as $C_i(\theta_i, e_i)$ where e_i is the emissions by firm i (for the Neuse, delineated in pounds of nitrogen per day) and θ_i is a parameter that influences costs. We assume that θ_i is known to firm i. To the regulator, and, in the case of private information, to the other firms, θ_i is random variable with expected value $\bar{\theta}_i$ and variance σ_i^2. Following Malueg and Yates (2009), the function C_i is quadratic and has the form

$$C_i(\theta_i, e_i) = \frac{\lambda}{2}\left(\frac{\theta_i}{\lambda} - e_i\right)^2. \tag{1}$$

acf

We interpret C_i as a total annual cost, and thus it includes both annual operation and maintenance costs as well as annualized capital costs. The marginal abatement cost function is

$$\frac{-\partial C_i}{\partial e_i} = \theta_i - \lambda e_i.$$

We interpret θ_i as the intercept of the marginal abatement cost function and λ as it's slope. The level of emissions that minimizes abatement costs is $\frac{\theta_i}{\lambda}$. This is sometimes referred to as the business-as-usual emission level.

To use these abatement cost functions in an actual market design problem, we specify a value for λ by utilizing an engineering cost study.[6] The engineering cost study is based on an analysis of inputs of energy, materials, and technology for a generic WWTP. The basic idea that underlies this approach is that all WWTP have access to the same general methods for

[6] Alternatively, one might conduct an econometric analysis. Examples of this approach are found in include Sado et al (2010), Fraas and Munley (1984), and McConnell and Schwarz (1992). Similar data is not readily available for the Neuse WWTP.

removing nitrogen, but abatement costs differ due to ideosyncratic differences in technical skills, input prices, and management at the various WWTP. This dovetails nicely with our theoretical model in which the regulator has imperfect information about firms abatement costs. Incorporating the ideosyncratic differences as random variables leads to a specification of $\bar{\theta}_i$. A value for σ_i^2 is not determined, but rather σ_i^2 is treated as a free parameter in the model. See the Appendix for the details of the engineering cost study.

3.2 Damages

Damages from pollution are quantified at measurement sites. Let the total pollution level at site j be given by z_j. The damage function is written as $D(z)$ where $z = (z_1, z_2, \ldots, z_n)$ is a $1 \times n$ vector of pollution levels. We assume that D is quadratic so it can be written as

$$D(z) = \frac{1}{2} z B z^t,$$

where B is a diagonal matrix. We interpret each element b_{jj} as the slope of marginal damage at measurement site j. Total pollution at site j is has two components. The first component x_j is due to the activity of the ten WWTP (as described below). The second component y_j is due to other point and non-point sources of pollution, which we assume are exogenous. Collecting the x_j and y_j into the vectors x and y, we have

$$D(x+y) = \frac{1}{2}(x+y)B(x+y)^t = \frac{1}{2} x B x^t + y B x^t + \frac{1}{2} y B y^t. \tag{2}$$

damm

Next we define a mapping of the spatial distribution of pollution. This mapping converts emissions of pollution at the polluting sources into levels of pollution at measurement sites. Let e be $1 \times m$ vector of emissions. We need to determine the $m \times n$ transfer matrix A such that

$$x = eA. \tag{3}$$

tfer

For our study of the Neuse, the transfer matrix A is based on a watershed-scale water quality model. In particular, we use a first-order nitrogen attenuation model based on the Spatially-Referenced Regression on Watershed attributes (SPARROW) model maintained by the USGS for southeastern river basins (Hoos and McMahon, 2009). The SPARROW model is a nonlinear regression which uses spatially-referenced watershed and stream channel characteristics to predict in-stream nutrient loads. The details of the calculation of A are given in the Appendix.[7]

Substituting (3) into the damage function (2) and simplifying gives

$$D(e) = \frac{1}{2}e(ABA^t)e^t + y(BA^t)e^t + \frac{1}{2}yBy^t.$$

This function is defined with respect to emissions of pollution at the polluting sources, and hence is commensurate with the abatement cost functions.

Finally, we need to determine explicit values for the b_{jj}. To keep the analysis simple, we assume that the $b_{jj} = b$ for every j. This implies that the damage function is the same at each measurement site, whether it be a location in the river, at a reservior, or at the estuary. Although recent regulation has focused solely on emissions in the estuary, there is evidence that emissions at other sites are now a matter of concern to regulators as well. Our model assumes that emissions at these other sites are of equal concern as the estuary, but it is easy to adapt it if these sites are of lesser concern. There are two methods for determining b. First, one could turn to the large literature on stated and revealed preferences and try to estimate a specific value for b. The second approach, and the one we adopt, is to use the context of the overall model to determine upper and lower bounds for b

First consider an upper bound for b. Let w be a $1 \times n$ vector of pollution emissions.

[7]Although we do not pursue this approach here, it is conceptually easy to add uncertainty about the spatial distribution of pollution. In this case, the elements of A are random variables. For the Neuse, one could use another model, called SWAT (see Gassman et al 2007), as the outputs of this model are indeed stochastic.

Consider the problem

$$\max_{w} E[\sum C_i(\theta_i, w_i) + D(w)], \tag{4}$$

where E indicates the expectation operator. As the magnitude of b increases, damages become increasingly more severe, and correspondingly, the magnitude of the elements of the optimal w decrease. Because C_i and D are quadratic, for some large value of the b, the magnitude of at least one of the elements of the optimal w will become zero. The critical value of b that causes this to happen represents the maximum value for the damage parameters such that it is economically efficient to allow all plants to emit positive amounts of Nitrogen. If damages were any more severe, then at least one firm should completely eliminate all emissions. As such, this critical value of b represent a useful upper bound on the magnitude of damages.

Next consider a lower bound for b. This is derived from the current regulatory environment. The Neuse River Estuary is classified as a 303(d) impaired water under the CWA, specifically as a nutrient sensitive water. In response, the state of NC in combination with the EPA constructed a limit on the total load of nitrogen at the estuary. The lower bound for b is constructed such that the total load of nitrogen at the estuary, as calculated from our model, is equal to the actual regulatory limit. In particular, for a given b, (4) specifies a w. Mapping the flow of emissions from this w through the river system gives a total load of nitrogen at the estuary. When this matches the actual regulatory limit, we have the lower bound for b.

3.3 Market equilibria

We now describe the market equilibria in the various trading zones.

Before trading occurs, firms are given an initial endowment of permits. As discussed above, we simply give the firms the optimal endowments for the no-trading partition. This is equal to the optimal w from the problem (4) for a specific value of b.[8]

[8] We have $w = (E[\theta] - yBA^t)(L + ABA^t)^{-1}$ where θ is the vector $(\theta_1, \theta_2, \ldots, \theta_m)$ and L is a diagonal

Consider a zone with $\ell > 1$ firms. Following Malueg and Yates (2009), market equilibria are defined through a net-trade function. The net-trade function is linear in the market price and contains one parameter a_i that is selected by the firms. The net-trade function is

$$v_i = \frac{1}{\lambda}(a_i - p) \tag{5}$$

val

Equation (5) specifies how many permits firm i is willing to buy (sell) as a function of the market price p. So firms select the intercept of the net-trade function, but the slope is fixed at $\frac{1}{\lambda}$. The firms report the net-trade functions to the market maker. The market maker selects the equilibrium price such that aggregate net trades are zero. So the equilibrium price is the solution to

$$0 = \sum_i \frac{1}{\lambda}(a_i - p).$$

Solving this equation for p yields the equilibrium price

$$p = \tilde{a}$$

where the "tilde" denotes the average across firms in the zone, so that $\tilde{a} = \frac{1}{\ell}\sum a_i$. In other words, the equilibrium price is equal to the average of the selected parameters.

Once the equilibrium price is specified, then the net-trade function combined with the permit endowment determines the emissions of pollution:

$$e_i = v_i + w_i.$$

For example, if the net-trade function, evaluated at the equilibrium price, is positive, then the firm buys permits in the market and thus they are able to emit more pollution than their initial endowment.

The details of how the firms select the parameters a_i depend on the market structure.

matrix with entries equal to λ.

3.4 Competition

The competitive case in not realistic when ℓ is small, but it serves as a useful baseline. Competitive firms take the market price as given. Firm i selects a_i to minimize the sum of total costs (abatement costs plus permit expenditures):

$$\min_{a_i} C_i(v_i + w_i) + pv_i.$$

The solution to this problem is

$$a_i^c = \theta_i - \lambda w_i,$$

where we use the superscript c to denote the competitive case. It follows that the equilibrium price is

$$p^c = \tilde{\theta} - \lambda \tilde{w},$$

where once again the "tilde" denotes the average across firms in the zone.

In equilibrium, then, the emissions of pollution for firm i are

$$e_i^c(\theta) = v_i + w_i = \frac{1}{\lambda}(a_i^c - p^c) + w_i = \frac{1}{\lambda}(\theta_i - p^c). \tag{6}$$

compete

The emissions of pollution depend on the equilibrium price, which depends on all of the θ_i's in the zone. So we use the notation $e_i(\theta)$ because the emissions of firm i may ultimately depend on the entire vector $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ (in the case in which there is only a single zone).

3.5 Market Power

It is more realistic to assume that firms have market power, because there may be a small number of both buyers and sellers in the zone. In this case, firms realize that the value of a_i they select will influence the market price (through the relation $p = \tilde{a}$.) Firm i selects a_i to minimize the sum of total costs :

$$\min_{a_i} C_i(v_i + w_i) + \tilde{a} v_i.$$

The optimal choice of a_i depends on choice of a's by other firms in the zone. So we specify a Nash equilibrium. Assuming an interior solution, Malueg and Yates (2009) show that the Nash equilibrium value for a_i is given by

$$a_i^{mp} = a_i^c + \frac{1}{\ell}\left[(\tilde{\theta} - \lambda\tilde{w}) - (\theta_i - \lambda w_i)\right],$$

where the superscript mp denotes market power. Starting from the competitive solution, the firm adds an adjustment according the difference between their own marginal abatement costs at the permit endowment and the average marginal abatement cost at the permit endowment. Firms that have higher-than-average marginal costs expect to purchase permits, so they want to reduce their net-trade function to put downward pressure on prices. Hence they adjust a_i below the competitive solution. The magnitude of this adjustment decreases in ℓ.

It turns out that the equilibrium price is the same as under competition:

$$p^{mp} = \tilde{\theta} - \lambda\tilde{w}.$$

The equilibrium emissions of pollution are not the same, however, as we have

$$e_i^{mp}(\theta) = v_i + w_i = \frac{1}{\lambda}(a^{mp} - p^{mp}) + w_i = \left(\frac{\ell - 1}{\ell}\right) e_i^c(\theta) + \frac{1}{\ell} w_i. \tag{7}$$

market

3.6 Private information

In the market power case, the solution implicitly assumes that a given firm i knows the values for θ_j for all the other firms j. For the private information case, we relax this assumption.

Here firm i has the same information as the regulator about the θ_j. So firm i treats θ_j as a random variable with expected value $\bar{\theta}_j$ and variance σ_j^2. Let the average of the expected values for the firms in a zone be denoted by $\tilde{\bar{\theta}}$. A Bayesian-Nash equilibrium is now the relevant solution concept. Firm i selects a_i to minimize the sum of total costs :

$$\min_{a_i} E[C_i(v_i + w_i) + \tilde{a}v_i \mid \theta_i.]$$

Malueg and Yates (2009) show that the Bayesian-Nash equilibrium value for a_i is given by

$$a_i^{pi} = a^c + \frac{1}{\ell}\left[(\tilde{\bar{\theta}} - \lambda\tilde{w}) - (\bar{\theta}_i - \lambda w_i)\right] + \frac{1}{\ell+1}(\bar{\theta}_i - \theta_i). \tag{8}$$

tisp

The adjustment from the competitive case now has two terms. The first term is analogous to the market power adjustment. It is the difference, at the endowment, between the average expected marginal abatement cost and the firm's own expected marginal abatement cost The second term is the difference between the expected and actual realization of its cost parameter.

The equilibrium price,

$$p^{pi} = p^c + \frac{1}{\ell+1}(\tilde{\bar{\theta}} - \tilde{\theta}),$$

is not necessarily equal to the equilibrium price under competition. For example, if the average of the realized cost parameters exceeds the average of the expected values, then the equilibrium price is lower than the equilibrium price under competition. The equilibrium emissions of pollution are

$$e_i^{pi}(\theta) = v_i + w_i = \frac{1}{\lambda}(a^{pi} - p^{pi}) + w_i = e_i^{mp}(\theta) + \frac{1}{\lambda}\left(\frac{1}{\ell(\ell+1)}\right)((\tilde{\bar{\theta}} - \tilde{\theta}) - (\bar{\theta}_i - \theta_i)). \tag{9}$$

private

3.7 Optimal Design

For a given market design, we use the model components to determine the expected total costs associated with that design. These calculations are based on the expressions for equilibrium levels of emissions $e_i(\theta)$ derived above. We can think of these as describing the firms' responses to the the given market design.

For abatement costs, we substitute the firms' responses into the abatement cost functions (1). We have

$$E[C_i(\theta_i, e_i(\theta))] = E\left[\frac{\lambda}{2}\left(\frac{\theta_i}{\lambda} - e_i(\theta)\right)^2\right].$$

Using the rules of expectations, we can reduce this to an algebraic equation involving the expected values and variances of the random variables as well as the other parameters of the model. (See the Appendix.) We are able to do this because of the assumptions that the abatement cost functions are quadratic and have a common slope. If these assumptions did not hold, then we would have to use numerical methods to solve for the $e_i(\theta)$. This in turn would require the use of Monte Carlo methods to evaluate expected costs.

For damages, we collect the firms' responses into the vector function $e(\theta) = (e_1(\theta), e_2(\theta), \ldots, e_m(\theta))$. Expected damages are given by

$$E[D] = \frac{1}{2}e(\theta)(ABA^t)e^t(\theta) + y(BA^t)e^t(\theta) + \frac{1}{2}yBy^t$$

$$= \frac{1}{2}\sum\sum E[e_i(\theta)e_j(\theta)]\nu_{i,j} + \sum E[e_i(\theta)]\phi_i + \frac{1}{2}yBy^t.$$

where $\nu_{i,j}$ is the i,j'th element of ABA^t and ϕ_i is the i'th element of yBA^t. As with abatement costs, we can reduce this to an algebraic equation involving the expected values and variances of the random variables as well as the other parameters (see the Appendix.)[9]

[9] If the elements of A or B are stochastic, then we can simply replace the random variables with their expected values, under the assumption that these random variables are independent from θ.

The total expected costs of a given market design is

$$\sum(E[C_i] + E[D]).$$

It is then simply a matter of looping through all possible market designs and picking the one with the lowest total expected costs.

4 Results

We now determine the optimal market design for the Neuse River. Our model has two sets of free parameters, the variances of the regulator's uncertainty about abatement costs σ_i^2 and the damage parameter b. For the variance parameters, we scale them proportionately to the expected values of the random variables, so that we can use a single uncertainty parameter η. In particular, let $\sigma_i = \frac{\eta}{100}(\frac{\bar{\theta}_i}{2})$. We can interpret η as the "percent error" in the random variables. In other words, it is very likely[10] that a realization of the random variable θ_i lies within η percent of the expected value $\bar{\theta}_i$. As η increases, uncertainty about abatement costs increases, abatement cost concerns dominate, and the optimal design converges toward one single zone with all firms. As b increases, damages become more severe, hot spot concerns dominate, and the optimal design converges toward the no trading partition.

Our first analysis is to give a broad overview of the parameter values that lead to these extreme cases. The optimal design as a function of the parameters is illustrated in Figure 2. At the bottom of the figure is the region for which the optimal design is the no trading partition, and hence there are 10 zones. Notice that the boundary for this region occurs at very small levels for η. As soon as there is essentially any uncertainty about abatement costs, the optimal design allows for at least some limited trading. At the top of the figure is the region for which the optimal design is the full trading partition in which there is a single zone. The boundary for this region occurs at very large levels for η. There must be quite a

[10] For a normal random variable, the probability is 0.95.

lot of uncertainty about abatement costs before the optimal design allows full trading. For the subsequent analysis, we use more moderate values for η. In particular, we use 10 percent error as our base case uncertainty about abatement costs, and also consider 5 and 20 percent errors.

Consider next the distribution of expected total costs over all 115,975 possible market designs for one particular parameter combination ($\eta = 10$, $b = 30000$) and competitive market conditions (See Figure 3). The difference between expected total expected costs of the optimal design and the worst design is approximately 2.1 million dollars per year. An interesting feature of this distribution is the clump of outcomes isolated by itself on the right-hand side. Most of the market designs in this clump correspond cases in which WWTP number 4 is in its own zone. This particular plant has the largest outflow, and correspondingly the largest abatement costs. A market design in which this plant is not allowed to trade generally performs poorly because the plant is forced to reduce its emissions by incurring large abatement costs. Also of importance in the fact that the left tail distribution is very thin. Only a few market designs perform very well. The optimal market design in Figure 3 is described by the partition $\{1\}, \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$. In this design, there are two zones. Firm 1 is in it's own zone and the rest of the firms are in the other. It turns out this is the optimal market design for both the market power and private information cases as well. The corresponding histograms have a similar shape, although the range is not quite as large as for competition.

To obtain a more comprehensive study of the differences across market conditions, we vary the damage parameter from its lower to upper bound while keeping η fixed at 10. The corresponding optimal market designs are described in Table 2. For a given set of parameters, the optimal market design may vary depending on the structure of competition. For example, consider the $b = 55000$ row. The optimal design under competition is $\{1\}, \{2, 3, 4, 5, 6, 8\}, \{7, 9, 10\}$, but the optimal market design under market power and private information is $\{1\}, \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

In addition to the optimal design, it is also of interest to specify the cost savings associated

Table 2: Optimal Market Designs for Various Values of b with $\eta = 10$ opteight

b	Competition	Market Power	Private Information
30000	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
35000	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
40000	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
45000	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
50000	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
55000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
60000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$
65000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,7,8,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
70000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
75000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
80000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
85000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
90000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
95000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
100000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
105000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$

with allowing trading. Figure 4 shows the difference between the expected total costs under the optimal market design and the expected total costs under the no trading partition for the same set of parameters as in Table 2. The average cost savings over all values of the parameters and market structures is about 1.6 million dollars per year.

We also analyze the robustness of our results with respect to η. First consider $\eta = 5$. Figure 5 shows the cost savings in this case. For $\eta = 5$, the regulator's uncertainty about abatement costs are lower than for $\eta = 10$, and correspondingly the magnitude of total cost savings is smaller, averaging around 375 thousand dollars per year. The optimal designs are given in Table 3. The optimal design generally has more zones than for $\eta = 10$. For example, when $b = 105,000$ the optimal design is $\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$, which has four zones. When $b = 30,000$, the optimal design is $\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$, which has three zones. Next consider $\eta = 20$. Figure 6 shows the cost savings in this case. For $\eta = 20$, the regulator's uncertainty about abatement costs are higher than for $\eta = 10$, and correspondingly the magnitude of total cost savings is larger, averaging around 6.8 million dollars per year. There is no variation in the optimal design. For all values of b, the optimal

Table 3: Optimal Market Designs for Various Values of b with $\eta = 5$ optnine

b	Competition	Market Power	Private Information
30000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
35000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
40000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
45000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
50000	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
55000	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
60000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
65000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$
70000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6,8\},\{7,9,10\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$
75000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$
80000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$
85000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$	$\{1\},\{2,3,4,5,6\},\{7,8,9,10\}$
90000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$
95000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$
100000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$
105000	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$	$\{1\},\{2,3,4,5,6\},\{7,9,10\},\{8\}$

design is $\{1\},\{2,3,4,5,6,7,8,9,10\}$.

Based on these results, we delineate two observations about the optimal market designs. First, WWTP 1 is never allowed to trade with the other WWTP. It is not optimal to allow Firm 1 to trade with downstream firms, because the net result may be a large increase in pollution at an upstream location and this pollution is then routed downstream through large portion of the river system. Second, firms grouped together in the same zone are generally close together spatially with respect to the schematic diagram in Figure 1. There are exceptions, however, depending on the specific values of the parameters. For example, when there are three zones in an optimal design, WWTP 8 is sometimes paired with WWTP 2-6 rather than 7,9, and 10.

Next we delineate three observations about the effects of market structure. First, there is variation in the cost savings according to the type of market structure, but the magnitude of this variation is small relative the to the overall cost savings. For example, the biggest difference between the lines in Figure 4 is about 40,000 dollars per year. Second, the cost savings are not ordered with respect to market structure. For example, in Figure 6, competition yields greater cost savings than either private information or market power when b is small, but this is reversed when b is large.[11] This verifies the intuition discussed in the introduction that a competitive market may not dominate a market with market power. Even though both market power and private information lead to higher abatement costs relative to competition, once we account for damages, the optimal market design for both market power and private information may lead to lower total expected costs than competition. Third, the number of zones under competition is greater or equal to the number of zones under market power and private information.

Our final analysis is to match the results of the model with anticipated changes to the regulatory environment, thus providing an explicit policy recommendation. The next phase in Nitrogen regulation is likely to involve a large decrease in emissions relative to current limits. This can be simulated in our model by selecting the upper bound on the damage parameter $b = 105,000$. This corresponds to an approximately 25 percent decrease in Nitrogen as measured at the estuary.[12] Using the results for the base case value of $\eta = 10$, this indicates the optimal market design should be selected from the bottom row in Table 2. This implies a market design consisting of three zones. WWTP 7,9, and 10 are in the first zone, WWTP 1 is in the second zone, and the rest are in the third zone. This optimal design leads to approximately 1.5 millions dollars per year of cost savings relative to the no-trade partition.

5 Conclusion

Our analysis of market structure and optimal scale for North Carolina's Neuse River offers an economic assessment of 303 (d) regulation. Under the most likely regulatory scenario, we find cost savings of 1.5 million dollars per year under the optimal market design relative to

[11] For the specific model and values of the parameters considered in this study, the private information case always leads to greater cost savings than the market power case, but this is not a general property.

[12] In the context of our model, an even greater reduction would fundamentally alter the behavior of the WWTP, as some would have to essentially eliminate emissions altogether. Practically speaking, this would involve sending sewage to other plants for treatment.

the typical 303 (d) regulation in which the WWTP are not allowed to trade. The markets we analyze are relatively simple to implement in that firms need only trade in a single market and permits trade on a one-for-one basis in that market. As such, it is our hope that trading of this type might find increasing application in the Neuse as well as other watersheds.

The traditional distinction between command-and-control and incentive-based regulation breaks down in an analysis of optimal scale. In fact, the optimal design for the Neuse River has elements of both types of regulation. Because WWTP 1 is in its own zone, the regulator is essentially exerting command-and-control over WWTP 1. The regulator gives an endowment of permits to this WWTP but it is not allowed to adjust its emissions level through trade. The other WWTP's are subject to incentive-based regulation. They are given an endowment of permits and are allowed to adjust their emission levels through trade with other firms in their zones.

We made several assumptions so that an exhaustive search of the feasible set of market designs could be conducted in a reasonable time. To analyze a market with much more than 10 firms, or to consider more general functional forms for abatement costs, one must consider a more sophisticated search technique. In particular, a nested partition algorithm seems to be a promising way to identify optimal market designs in these more complicated cases. With a properly tested and validated search algorithm, the methodology described in this paper can easily be "scaled". For example, SPARROW has been used to analyze the flow of nitrogen through watersheds in the entire southeastern US. One could conduct similar analysis in each of these watersheds to make recommendation for the optimal market. A more ambitious approach would be to combine many of the watersheds together and analyze the optimal market design on a regional basis.

Appendix

Engineering Cost Analysis

Consider a generic WWTP. The primary variables in an engineering analysis are the size of the outflow of the plant f (in millions of gallons per day) and concentration of pollution emitted from the plant c (in mg/l). To covert to the units of emissions, we have

$$e = \alpha c f \tag{10}$$

conv

where $\alpha = 8.3431$ is the appropriate unit conversion so that e is in units of pounds per day. We assume a generic business-as-usual concentration κ, and then base the engineering calculations on reducing pollution below this level. From (10), the business-as-usual emissions corresponding to κ are $\alpha\kappa f$. Consistent with the abatement cost function specification (1), we consider an engineering abatement cost function for a generic WWTP with outflow f and concentration c as

$$C = \frac{\lambda}{2}\left(\alpha\kappa f - \alpha c f\right)^2. \tag{11}$$

estimate

Now suppose we have engineering data which contains several different concentrations, several different outflows for each concentration, and cost data for each combination of concentration and outflow. Using a simple regression (without a constant) based on (11) gives us an estimate of λ.[13]

The appropriate data comes from the Nutrient Reduction Technology Cost Task Force (2002). Using Table 4c, we have one value for c (3 mg/L) and four values for f (.1MGD, 1MGD, 10 MGD, 30 MGD) The tables report both capital costs (CP) as well as yearly operations (OM) costs in year 2000 dollars. Following Tsagavakis et al (2003) we convert these into total yearly costs according to the formula

$$C = OM + CP \times CRF,$$

[13]The error in this regression is interpreted as the error in the engineering study, and is not the same thing as σ_i^2, which is firm specific error. Error in the engineering study is not explicitly utilized.

where CRF is the capital recovery factor. It is defined as

$$CRF = \frac{r(1+r)^t}{(1+r)^t - 1},$$

where t is the economic life (assumed to be 20 years) and r is the cost of capital (assumed to be six percent). Based on the current regulatory requirements for the WWTP's in the Neuse, we select a value for κ of 3.7 mg/L. With these inputs, the simple regression yields an estimated value for λ of 243.

To determine a value for $\bar{\theta}_i$, we assume that the ideosyncratic differences in WWTP i effect the actual business as usual level of pollution at that plant. In particular, for a plant with outflow f_i, let

$$\frac{\theta_i}{\lambda} = \alpha\kappa f_i + \varepsilon_i$$

where ε_i is a random variable with expected value zero and variance $\hat{\sigma}_i^2$. It follows that the expected value of θ_i is $\lambda\alpha\kappa f_i$ (and the variance is $\lambda^2\hat{\sigma}^2$).

Calculation of A using SPARROW

To minimize computation we utilize only a portion of the SPARROW model. The matrix A is simply a matrix of in-stream N delivery ratios (from source to measurement site) derived from the SPARROW model as calibrated for the Neuse River Basin. In SPARROW the in-stream nitrogen load for a stream reach is calculated as the sum of contributions from the reach immediately upstream and the runoff from the reach catchment. Within the stream channel the model assumes a first-order attentuation model for nitrogen as it travels through the network as a result of ecological processes. Thus the load reaching a down-stream end of a reach is modeled in SPARROW as

$$L_{downstream} = L_{upstream} * e^{-k_i T_i},$$

where k_i is a nitrogen uptake rate with units of inverse time and T_i represents the average travel time of water, both indexed for reach i. The parameters k_i and T_i are identified from hydrologic and water quality data via the regression. For our purposes, we estimated the stream-wise distances between point sources and measurement sites using GIS software. We then use the SPARROW estimate T_i to calculate an average water velocity between each point source and measurement site. When the path from source to site included multiple SPARROW reaches, the average velocity was computed based on the average of the T_i weighted by the proportion of the path length spent in each reach. These average velocities are multiplied by the corresponding weighted average k_i to get a first-order decay rate in units of inverse distance. The elements of our N delivery matrix A as computed as

$$A_{i,j} = e^{-\delta_{i,j} L_{i,j}},$$

where $A_{i,j}$ represents the delivery ratio from point source i to measurement site j, $\delta_{i,j}$ is the nitrogen decay rate per unit distance taken as a weighted average of the decay rates for the SPARROW reaches traversed, and $L_{i,j}$ is the in-stream distance from source i to measurement site j. If a reservoir is present along the stream path, SPARROW introduces a further factor to represent reservoir processing of nitrogen given by

$$\frac{1}{1 + Z^R \theta_R},$$

where θ_R is the reservoir loss coefficient estimated by SPARROW ($\theta_R = 10.7$) and Z_R is the quotient of the mean flow into the reservoir and the average reservoir surface area.

This approach only takes into account the routing and decay of point source nitrogen loads. To model the background non-point source loading we used the SPARROW predicted loads for each measurement site and then, using the nitrogen delivery matrix A, subtracted the contributions to the predicted loads due to the wastewater treatment plants.[14]

[14]SPARROW applies a fitted loss coefficient to the emissions of all point sources in the Neuse basin such that only 79% of point source loads are modeled as reaching the stream channel for transport. To calculate the N background loads we applied this factor. However, for subsequent calculations of N delivery we assume

Details of Expected Total Cost Calculation

Each θ_i is a random variable with expected value $\bar{\theta}_i$ and variance σ_i^2. Thus $E[\theta_i^2] = \sigma_i^2 + \bar{\theta}_i^2$. The random variables are independent. It follows that $E[\theta_i\theta_j] = \bar{\theta}_i\bar{\theta}_j$. Because the subsequent calculations may involve firms in different zones, we introduce a superscript for zones. The superscript i refers to the zone of which firm i is a member. As before, the subscript i refers to firm i. We modify the notation used in the main text to account for this new notation as shown in Table 4.

Table 4: Variable definitions vardef

Definition	Description
ℓ^i	Number of firms in i's zone
Δ^i	Set of firms in i's zone
$\tilde{\bar{\theta}}^i = \frac{1}{\ell^i}\sum_{k\in\Delta^i}\bar{\theta}_k$	Average expected values in i's zone.
$\tilde{\sigma}^i = \frac{1}{\ell^i}\sum_{k\in\Delta^i}\sigma_k^2$	Average variance in i's zone.
$\tilde{w}^i = \frac{1}{\ell^i}\sum_{k\in\Delta^i}w_k$	Average endowment in i's zone.
$\tilde{\theta}^i = \frac{1}{\ell^i}\sum_{k\in\Delta^i}\theta_k$	Average of actual cost parameters in i's zone.
$p^i = \tilde{\theta}^i - \lambda\tilde{w}^i$	The competitive price in i's zone.

Using these definitions, we can calculate the expected values of various quantities as shown in Table 5. These quantities determine expected abatement costs and damages, according to the type of market structure.

that 100% of WWTP nitrogen loads reach the stream channel for transport.

Table 5: Expected values of various quantities expqs

$$E[p^i] = \tilde{\bar{\theta}}^i - \lambda\tilde{w}^i$$
$$E[(p^i)^2] = \frac{1}{\ell^i}\tilde{\sigma}^i + (\tilde{\bar{\theta}}^i - \lambda\tilde{w}^i)^2$$
$$E[p^ip^j] = (\tilde{\bar{\theta}}^i - \lambda\tilde{w}^i)(\tilde{\bar{\theta}}^j - \lambda\tilde{w}^j) \text{ for } i \text{ and } j \text{ in different zones}$$
$$E[\theta_ip^i] = \frac{\sigma_i^2}{\ell^i} + \bar{\theta}_i(\tilde{\bar{\theta}}^i - \lambda\tilde{w}^i)$$
$$E[\theta_jp^i] = \frac{\sigma_j^2}{\ell^j} + \bar{\theta}_j(\tilde{\bar{\theta}}^i - \lambda\tilde{w}^i) \text{ for } i \text{ and } j \text{ in the same zone}$$
$$E[\theta_jp^i] = \bar{\theta}_j(\tilde{\bar{\theta}}^i - \lambda\tilde{w}^i) \text{ for } i \text{ and } j \text{ in different zones}$$

Competition

First consider the calculation under the assumption that the market is competitive. From (6) we have

$$e_i^c(\theta) = \frac{1}{\lambda}(\theta_i - p^i).$$

We substitute this into the abatement cost functions and damage function. For the cost functions we have

$$E[C_i(\theta_i, e_i^c(\theta))] = E\left[\frac{\lambda}{2}\left(\frac{\theta_i}{\lambda} - e_i^c(\theta)\right)^2\right] = \frac{1}{2\lambda}E[(p^i)^2].$$

For the damage function the calculation for the linear and constant term

$$\sum E[e_i(\theta)]\phi_i + \frac{1}{2}yBy^t$$

is straightforward (for competition as well as market power and private information). For the quadratic term we have

$$\frac{1}{2}\sum\sum E[e_i^c e_j^c]\nu_{i,j}.$$

For the diagonal elements $i = j$ we have

$$E[(e_i^c)^2] = \frac{1}{\lambda^2}(E[(\theta_i)^2] - 2E[\theta_i p^i] + E[(p^i)^2]).$$

For the off diagonal elements we have

$$E[e_i^c e_j^c] = \frac{1}{\lambda^2}(E[(\theta_i - p^i)(\theta_j - p^j)]) = \frac{1}{\lambda^2}(E[\theta_i\theta_j] - E[\theta_i p^j] - E[\theta_j p^i] + E[p^i p^j]).$$

To use this expression we have to be careful to use the right expression for $E[\theta_j p^i]$ and $E[\theta_i p^j]$ depending on whether or not i and j are in the same zone.

Market Power

From (7), we have

$$e_i^{mp}(\theta) = f(\ell^i)e_i^c(\theta) + (1 - f(\ell^i))w_i.$$

where $f(\ell^i) = \frac{\ell^i - 1}{\ell^i}$. For the cost functions we have

$$E[C_i(\theta_i, e_i^{mp}(\theta))] = E\left[\frac{\lambda}{2}\left(\frac{\theta_i}{\lambda} - e_i^{mp}(\theta)\right)^2\right] = \frac{1}{2\lambda}E[\theta_i^2] - E[\theta_i e_i^{mp}] + \frac{\lambda}{2}E[(e_i^{mp})^2].$$

Substituting in and simplifying gives

$$E[C_i(\theta_i, e_i^{mp}(\theta))] = \frac{1}{2\lambda}E[\theta_i^2] - f(\ell^i)\frac{1}{\lambda}(E[\theta_i^2] - E[\theta_i p^i]) - (1 - f(\ell^i))w_i E[\theta_i]$$

$$+\frac{\lambda}{2}\left(f(\ell^i)^2 E[(e_i^c)^2] + 2f(\ell^i)(1 - f(\ell^i))w_i E[e_i^c] + (1 - f(\ell^i))^2 w_i^2\right).$$

For the quadratic term of the damage function we have

$$\frac{1}{2}\sum\sum E[e_i^{mp}e_j^{mp}]\nu_{i,j} = \frac{1}{2}\sum\sum E[(f(\ell^i)e_i^c + (1 - f(\ell^i))w_i)(f(\ell^j)e_j^c + (1 - f(\ell^j))w_j)]\nu_{i,j}.$$

For the diagonal elements $i = j$ the expected value becomes

$$E[(e_i^{mp})^2] = f(\ell^i)^2 E[(e_i^c)^2] + 2f(\ell^i)(1 - f(\ell^i))w_i E[e_i^c] + (1 - f(\ell^i))^2 w_i^2.$$

For the off diagonal elements we have

$$E[e_i^{mp}e_j^{mp}] = f(\ell^i)f(\ell^j)E[e_i^c e_j^c] + f(\ell^i)(1-f(\ell^j))w_j E[e_i^c] + f(\ell^j)(1-f(\ell^i))w_i E[e_j^c] + (1-f(\ell^i))(1-f(\ell^j))w_i w_j,$$

where again we must use the right expressions depending on whether or not i and j are in the same zone.

Private Information

The final case is private information. From (9) we have

$$e_i^{pi}(\theta) = e_i^{mp}(\theta) + \frac{1}{\lambda} g(\ell^i) \left((\tilde{\bar{\theta}}^i - \tilde{\theta}^i) + (\theta_i - \bar{\theta}_i) \right),$$

where $g(\ell^i) = \frac{1}{\ell^i(\ell^i+1)}$. It can be shown that

$$E[C_i(\theta_i, e_i^{pi}(\theta))] = E[C_i(\theta_i, e_i^{mp}(\theta))] + \frac{1}{\lambda} \left(\frac{2(\ell^i)^2 - 1}{2(\ell^i)^3(\ell^i+1)^2} \right) \tilde{\sigma}^i + \frac{1}{\lambda} \left(\frac{-4(\ell^i)^2 + \ell^i + 2}{2(\ell^i)^3(\ell^i+1)^2} \right) \sigma_i^2.$$

For the quadratic term of the damage function, the diagonal elements are given by

$$E[(e_i^{pi})^2] = E[(e_i^{mp})^2] + \frac{1}{\lambda^2} \left(\sigma_i^2 - 2\frac{\sigma_i^2}{\ell^i} + \frac{\tilde{\sigma}^i}{\ell^i} \right) \left(\frac{2(\ell^i)^2 - 1}{(\ell^i)^2(\ell^i+1)^2} \right).$$

For the off-diagonal terms, when firms i and j are in the same zone we have

$$E[e_i^{pi} e_j^{pi}] = E[e_i^{mp} e_j^{mp}] + \frac{1}{\lambda^2} \left(\frac{1}{\ell^i} (\tilde{\sigma}^i - \sigma_i^2 - \sigma_j^2) \right) \left(\frac{2(\ell^i)^2 - 1}{(\ell^i)^2(\ell^i+1)^2} \right).$$

Finally, if i and j are in different zones we have

$$E[e_i^{pi} e_j^{pi}] = E[e_i^{mp} e_j^{mp}].$$

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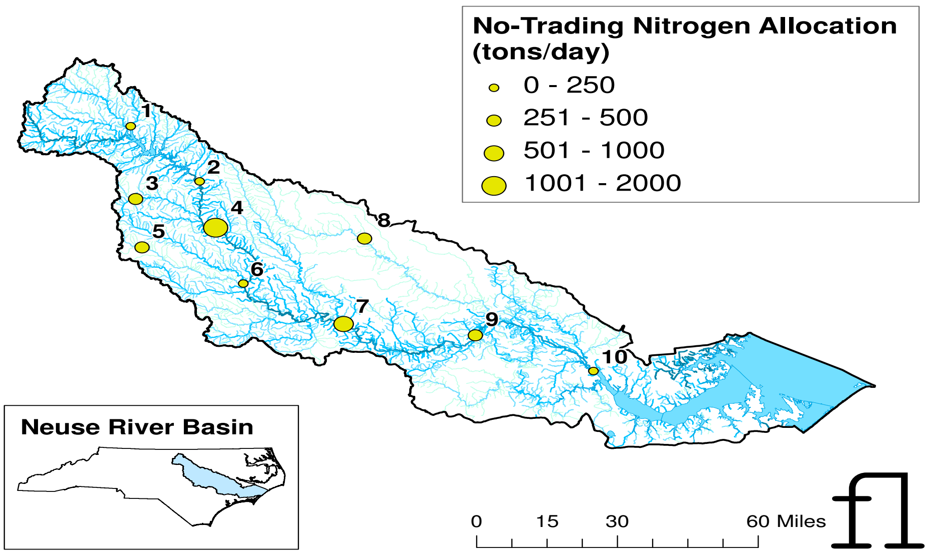
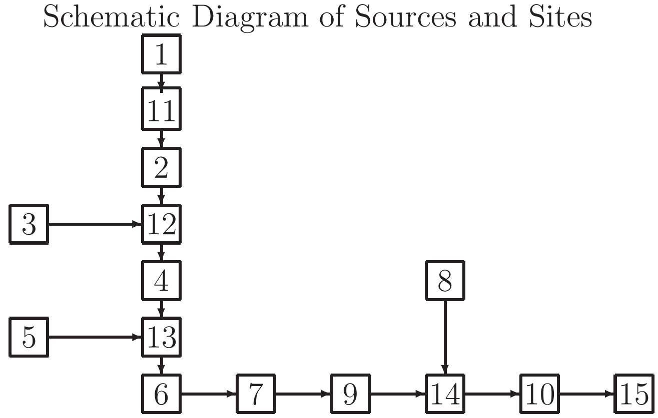
Figure 1: Neuse River: WWTP Sources and Measurement Sites

map

Location of WWTP's



Schematic Diagram of Sources and Sites



Both Source and Site		Site Only	
Number	Name	Number	Characteristic
1	SGWASA WWTP	11	Falls Lake reservoir
2	Smith Creek WWTP	12	Confluence
3	North Cary WRF	13	Confluence
4	Neuse River WWTP	14	Confluence
5	South Cary WRF	15	Estuary
6	Central Johnston County WWTP		
7	Goldsboro WWTP		
8	Wilson WWTP		
9	Kinston Regional WRF		
10	New Bern WWTP		

Figure 2: Optimal Design as function of b and η

over

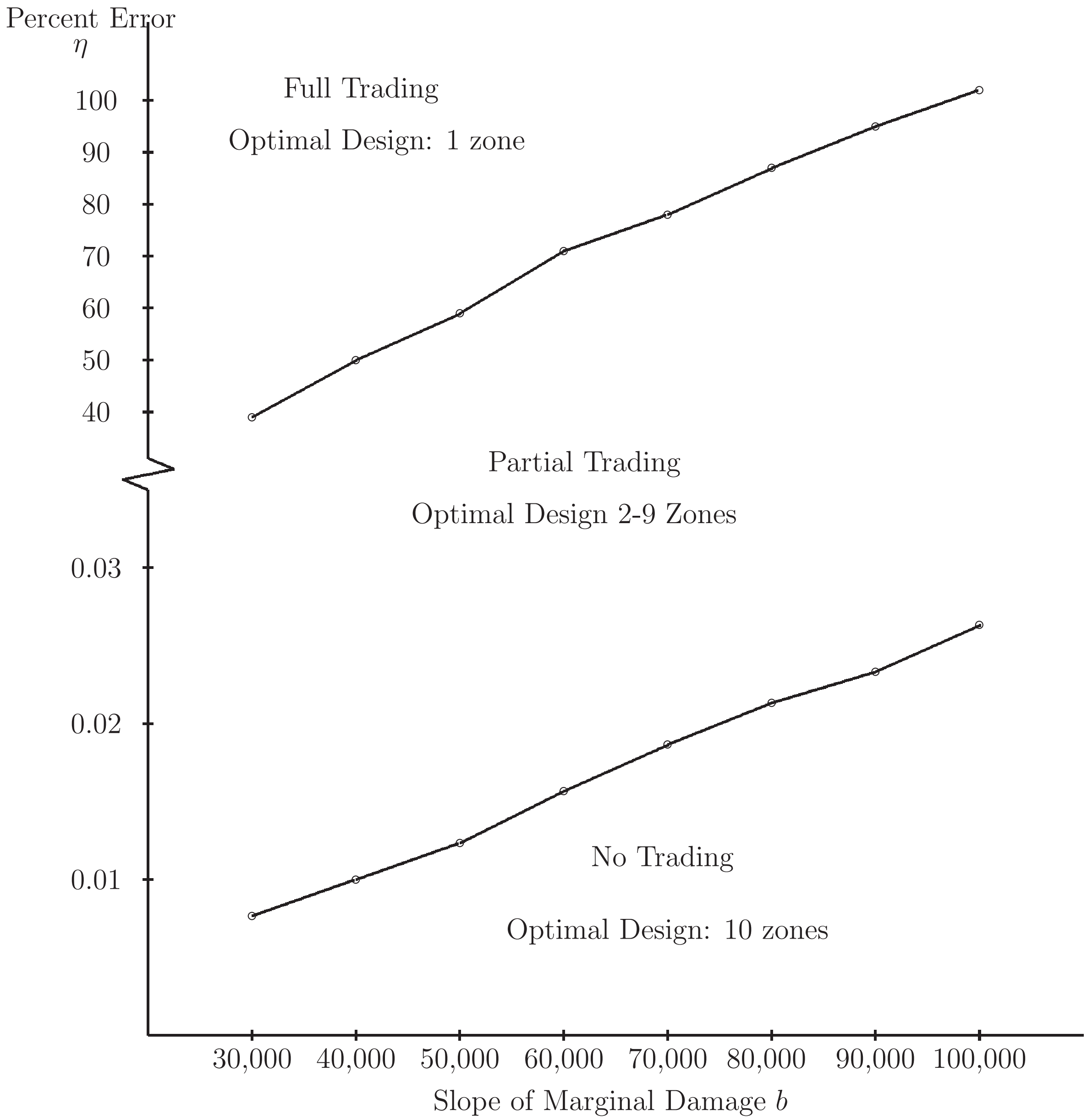


Figure 3: Distribution of Total Expected Costs (millions of dollars per year): Competition, $b = 30000$, $\eta = 10$ hist

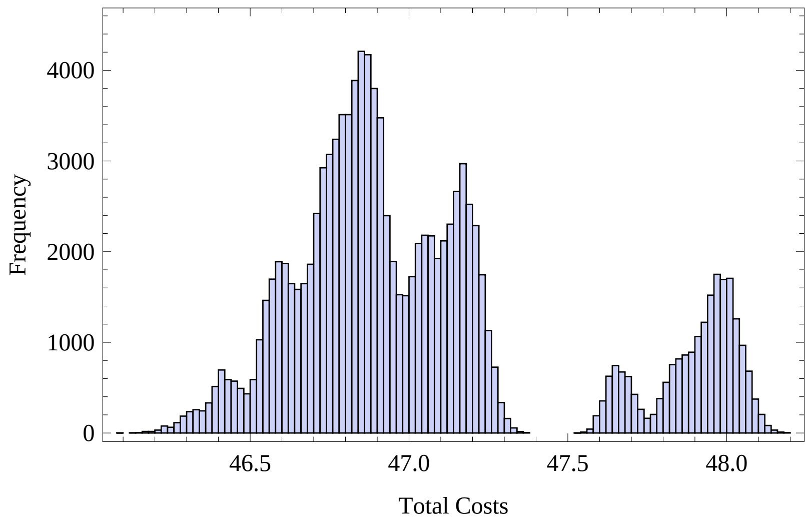


Figure 4: Expected Cost Savings Relative to No Trade (millions of dollars per year): $\eta = 10$ ploteight

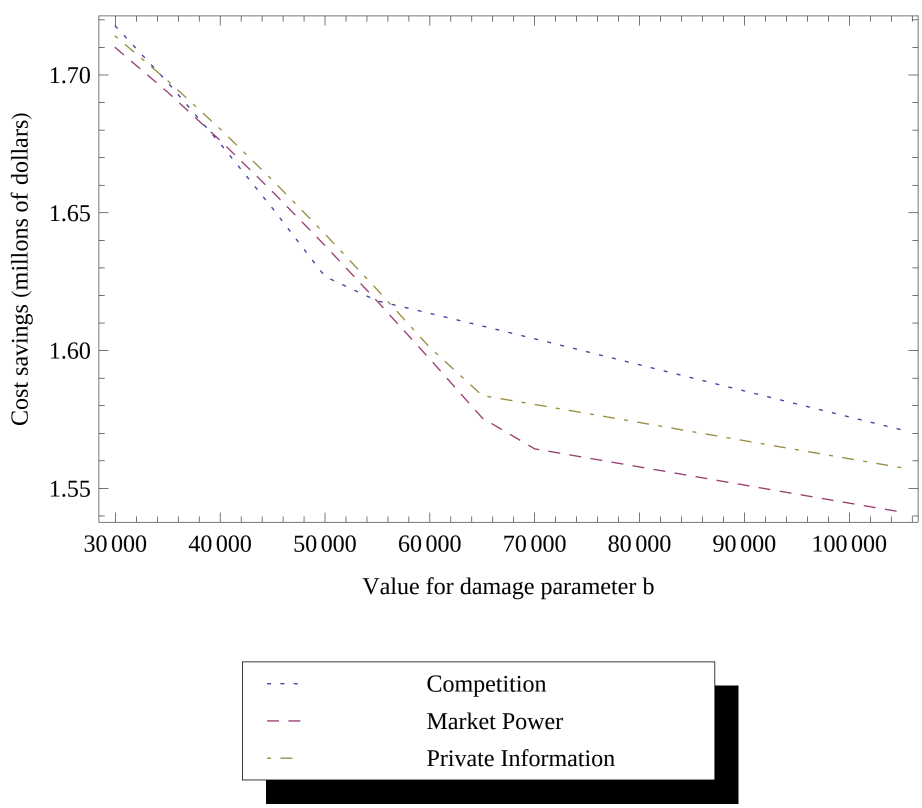


Figure 5: Expected Cost Savings Relative to No Trade: $\eta = 5$

plotone

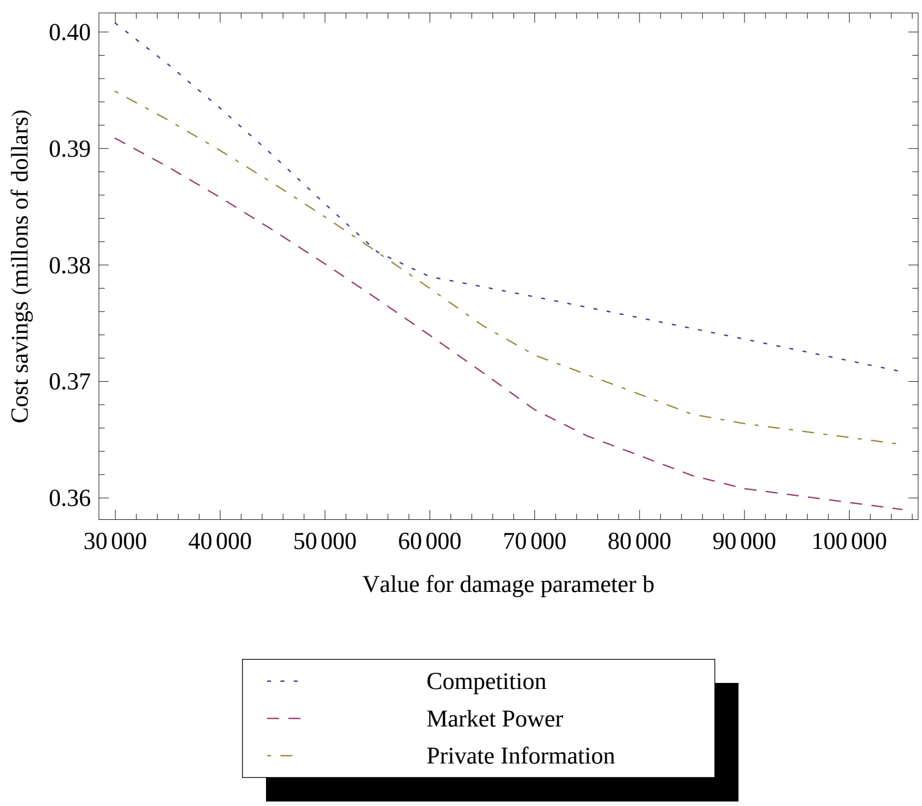


Figure 6: Expected Cost Savings Relative to No Trade: $\eta = 20$ plottwo