BRIEF NOTES ON INTERGENERATIONAL DISCOUNTING

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- But for intergenerational problems like climate change, uncertainty is at the heart of the matter. What kind of uncertainty is relevant?
- My main message: Beliefs about possible catastrophic outcomes drive everything.

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- "Normal" stochastic fluctuations in g are unimportant for estimating r. I claim only catastrophic uncertainty matters.
- What is a catastrophe? Any major event that will:
  - Substantially reduce the capital stock; and/or
  - Reduce the efficiency of production (reduce productivity, increase aggregate operating costs).
- Example: Runaway warming with global mean temperatures exceeding 7°C, large increases in sea levels, etc., etc.
- You can substitute your own favorite catastrophe.
- Want to evaluate a costly policy (e.g., stringent GHG abatement) to reduce or eliminate likelihood of catastrophe.

General Equilibrium Model

- Pindyck and Wang (2011): tractable GE model with possible arrival of catastrophic events.
  - Recursive preferences (E-W-Z), production, capital accumulation, and adjustment costs (so $q \neq 1$).
  - Catastrophes: Poisson events with a mean arrival rate λ, results in loss of fraction $1 - Z$ of capital stock, where Z is stochastic, follows one-parameter (α) power distribution.
  - Calibrate to basic data for C/I, q, risk-free rate, equity premium, and "normal" growth rate.
- PW use calibrated model to estimate WTP: permanent consumption tax society would accept to reduce λ.
- I will focus on discount rate.

Model

- Representative household has recursive preferences:

$$V_t = \mathcal{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right], \tag{1}$$

where

$$f(C, V) = \frac{\rho}{1-\psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\eta)V)^\omega}{((1-\eta)V)^{\omega-1}}, \tag{2}$$

and:

  - $\rho > 0$ is rate of time preference
  - ψ is elasticity of intertemporal substitution
  - η is coefficient of relative risk aversion.
- Define $\omega \equiv (1-\psi^{-1})/(1-\eta)$.
- If $\eta = \psi^{-1}$ so that $\omega = 1$, get standard CRRA utility.

Model (Con't)

- CRTS AK production technology, where K is *total* capital:

$$Y(t) = AK(t)\,, \tag{3}$$

- Catastrophe: Poisson arrival at mean rate λ.
- Shock destroys fraction $1 - Z$ of capital stock K.
- Remaining fraction Z a random variable. Z follows a power distribution over (0,1) with parameter $\alpha > 0$:

$$f_Z(z) = \alpha z^{\alpha-1} \;\; ; \;\; 0 \leq z \leq 1 \;\; , \tag{4}$$

so $\Pr(Z \geq z) = 1 - z^{\alpha}$, and $\mathcal{E}(Z) = \alpha/(\alpha+1)$.

Model (Con't)

- Capital stock K evolves as:

$$dK(t) = \phi(i)Kdt + \sigma K(t)dW(t) - (1 - Z)K(t)dJ(t)\,, \tag{5}$$

  where $i = I/K$, σ is "normal" volatility and $J(t)$ is a jump process.
- $\phi(i)$ captures depreciation and costs of installing capital.
- In equilibrium, $\phi(i) = g$, where g is normal (no catastrophe) growth rate.

Key Result

- Solution of model yields equilibrium interest rate:

$$r = \rho + \psi^{-1} g - \frac{\eta(\psi^{-1}+1)\sigma^2}{2} - \lambda \mathcal{E}\left[(\psi^{-1}-\eta)\left(\frac{1-Z^{1-\eta}}{1-\eta}\right) + (Z^{-\eta}-1)\right] \quad (6)$$

- Simpler with power distribution for Z: $\mathcal{E}(Z^n) = \alpha/(\alpha+n)$.
- Eqn. (6) is generalized Ramsey rule.
  - If $\psi^{-1} = \eta$ (CRRA expected utility) and if no stochastic changes in K, get $r = \rho + \eta g$.
  - With CRRA, $\eta(\eta+1)\sigma^2/2$ is standard adjustment for continuous fluctuations in K.
  - Last term adjusts for catastrophic shocks.

Key Result (Con't)

- Assume $\psi^{-1} = \eta = 2$ (CRRA expected utility) for simplicity.
- For US over past 70 years, $\sigma = .025$ and $g = .02$.
- So $\eta(\eta + 1)\sigma^2/2 < .002$. Not important.
- What is important? Catastrophic risk.

What is Important?

- Suppose $Z = 0.8$ with certainty, and $\lambda = .05$
- Then last term is $(.05)(.56) = .028$.
- Suppose $Z = 1.0$, 0.8, or 0.6, each with probability $1/3$.
- Then last term is $(.05)(.780) = .039$.
- In this last case, if $\rho = .02$, discount rate is reduced from 6% to about 2%.
- This has a huge impact. Put in your beliefs about λ and distribution for Z and you could easily get a negative r.
- Problem: What are "correct" beliefs about λ and distribution for Z?

How to Determine η and ρ? (Question 1b)

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- These are *not* "ethical" parameters. No support for claiming $\rho = 0$ on ethical grounds.
- These are behavioral or preference parameters.
- If $\rho = .05$ for society as a whole, policy-makers should not over-ride that preference on the grounds of (someone's) ethics.
  - There could be other reasons for over-riding $\rho = .05$.
  - Perhaps society doesn't understand discounting (but then actual $\rho \neq .05$).
  - Perhaps time preferences are more complex, and possibly time-inconsistent.
  - But then $\rho = .05$ (and constant) does not truly reflect society's preferences.
- Putting these issues aside, and assuming η and ρ are constant, how to estimate their values?

Determining η, ρ, and Maybe ψ

- What about using market data for economic and financial variables, as done in finance and macro? Get numbers all over the place.
- Majority of studies put η in range of 1.5 to 4, which is a big range. But some recent studies extend range to 10.
- There was once a "'consensus" that $\rho \approx .02$ or .03, but lately macro studies use numbers around .05.
- Estimates of ψ in literature range from .2 (Hall) to 2 (Barro, Bansal). (Note that if $\eta \geq 2$, CRRA utility requires $\psi \leq .5$.)
- In any case, these studies reflect short-run values.

Determining η, ρ, and ψ (Con't)

- I suggest a conjoint study done over a large sample (at least 1000) people.
- Presents sets of choices, where each choice is a "'product" with 4 or 5 attributes, e.g.,
  - Fixed monetary award at different dates.
  - Lotteries at those same dates.
  - Combinations of lotteries and fixed amounts.
- Applications in marketing suggest this may be informative.
- *But*, requires respondents to understand lotteries, etc.

Summary

- For construction project (highway, bridge), even one lasting 100 years, Ramsey equation, or CAPM, may be fine. Long horizon by itself not a problem.
- Problem arises with projects directed at averting catastrophes.
- Then simple cost/benefit analysis misleading. Risk premium, discount rate, etc. all endogenous.
- We saw discount rate depends critically on nature of catastrophic risk. Can lead to very low discount rate.

Summary

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- We saw discount rate depends critically on nature of catastrophic risk. Can lead to very low discount rate.

- Can also justify low discount rate using option-based arguments:
- "Project" (investment to reduce λ) has negative risk premium.
- Like buying a deeply out-of-the-money put. Downside risk so large that put is valuable.
- Instead of standard cost/benefit analysis, must value "real" put in GE with production and illiquid capital.