A Numerical General Equilibrium Model for Evaluating U.S. Energy and Environmental Policies

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This paper describes in full detail the Goulder–Hafstead Energy-Environment-Economy (E3) computable general equilibrium (CGE) model of the U.S. economy. This paper is divided into two main sections. The first section fully describes the model and its numerical solution. The second section describes how the data and parameter inputs into the model are derived. A third section lists the industry and consumer goods aggregation used in this version of model.

A. Model Description

The E3 CGE model is an intertemporal general equilibrium model with international trade. The model combines a fairly realistic treatment of the U.S. tax system with a detailed representation of domestic energy production and demand. The agents in the model are domestic and foreign producers of various goods and services, a domestic and foreign representative household, and a domestic and foreign government. The model captures the interactions among these agents, whose actions generate supplies and demands for various commodities and productive factors. The model is solved in yearly intervals beginning in a benchmark year.[1]

The model description is organized as follows: the baseline E3 CGE model is described for the domestic producers, households, and government. Except where noted, the structure of the foreign economy matches that of the domestic economy. Next, the incorporation of environmental policies such as carbon pricing and direct regulation is described. The model description concludes with a discussion on how the E3 CGE model is numerically solved.

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[1]The benchmark year is 2010.

1   Producers

1.1   Technology

1.1.1   General Production

Production in each industry is given by a nested production function structure. Gross output, X_i, is produced using capital (K_i), labor (L_i), an energy composite ($\bar{E}_i$), and a materials composite ($\bar{M}_i$) such that

$$X_i = f_i(K_i, \hat{f}(L_i, \tilde{f}(\bar{E}_i, \bar{M}_i))). \tag{1}$$

The functions f_i, $\hat{f}_i$, and $\tilde{f}_i$ are CES functions:

$$f(x, y) = \gamma_f \left[\alpha_f x^{\rho_f} + (1 - \alpha_f)y^{\rho_f}\right]^{\frac{1}{\rho_f}}. \tag{2}$$

Note that the industry subscript i has been suppressed and γ_f is the CES scaling parameter, α_f the CES share parameter, and σ_f is the elasticity of substitution ($\rho_f = (\sigma_f - 1)/\sigma$).[2]

Net output, Y_i, is equal to gross output minus the loss of output associated with installing new capital (or dismantling old capital). Per-unit adjustment costs, ϕ, are a quadratic function of the rate of net investment:

$$\phi(I/K) = \frac{(\zeta/2)(I/K - \delta)^2}{I/K} \tag{3}$$

where I represents gross investment and ζ is the marginal adjustment cost parameter and δ is the rate of economic depreciation of the capital stock. Net output in industry i is therefore given by

$$Y_i = X_i - \phi_i(I_i/K_i)I_i. \tag{4}$$

The energy composite $\bar{E}_i$ is a CES function of the products of the different energy industries,[3]

$$\bar{E} = \gamma_{\bar{E}} \left[\sum_{j=1}^{n_E} \alpha_{\bar{E}_j} E_j^{\rho_{\bar{E}}}\right]^{\frac{1}{\rho_{\bar{E}}}} \tag{5}$$

where $\sum_{j=1}^{n_E} \alpha_{\bar{E}_j} = 1$.

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[2]Analogous expressions apply for $\hat{f}$ and $\tilde{f}$.

[3]Energy industries and material industries are listed in Section C. See the Section B for complete data documentation.

Likewise, the materials composite $\bar{M}_i$ is also a CES function of the products of the non-energy industries,

$$\bar{M} = \gamma_{\bar{M}} \left[\sum_{j=1}^{n_M} \alpha_{\bar{M}_j} M_j^{\rho_{\bar{M}}} \right]^{\frac{1}{\rho_{\bar{M}}}} \tag{6}$$

where $\sum_{j=1}^{n_M} \alpha_{\bar{M}_j} = 1$.

The elements E_j $(j = 1, ..., n_E)$ and M_j $(j = 1, ..., n_M)$ in the composite functions $\bar{E}$ and $\bar{M}$ are themselves CES composites of domestically produced and foreign made goods:[4]

$$E_j = \gamma_{E_j} \left[\alpha_{E_j} (E_j^d)^{\rho_{E_j}} + (1 - \alpha_{E_j})(E_j^f)^{\rho_{E_j}} \right]^{\frac{1}{\rho_{E_j}}} \tag{7}$$

$$M_j = \gamma_{M_j} \left[\alpha_{M_j} (M_j^d)^{\rho_{M_j}} + (1 - \alpha_{M_j})(M_j^f)^{\rho_{M_j}} \right]^{\frac{1}{\rho_{M_j}}} \tag{8}$$

where E_j^d and E_j^f (M_j^d and M_j^f) represent domestic and foreign energy (material) inputs of type j.

The total number of industries is denoted by $n_I = n_E + n_M$ and industries are ordered such that $(E_1, ..., E_{n_E}, M_1, ..., M_{n_M})$. The prices $\{p_{ij}^d\}$ and $\{p_{ij}^f\}$ paid for domestic and foreign goods i by industry j are given by

$$p_{ij}^d = p_i(1 + \tau_{ij}^I) \tag{9}$$

$$p_{ij}^f = (p_i^f/e)(1 + \tau_{ij}^I) \tag{10}$$

where p_i and p_i^f are the gross-of-tax unit prices for domestic and foreign industry goods i, e is the exchange rate converting foreign currency into the domestic currency and τ_{ij}^I represents (pre-existing) intermediate input taxes on industry j's use of industry good i.

1.1.2 Oil&Gas Industry

In industries other than oil&gas, γ_f is a fixed and exogenous parameter. In the oil&gas industry, γ_f is a decreasing function of cumulative extraction:

$$\gamma_{f,t} = \epsilon_1 \left[1 - (Z_t/Z)^{\epsilon_2} \right] \tag{11}$$

where ϵ_1 and ϵ_2 are parameters. Z_t represents cumulative extraction as of the beginning of period t, and Z is the original estimated total stock of recoverable reserves of oil&gas (as estimated from the benchmark year). The following equation of motion specifies the evolution of Z_t:

$$Z_{t+1} = Z_t + Y_t. \tag{12}$$

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[4]There is one exception. Domestic and Foreign Oil&Gas are perfect substitutes in the model.

Equation (11) implies that the production function for oil&gas shifts downward as cumulative extraction increases. This addresses the fact that as reserves are depleted, remaining reserves become more difficult to extract and require more inputs per unit of extraction.

1.2 Behavior of Firms

In each industry, firms' choices of input quantities and investment levels in each period of time are made to maximize the value of the firm.

The value of the firm can be expressed in terms of dividends and new share issues, which in turn depend on profits in each period. The firms' earnings before tax in any given period are given by, suppressing time and industry subscripts,

$$\pi^b = p^n Y - w(1 + \tau_L)L - p_{\bar{E}}\bar{E} - p_{\bar{M}}\bar{M} - iDEBT - TPROP \tag{13}$$

where p_n is the output price net of output taxes, $w(1 + \tau_L)$ is the total cost of labor, wages plus labor taxes, $p_{\bar{E}}$ is the unit price of the composite $\bar{E}$, $p_{\bar{M}}$ is the unit price of the composite $\bar{M}$, i is the gross of- tax interest rate paid by the firm, $DEBT$ is the firm's current debt and $TPROP$ is the property tax payments. $TPROP = \tau_p p_{K,s-1} K_s$ where τ_p is the property tax rate, and $p_{K,t}$ is the purchase price of a new unit of capital at time t.

Firms' before tax earnings are taxed at the corporate tax rate τ_a. After-tax profits are equal to taxed earnings plus tax allowances,

$$\pi = (1 - \tau_a)\pi^b + \tau_a(DEPR + DEPL) \tag{14}$$

where $DEPR$ is the current gross depreciation allowance and $DEPL$ is the current gross depletion allowance. Current depreciation allowances, $DEPR$, can be expressed as $\delta^T K^T$ where K^T is the depreciable capital stock basis and δ^T is the depreciation applied for tax purposes. Current depletion allowances, $DEPL$, which only apply to the oil&gas industry, are a constant fraction β of the value of current extraction, $\beta p^n Y$.

The following accounting or cash-flow identity links the firm's sources and uses of revenues:[5]

$$\pi + BN + VN = DIV + IEXP. \tag{15}$$

The left-hand side is the firm's source of cash: profits, new debt issue (BN), and new share issues (VN). The uses of cash on the right-hand side are investment expenditure ($IEXP$) and dividend payments (DIV). Negative share issues are equivalent to share repurchases, and represent a use rather than source of cash.

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[5]For a discussion of alternative specifications, see Poterba and Summers (1985).

Firms pay dividends equal to a constant fraction, a, of profits gross of capital gains on the existing capital stock and net of economic depreciation. They also maintain debt equal to a constant fraction, b, of the value of the existing capital stock;[6]

$$DIV_s = a[\pi_s + (p_{K,s} - p_{K,s-1})K_s - \delta p_{K,s}K_s] \tag{16}$$

$$BN_s \equiv DEBT_{s+1} - DEBT_s = b(p_{K,s}K_{s+1} - p_{K,s-1}K_s). \tag{17}$$

Investment expenditures are expressed as

$$IEXP_s = (1 - \tau_k)p_{K,s}I_s \tag{18}$$

where τ_k is the (potential) investment tax credit rate. Of the elements in equation (15), new share issues, VN, are the residual, making up the difference between $\pi + BN$ and $DIV + IEXP$.

Arbitrage possibilities compel the firm to offer its stockholders a rate of return comparable to the rate of interest on alternative assets,[7]

$$(1 - \tau_e)DIV_s + (1 - \tau_v)(V_{s+1} - V_s - VN_s) = (1 - \tau_b)i_s V_s \tag{19}$$

where τ_e, τ_v, and τ_b are the personal tax rates on dividend income (equity), capital gains, and interest income (bonds) respectively. The return to stockholders consists of the current after-tax dividend plus the after-tax capital gain (accrued or realized) on the equity value (V) of the firm net of the value of new share issues. This return must be comparable to the after-tax return from an investment of the same value at the market rate of interest, i. Recursively applying equation (19) subject to the usual transversality condition ruling out eternal speculative bubbles yields the following expression for the equity value of the firm:

$$V_t = \sum_{s=t}^{\infty} \left[\frac{1 - \tau_e}{1 - \tau_v} DIV_s - VN_s \right] d_t(s) \tag{20}$$

where

$$d_t(s) = \prod_{u=t}^{s} \left[1 + \frac{r_u}{1 - \tau_v} \right]^{-1}. \tag{21}$$

The transversality condition is given by

$$\lim_{s \to \infty} V_s d_t(s) = 0. \tag{22}$$

Equation (20) indicates that the equity value of the firm is the discounted sum of after-tax dividends net of new share issues. The firm's problem is completed by adding the equation of motion for the capital stock:

$$K_{s+1} = I_s + (1 - \delta)K_s. \tag{23}$$

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[6]For a survey and analysis of these issues, see Sinn (1990) and Bernheim and Shoven (1989).

[7]This abstracts from uncertainty and, therefore, risk. It is possible to modify the arbitrage equation to account for risk differentials across assets. See Goulder (1989).

1.3 Optimal Production, Input Use, and Investment

In each period of time, the manager of the firm chooses inputs of labor, energy inputs, material inputs and investment to maximize the value of the firm. Due to the linear homogeneity of the nested production function, optimal input intensities can be solved for independently of the decision for total production and investment. Therefore, the producer problem can be divided into two problems: the inner nest problem to decide optimal input intensities and the outer nest production and investment problem.

1.3.1 Optimal Input Intensities

For any CES function of the form

$$X = \gamma \left[\sum_{i=1}^{n} \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}}$$

where the price of input x_i is p_i, the Lagrangian expression for obtaining the composite X at minimum cost is given by

$$L = \sum_{i=1}^{n} p_i x_i + \lambda \left\{ \gamma \left[\sum_{i=1}^{n} \alpha_i x_i^{\rho} \right]^{\frac{1}{\rho}} - X \right\}.$$

The first order conditions can be summarized as

$$\frac{x_i}{x_j} = \left[\frac{\alpha_j \, p_i}{\alpha_i \, p_j} \right]^{\frac{1}{\rho-1}}$$

for all i and j in $(1, ..., n)$. The minimum cost for composite X is P:

$$P = \gamma^{\frac{\sigma}{\sigma-1}} \left[\sum_{i=1}^{n} \alpha_i^{\sigma} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{24}$$

The optimal demand intensity for input x_i is

$$\frac{x_i}{X} = \left[\frac{\alpha_i}{\gamma} \right]^{\sigma} \left[\frac{p_i}{P} \right]^{-\sigma} \tag{25}$$

where σ is the constant elasticity of substitution equal to $1/(1-\rho)$.

Given prices paid for domestic and foreign goods, p_{ij}^d and p_{ij}^f, equations (24) and (25) can be applied to all inner nests of the production function to solve for

1. The optimal composites, E_i and M_i, of domestic and foreign energy and material goods and their unit prices p_{E_i} and p_{M_i}.

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2. The optimal energy and materials composites, $\bar{E}$ and $\bar{M}$, and their unit prices $p_{\bar{E}}$ and $p_{\bar{M}}$.

3. The optimal composite of intermediate inputs, $\tilde{F} = \tilde{f}(\bar{E}, \bar{M})$ and its unit price $p_{\tilde{F}}$.

4. The optimal composite of intermediate inputs and labor, $\hat{F} = \hat{f}(L, \tilde{F})$ and its unit price $p_{\hat{F}}$.

1.3.2 Optimal Production and Investment

Let $p_{\hat{F}}$ denote the unit price of the composite of intermediate inputs and labor, $\hat{F}$, and let p^n denote the net producer price of its output. For all industries other than oil&gas, the optimal production and investment decision can be summarized with the following Lagrangian:

$$
\begin{aligned}
L \;=\; & \sum_{s=t}^{\infty} \Big[\alpha_1 p_s^n f(K_s, \hat{F}_s) - \alpha_1 p_{\hat{F},s} \\
& + (\alpha_2 p_s^K - \alpha_3 p_{s-1}^K) K_s - (\alpha_4 p_s^K + \alpha_1 p_s^n \phi_s) I_s \Big] d_t(s) \\
& + B_t - \sum_{s=t}^{\infty} \lambda_s \left[K_{s+1} - (1-\delta) K_s - I_s \right] d_t(s),
\end{aligned}
\tag{26}
$$

where
$\alpha_1 = (1-\nu)(1-\tau_a)$;
$\alpha_2 = (b-\nu)(1-\delta)$;
$\alpha_3 = b - \nu + \alpha_1(ib + \tau_p)$;
$\alpha_4 = 1 - b - \tau_k - (1-\nu)DD_s$;
$\nu = a[1 - (1-\tau_e)/(1-\tau_v)]$;
DD_s =present value of depreciation deductions on a dollar of investment taken at time s;
B_t =present value of depreciation deductions attributable to capital at time t.

Given the current stock of capital, the manager chooses production by deciding on $\hat{F}_s$, the labor and intermediate input composite, and I_s, optimal investment, to maximize this Lagrangian. The first-order conditions for this problem are

$$
\frac{\partial L}{\partial \hat{F}_s} \;:\; p_s^n f_{\hat{F},s} = p_{\hat{F},s}
\tag{27}
$$

$$
\frac{\partial L}{\partial I_s} \;:\; \alpha_4 p_s^K + \alpha_1 p_s^n \left[\phi_s + \phi_s' \frac{I_s}{K_s} \right] = \lambda_s
\tag{28}
$$

$$
\frac{\partial L}{\partial K_{s+1}} \;:\; \lambda_s \left[1 + \frac{r_{s+1}}{1-\tau_v} \right] = \Big[\alpha_1 p_{s+1}^n f_{K,s+1} + \alpha_2 p_{s+1}^K - \alpha_3 p_s^K
\tag{29}
$$

$$
+ \alpha_1 p_{s+1}^n \phi_{s+1}' \left(\frac{I_s}{K_s} \right)^2 \Big] + \lambda_{s+1}(1-\delta)
$$

where $f_{\hat{F},s}$ and $f_{K,s}$ denote the derivative of f with respect to $\hat{F}$ and K at period s, respectively.

Equation (27) equates the marginal product of $\hat{F}$ with its marginal cost. Equation (28) equates the marginal cost, inclusive of adjustment costs, of a new unit of capital with its marginal shadow value, λ_s. Equation (29) is the intertemporal Euler condition for the shadow value of capital.

The production and investment problem for the oil&gas industry must be augmented to include the effect of cumulative extraction on productivity. For this industry, the Lagrangian is

$$
\begin{aligned}
L \;=\; & \sum_{s=t}^{\infty} \Big[\alpha_5 p_s^n f(K_s, \hat{F}_s, Z_s) - \alpha_1 p_{\hat{F},s} \\
& + (\alpha_2 p_s^K - \alpha_3 p_{s-1}^K) K_s - (\alpha_4 p_s^K + \alpha_5 p_s^n \phi_s) I_s \Big] d_t(s) \\
& + B_t - \sum_{s=t}^{\infty} \lambda_s \left[K_{s+1} - (1-\delta) K_s - I_s \right] d_t(s) \\
& - \sum_{s=t}^{\infty} \mu_s \left[Z_{s+1} - Z_s - \Big(f(K_s, \hat{F}_s, Z_s) - \phi_s I_s \Big) \right] d_t(s),
\end{aligned}
\tag{30}
$$

where $\alpha_5 = (1-\nu)(1 - (1-\beta)\tau_a)$.

The additional summation term corresponds to the equation related to the change in reserves. The factor α_5 accounts for the potential presence of depletion allowances in this industry. Note that if depletion allowances β are equal to zero, then $\alpha_5 = \alpha_1$. The first-order equations are:

$$
\frac{\partial L}{\partial \hat{F}_s} \;:\; f_{\hat{F},s}(\alpha_5 p_s^n + \mu_s) = \alpha_1 p_{\hat{F},a}
\tag{31}
$$

$$
\frac{\partial L}{\partial I_s} \;:\; \alpha_4 p_s^K + \alpha_5 p_s^n \left[\phi_s + \phi_s' \frac{I_s}{K_s} \right] = \lambda_s
\tag{32}
$$

$$
\frac{\partial L}{\partial K_{s+1}} \;:\; \lambda_s \left[1 + \frac{r_{s+1}}{1-\tau_v} \right] = \Big[\alpha_5 p_{s+1}^n f_{K,s+1} + \alpha_2 p_{s+1}^K - \alpha_3 p_s^K
\tag{33}
$$
$$
+ \alpha_5 p_{s+1}^n \phi_{s+1}' \left(\frac{I_s}{K_s} \right)^2 + \mu_{s+1} f_{K,s+1} \Big] + \lambda_{s+1}(1-\delta)
$$

$$
\frac{\partial L}{\partial Z_{s+1}} \;:\; \mu_s \left[1 + \frac{r_{s+1}}{1-\tau_v} \right] = \Big[\alpha_5 p_{s+1}^n f_{Z,s+1} + \mu_{s+1}(1 + f_{Z,s+1}) \Big]
\tag{34}
$$

Again, the first equation equates the marginal value of more labor and intermediate input composite with its marginal cost. The marginal value now includes the shadow value of cumulative extraction; each marginal increase in input use causes an incremental increase in output and causes a negative contribution to future production through the resource stock

effect. Equation (32) is the same as equation (28); it equates the marginal cost, inclusive of adjustment costs, of a new unit of capital to the shadow value of capital. Equation (33) is the firm's intertemporal Euler condition for the shadow value of capital. In the oil&gas industry, this equation must include the stock effect, μ_s, of an additional unit of capital on future productivity.

Equation (34) is unique to the oil&gas industry. It is the intertemporal Euler condition for μ_s, the shadow value of cumulative extraction. This equation states that current shadow value of reserves must equal the shadow value of the reserves in the next period, discounted to the current period, plus the discounted contribution to future output and productivity of a marginal reduction in reserves.

As discussed in Goulder (1991), the optimal path of output and investment for the oil&gas industry is quite different than industries with no stock effects. In the oil&gas industry, the levels of output and capital ultimately approach zero. In general, it is not optimal for firms to deplete the entire stock of reserves; rising extraction costs ultimately make it uneconomic to reduce reserves any further. Cumulative extraction asymptotically approaches a total which is less than the original stock of reserves.

2   Households

2.1   Utility

Households are modeled as an infinitely-lived representative agent that chooses consumption, leisure, and savings each period to maximize its intertemporal utility subject to its budget constraint. In year t the household chooses a path of "full consumption" C to maximize

$$U_t = \sum_{s=t}^{\infty} (1+\omega)^{t-s} \frac{\sigma}{\sigma - 1} C_s^{\frac{\sigma-1}{\sigma}} \tag{35}$$

where ω is the subjective rate of time preference and σ is the intertemporal elasticity of substitution in full consumption. C is a CES composite of consumption of goods and services, $\bar{C}$, and leisure, l:

$$C_s = \left[\bar{C}_s^{\frac{v-1}{v}} + \alpha_l^{\frac{1}{v}} l_s^{\frac{v-1}{v}} \right]^{\frac{v}{v-1}}, \tag{36}$$

where v is the elasticity of substitution between goods and leisure and α_l is a leisure intensity parameter.

Consumers use Cobb-Douglas spending shares, α_{C_i}, to divide their consumption of goods

and services across n_c consumer goods and services,[8]

$$\bar{C} = \prod_{i=1}^{n_c} \tilde{C}_{i,s}^{\alpha_{C_i}}. \tag{37}$$

Consumer goods are produced domestically and abroad. Domestically and foreign produced goods are imperfect substitutes; each consumer good $\tilde{C}_i$ is a CES aggregate of domestic ($\tilde{C}_i^d$) and foreign ($\tilde{C}_i^f$) produced goods. Omitting subscripts for type of consumer good and time period, $\tilde{C}$ is given by

$$\tilde{C} = \gamma_{\tilde{C}} \left[\alpha_{\tilde{C}} (\tilde{C}^d)^{\rho_{\tilde{C}}} + (1 - \alpha_{\tilde{C}})(\tilde{C}^f)^{\rho_{\tilde{C}}} \right]^{\frac{1}{\rho_{\tilde{C}}}}. \tag{38}$$

The household's intertemporal budget constraint is

$$W_{t+1} - W_t = \bar{r}_t W_t + Y_t^l + G_t^T - \bar{p}_t \bar{C}_t \tag{39}$$

where W_t is the household's financial wealth at time t, $\bar{r}$ is the average after-tax return on the household's financial wealth holdings, Y^l is the household's after-tax labor income, G^T is transfer income from the government, and $\bar{p}$ is the unit price of the consumption composite $\bar{C}$. The intertemporal budget constraint can be written recursively by applying the transversality condition that rules out speculative bubbles in household debt such that

$$\sum_{s=t}^{\infty} \bar{p}_s \bar{C}_s d_t^C(s) = \sum_{s=t}^{\infty} \left[Y_t^l + G_t^T \right] d_t^C(s) \tag{40}$$

where

$$d_t^C(s) = \prod_{u=t}^{s} [1 + \bar{r}_u]^{-1}. \tag{41}$$

The transversality condition is given by

$$\lim_{s \to \infty} W_s d_t^C(s). \tag{42}$$

Equation (40) shows that the present value of household consumption must equal human wealth, the present value of after-tax labor income plus transfers.

2.2 Solving for Optimal Consumption, Labor Supply, and Savings

Similar to the producer problem, the linear homogeneity of the nested utility function implies that aggregate consumption and leisure can be solved independent of optimal consumption composites. Household labor income, Y^l, is equal to total time worked (total potential labor

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[8]See Section 3 for list of consumer goods

time, $\bar{l}$, minus leisure), times the net-of-tax wage rate, $\bar{w}$. The household maximizes utility by choosing $\bar{C}_s$, l_s, and W_{s+1}. The Lagrangian for the household problem is

$$L^C = \sum_{s=t}^{\infty}(1+\omega)^{t-s}\frac{\sigma}{\sigma-1}\left[\left[\bar{C}_s^{\frac{v-1}{v}} + \alpha_l^{\frac{1}{v}}l_s^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}}\right]^{\frac{\sigma-1}{\sigma}} \tag{43}$$
$$\sum_{s=t}^{\infty}\lambda_{C,s}\left[(1+\bar{r}_s)W_s + \bar{w}(\bar{l}_s - l_s) + G_s^T - W_{s+1} - \bar{p}_s\bar{C}_s\right].$$

The first-order conditions with respect to $\bar{C}_s$, l_s, and W_{s+1} are

$$\frac{\partial L^C}{\partial \bar{C}_s} = (1+\omega)^{t-s}\left[\bar{C}_s^{\frac{v-1}{v}} + \alpha_l^{\frac{1}{v}}l_s^{\frac{v-1}{v}}\right]^{\frac{\sigma-v}{\sigma(v-1)}}\bar{C}_s^{\frac{-1}{v}} = \lambda_{C,s}\bar{p}_s \tag{44}$$

$$\frac{\partial L^C}{\partial l_s} = (1+\omega)^{t-s}\left[\bar{C}_s^{\frac{v-1}{v}} + \alpha_l^{\frac{1}{v}}l_s^{\frac{v-1}{v}}\right]^{\frac{\sigma-v}{\sigma(v-1)}}\alpha_l^{\frac{1}{v}}l_s^{\frac{-1}{v}} = \lambda_{C,s}\bar{w}_s \tag{45}$$

$$\frac{\partial L^C}{\partial W_{s+1}} : \quad \lambda_{C,s} = (1+\bar{r}_{s+1})\lambda_{C,s+1} \tag{46}$$

Equations (44) and (45) equate the marginal value of consumption to the marginal cost of consumption and the marginal value of leisure to the marginal cost of leisure. Conditional on the shadow value of wealth, $\lambda_{C,s}$, these equations combine to identify optimal leisure and aggregate consumption. Equation (46) is the intertemporal Euler equation of the shadow of wealth.

Consumption composite intensities are optimally solved for independent of aggregate consumption, $\bar{C}$. Optimal composite intensities are the intensities that obtain one unit of the composite at the lowest cost. Because $\tilde{C}_i$ is a CES composite of a domestic and foreign consumer good or service, equations (24) and (25) from section (1.3.1) can be applied to solve for the optimal domestic and foreign good intensity. If for a given composite, $\tilde{C}_i$, the prices of the domestic and foreign good are p_d^c and p_f^c, then the minimum unit price $\tilde{p}_i$ is, suppressing industry subscripts,

$$\tilde{p} = \gamma_{\tilde{C}}^{\frac{\sigma_{\tilde{C}}}{\sigma_{\tilde{C}}-1}}\left[\alpha_{\tilde{C}}^{\sigma_{\tilde{C}}}(p_d^c)^{1-\sigma_{\tilde{C}}} + (1-\alpha_{\tilde{C}})^{\sigma_{\tilde{C}}}(p_f^c)^{1-\sigma_{\tilde{C}}}\right]^{\frac{1}{1-\sigma_{\tilde{C}}}} \tag{47}$$

and the optimal demand intensity for the domestic good is

$$\frac{\tilde{C}^d}{\tilde{C}} = \left[\frac{\alpha_{\tilde{C}}}{\gamma_{\tilde{C}}}\right]^{\sigma_{\tilde{C}}}\left[\frac{p_d^c}{\tilde{p}}\right]^{-\sigma_{\tilde{C}}} \tag{48}$$

The Cobb-Douglas composite function for $\bar{C}$ implies that overall goods and services expenditures in period s are allocated across the individual goods and services, $\tilde{C}_{i,s}$ ($i = 1, ..., n_c$) according to the fixed expenditure shares $\alpha_{C,i}$,

$$\frac{\tilde{C}_i}{\bar{C}} = \alpha_{C,i}\left[\frac{\tilde{p}_i}{\bar{p}}\right]^{-1} \tag{49}$$

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where the optimal price of the aggregate consumption composite $\bar{p}$ is

$$\bar{p} = \prod_{i=1}^{n_c} \tilde{p}_i^{\alpha_{C,i}}. \tag{50}$$

3 Government

There is a single government to represent the combination of federal, state, and local government. The government levies taxes and issues debt to raise revenues and uses that revenue to finance government spending on goods and services (including labor), government capital accumulation, and transfers.

3.1 Government Expenditure

Government expenditures, G, are split between nominal purchases of goods and services, G^P, nominal investment expenditures, G^I, nominal transfers, G^T, and nominal labor expenditures, G^L,

$$G_t = G_t^P + G_t^I + G_t^T + G_t^L \tag{51}$$

Real investment expenditures and real transfers are assumed to grow exogenously at the steady state growth rate. Expenditures on goods and service and labor are described below.

3.1.1 Government Purchases

Total real government expenditures on goods and services grows exogenously at the steady state growth rate. Nominal government purchases of industry output, however, are determined by fixed Cobb-Douglas expenditure shares, $\alpha_{G,i}$, such that

$$\bar{G}_i^P p_i = \alpha_{G,i} G^p \tag{52}$$

where $\bar{G}^P$ is the real quantity demanded by government and p is the price of output.

3.1.2 Government Labor

The government wage bill, G^L, is nominal net of tax expenditures on labor; $G^L = w(1 + \tau_{L^g})L^g$ where w is the nominal wage rate, τ_{L^g} is the tax rate the government levies on its own use of labor, and L^g denotes the quantity of labor purchased. Real government spending on labor, exclusive of taxes, grows exogenously at the steady state growth rate.

3.2 Government Financing

3.2.1 Government Budget Constraint

The government finances its expenditures through taxes and debt issue. Taxation and borrowing decisions are linked through an intertemporal budget constraint, derived from the government's cash-flow equation:

$$G_s + r_s D_s = T_s + D_{s+1} - D_s \tag{53}$$

The left-hand side of the equation represents the government's use of revenues, expenditures, G, and interest payments $r_s D_S$. r_s represents the gross interest rate paid on government debt in period s, (the same rate offered by private firms in equilibrium) and D_s represents the stock of outstanding debt at the beginning of period s. The right-hand side of the cash-flow equation is the government's source of revenues: tax receipts, T_s and new debt issue, $D_{s+1} - D_s$. Tax revenue is collected from taxes on households, T^H (includes income, dividend, capital gains taxes, and lump-sum taxes), sales taxes on consumption, T^S taxes on firms, T^F (includes corporate taxes, output taxes, property taxes, and intermediate input taxes), and net border taxes, T^B (tariff revenues minus export subsidies):

$$T = T^H + T^S + T^F + T^B \tag{54}$$

Applying equation (53) recursively from time period t to t' $(t' > t)$ yields the following equation of motion for government debt:

$$D_{t'+1} d_t(t'+1) = D_t + \sum_{s=t}^{t'} (G_s - T_s) d_t(s) \tag{55}$$

where $d_t(s)$ is defined in equation (21). The following government intertemporal budget constraint

$$\sum_{t=s}^{\infty} (T_s - G_s) d_t(s) = D_s \tag{56}$$

is derived by applying the following government transversality condition to the equation of motion for government debt:

$$\lim_{t \to \infty} D_{t'+1} d_t(t'+1) = 0. \tag{57}$$

Equation (56), the government's intertemporal budget constraint, states that the present value of future tax revenues must exceed the present value of government expenditure (net of interest payments) by an amount equal to the level of outstanding government debt at the beginning of the period. The transversality condition prevents debt from growing faster than the rate of interest.

3.2.2 Marginal Financing

Real government debt grows exogenously at the steady state growth rate. Since real government expenditure is exogenous as well, marginal financing must be done through taxes. The government's cash-flow constraint must be satisfied through adjustments in the taxes. The model allows for adjustments in lump-sum taxes from the household and/or changes in the marginal income tax rates on labor and capital.

4 Foreign Trade

The model treats the rest of the world as a single economy. This foreign economy has industry, households, and government that mirror the domestic economy: the foreign household and government mirror the domestic household and government exactly while foreign industry is modeled slightly different than the domestic set of industries.

4.1 Foreign Industry

There is one foreign industry that can exactly produce all industry goods using just capital and labor (there is no foreign industry specific capital or labor). Foreign gross output is given by

$$X^f = f(K^f, L^f) \tag{58}$$

where f is a CES aggregation function. The foreign sector faces the same per unit adjustment cost function as the domestic industries, therefore foreign net output Y_f is

$$Y^f = X^f - \phi(I^f/K^f)I^f. \tag{59}$$

While intermediate inputs do not directly contribute to production, for each unit of output the foreign industry is required to use intermediate input IN_i per unit of output for industry good i. Intermediate input IN_i is a CES aggregate of domestically-produced, IN_i^d, and foreign-produced IN_i^f industry good i, which can be obtained at minimum cost p_i^{IN} using equation (24) given the price p_i^f, the price of foreign industry goods, and p_i^{df}, the price of domestic (U.S.) industry goods to the foreign industry. The price p_i^{df} is given by

$$p_i^{df} = p_i(1 - \tau^{EX_p})e \tag{60}$$

where τ^{EX_p} is a per unit export subsidy on producer goods in the domestic (U.S) economy.

Before tax profits of the foreign industry are, ignoring the foreign superscript, given by

$$\pi^b = p^n Y - w(1 + \tau_L)L - \sum_{j=1}^{n_I} p_j^{IN} IN_j X - iDEBT - TPROP \tag{61}$$

The remainder of the foreign firm problem is identical to the domestic firm problem. In fact, the first-order conditions with respect to capital and investment are identical to equations (28) and (29). The first order condition with respect to labor becomes

$$\left(p_s^n - \sum_{i=j}^{n_I} p_j^{IN} IN_j \right) f_{L,s} = p_{L,s} \tag{62}$$

5 Price Relationships

5.1 General

The numeraire in the model is the first-period nominal wage. The nominal wage grows exogenously at a rate specified by the modeler. Domestic prices are endogenously determined such that supply equals demand in each industry with the exception of the oil&gas industry (more on this below). The producer prices of foreign goods are also exogenous (in the foreign currency) and grow at the same rate as the nominal wage. However, the prices of foreign goods in the domestic currency are endogenous because the nominal exchange rate is endogenous; the equilibrium nominal exchange rate generates balanced trade.

5.2 Producer Prices

The gross-of-tax price of output for domestic industry i is denoted by p_i. The net-of-tax producer price, p_i^n, is given by

$$p_i^n = (1 - \tau_{x,i})p_i \tag{63}$$

where $\tau_{x,i}$ is the (potential) ad valorem tax rate on the output of industry i. As noted above, producer prices of foreign goods, p_i^f are exogenous and grow at the same rate as the nominal wage.[9]

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[9]Because there is no industry good specific capital and labor in the foreign industry, $p_i^f = p^f$ for all i.

5.2.1 Oil&Gas and Synthetic Fuels Prices

The world price of oil&gas is exogenously specified in real terms.[10] At this price, the foreign oil&gas producers can supply an infinitely elastic supply of oil&gas. The domestic price equals the world price plus any applicable tariffs.

Oil&gas and synthetic fuels are perfect substitutes in demand; the equilibrium price of the industry output of both goods must be equal. The equilibrium price, however, will depend on supply circumstances. Let p_f^O denote the exogenous price of imported oil&gas and let p^S denote the synfuels price that would be necessary to generate a quantity of synfuels such that oil&gas imports are zero. The equilibrium price of oil&gas and synfuels, p^O, must satisfy

$$p^O = \left[\begin{array}{ll} p_f^O(1+\tau_O) & (p_f^O < p^S) \\ p^S & (p_f^O > p^S) \end{array} \right. \tag{64}$$

When p^S exceeds $p_f^O(1+\tau_O)$, domestic production of oil&gas and synfuels are not sufficient to satisfy the domestic demand for their products. In this case, imports of oil&gas, O^I become the marginal source of supply, and $p^O = p_f^O(1 + \tau_O)$ where τ_O represents (potential) tariffs on oil imports. When imports are the marginal source of supply, the quantities of imports are adjusted to balance the domestic supply and demand for oil&gas and synfuels output. In contrast, when domestic oil&gas and synfuels production can meet the entire domestic demand, synfuels production represents the marginal source of supply and $p^O = p^S$. Under these circumstances, as in all other markets, the price p^S adjusts such that domestic supply of oil&gas and synfuels equals total demand for the domestic good.

5.3 Consumer Prices

Consumer goods are produced from industry goods using a Leontiff production function. The matrix IC, which is fixed across all periods, indicates the intensities of industry goods used to produce each consumer good, domestic and foreign; the matrix has dimension $(n_I) \times n_c$. The production of consumer goods from industry goods requires no labor or capital, therefore the price of domestic consumer goods, $p_{d,i}^p$, is simply

$$p_{d,i}^p = \sum_{j=1}^{n_I} IC_{ji}p_i. \tag{65}$$

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[10]In general, any path for the real price of oil is possible. In practice, a constant increment in the real price is used to hit a chosen (real price-year) forecast.

Since IC is the same for the production of foreign consumer goods, an analogous equation holds for foreign consumer goods.[11]

Net-of-tax prices paid by domestic consumers for goods, p_d^c and p_f^c, will then simply be the prices of produced consumer goods adjusted by taxes:

$$p_{d,i}^c = p_{d,i}^p(1 + \tau_{c,i}) \tag{66}$$

$$p_{f,i}^c = (p_{f,i}^p/e)(1 + \tau_{c,i})(1 + \tau_t) \tag{67}$$

where e is the exchange rate, $\tau_{c,i}$ is an ad valorem sales tax, and τ_t is an ad valorem tariff. The net-of-tax prices paid by foreign consumers for goods, p_{df}^c and p_{ff}^c, are

$$p_{ff,i}^c = p_{f,i}^p(1 + \tau_{cf,i}) \tag{68}$$

$$p_{df,i}^c = ep_{d,i}^p(1 + \tau_{cf,i})(1 - \tau^{EX_c}) \tag{69}$$

where $\tau_{cf,i}$ is the foreign ad valorem sales tax and τ^{EX_c} is an ad valorem export subsidy on consumer goods.

5.4 Capital Prices

New capital goods are produced using producer goods from domestic and foreign industries. Total domestic investment demand is $\hat{I} = \sum_i I_i + I^g$ where I_i is the demand for new capital good from industry i and $I^g = G^I/p^K$ represents real government purchases of new capital.[12] There are both domestic and foreign capital good producers that produce domestic and foreign capital goods supplies, $\hat{I}^d$ and $\hat{I}^f$ respectively, by combining producer goods in a CES aggregation function. Capital goods producers use no labor or capital to produce new capital goods from producer goods and therefore seek to provide domestic (or foreign) capital goods at a minimum cost.

By applying equations (24) and (25), the unit price of new domestic capital goods is given by

$$p_d^K = \gamma_{I^d}^{\frac{\sigma_{I^d}}{\sigma_{I^d}-1}} \left[\sum_{i=1}^{n_I} \alpha_{I^d,i} p_i^{1-\sigma_{I^d}} \right]^{\frac{1}{1-\sigma_{I^d}}} \tag{70}$$

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[11]And because in practice foreign producer goods are constant across all industries, ($p_j^f = p^f$ for all industries j), $p_{f,i}^p = p^f$ for all consumer goods i.

[12]The model assumes that capital goods used in the housing services industry differ from those used by other domestic industries. Therefore there are two forms of capital each with its own price in the model, one applying to housing services and the other applying to other industries.

and the optimal capital good demand for industry good i is

$$\hat{I}_i^d = \hat{I}^d \left[\frac{\alpha_{I^d,i}}{\gamma_{I^d}}\right]^{\sigma_{I^d}} \left[\frac{p_i}{p_d^k}\right]^{-\sigma_{I^d}} \tag{71}$$

where σ_{I^d} is the constant elasticity of substitution between industry outputs in the domestic new capital good composite, γ_{I^d} is the CES scaling parameter and $\alpha_{I^d,i}$ is the CES share parameter. Analogous expressions and parameters hold for the foreign new capital good composite $\hat{I}^f$. The total supply of capital goods for use by domestic users is divided between domestic and foreign sources with a fixed share, α_I such that $\hat{I}^d = \alpha_I \hat{I}$ and $\hat{I}^f = (1 - \alpha_I)\hat{I}$; therefore the unit price of new capital goods is

$$p^K = \alpha_I p_d^K + (1 - \alpha_I) p_f^K. \tag{72}$$

The demand for new capital goods for the foreign industry is much simpler. The foreign industry uses no domestic new capital goods. And because there is no industry-specific capital and labor, it must be the case that $p^{K,f} = p^f$.

6 Environmental Policies

6.1 Carbon Pricing

The model can incorporate both a carbon tax and a cap-and-trade program on carbon emissions. Carbon taxes and cap-and trade programs both introduce a price on carbon, p_c, into the model. Regardless of policy, the price of carbon is introduced into the model identically. The difference between the carbon tax and the cap-and-trade program is how the price on carbon is introduced into the model. Carbon taxes introduce this price through an exogenous profile of prices through time; cap-and-trade programs introduce an exogenous limit, A, on the quantity of carbon emissions in each period. In equilibrium, the price of carbon, p_c, will be the price such that total emissions, H, are equal to the total number of allowances,A, in each period (provided there is no banking and borrowing provision).

Given an exogenous path of carbon prices (carbon tax) or an exogenous path of emissions limits (cap and trade), carbon policies in the model are differentiated by (1) Point of Regulation, (2) Industry Coverage, (3) Tax Exemptions (carbon tax) or Allowance Allocation (cap and trade), (4) Revenue Neutrality, and (5) Special Cap and Trade Provisions.

6.1.1 Points of Regulation

The points of regulation represent a key design element of any carbon tax or cap-and-trade program. Under a fully upstream policy, the points of regulation are the entry points of carbon into the economy, the mine mouth for coal and the wellhead for oil and gas, and the port of entry for imported fossil fuels. Here, the price of carbon is introduced as an output tax on producers with goods with a positive carbon content and a tariff on the imported fossil fuels. The net producer price received by producers of goods with a positive carbon content (in practice, fossil fuel producers oil&gas, synfuels, and coal mining) is:

$$p_i^n = (1 - \tau_{x,i})p_i - c_i p_c \tag{73}$$

where c_i is the carbon content of one unit of output.[13] Additionally, any tariffs on the import of industry i goods will be incremented by $p_c c_i$ under a fully upstream policy.

Alternatively, the carbon policy could be implemented further down the production stream at the industrial user's gate. This point of regulation will be referred to as *Modified Upstream*. In a modified upstream carbon policy, industrial users face intermediate input taxes on their use of goods with carbon contents (i.e. fossil fuels), regardless of the source, domestic or foreign; the unit price paid for domestic and foreign industry good i by industry j, p_{ij}^d and p_{ij}^f, becomes

$$p_{ij}^d = p_i(1 + \tau_{ij}^I) + p_c c_i \tag{74}$$
$$p_{ij}^f = (p_i^f/e)(1 + \tau_{ij}^I) + p_c c_i. \tag{75}$$

Because industrial users must pay the intermediate input tax on foreign produced fossil fuels, no carbon tariff is applied in the modified upstream point of regulation policy to prevent double taxation of fossil fuels.

6.1.2 Industry Coverage

Carbon policies can be differentiated by which industries are covered by the policy. The model can study policies that restrict the coverage to specific fossil fuel suppliers (fully upstream point of regulation) or specific fossil fuel users (modified upstream point of regulation). To do this, equations (66) - (68) are modified such that

$$p_i^n = (1 - \tau_{x,i})p_i - c_i p_c u_i \tag{76}$$

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[13]Carbon prices are denoted per metric ton of emissions. Therefore, the carbon coefficient is equal to the metric tons of carbon released per unit of output of the fuel.

$$p_{ij}^d = p_i(1 + \tau_{ij}^I) + p_c c_i d_j \tag{77}$$

$$p_{ij}^f = (p_i^f/e)(1 + \tau_{ij}^I) + p_c c_i d_j. \tag{78}$$

where u_i is a dummy variable equal to one if industry i is a covered supplier industry in the fully upstream point of regulation and d_j is a dummy variable equal to one if industry j is a covered industrial user under the modified upstream point of regulation.

6.1.3 Tax Exemptions/Allowance Allocation

Tax Exemptions in the Carbon Tax Policy   The model allows for tax exemptions in the carbon tax policy through lump-sum rebates on a proportion of the tax bill. Abstracting from point of regulation, if industry i owes a carbon tax payment of T_i^C, the policy can specify the fraction of the bill, $\alpha_i^{T^C}$, that is repaid to the producer at the end of the period. In such a policy, profits are given by

$$\pi_i = (1 - \tau_a)\pi_i^b + \tau_a(DEPR_i + DEPL_i) + (1 - \tau^R)(\alpha_i^{T^C} T_i^C) \tag{79}$$

where τ^R represents a (potential) tax rate on lump-sum receipts. Note that the carbon tax itself is implicit in before tax profits, π^b, because before tax profits include the net-of-tax price p^n, not gross-of-tax price p, and p^E is a function of p_{ij}^d and p_{ij}^f which include the modified upstream carbon tax. Total carbon tax receipts received by the government will be $T^C = (1 - \tau^R)\sum_i \alpha_i^{T^C} T_i^C$.

Allowance Allocation in a Cap-and-Trade Program   In the cap-and-trade program, the model can consider any combination of auctioning and free allocation of allowances, including the limiting cases of 100% auctioning and 100% free allocation. In addition, it is flexible as to how the free allowances are allocated across various industries. Let α_i^A denote the fraction of total allowances, A, allocated to industry i.[14] Similar to tax exemptions in a carbon tax policy, free allowances in the cap-and-trade program augment the firm's profits such that

$$\pi_i = (1 - \tau_a)\pi_i^b + \tau_a(DEPR_i + DEPL_i) + (1 - \tau^R)(\alpha_i^A A) \tag{80}$$

Under a cap-and-trade program, the government will receive total carbon payments of $T^C = p^c A$. However, with free allowance allocation, the total carbon receipts must be reduced by the value of the allowances given away free, $T^C = p^c(A - \bar{A})$ where $\bar{A} = \sum_{i=1}^{n_I} \alpha_i^A A$.

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[14] Allowance allocations do not have to be limited to industries covered by the cap-and-trade program. Any industry not covered will sell its allocated allowances to a covered industry.

6.1.4 Revenue Neutrality

A climate policy alters the government tax base and tax revenue through its effects on economic output and income. Additionally, the government receives an additional source of tax revenue, T^C, from carbon tax/allowance receipts. In a cap-and-trade policy, total potential (before free allowance allocation) carbon revenue is simply $p^c A$. Under a carbon tax, total carbon taxes due by industry, T_i^C are equal to $Y_i c_i$ under an upstream program and are equal to $\sum_j (E_{ji}^d + E_{ji}^f) c_j$ under a modified upstream program. Depending on the policy and tax exemptions/allowance allocation, the net change in tax revenue could be positive or negative. The model allows for the option of imposing revenue-neutrality (net change in tax revenue of zero) through adjustments in marginal tax rates or lump-sum taxes.

6.1.5 Special Cap-and-Trade Provisions

The model allows for two special provisions under a cap-and-trade program: banking and borrowing and offsets.

Banking and Borrowing Banking and borrowing allows firms to equate marginal abatement costs (in present value) over time, which reduces the present value of abatement costs. Under a system with fully functioning banking and borrowing, in market equilibrium, the discounted allowance price will be the same in every period within the interval where banking and borrowing is allowed. Therefore, banking and borrowing imposes the following requirement on allowance prices

$$p_s^c = (1 + r_{s+1})^{-1} p_{s+1}^c. \tag{81}$$

where r_{s+1} is the risk-free interest rate from the firm problem. Additionally, banking and borrowing relaxes the condition that covered emissions, $\bar{H}_t$, must equal total allowances, $\bar{H}_t = A_t$, for all t. Banking and borrowing instead imposes the intertemporal condition such that

$$\sum_{s=t}^{t'} \bar{H}_s = \sum_{s=t}^{t'} A_s \tag{82}$$

for the fixed interval $[t, t']$

Offsets The model allows for emissions offsets in a cap-and-trade program. By allowing for offsets, firms that are required to hold allowances are allowed to substitute allowances for carbon offsets from a certified offset supplier. In equilibrium, the entity must be indifferent

between holding allowances and offsets (provided offsets provide the same rights to emit as allowances); it must be the case that the price of offsets equals the allowance price, p^c.

Adding offsets allows the entities to produce more emissions than the total emissions allowances each period. Therefore, the constraint $\bar{H}_t = A_t$ is replaced by $\bar{H}_t = A_t + Q_t^o$ where Q_t^o is the quantity of offsets supplied. Offsets are supplied exogenously through an offset supply curve $Q^o(p^c)$ with an inverse price curve $P^o(Q^o)$.[15] The price of allowances/offsets adjusts such that the new emissions constraint binds in each period. The government still receives allowance revenue, $p^c(A_t - \bar{A}_t)$, but offset suppliers receive offset revenue $p^c Q^o(p^c) - P^o(Q^o)$.[16]

6.2 Command and Control

The model offers a flexible treatment of command and control policies that put restrictions on intermediate inputs for a given industry. For the purposes of this model, the command and control policies are restricted to policies that govern the input of electricity from a given generator for the retail electricity sector. Two prominent command and control policies for the retail electricity sector are a clean energy standard and emissions standards.

6.2.1 Clean Energy Standard

Clean Energy Standards are command and control style policies that place a minimum floor on a specific type of electricity (electricity produced using "clean" technologies). Consider more generally minimum input constraints such that

$$\frac{\sum_{i=1}^{N} a_i \beta_i x_i}{\sum_{i=1}^{N} \beta_i x_i} \geq \bar{a} \tag{83}$$

where β_i is a scalar (note: there is no constraint on the β's, they are just used to transform the measure of input if necessary), a_i is the share of the input's use that can be applied toward the requirement, and $\bar{a}$ is the minimum input requirement. Recall from Section 1.3.1, for a CES function of the form

$$X = \gamma \left[\sum_{i=1}^{n} \alpha_i x_i^\rho \right]^{\frac{1}{\rho}},$$

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[15]Currently, offsets are produced using no labor or capital and simply cost $P^o(Q^o)$ to the certified offset producer.

[16]In general, any industry could be an offset supplier, but in practice offset suppliers are restricted to domestic and foreign agriculture and forestry industries.

the minimum cost for composite X is given by

$$P = \gamma^{\frac{\sigma}{\sigma-1}} \left[\sum_{i=1}^{n} \alpha_i^{\sigma} p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{84}$$

and the optimal demand intensity for input x_i is given by

$$\frac{x_i}{X} = \left[\frac{\alpha_i}{\gamma} \right]^{\sigma} \left[\frac{p_i}{P} \right]^{-\sigma} \tag{85}$$

where σ is the constant elasticity of substitution equal to $1/(1 - \rho)$.

The minimum input constraint (83) turns the unconstrained minimization problem above into a constrained minimization problem. One can show that the solution to the constrained minimization problem is equal to a unconstrained minimization problem with input taxes and subsidies. Consider the unconstrained problem with taxes/subsidies. Let $\hat{p}_i = p_i + \tau_i$ and let τ represent the vector of taxes/subsidies. Following equations (84) and (85), then the minimum cost P is

$$P(\tau) = \gamma^{\frac{\sigma}{\sigma-1}} \left[\sum_{i=1}^{n} \alpha_i^{\sigma} \hat{p}_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{86}$$

and the optimal demand intensity for input $x_i(\tau)$ is given by

$$\frac{x_i(\tau)}{X} = \left[\frac{\alpha_i}{\gamma} \right]^{\sigma} \left[\frac{\hat{p}_i}{P(\tau)} \right]^{-\sigma}. \tag{87}$$

It can be easily verified that if $\tau_i = \mu^*(\bar{a} - a_i)\beta_i$ and $\sum_{i=1}^{N} \tau_i x_i(\tau) = 0$, then the solution to the unconstrained minimization with taxes/subsidies is exactly equal to the constrained minimization problem with constraint (83). In other words, the constrained input intensity minimization problem can be replicated with a revenue neutral set of taxes and subsidies on inputs.

6.2.2   Emissions Standards

In the electric power sector, an emissions standard is a restraint on the quantity of emissions that can be produced per megawatt hour. More generally, emissions standards take the form of a constraint on inputs such that

$$\frac{\sum_i e_i \beta_i x_i}{\sum_i \beta_i x_i} \leq \bar{e} \tag{88}$$

where e_i is the emissions per unit output of industry i, β_i is again a scalar (to convert units of output into megawatt hours), and $\bar{e}$ is the maximum emissions per megawatt hour imposed by the Emissions Standard policy.

Following the same argument in section 6.2.1, the constrained minimization problem with constraint (88) can be replicated with equations (86) and (87) and a revenue neutral set of taxes and subsidies on inputs such that $\tau_i = \mu * \beta_i(e_i - \bar{e})$ and $\sum_{i=1}^{N} \tau_i x_i(\tau) = 0$.

7 Equilibrium Conditions

7.1 Intratemporal Equilibrium

Equilibrium in each period satisfies five types of conditions:

1. The aggregate demand for labor equals aggregate supply.
2. The demand for each industry output equals its supply.[17]
3. The aggregate demand for loanable funds by firms and government equals the aggregate supply provided by households.
4. Government tax revenues equal government spending less the government deficit
5. The value of exports equals the value of imports.

These conditions are met through adjustment in output prices, in the market interest rate, tax adjustments, and the exchange rate. Conditions (1)-(4) must also hold for the foreign economy. More detail on the supply and demand equations are provided below.

Aggregate labor demand, L^d, and aggregate labor supply, L^s, are given by

$$L^d = \sum_{i=1}^{n_I} L_i + L^g \tag{89}$$

$$L^s = \bar{l} - l \tag{90}$$

Final goods supply for industry i, FG_i^S, is net output less intermediate input use:

$$FG_i^S = Y_i - \sum_{j=1}^{n_I} a_{ij} X_j \tag{91}$$

where a_{ij} is the optimized per unit demand for industry good i per unit of gross output of industry good j and therefore $a_{ij} X_j$ is the total quantity of industry good i used by industry j.

The final good demand for industry i, FG_i^D consists of consumption demand, investment

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[17]Oil&gas and synfuels are perfect substitutes and generate a single supply-demand equation.

demand, government demand and export demand:

$$FG_i^D = \sum_{j=1}^{n_c} IC_{ij}(\bar{C}_j^{dd} + \bar{C}_j^{fd}) + \hat{I}_i^d + G_i^P + EXP_i \tag{92}$$

where $\bar{C}_j^{dd}$ and $\bar{C}_j^{fd}$ represent domestic and foreign demands for the domestically produced consumer good j and $EXP_i = IN_i^d$ represents foreign industry purchases of domestic industry good i.

$BORROW$ represents the aggregate demand for loanable funds,

$$BORROW = \sum_{i=1}^{n_I}(BN_i + VN_i) + BN^G. \tag{93}$$

Firm's demand for loanable funds come from new debt issues and new share issues. Government demand for loanable funds is through new government debt issue, $BN^G = D_{s+1} - D_s$.

Domestic savings, $SAVE$, equals gross income plus government transfers less household taxes and consumption expenditures,

$$SAVE = Y^I + G^T - T^H - \tilde{p}\tilde{C} \tag{94}$$

where gross income[18] is

$$Y^I = Y^L + i(\sum_{j=1}^{n_I} DEBT_j + D) + \sum_{j=1}^{n_I} DIV_j. \tag{95}$$

Government expenditures and government revenues are described in section (3). If a carbon tax or a cap-and-trade program is used, then government revenues are augmented by T^C.

The total value of exports, $VALEXP$, is simply the sum of the values of exports of consumer goods and exports of producer goods,

$$VALEXP = \sum_{j}^{n_c} \bar{C}_j^{fd} p_{d,j}^p e(1 - \tau_j^{EX_c}) + \sum_{j}^{n_I} IN_j^d p_j(1 - \tau_j^{EX_p})e. \tag{96}$$

The total value of imports, $VALIMP$, is equal to the sum of the values of exports of consumer goods, producer goods, new capital goods, and oil imports

$$\begin{aligned} VALIMP \;=\; & \sum_{j}^{n_c} \bar{C}_j^{df}(p_{f,j}^p/e)(1 + \tau_{c,j}) \\ & + \sum_{j=1}^{n_I}\left[\sum_{i=1}^{n_E}(p_i^f/e)E_{ij}^f + \sum_{i=1}^{n_M}(p_i^f/e)M_{ij}^F\right] \\ & + \hat{I}^f p_f^K + O^I p_f^O. \end{aligned} \tag{97}$$

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[18]Capital gains are not included in gross income, because gross income measures are concerned with the nominal income flow and the flow of funds offered by households to firms. In the model, capital gains are taken into account in updating the household's wealth from period to period.

7.2   Intertemporal Equilibrium

Agents in the model are forward-looking with perfect foresight. This implies that the following condition is imposed on both households' and firms' time t expectation of variable x_{t+1}:

$$E_t[x_{t+1}] = x_{t+1}. \tag{98}$$

Equation (98) implies that an agent's expectations must be exact for all variables in all periods and the model abstracts away from any uncertainty over future prices or quantities. See section (8.3) for a discussion on how to solve for this intertemporal equilibrium.

8   Solution Method

The equilibrium solution is intertemporal because market-clearing at any point in time depends on both current prices and future prices. To solve for such an equilibrium, we apply an algorithm similar to that of Fair and Taylor (1983).

A sketch of the solution procedure is as follows. First, the model is solved with steady-state constraints to obtain terminal values for "dynamic variables." Next, a path of posited values for the dynamic variables that embody agents' expectations are specified. The posited dynamic variables include shadow values for human wealth and for capital in each industry. The model is solved using the posited dynamic variables as inputs. Next, the values of these dynamic variables are compared with the values of dynamic variables that result from the simulation. If the values do not match, the posited dynamic variables are adjusted and the model is solved again. The process is repeated until the posited and derived dynamic variables match, at which point the model has attained the perfect foresight intertemporal equilibrium.

8.1   Steady-State

Agents in the model have infinite horizons. Obviously, it is only possible to perform simulations over a finite number of periods. To account for the performance of the model after the last simulation period, the model imposes a regularity condition - steady-state growth - in the final simulation period. To impose the regularity condition, the terminal values for the dynamic variables are set equal to their steady state (with balanced growth) values, adjusted for growth.

8.2 Dynamic Variables

Equations (29), (33), (34), and (46) express the optimal equations of motion of shadow values across successive periods. These equations can be written as

$$\lambda_{i,s} = (F_{\lambda,i,s+1} - \alpha_3 p_s^K)(1 + r_{s+1})^{-1} \qquad (99)$$

$$\mu_s = F_{\mu,s+1}(1 + r_{s+1})^{-1} \qquad (100)$$

$$\lambda_{C,s} = F_{C,s+1}(1 + \bar{r}_{s+1})^{-1} \qquad (101)$$

where

$$F_{\lambda,i,s+1} = \alpha_1 p_{s+1}^n f_{K,s+1} + \alpha_2 p_{s+1}^K + \alpha_1 p_{s+1}^n \phi_{s+1}' \left[\frac{I_{s+1}}{K_{s+1}} \right]^2$$

$$+ \lambda_{i,s+1}(1 - \delta) + \Psi_i \qquad (102)$$

$$F_{\mu,s+1} = \alpha_5 p_{s+1}^n f_{Z,s+1} + \mu_{s+1}(1 + f_{Z,s+1}) \qquad (103)$$

$$F_{C,s+1} = \lambda_{C,s+1} \qquad (104)$$

with

$$\Psi_i = \left[\begin{array}{ll} 0 & i \neq og \\ \mu_{s+1} f_{K,s+1} & i = og. \end{array} \right. \qquad (105)$$

The variables F_λ, F_μ, and F_C are the dynamic variables. These variables consolidate all of the information about the future that is critical to optimal decision making in the current period.[19]

8.3 Solving for the Conditional Equilibrium

The optimal values for the dynamic variables during the transition to the steady-state cannot be obtained directly. To solve the model, paths of the *posited* dynamic variables are defined where each path terminates in the (known) steady-state values. The paths of the posited dynamic variables are represented by $\tilde{F}_{\lambda,i,s}$ ($s = 2, ..., \bar{t}, i = 1, ..., n_I$), $\tilde{F}_{\mu,s}$ ($s = 2, ..., \bar{t}$), and $\tilde{F}_{C,s}$ ($s = 2, ..., \bar{t}$) where $\bar{t}$ is the final simulation period and terminal values are given by $\tilde{F}_{\lambda,i,\bar{t}+1}$, $\tilde{F}_{\mu,\bar{t}+1}$, and $\tilde{F}_{C,\bar{t}+1}$.[20]

Given the paths of the posited dynamic variables, it is possible to solve the model sequentially beginning in the first period. In each period, given the values of the posited

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[19]The foreign economy will have one dynamic variable to represent the shadow value of its capital and one dynamic variable to represent its shadow value of consumption.

[20]Without loss of generality, the posited and terminal values for the foreign economy are not explicity defined here.

dynamic variables, current prices are sufficient to determine optimal agent behavior. There-fore, given the posited dynamic variable paths, the solution requirement each period is to obtain prices each period that yield excess demands of zero.

To obtain the market clearing prices each period conditional on the posited paths of dynamic variables, the algorithm of Powell (1978), designed for systems of nonlinear equations, is used. The excess demand conditions are given in section (7.1). The equilibrium prices are the before-tax industry prices, p_i and p^f, the domestic and foreign before-tax interest rate, r, r^f, the nominal exchange rate e, adjustments to domestic and foreign lump-sum tax levels, and either the level of oil&gas imports or the synfuels price, as discussed earlier. All other prices are either exogenously specified or functions of the equilibrium prices.

8.4   Solving for the Intertemporal Equilibrium

The conditional equilibrium solution generates time paths for all the prices and quantities in the model. These prices and quantities can be substituted into the right-hand side of equations (102) - (105) to generate derived dynamic variables $\hat{F}_{\lambda,i}$, $\hat{F}_\mu$, and $\hat{F}_C$.

For the conditional equilibrium solution to correspond to the perfect-foresight intertemporal equilibrium, it is necessary that posited and derived dynamic variables match in each period:

$$\hat{F}_{\lambda,i,s} = \tilde{F}_{\lambda,i,s} \quad s = 2, ..., \bar{t}; i = 1, ..., n_I \tag{106}$$

$$\hat{F}_{\mu,s} = \tilde{F}_{\mu,s} \quad s = 2, ..., \bar{t} \tag{107}$$

$$\hat{F}_{C,s} = \tilde{F}_{C,s} \quad s = 2, ..., \bar{t}. \tag{108}$$

Solving for the intertemporal equilibrium to the model hinges on revising the posited values until conditions (101) - (103) are met. After each iteration, the posited dynamic variables are revised by taking a weighted average of the posited and derived dynamic variable using a weighting parameter η. Iterations on the conditional equilibrium are repeated until, for all dynamic variables in every period, the posited values are within a desired tolerance of the corresponding derived value.

B. Data and Parameters

The model requires both data and parameter values as inputs into the model. Primary data is used as initial values where appropriate, such as for industry capital stocks, to benchmark the model to the benchmark year, 2010. Parameter values for the model are both imposed directly, primary parameters, and calibrated from primary data, derived parameters.

A three-step process is required to derive the model consistent data and parameters (referred hereafter as E3 model input) from primary data collected from public sources. First, the necessary primary data is collected and re-aggregated to the E3 industry aggregation. The list of E3 industries is listed in section C.[21] Second, the re-aggregated primary data needs to be "washed" to make it fully consistent. Because the primary data comes from multiple data sources with varying levels of industrial aggregation, the primary data needs to be scaled or washed such that total expenditures equal total receipts in each E3 industry. Finally, the washed data is combined with the primary parameters to calibrate the remainder of the model parameters, the derived parameters. The washed data is combined with the primary and derived parameters to complete the E3 model input.

1   Primary Data Collection

Primary, or raw, data provides the backbone for the accurate calibration of the E3 CGE model. In the primary data collection step, raw data for the benchmark year, 2010 in this case, is collected from various sources and transformed into a single consolidated dataset consistent with E3 industrial aggregation levels.

The biggest challenge to constructing the consolidated primary dataset is successfully aggregating and disaggregating the raw data into the E3 24 industry classification. This aggregation and disaggregation requires the use of simplifying assumptions. All such simplifying assumptions will be clearly defined below.

The majority of the raw data used to compile the consolidated primary dataset is from the Bureau of Economic Analysis (BEA), except where noted. Hyperlinks will be provided to original sources where possible.

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[21]There are 25 E3 industries in the model, but one is the backstop industry, so at the primary data collection step, the data needs to be re-aggregated into 24 industries

1.1 Producer Data

1.1.1 BEA Annual Industry Accounts

The primary source of data for the E3 CGE model is the BEA Annual Industry Input-Output (I-O) Accounts. These I-O accounts provide a detailed view of the relationships between U.S. producers and users and the contribution to production across industries by documenting how industries interact as they provide inputs to, and use outputs from, each other to produce GDP.[22] From these accounts, we can extract usable data on IO tables, labor demand, government purchases, personal consumption expenditures, and personal fixed investment by industry.

BEA I-O data is presented in Make and Use tables. The make table reports the quantity of each commodity that is produced by each industry while the use table reports the quantity of commodity goods used as inputs to industry production and the commodities that are consumed by final users.[23] At the summary level, we can derive IO tables, labor demand, government purchases, personal consumption expenditures, and personal fixed investment for 65 industries.

At the summary level, the BEA Make table is a 67-row by 68-column table with commodity codes listed in rows and industry codes listed in columns. Each cell in the table lists the value of the commodity produced by the domestic industry, in producers' prices (millions of 2010 dollars). For example, below is the top left corner of the commodity-by-industry table; the top row lists the value of commodities produced by the BEA industry "Farms".

PubIndCode	indLabel	111CA	113FF	211
111CA	Farms	324928	4668	
113FF	Forestry, fishing, and related activities	13	41980	
211	Oil and Gas Extraction			209458

Similarly, the summary-level BEA Use table is a 75-row by 81-column table with commodity codes listed in rows and industry codes listed in columns. Each cell in the table lists the value of the commodity produced by the domestic industry, in producers' prices (millions of 2010 dollars). For example, below is the top left corner of the final commodity user-

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[22]For more information on the BEA I-O accounts, please see the guide at http://www.bea.gov/industry/iedguide.htm#IO.

[23]Both tables can be found in the 1998-2011 Supplementary Make and Use Tables (after redefinitions at the summary level) (downloadable from http://www.bea.gov/industry/io_annual.htm).

by-industry table; the top row lists the value of the intermediate inputs used by the BEA industry "Farms".

PubIndCode	indLabel	111CA	113FF	211
111CA	Farms	33324.2	950.1	0.1
113FF	Forestry, fishing, and related activities	14377.1	23030.4	
211	Oil and Gas Extraction		5.7	41607.7

1.1.2 Input-Output Tables

An input-output matrix indicates how much of commodity i is used in the production of commodity j (i = row, j = column). The IO matrix is necessary to derive production function parameters at each production nest. The raw IO matrix is derived from the BEA Make and Use tables referred to above. Make and Use matrices are derived from the first 65 rows and columns of the Make and Use tables. The input-output matrix is then constructed by multiplying the Use matrix by a "Percentage Make" matrix. The percentage make matrix is the Make matrix expressed in terms of percentage, by commodity, of each industry's output.

As an example, imagine a simple two-good economy, where commodities A and B are made and used by industries Y and Z. Imagine further that 100 units of A are used by industry Y and 50 by industry Z, 20 units of B are used by industry Y and 40 by industry Z. The use matrix (U) for this economy is

	Y	Z
A	100	50
B	20	40

Now let industry Y's output be composed of 50% of commodity A (and 50% of commodity B) and Z's output be composed of 20% of commodity A (and 80% of commodity B). The percentage make (M) matrix for this economy is

	Y	Z
A	.5	.5
B	.2	.8

To calculate the amount of commodity A used up in the production of commodity A requires multiplying the number of units of A used by each industry by the percentage that A

comprises of each industry's output. In other words, we calculate the amount of A that each industry uses up in the production of A and then sum up over each industry that produces A. In this case, the calculation is 100*.5 + 50*.2, which is the same calculation as multiplying the first row of matrix U by the first column of matrix M. In fact, the entire IO table is derived by multiplying the use matrix by the percentage make matrix, IO = U*M. For this simple economy, the IO table is

	Y	Z
A	60	90
B	18	42

Industry Y uses 60 units of commodity A and 18 units of commodity B to produce its output while industry Z uses 90 units of commodity A and 42 units of commodity B.

1.1.3 Aggregation and Disaggregation

From the BEA Annual Industry Accounts, we derive an IO matrix and "Use vectors" (labor demand, government purchases, personal consumption expenditures, and personal fixed investment) for the 65 industries in the summary level accounts. The IO matrix and Use vectors need to be aggregated and disaggregated using four steps to be consistent with the E3 aggregation level. Notably, the "Mining, except oil and gas" industry must be disaggregated to "coal mining" and "mining (excl. coal)" and the "Utilities" industry must be disaggregated into 6 industries, including four electricity related industries. The electricity sector disaggregation is necessary to model the interactions between electricity providers under a carbon policy, but unfortunately involves a significant amount of disaggregation from the primary data.

See Section C for the mapping from BEA summary industries to the E3 industries.

Step 1: Aggregation: The BEA data is aggregated into the 19 most broadly defined E3 categories possible without the use of disaggregation. This aggregation procedure is straightforward. Consider BEA sub-industries A and B that are both in E3 parent industry G. The non-aggregated IO matrix has cells:

a,a	a,b
b,a	b,b

The aggregated matrix will then have the following cell for industry G

(a,a) + (a,b) + (b,a) + (b,b)

For a Use vector, aggregation from sub-industries to a parent industry is simply a summation of sub-industry elements.

Step 2: Disaggregation: The BEA industries that are partitioned into multiple E3 industries are disaggregated in this step. This step disaggregates the data from 19 industries to 25 industries. Disaggregation is much less straightforward than aggregation. To aid in the disaggregation, we use the BEA Detailed 2002 Benchmark Make and Use Tables.[24] The 2002 benchmark data is much more disaggregated than the summary data; there are 426 industries as opposed to 65. The 2002 IO tables and Use vectors with 426 industries are aggregated into 24 industries that correspond with the E3 industries that will allow us to disaggregate the 2010 19 industry data we aggregated in the first step.

To see how this was done, take 2010 BEA industries A and B and C. Industries A and C use the output of industry B as inputs into their production. However, we want to look at the inputs of E3 industries B_1 and B_2 where industry B = industry B_1 + industry B_2, and industry B_1 and B_2 are included in the 2002 data.

Imagine the following 2010 input output tables with only the relevant fields filled in

	A	B	C
A		w	
B	x	z	y
C		u	

Now imagine the 2002 input output table with industries A, B_1, B_2, and C:

	A	B_1	B_2	C
A		W1	W2	
B_1	X1	Z11	Z12	Y1
B_2	X2	Z21	Z22	Y2
C		U1	U2	

Using shares calculated from the 2002 data (i.e $S\_x = X1/(X1 + X2)$), the 2010 input output table with industries A, B_1, B_2, and C becomes

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[24]The BEA Detailed 2002 Benchmark Make and Use Tables are available at http://www.bea.gov/industry/io_benchmark.htm#2002data.

	A	B_1	B_2	C
A		$S_w w$	$(1 - S_w)w$	
B_1	$S_x x$	$S_{z11} z$	$S_{z12} z$	$S_y y$
B_2	$(1 - S_x)x$	$S_{z21} z$	$S_{z22} z$	$(1 - S_y)y$
C		$S_u u$	$(1 - S_u)u$	

Applying this algorithm to the 2010 19 industry aggregated IO matrix creates a 2010 24 industry IO matrix. Disaggregating the BEA parent industry into sub-industries involves three assumptions (D1-D3)

ASSUMPTION D1: Sub-industry input-output shares are constant between 2002 and 2010.

For the Use vectors (labor demand, government purchases, personal consumption expenditures, and personal fixed investment), the same algorithm is applied to disaggregate the data. Sub-industry shares are calculated using the BEA 2002 426 Sector Use tables and these shares are used to disaggregate the Use vectors into 24 x 1 length vectors.

ASSUMPTION D2: Sub-industry shares for variables labor demand, government purchases, personal consumption expenditures, and personal fixed investment are constant between 2002 and 2010.

Unfortunately, there is one sector that still needs to be disaggregated further. In 2002, one of the 426 sectors is mining services, which is a sum of three 6-digit NAICS industries, 213113, 213114, 213115. For the E3 industry classification, we want to classify 213113 (Coal Mining Support Activities) as Coal mining (3) and the other two as Mining (Except Coal) (4). To do this, we use revenue shares from the 2007 Economic Census[25]. The calculated revenue share of 213113 for the Support Activities for Mining (Not Oil/Gas) is 0.5067. This share is used to disaggregate the Support Activities for Mining (Not Oil/Gas) (BEA 2002 industry # 30).

ASSUMPTION D3: For both the IO matrix and the Use vectors, the industry Support Activities for Mining (Not Oil/Gas) can be disaggregated using a constant 2002 revenue share across all inputs and outputs.

Step 3: Re-aggregation: The 25 sectors obtained by the disaggregation from Step 3 do not correspond one-to-one with the E3 industries. There are a number of disaggregated sectors that can now be re-aggregated to obtain E3 industry classification. For example, one of the 25 sectors is Non Refining Petroleum and Coal Products; this sector can be aggregated into the E3 industry Chemicals and Misc. Non Metal Products. In this step, the 25 industries

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[25]The 2007 Economic Census is available at http://www.census.gov/econ/census07/

are aggregated into 21 industries, 20 E3 industries and 1 electric power industry. Recall that there are 24 primary data E3 industries; at this stage of aggregation and disaggregation there is only one industry for electricity because the 2002 table does not disaggregate further than the electric power industry.

Step 4: Disaggregating the Electricity Sector: After step 3, we have an IO matrix (size 21x21) and Use vectors (size 21x1) for 20 E3 industries and 1 electric power industry. The electricity power sector needs to be disaggregated into the Electricity Transmission and Distribution (Retail), Coal Fired Electric Generation, Other Fossil Electric Generation, and Non-Fossil Electric Generation E3 industries. The disaggregation of the IO matrix and Use vectors is done by making six assumptions (E1-E6).

ASSUMPTION E1: Non electric power industries buy their electricity only from the Transmission/Distribution industry.

ASSUMPTION E2: All consumption (pce, pfi, or government) of electricity is from the Transmission/Distribution industry.

ASSUMPTION E3: For the IO matrix, only Coal Fired Generators use coal as an input, only Other Fossil Generators use Oil/Gas and Petroleum Refining as an input, and only Non Fossil Generators use Mining, Excl. Coal (i.e. uranium) as an input.

ASSUMPTION E4: The use of non-fuel inputs can be divided between generators and transmission/distribution using 2007 revenue shares. The use of non-fuel inputs for generators can be divided between generators using 2010 megawatt hours.

ASSUMPTION E5: If any electricity is imported or exported, that is done through the E3 industry Electricity Transmission and Distribution.

ASSUMPTION E6: The labor input is disaggregated using 2007 payroll shares from the 2007 Economic Census.

1.1.4 Capital

Capital in the model is divided between capital structures and capital equipment. The BEA publishes data on both Current-Cost Net Stock of Private Structures by Industry and Current-Cost Net Stock of Private Equipment and Software by Industry. The data is available through the Fixed Asset Interactive Tables on the BEA website. The capital stock data is aggregated and disaggregated into E3 industries. The following sectors in the BEA capital stock data need to be disaggregated to match E3 industries: Mining, Support

Activities for Mining, Utilities, and Petroleum and Coal Products. To disaggregate the capital stocks for the Mining and Petroleum and Coal products (not Utilities), we use revenue shares from the 2007 Economic Census. Explicitly, we are making one assumption (C1).

ASSUMPTION C1: The ratio of capital (structures and equipment) for the sub-industry relative to the parent industry is equal to the ratio of total revenue for the sub-industry relative to the parent industry in 2007.

Disaggregating the capital stocks for the utility sector is more complicated. The BEA Utility sector capital stocks are disaggregated into capital stocks for the Other (Non-electric) Utilities, Electric Power, and Natural Gas Distribution industries using 2007 revenue shares as above using assumption C1.

For the four electricity E3 industries, we employ a different disaggregation algorithm. To disaggregate transmission and distribution capital from generator capital, we use a capital share variable derived from the input-output table of Sue Wing (2006) such that 53.34 percent of the capital assigned to the electric power sector using the previous assumption is allocated to the Electric Transmission and Distribution industry. The remaining generator capital is allocated in proportion to the overnight installation cost of each type of generation (weighted if using multiple types of generators within one E3 generator). This implies 17.6, 9.5 and 19.5 percent of electric power sector capital is allocated to the Coal-Fired Generators, Other-Fossil Generators, and Non-Fossil Generators.

1.1.5 Other Producer Data

Depreciation Rates: Depreciation rates for each E3 industry is calculated from BEA data on levels of capital and depreciation.[26] For both structures and equipment, the rate of depreciation is derived by dividing the 2010 depreciation level by the 2009 level of capital. The data on 2010 depreciation levels and 2009 capital stocks is aggregated and disaggregated the same way as the 2010 capital stocks described in the previous section. The exception is the electric utilities. The BEA utility sector is disaggregated into the Other (Non-electric) Utilities, Electric Utilities, and Natural Gas Distribution industries using 2002 revenue shares as before. The depreciation rate calculated by dividing 2010 depreciation levels by 2009 capital stocks of the Electric Utilities sector is then applied to all four electricity sectors by assuming that each E3 electricity industry has the same rate of depreciation.

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[26]Depreciation levels are published as Current-Cost Depreciation of Private Equipment and Software by Industry and Current-Cost Depreciation of Private Structures by Industry. The same source for 2009 capital stocks is used as in the capital section.

ASSUMPTION E7: The three electric generators and the transmission/distribution sector have the same depreciation rates for structures and the same depreciation rates for equipment.

Debt Ratio and Payout Ratio: Debt ratio is defined as the ratio of debt to capital (variable b in the model). Payout ratio is defined as the ratio of dividends to net income ratio (variable a in the model). Both variables are constructed for E3 industries from data on publicly traded firms. The data is a merged dataset from Value Line Database which tracks 7000+ firms and Morningstar which tracks 8000+ publicly traded firms. From this dataset, average levels of debt, dividend payments, and net income are calculated for each E3 industry.

Due to the vertical integration within the electric utility industries, it is difficult to calculate separate ratios for the four distinct E3 electricity industries. Therefore, we calculate one debt ratio and one payout ratio for the electric utility industry and apply it to all four E3 electricity industries.

ASSUMPTION E8: The three electric generators and the transmission/distribution sector all have the same dividend to net income ratio and debt to capital ratio.

1.1.6   Consumer Good Production

In the model, consumer goods are produced from industry goods using a fixed Leontieff production function. The Leontieff production function is summarized by the matrix IC which indicates the intensity of each industry good used to produce one consumer good. Given n_i industry goods and n_c consumer goods, the matrix IC has dimension $n_i \times n_c$. The BEA publishes Input Output Commodity Composition of NIPA Personal Consumption Expenditures in benchmark years. The table lists the value of each commodity used in producing a NIPA consumption good in millions of dollars. The table is re-aggregated to the E3 industries and E3 consumption goods. This re-aggregated table is then converted into intensities to obtain the Leontieff production matrix IC.

1.2   Household Data

Household data is required on total number of households, the ownership share in the Housing Services Industry, and capital and labor income.

Number of Households: The total number of households is necessary to scale consumption

found by solving the representative household to get the correct level of consumption. The total number of households is from the U.S. Census Bureau's Housing Unit Estimates (for April 1, 2010).[27]

Ownership of Housing Services Industry: By adjusting the tax rates that each industry faces, the model is able to capture the fact that the housing services industry faces different tax treatment by type of owner. Therefore, we require the ownership shares of the Housing Services Industry for Owner-occupied, Non-corporate Rentals, and Corporate Rentals as an input into the model. The BEA publishes the Current-Cost Net Stock of Residential Fixed Assets by Type of Owner, Legal Form of Organization, Industry, and Tenure Group (A) as a table within the section on Fixed Assets. From this table, we derive the ownership shares for the housing services industry.

Capital and Labor Income: Data on capital and labor income are obtained from BEA Form SA05.[28] This form publishes data on aggregate personal income from the following sources:

1. Dividends, Interest, and Rents
2. Personal Current Transfer Receipts
3. Wage and Salary Disbursements
4. Supplements to wages and salaries
5. Proprietor's Income

Capital income is composed of (1) and some of (5) and labor income is composed of (3), (4) and some of (5). Proprietors Income, (5), is allocated between capital and labor income by calculating the share of capital income to labor income in the other categories (i.e. (1)/(1) + (3) + (4)).[29]

1.3   Government Data

The government plays a key role in the model. A fairly realistic treatment of the U.S. tax system is necessary to capture tax interaction effects as well as the significance of recycling carbon revenues to agents in the model. Accurate data on government taxes and transfers is thus essential to the E3 CGE model.

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[27]http://www.census.gov/popest/data/housing/totals/2011/tables/HU-EST2011-01.csv

[28]Using the Interactive Data, select Regional Data, GDP and Personal Income, Annual State Personal Income and Employment, Personal Income and Earnings by Industry. Select United States and 2010.

[29]In the model, we solve the representative household and scale up to get total consumption. Therefore, as an input in the model, we need capital and labor income per household as the data input as opposed to aggregate capital and labor income.

1.3.1 Government Taxes and Transfers

Tax rates in the model, except where noted, are derived from levels of taxes paid. Because the government in the model represents federal, state, and local government, the taxes collected must reflect the levels of taxes collected at all three levels of government.

Income Taxes Federal income tax receipts for a given year are available from the annual Budget of the United States Government Historical Tables. State and local income tax receipts are available from the U.S. Census' Quarterly Summary of State and Local Tax Revenue.

Property Taxes The BEA publishes the Survey of Current Business. In the National Income and Product Account (NIPA) Tables contains a table on Taxes on Production and Imports. On this table is the aggregate property taxes. Because this data is disaggregated by industry, we assume that all industries face the same property tax rate. This rate is calculated by dividing the aggregate property tax payments by aggregate private structural capital.

Indirect Labor Taxes Firms must pay payroll taxes (Medicare and Social Security) for each employee. The government defines these payments as "Contributions to Government Social Insurance." Data on aggregate Contributions to Government Social Insurance is available from the BEA Survey of Current Business NIPA tables under the category 'Government Current Receipts and Expenditures.' This total amount of indirect labor taxes are allocated to E3 industries by shares of labor demand (from the BEA Use tables). The proportional allocation assumes that indirect labor taxes are proportional to wage and salary disbursements by industry.

Consumer Good Excise Taxes The government charges excise taxes on specific consumer goods (i.e. state and federal gas taxes). Data on the level of tax revenue for excise taxes (also called specific taxes) is available from the BEA Survey of Current Business NIPA tables under the category 'Government Current Receipts and Expenditures.'

Ad Valorem Consumer Good Taxes Levels of Ad Valorem tax by E3 consumer good are calculated by multiplying the aggregate (across consumer goods) national sales tax level by estimated sales tax shares by commodity using sales tax data from BEA Survey of Current Business NIPA tables (categories 'Government Current Receipts and Expenditures' and 'Personal Income and Outlays').

Intermediate Input Taxes Firms must pay state and federal taxes on goods that they use (but not goods used as inputs into production). For example, an auto manufacturer will not

pay taxes on tires but truck drivers (the transportation industry) must pay taxes on tires purchased to replace worn tires. Intermediate input taxes are calculated as the amount paid by each E3 industry for each E3 industry commodity good (similar to input-output matrix). The primary source for this data is the IRS Statistics of Income Excise Tax Statistics.[30] Given this data on excise taxes by commodity, the consumer portion is removed, and the remaining level is allocated across E3 industries. Where possible, exemptions for inputs into production are made; otherwise, the tax level is allocated across E3 industries according to the Input-Output matrix (i.e. the tax on a commodity is paid by E3 industries in the same proportion as the commodity is used by E3 industries).

Marginal Income Tax Rates Marginal income tax rates in the model are taken from the NBERs TAXSIM model for 2010. The rates are the sum of federal and state dollar weighted average marginal income tax rate. The rates are 0.2582, 0.2806, 0.2344, and 0.1949 for labor income, interest income, dividend income, and long-term capital gains.

Corporate Income Tax Rates Corporate Tax Rates are calculated by adding weighted average state rates to the Federal rate of 0.35. Thirteen states use tax brackets to tax firms by firm size. Weighted averages are used to calculate corporate taxes for these states. Data on personal income and employment by state is available from the BEA Survey of Current Business NIPA tables. Data on firm pre-tax profits by state (used to weight state corporate tax rates) are calculated by multiplying the national ratio of pre-tax profits to wage and salary disbursements across firm size classes (available through BEA Survey of Current Business NIPA tables) by average payrolls with each firm size class in each state (available from U.S. Census, Statistics of U.S. Businesses). State tax rates are publicly available through tax centers such as the Tax Policy Center and the Tax Foundation.

Transfers Federal, state, and local governments transfer large sums of government revenues to households in the form of disbursements for old-age survivors, disability, health insurance, unemployment insurance, veterans benefits, retirement benefits and "other transfers." Total transfer payments are available in the Economic Report of the President in the "Federal and State and Local Government Receipts and Current Expenditures, NIPA" table.

1.3.2 Government Spending and Financing

The model assumes that overall government spending is exogenous and increases at a constant rate. The initial level of government spending by industry is calculated from the "Use"

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[30]Table 20: Federal Excise Taxes Reported to or Collected by the Internal Revenue Service, Alcohol and Tobacco Tax and Trade Bureau, and Customs Service, by Type of Excise Tax, Fiscal Years 1999-2011.

vectors derived from the BEA Use tables. Government investment also grows at a constant rate. Initial government capital stocks are obtained from BEA estimates of government stocks of structures and equipment and software from the Fixed Asset Table for 2010. Also, the model assumes the the level of debt stays approximately constant relative to GDP over time. The level of debt in 2010 is available from the U.S. Treasury and is initialized as the initial level of debt for the government in the model.

2 Primary Parameters

Primary parameters are defined as the parameters in the model that are independent of the primary raw data. Elasticities of substitution in the CES production functions are an example of primary parameters. Other parameters in the CES production functions, however, are derived parameters because we calibrate them to match the data, given the elasticities of substitution.

2.1 Producer Primary Parameters

Production function elasticities of substitution at all production nests are derived from estimates by Dale Jorgenson and Peter Wilcoxen. The Jorgenson-Wilcoxen estimates of parameters for translog cost functions are translated into elasticities of substitution parameters to make them compatible with our model. When the E3 industries are more disaggregated than the comparable Jorgenson-Wilcoxen sectors, we simply impose the same elasticities of substitution across sub-industries. For example, we must assume that all utility E3 industries (Other (Non-electric) Utilities, Electricity Transmission and Distribution, Coal Fired Electricity Generators, Other Fossil Electricity Generators, Non-Fossil Electricity Generators, and Natural Gas Distribution) have the same elasticities of substitution. However, we do impose a higher elasticity of substitution for energy inputs for the Electricity Transmission and Distribution industry to reflect the high substitutability between electricity generated by different fuel sources.[31]

The marginal adjustment cost parameter ζ is based on Summers (1981). Summers's adjustment cost estimates are based on the assumption that adjustment costs are variable costs that are convex in the rate of investment, as in our model. More recent capital adjustment cost estimates downplay the variable cost component and emphasize fixed costs of adjustment.

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[31]Due to the geographic constraints, it is not the case that in a representation of the entire U.S. that electricity generated by different fuel sources will be perfectly substitutable.

At the time, some economists thought that Summerss estimates adjustment cost estimates were high, so we use the lower bound of his adjustment cost estimates. Further research in the 1990's using similar methods as Summers found similar adjustment cost estimates.

2.2   Household Primary Parameters

Household utility parameters are generally taken from relevant economic literature. The elasticity of substitution in consumption between goods and leisure, v, is set to yield a compensated elasticity of labor supply of 0.4. This value lies in the middle of the range of estimates displayed in the survey by Russek. The intertemporal elasticity of substitution in consumption, σ, is set to 0.5. This value falls between the lower estimates from time-series analyses (e.g. Hall) and higher ones from cross-sectional studies (e.g. Lawrance). The labor intensity parameter α_l is set to generate a ratio of labor time to the total time endowment equal to 0.44.

2.3   Carbon Coefficients

Carbon Dioxide emissions coefficients are set to match the distribution of emissions from energy consumption by source in 2010.

		Source			
		Electric Power	Transportation	Residential	Commercial	Industrial
	Coal	1828	0	1	6	154
Fuel	Natural Gas	399	38	259	168	401
	Motor Gasoline	0	1124	0	4	19
	Other Petroleum	33	712	78	45	333

Coefficients convert the input of coal and oil into emissions. Due to differences between the aggregation of reported emissions sources (electric power, transportation, commercial, residential, and transportation), we make the following assumptions: a) the coal coefficient for all industries except coal fired electricity generators is equal; b) the oil coefficient for all industries except petroleum refining, other fossil electricity generation, and natural gas distribution are equal. Importantly, we assume that the carbon coefficients converting inputs of the backstop technology for the oil&gas sector into emissions are the same as the oil&gas sector.

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3   Washing the Primary Data

Once the consolidated primary dataset is completed, the data needs to be "washed" or scaled to make it fully consistent; consistency is defined such that total expenditures equal total receipts for each industry. Due to multiple data sources and aggregation and disaggregation, the primary dataset will not be consistent. The following ratios are used to scale the data to make it consistent.

RATIOG: Scales government purchases so that purchases agree with the sum of expenditures and new debt issue.

RATIOC: Scales consumption so that it is fully consistent with the level of consumption and savings required for steady-state growth.

RATIOL: Scales aggregate household labor income to agree with total labor payments by firms.

RATIOK: Scales aggregate household capital income to agree with total capital income generated by firms.

RATIOT: Scales exports to assure current account balance.

4   Calibrating the Derived Parameters

Given the primary scaled data and the primary parameters, the remaining parameters (CES scale and share parameters in production and consumption nests) are derived by calibrating the model to the scaled primary data. To do this, we assign baseline calibration prices (normally unit prices are assumed) to the equilibrium prices and derive the remaining prices in the model. From these prices, we "invert" the model to solve for the derived parameter values such that are consistent with the primary scaled data. In other words, we look for derived parameters such that given the baseline prices and parameters, both primary and derived, the model exactly replicates the scaled data in the benchmark period.

C. Industry and Consumer Good Classification

1 E3 Industry Classification

Energy Industries	NAICS Industry
Oil&Gas Extraction	211, 213111-213112
Synthetic Fuels	n/a (Backstop Industry)
Coal Mining	2121, 213113
Electricity Transmission and Distribution	22112
Coal Fired Electricity Generation	221112
Other Fossil Fuel Electricity Generation	221112
Non-Fossil Fuel Electricity Generation	221111, 221113-221119
Natural Gas Distribution	2212
Petroleum Refining	32411

Material (Non-Energy) Industries	NAICS Industry
Agriculture and Forestry	11
Non-Coal Mining	2122-2123, 213114-213115
Other (Non-Electric) Utilities	2213
Construction	23
Food and Tobacco	311-312
Textiles, Leather, ad Footware	313-316
Wood and Paper Products	321-323
Chemicals and Misc. Non-Metal Products	32412, 32419, 325-327
Primary Metals	331
Machinery and Metals Manufacturing	332-335, 3364-3369, 337, 339
Motor Vehicle Production	3361-3363
Transportation	481, 483-488, 493
Railroads	482
Information and Communications	51
Services	42, 44, 52-81

2 E3 Consumer Good Classification

Consumer Goods	BEA NIPA PCE Category
Food	08
Alcohol	09-10
Tobacco	07
Utilities	37-41
Housing	24-27
Furnishings	29, 32-33
Appliances	30-31, 35, 91
Clothing	12, 14-16, 18
Vehicle Use	70-73
Other Transportation Services	74, 76-77, 79-80, 82-85
Financial Services	61-64
Other Services	42-43, 65-67, 108
Recreation	87-90, 94-96, 100-103, 110
Personal Care	17, 19, 21-22, 34
Health Care	45-50, 56
Gas	75
Education	105-107

3 BEA Summary Industry to E3 Industry Mapping

		BEA Summary Industry		Aggregate to E3 Industry
1	111CA	Farms	1	Agriculture
2	113FF	Forestry fishing and related activities	1	Agriculture
3	211	Oil and gas extraction	2	Petroleum and Natural Gas Extraction
4	212	Mining, except oil and gas		
		— PARTITION 1	4	Mining (Excl. Coal)
	— 2121	— PARTITION 2:Coal Mining	3	Coal Mining
5	213	Support activities for mining		
		— PARTITION 1	2	Petroleum and Natural Gas Extraction
		— PARTITION 2	4	Mining (Excl. Coal)
		— PARTITION 3	3	Coal Mining
6	22	Utilities		
	— 2213	— PARTITION 1: Water, sewage other system	5	Other (Non-Electric) Utilities
		— PARTITION 2	6	Elec Transmission/Distribution
		— PARTITION 3	7	Coal-Fired Electric Generation
		— PARTITION 4	8	Other Fossil Fuel Electric Generation
		— PARTITION 5	9	Non Fossil Fuel Electric Generation
	— 2212	— PARTITION 6: Natural gas distribution	10	Natural Gas Distribution
7	23	Construction	11	Construction
8	311FT	Food and beverage and tobacco products	12	Food and Tobacco
9	313TT	Textile mills and textile product mills	13	Textile Products, Leather and Footwear
10	315AL	Apparel and leather and allied products	13	Textile Products, Leather and Footwear
11	321	Wood products	14	Wood and Paper Products
12	322	Paper products	14	Wood and Paper Products
13	323	Printing and related support activities	14	Wood and Paper Products
14	324	Petroleum and coal products		
	— 32411	— PARTITION 1: Petroleum Refinery	15	Petroleum Refining
		— PARTITION 2	16	Chemicals and Misc. Non-Metal Products
15	325	Chemical products	16	Chemicals and Misc. Non-Metal Products
16	326	Plastics and rubber products	16	Chemicals and Misc. Non-Metal Products
17	327	Nonmetallic mineral products	16	Chemicals and Misc. Non-Metal Products
18	331	Primary metals	17	Primary Metals
19	332	Fabricated metal products	18	Machinery and Metals Manufacturing
20	333	Machinery	18	Machinery and Metals Manufacturing
21	334	Computer and electronic products	18	Machinery and Metals Manufacturing
22	335	Electrical equipment, appliances, and components	18	Machinery and Metals Manufacturing
23	3361MV	Motor vehicles, bodies and trailers, and parts	19	Motor Vehicle Production
24	3364OT	Other transportation equipment	18	Machinery and Metals Manufacturing
25	337	Furniture and related products	18	Machinery and Metals Manufacturing
26	339	Miscellaneous manufacturing	18	Machinery and Metals Manufacturing
27	42	Wholesale trade	23	Services (Including Rental Housing)
28	44RT	Retail trade	23	Services (Including Rental Housing)
29	481	Air transportation	20	Transportation
30	482	Rail transportation	21	Railroads Transportation

31	483	Water transportation	20	Transportation
32	484	Truck transportation	20	Transportation
33	485	Transit and ground passenger transportation	20	Transportation
34	486	Pipeline transportation	20	Transportation
35	487OS	Other transportation and support activities	20	Transportation
36	493	Warehousing and storage	20	Transportation
37	511	Publishing industries (includes software)	22	Information (Communications)
38	512	Motion picture and sound recording industries	22	Information (Communications)
39	513	Broadcasting and telecommunications	22	Information (Communications)
40	514	Information and data processing services	23	Services (Including Rental Housing)
41	521CI	Federal Reserve banks, credit intermediation, and related activities	23	Services (Including Rental Housing)
42	523	Securities, commodity contracts, and investments	23	Services (Including Rental Housing)
43	524	Insurance carriers and related activities	23	Services (Including Rental Housing)
44	525	Funds, trusts, and other financial vehicles	23	Services (Including Rental Housing)
45	531	Real estate	24	Owner-Occupied Housing Services
46	532RL	Rental and leasing services and lessors of intangible assets	21	Services (Including Rental Housing)
47	5411	Legal services	23	Services (Including Rental Housing)
48	5412OP	Miscellaneous professional, scientific and technical services	23	Services (Including Rental Housing)
49	5415	Computer systems design and related services	23	Services (Including Rental Housing)
50	55	Management of companies and enterprises	23	Services (Including Rental Housing)
51	561	Administrative and support services	23	Services (Including Rental Housing)
52	562	Waste management and remediation services	23	Services (Including Rental Housing)
53	61	Educational services	23	Services (Including Rental Housing)
54	621	Ambulatory health care services	23	Services (Including Rental Housing)
55	622HO	Hospitals and nursing and residential care facilities	23	Services (Including Rental Housing)
56	624	Social assistance	23	Services (Including Rental Housing)
57	711AS	Performing arts, spectator sports, museums, and related activities	23	Services (Including Rental Housing)
58	713	Amusements, gambling, and recreation industries	23	Services (Including Rental Housing)
59	721	Accommodation	23	Services (Including Rental Housing)
60	722	Food services and drinking places	23	Services (Including Rental Housing)
61	81	Other services, except government	23	Services (Including Rental Housing)
62	GFE	Federal government enterprises	23	Services (Including Rental Housing)
63	GFG	Federal general government	23	Services (Including Rental Housing)
64	GSLE	State and local government enterprises	23	Services (Including Rental Housing)
65	GSLG	State and local general government	23	Services (Including Rental Housing)