Ex-vessel Pricing and IFQs: A Strategic Approach

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Abstract

In this paper, intraseasonal fishing is modeled as a differential game between fishermen in a total allowable catch–regulated fishery with and without individual fishing quotas (IFQs). Heterogeneous harvest values are included by incorporating time-specific harvest costs and a stock effect into fishermen’s profit functions. I also allow for strategic interaction among fishermen via ex-vessel price dynamics. The equilibrium harvest strategies of the differential games are solved numerically through the use of a genetic algorithm. I demonstrate how different harvesting sector environments lead to varying degrees of ex-vessel price increases when IFQs are implemented. The primary result shows that possible margins for competition among fishermen, beyond competition for a greater share of the total allowable catch, can still exist under IFQ management and may be substantial enough to be able to prevent sizeable rent transfers from the processing sector to the harvesting sector.

Key Words: individual fishing quotas, property rights, differential games, genetic algorithm

JEL Classification Numbers: Q22, C73, C61
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1. Introduction

An individual fishing quota (IFQ) system typically allocates a specific portion of the total allowable catch (TAC) to an individual entity with the provision that the quota may be caught at any time throughout a prespecified time period. IFQs, also referred to as individual transferable quotas (ITQs), have been supported by both economists and ecologists as a management system that addresses the causes of inefficient resource use, as opposed to other management regulations such as TAC restrictions, season length restrictions, and limited-entry programs, which focus more on symptoms of inefficient resource use. These systems have been implemented in several parts of the world and have generally proved more successful in terms of economic efficiency and resource sustainability than their management system counterparts devoid of rights-based approaches.¹ However, IFQs have been resisted in many other places and currently only about 15 percent of the global marine catch is taken under IFQs (Arnason 2007).

One of the major obstacles associated with implementing an IFQ system is the resulting distributional effects. In particular, the distributional impacts of IFQs in ex-vessel markets, the market between fishermen and processors, have garnered much attention recently. The concern is that as IFQs give fishermen the freedom to harvest at a slower rate an increase in competition for raw fish among processors could occur. This could result in higher ex-vessel prices and a disproportionate transfer of rents to the harvesting sector. This concern was expressed in the theoretical model of Matulich, Mittelhammer and Reberte (1996) and has been reiterated in Matulich and Sever (1999) and Hackett et al. (2005).

In considering how IFQs give fishermen the ability to force higher ex-vessel prices (i.e., extract rents from the processing sector), one must keep in mind that while IFQs do give fishermen secured harvest rights, these rights are not perfectly delineated. For instance, standard

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¹ Among others, see Annala (1996) for a review of the New Zealand ITQ experience and Arnason (1993) for an account of the Icelandic ITQ system.
IFQs often do not impose overly restrictive time and spatial stipulations on harvests. Given that fish stocks can exhibit economic heterogeneity across time and space and that production externalities, such as congestion costs and stock-depletion costs, can cause a fisherman’s effort to directly impact the costs of others, it is easy to see how competitive behavior among fishermen can still exist even with IFQs. Issues of how IFQs relate to these instances of economic heterogeneity in fish stocks and production externalities have been discussed in the literature (e.g., Boyce 1992; Wilen 2002; Costello and Deacon 2007). In this paper, we continue to explore these themes. More specifically, we use a strategic interaction setting to investigate how incorporating time-specific harvesting costs and cumulative stock-depletion costs affects harvesting paths and resulting ex-vessel prices under a TAC-regulated fishery with and without IFQs.

The paper makes several contributions to the existing literature of modeling within-season harvesting behavior. With respect to the modeling of a TAC-regulated fishery without IFQs, it is often the case that season length is exogenously imposed (e.g., Matulich, Mittelhammer and Reberte 1996; Homans and Wilen 1997, 2005; Hannesson and Kennedy 2005). Generally this is done for mathematical convenience, but in reality season lengths for TAC-regulated fisheries are not prespecified. Fishing seasons in TAC-regulated fisheries are usually open until the cumulative harvest equals the TAC. Through the use of nonlinear computational techniques in numerical simulations we are able to model the season length as a free time horizon. This allows us to more accurately portray the “race for fish” that occurs without rights-based management and the consequences this harvesting practice can have on ex-vessel prices.

With respect to the modeling of the fishery with IFQs this work most closely resembles that of Costello and Deacon (2007). Both models use a game-theoretic setting to highlight how the implementation of standard IFQs does not fully remove competitive harvesting pressures. However, the model assumptions and respective goals of the research are quite different. Costello and Deacon assume that the fishery is spatially heterogeneous and that each “patch” has an exogenously determined, time-dependent harvest value. They conclude that, in the absence of a fleet coordinating its harvesting strategies, standard IFQs will not result in the rent-maximizing

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2 IFQs generally stipulate that the granted harvest right must be utilized in some extended period of time, but do not generally give specific, binding time periods to harvest. Also, some IFQ programs (e.g., New Zealand and Alaska) include broad geographic stipulations on where quotas can be landed.
harvest strategy. In contrast, we do not model spatial heterogeneity specifically, but make the value of the harvest dependent upon the total quantity harvested, the time in which it is harvested, and the perceived competitiveness of the processing sector. This in turn adds another dimension of externalities as the benefits from slow harvesting in an effort to increase ex-vessel prices cannot be internalized. Furthermore, by explicitly modeling the ex-vessel pricing dynamics in settings with and without IFQs, we can make some general predictions about how the parameter settings and harvesting environment affect the ex-vessel price increases associated with IFQs.

The organization of the paper is as follows. In the next section we discuss the individual fisherman’s maximization problem under both management systems considered. We then briefly discuss the solution concept used to identify the equilibrium harvest rates from the simulations. The results of the simulations, with discussion, are then presented. The final section concludes the paper and provides possible extensions of this work.

2. Maximization Problem

The models considered here are for a single season (i.e., an intraseasonal model) in which cumulative harvest is regulated by an exogenously determined TAC and no appreciable fish stock regeneration occurs. Consequently, the harvesting decision is not impacted by any perceived fish population dynamics as is typically the case. Given this framework, the model more closely resembles those considering the strategic extraction of a common pool nonrenewable resource, where the TAC is the common pool resource, than a renewable resource extraction model.

We first consider a model of strategic harvesting in which the \( n \) fishermen participating in the fishery do not have IFQs. The number of participants, \( n \), is assumed to be fixed throughout the season and is not necessarily at the level which produces zero rents. While this may appear to be a limiting restriction, provisions such as limited-entry management and/or prohibitively

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3 By standard IFQ it is meant that the IFQ does not include binding time and/or space-specific harvest stipulations. Costello and Deacon (2007) do analyze the efficiency under the assumption “full assignment of property rights,” concluding that complete rent capture can occur in this setting.
expensive start-up costs may be sufficient to deter entry up to the zero-rent level. The individual, \(i\), chooses instantaneous harvest levels, \(y_i(t)\), to maximize seasonal profits:

\[
\max_{y_i} \int_0^T p(t) y_i(t) - c(y_i(t), X(t), t) \, dt
\]

subject to:

\[\dot{p}(t) = f(Y, y_i(t), y_{-i}(t))\]

\[\dot{X}(t) = y_i(t) + y_{-i}(t)\]

\[0 \leq y_i(t) \leq y_i^{\text{max}}, \quad 0 \leq p(t) \leq p_{\text{max}}, \quad 0 < T \leq T_{\text{max}}, \quad X(0) = 0, \quad X(T) \leq TAC, \quad p(0) = p_0\]

In this setup, \(y_{-i}(t)\) represents the harvest rates of the other \((n-1)\) participants. Fishermen do not take the ex-vessel price, \(p(t)\), as given, but rather are aware that the price dynamics are affected by the processing sector’s desired level of throughput, \(Y\), their own production, \(y_i(t)\), and the production of others, \(y_{-i}(t)\). The price dynamics are such that when \(Y \geq (y_i(t) + y_{-i}(t))\), \(\dot{p}(t) \geq 0\) and \(Y < (y_i(t) + y_{-i}(t))\), \(\dot{p}(t) < 0\). The cost of harvesting is dependent on the harvester’s own production such that \(\frac{\partial c(\bullet)}{\partial y_i} > 0\), \(\frac{\partial^2 c(\bullet)}{\partial y_i^2} \geq 0\). A production externality can also exist as the individual’s cost of harvesting depends on the harvest rate of the others through the state variable \(X(t)\), such that \(\frac{\partial c(\bullet)}{\partial X_i} > 0\), \(\frac{\partial^2 c(\bullet)}{\partial X_i^2} \geq 0\). We refer to the cost associated with \(X(t)\) as a stock effect cost. In addition to the harvest levels and stock effect, we also consider the cost impacts of the time period \(t\) in which harvesting occurs. This consideration is particularly relevant for migratory species and/or in areas where seasonal weather conditions affect harvesting costs.

Other model features should also be noted. First, this model incorporates a maximum price that fishermen can obtain for their harvest, \(p_{\text{max}}\). The price \(p_{\text{max}}\) represents the reservation price of the processors. Given this, \(p_{\text{max}}\) can be interpreted as the price at which the average revenue generated from processing fish is equal to the processing sector’s average cost. Thus, it is the price that extracts all rents from the processing sector. Second, when the boundary

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4 See Kennedy and Hannesson (2005) for more on the argument of assuming a fixed number of fishermen and the zero-rent assumption versus the rent-maximization assumption.
constraint on $X(T)$ is binding and $T < T_{\text{max}}$, then the terminal time $T$ is a free variable that must be solved for in the optimization problem. This marks another departure from other within-season fishing models, which typically assume a predetermined fixed season length. The appropriateness of the free $T$ specification or the fixed $T$ specification is dependent upon whether the binding regulation on the fishery is a TAC or a season-length restriction. Finally, because we are dealing with intraseasonal harvesting, time discounting is ignored.

The trade-offs facing an individual harvester under the no-IFQ scenario are quite apparent. The individual has an incentive to have a low harvest rate in order to induce higher ex-vessel prices and to decrease costs. On the other hand, the lack of property rights puts the TAC in the public domain of the participants, giving the individuals the incentive to increase their harvest rates to capture a higher share of the TAC. This is the commonly referenced incentive that leads to a race for fish. The stock effect and potentially increasing harvest costs over time give further incentive for harvesting earlier.

The second general model considered is one in which the individual harvester is granted an IFQ. This maximization problem is therefore similar to a strategic extraction problem of a nonrenewable resource in which each participant has its own exclusive resource stock. The actions of the participants are, however, still linked by the price dynamics and the stock effect. The individual’s maximization problem in this model becomes:

$$\max_{y_i} \int_0^{T_i} p(t)y_i(t) - c(y_i(t), X(t), t) \, dt$$

subject to:

$$\dot{X}_i(t) = y_i(t)$$

$$0 \leq y_i(t) \leq y_{\text{max}}, \ 0 \leq p(t) \leq p_{\text{max}}, \ 0 < T_i < T_{\text{max}}$$

$$X_i(0) = 0, X_i(T_i) \leq TAC_i, \ p(0) = p_0$$

For this setup, $TAC_i$ is fisherman $i$’s share of the TAC. Assuming the time constraint is not binding, each individual in the IFQ model will in effect decide its own season length, $T_i$.

While the fishermen no longer face competition for the TAC under an IFQ system, they still have interdependent harvest strategies. Fishermen continue to have an incentive to harvest early assuming increasing costs over time and/or a stock effect. In this sense, IFQs do not completely remove the competition. Combining this with the fact the ex-vessel pricing benefits
from harvesting more slowly cannot be internalized, it becomes apparent that strategic
interaction among fishermen continues to exist with IFQs. Furthermore, the optimal harvest path
is not obvious in this setting. With the boundary constraints, the problem lends itself to nonlinear
and discontinuous harvest paths. Additionally, one can see how ignoring these interaction
subtleties can possibly be a significant oversimplification of the problem. The goal of this
research is to see how different parameter combinations affect the resulting optimal paths and
how those differing paths, in turn, affect the fishermen’s ability to extract rents from the
processing sector.

3. Solution Method

We begin by assuming that once fishing begins harvest rates are only individually known.
The solutions we are searching for are therefore open-loop Nash equilibria, as opposed to closed-
loop or feedback solutions, which allow players to form state-dependent strategies at each time
period. Whether or not it is appropriate to consider only open-loop equilibria is fishery
dependent.

In principle, given functional specifications one could solve for the open-loop Nash
equilibria of the maximization problems above by simultaneously solving \( n \) optimal control
problems. The problems can be further simplified if it is assumed that the fishermen are
identical, forcing symmetric harvest strategies. Even then, with the multiple boundary constraints
for the state and control variables, many possible solution paths can exist. Adding further
complications to finding the analytical solution is the fact that there is a possibility for a free time
horizon and strategic interaction elements in both the ex-vessel price and cost function, leading
to complex best-response functions. We therefore avoid attempting to analytically solve the
open-loop Nash equilibria and opt instead to computationally derive the equilibria for a series of
simulations.

With the possibility for highly nonlinear solutions, finding the equilibria via
computational methods is no trivial task. The genetic algorithm (GA) is one such algorithm that
is designed to handle such nonlinearities. GAs have also been used as the search algorithm in
several game-theoretic applications (e.g., Özyildirim 1997; Alemdar and Özyildirim 1998;
Alemdar and Sirakaya 2003).

GAs perform an iterative, multidirectional search procedure patterned after Darwinian
principles of reproduction and survival of the fittest. The basic operation of a GA is as follows.
First a random population of individuals is generated. In this case an “individual” constitutes a
possible harvest strategy. Then, each individual is evaluated by a fitness function, which is the seasonal profit for these problems. The population then undergoes a process of evolution in which the fit individuals (those returning a greater seasonal profit) survive and the unfit individuals are discarded. The fit individuals are saved, and some undergo crossover and mutation to form a new population of potential solutions, a new generation. The process then repeats itself until all of the individuals of the population converge to an identical solution, representing the optimal solution.5

To use the GA to solve for the open-loop Nash equilibria of the games described above we make several simplifications. First, the problems of section 2 must be discretized to be solved by a GA. The discrete time reformulation process used for this paper follows the style proposed by Mercenier and Michel (1994). The discrete time approximation is as follows:

\[
0 \max \sum_{m=0}^{M} \Delta_m \left[ p(t_m) y_i(t_m) - c(y_i(t_m), X(t_m), t_m) \right]
\]

subject to:

\[
p(t_{m+1}) - p(t_m) = \Delta_m \left[ f \left( Y, y_i(t_m), y_{-i}(t_m) \right) \right]
\]

\[
X(t_{m+1}) - X(t_m) = \Delta_m \left[ y_i(t_m) + y_{-i}(t_m) \right] \text{ without IFQs}
\]

or

\[
X_i(t_{m+1}) - X_i(t_m) = \Delta_m y_i(t_m) \text{ with IFQs}
\]

\[
X(t_0) = 0, X(t_M) \leq TAC, X_i(t_0) = 0, X_i(t_M) \leq TAC_i, p(t_0) = p_0
\]

\[
0 \leq y_i(t_m) \leq y_{max} \leq 0 \leq p(t_m) \leq p_{max}
\]

\[
M \text{ is a user-specified terminal period. For this particular problem, the optimal solution might be such that TAC is satisfied before } M \text{ is reached (i.e., } T < T_{max}). \text{ The step size, } \Delta_m, \text{ is also a programmer-specified value that converts the continuous harvest values into incremental values. As the step size decreases the GA evaluates the problem at a finer time scale, but then } M \text{ must be increased to keep the same terminal period, which corresponds to } T_{max}. \text{ An individual generated by the GA is then a vector of individual harvest values for each time period } m \text{ from } m = 0 \text{ to } m = M.
\]

5 For more details on GAs see Goldberg (1989) and De Jong (1993).
The second simplification made is to assume that all participants are homogeneous. This implies that in equilibrium $y_i = y_{-i}$, where $y_i$ and $y_{-i}$ denote harvest strategies over the entire differential game. While parallel GAs can be constructed for players maximizing multiple objective functions, computationally the problem becomes prohibitively expensive to solve as the number of objective functions goes above two. We therefore forego the reality of heterogeneous fishermen in favor of having a larger number of participants, each with the same maximization problem. Given this assumption, the general algorithm we implement to obtain the open-loop Nash equilibrium for a given simulation is as follows. 1) An initial harvest strategy is first supplied for $y_{-i}$. 2) This strategy, along with model parameters, is then supplied to the GA, which searches for the individual’s best response strategy, $y_i$, using the seasonal profit as the fitness function. 3) This strategy is then given as the new strategy for the “other” fishermen, $y_{-i}$. 4) Steps 2 and 3 are then repeated until $y_i = y_{-i}$. At this point the strategies are mutual best responses, thus constituting a pure-strategy Nash equilibrium.\(^6\)

4. Simulation Results

In order to run the simulations all general functions must be specified. This in turn forces us to model the conjectures fishermen may have about the ex-vessel price process. Intuitively, these conjectures will be dependent upon the perceived market structure of the associated processing sector, the level of processing capitalization, and the harvesting strategies of the other fishermen. If the processing sector is highly collusive (i.e., near a monopsony), then it is less likely to bid up prices in the face of raw fish supply shortages. However, as the processors become increasingly competitive, we would expect them to be highly responsive to supply shortages as depicted in Matulich Mittelhammer and Reberte (1996). Presumably, fishermen are aware of this structure, or have made some conjecture about it, and respond accordingly bearing in mind the harvest strategies of the entire fishing fleet. While explicitly modeling the market structure of the processing sector is beyond the scope of this paper, we do propose a price dynamics equation that does capture these intuitive characteristics. The price dynamics equation that the simulations are based on is:

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\(^6\) We consider only pure-strategy equilibria in our numerical simulations. Under certain parameter specifications, however, it is possible to have only mixed-strategy equilibria.
\[ \dot{p} = \begin{cases} s_u & \text{when } Y \geq y_i(t) + y_{-i}(t) \\ s_o & \text{when } Y < y_i(t) + y_{-i}(t) \end{cases} \]

The parameter \( s_u \) reflects how the processors respond to an “under” supply of raw fish and \( s_o \) represents the processors’ response to an “over” supply of raw fish. The parameters \( s_u, s_o, \) and \( Y \) are user-specified parameters such that \( s_o, s_u, Y > 0 \).

As stated above, \( Y \) denotes the optimal throughput for the processors. Thus, \( Y \) is associated with the processors’ level of capitalization. If the cumulative harvest is less (greater) than the processors’ desired level, prices rise (fall). The amount by which prices rise (fall) is determined by the degree of shortage (surplus) and the adjustment factor \( s \). The adjustment factor is used to proxy the competitiveness of the processing sector.\(^7\) If the processing sector is highly competitive we would expect a large increase in prices due to supply shortages and small decreases in prices due to supply surpluses (\( s_u > s_o \)). The opposite would be true for a highly collusive processing sector.

On the cost side, we run simulations under two distinct cost-function specifications: one in which the fishermen face increasing harvesting costs over time and one in which a stock effect is present such that harvesting costs increase as the cumulative harvest increases. Modeling a stock effect in the cost function is standard in most fishery economics models. Time-specific cost, however, have received less attention, even in models of intraseasonal harvesting. Time-specific costs are particularly relevant, especially for fisheries that experience inclement seasonal weather, such as those in the extreme north or south latitudes, and/or for fisheries of highly migratory species. In reality, fishermen may include both stock effects and time-specific costs in their cost function, but we model them separately to better understand each component’s effect on the resulting harvesting strategies and, consequently, the ex-vessel prices.

The convex cost specification with time-specific costs used in the simulations is:

\[ c_i(y_i(t), t) = \alpha_i y_i(t) + \beta_i y_i(t)^2 \]

where \( \alpha_i \) and \( \beta_i \) are user-specified parameters. The cost-function specification with the stock effect included is:

\(^7\) Similar price dynamics functions with adjustment factors have been used in sticky price studies (e.g., Fershtman and Kamien 1987, 1990).
where \( d(y_i(t) > 0) \) is an indicator function equal to one when \( y_i(t) > 0 \) and zero otherwise. Notice that in (14) the time-specific cost is dependent upon the present quantity caught, while in (15) the stock effect cost is only dependent on whether or not there is a present period catch. Admittedly, these cost functions are somewhat arbitrary. Cases can, however, be made for each of these specifications. For instance, if one is modeling a fishery where the weather becomes less hospitable as the season continues and this in turn increases the cost of catching each additional unit of fish, then (14) is an appropriate specification. The specification in (15) would be appropriate if the harvested species is spatially distributed evenly across a given area. In this case, costs would increase as cumulative harvest increases, since fishermen would first harvest in those areas closest to their home port. In reality, the fisherman’s cost function will be a complex mixture of these and other components, as well as being species- and location-dependent.

Accounting for all of these possible cost considerations is beyond the scope of this work. Using these specifications does not make this work without merit, as it does incorporate some of the more obvious cost determinants seen in many fisheries.

It is clear that these cost-function specifications, combined with the price dynamics equation in (13), capture the intuitive trade-offs fishermen face in determining their harvesting strategies. Slower-paced harvesting can reduce costs and increase ex-vessel prices, but the benefits from being patient cannot be fully internalized and there is additional incentive to harvest early through time-specific costs, stock effects, and/or unsecured harvest rights. It is also obvious that the shape of the harvest paths and the resulting ex-vessel prices will be highly dependent upon the user-specified parameter values. Thus, deriving comparative statics results can be complicated due to parameter interactions. We therefore pare down the investigation to look at only a subset of parameter interactions under both cost specifications considered in an effort to make some general conclusions about the impacts of imposing IFQs.

All simulation results shown are derived through the Genetic Algorithm and Optimization Toolbox (GAOT) for Matlab 5. See Houck, Joines, and Kay (1995) for specific details about this toolbox. The code that implements the GAOT and calculates the open-loop Nash equilibria can be provided upon request from the corresponding author.

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8 This harvesting behavior is described in Costello and Deacon (2007).

9 GAOT can be downloaded at http://www.ise.ncsu.edu/mirage/GAToolBox/gaot/
4.1 Harvest Paths and α’s

Of particular interest in this study is to see how the time-specific harvesting cost and stock effect cost affect fishermen’s behavior. We therefore begin by looking at how varying the α’s in (14) and (15) affects the open-loop Nash equilibrium harvest strategies. In the subsequent sections we explore how the interaction of changes in α’s and changes in other parameters affect ex-vessel prices.

The following parameter values are assumed for these simulations:

\[ n = 20, \quad s_u = s_o = 0.10, \quad Y = 100, \quad \beta = 2.75, \quad y_{\max} = 10, \quad P_0 = 50, \quad P_{\max} = 90, \quad TAC = 500, \quad T_{\max} = 20. \]

As in Costello and Deacon (2007), we make the additional assumption that under IFQ management, each fisherman has an identical quota allocation. Thus, \( TAC_i = TAC/n \) for all simulations with IFQs. For the purpose of comparison, we also conduct a simulation for each cost function in which the fishermen act as a cooperative entity and do not compete with one another. From a modeling standpoint this is done by maximizing the joint profit of the fleet. All of the parameter values are assumed for illustrative purposes and do not necessarily reflect a particular fishery. That is, we are not particularly concerned with the values of the harvest paths, but rather the shape of the path given the particular α value.

Figure 1 shows plots of the harvest paths using the cost specification given in (14) for \( \alpha_1 = 0, 1, 3, 6. \) With no IFQs, one can see that increasing the time cost of harvesting actually prolongs the season. This result may seem unintuitive at first, but upon further inspection the motivation is clear. As \( \alpha_1 \) increases, the cost of catching large quantities later in the season increases. Since the race for fish pushes ex-vessel prices down early in the season, fishermen respond to the higher per unit cost of harvesting late in the season by reducing late season harvest rates, resulting in prolonged seasons. In general, however, without IFQs the harvest strategies are quite similar regardless of the value of \( \alpha_1 \). This is expected because without IFQs the dominating factor in determining the harvest strategy is trying to acquire more of the TAC (i.e., the race for fish). In turn, this shortens the fishing season and reduces the impact of the increasing time cost of harvesting. Additionally, simulations with various other parameter values for the non-α parameters were conducted, resulting in similar harvest path shapes.

A much different harvesting scenario occurs when IFQs are implemented. As has been observed in practice, relative to the cases without IFQs, the season lengths with IFQs are significantly longer. However, unlike cases without IFQs, season lengths under IFQ management decrease as \( \alpha_1 \) increases, and the shape of the harvest path is highly dependent upon the \( \alpha_1 \) value. These results follow a more intuitive line of reasoning. With IFQs the race for a larger share of
the TAC is ended, which allows for slower-paced fishing relative to the no-IFQ case and consequently increases ex-vessel prices. However, as $\alpha_1$ increases, the cost of initially harvesting slowly increases. This gives the individual fisherman the incentive to increase harvest rates early in the season and to free-ride off the others’ early season slow harvesting. In equilibrium, all fishermen harvest more at the beginning of the season as $\alpha_1$ increases, thereby decreasing the season length. This behavior was consistently found using various parameter values for the other non-$\alpha$ parameters.

In the case where the fishermen act cooperatively to maximize the combined fleet’s profit, the fishermen are more patient in their harvesting than noncooperative fishermen, with or without IFQs, for all $\alpha_1$ values evaluated. This is because when the fishing effort is coordinated in order to jointly maximize the harvesting sector’s seasonal profit, the fishermen are not competing on any margin. The result is slower harvest rates in order to drive up ex-vessel prices and drive down harvesting costs.

Figure 2 plots the harvest paths for the different managerial scenarios and the cooperative case using the cost function given in (15) with $\alpha_2 = 0, 0.05, 0.10, \text{ and } 0.15$. These simulations are based on the same non-$\alpha$ parameters given above. The harvest paths for the case without IFQs show that varying stock effect cost has little impact on the resulting fisherman behavior. Additionally, the shapes of the harvest paths are similar to those with the increasing time costs—high harvest rates early that slowly taper off until the TAC is met. The fact that these two cost functions lead to similar harvest paths gives supporting evidence to the idea that the race for fish determines harvest strategies in the absence of property rights.

When IFQs are imposed, the harvest paths are highly dependent upon the $\alpha_2$ value. Similar behavior to the case of increasing time costs is derived—as the marginal stock effect cost increases, the fishermen’s ability to act collusively in an effort to force higher ex-vessel prices diminishes and more harvest effort is pushed to the beginning of the season. The shapes of the harvest paths based on (15) compared to those based on (14) are, however, different. With a stock effect cost, the equilibrium harvest path does not taper off toward the end of the season, unlike in the case of increasing time costs.

Comparing the cooperative case harvest paths to those with IFQs again highlights the concept that IFQs do not completely remove competition among fishermen. When acting cooperatively, fishermen are able to jointly hold out for higher ex-vessel prices before they begin harvesting. After that, the harvest path is dependent upon the stock effect cost.
4.2 Responses to Changes in \( \alpha \)'s and \( n \)

In this section and the subsequent sections we look at how varying the \( \alpha \)'s along with other parameters affects the average ex-vessel prices. Average ex-vessel price is calculated as:

\[
p_{avg} = \sum_{t=0}^{T} \left( \frac{p(t)y_i(t) + y_{-i}(t)}{\sum_{t=0}^{T}(y_i(t) + y_{-i}(t))} \right)
\]

Therefore, (16) is a harvest-weighted average price. This price allows us to see how effective the fishermen are at extracting rents from the processing sector under a variety of parameter settings. We look at these prices under a scenario in which \( n \) fishermen do not have IFQs, one in which they do have IFQs, and one in which the \( n \) fishermen act cooperatively. Table 1 provides the calculated \( p_{avg} \) values for the simulations with various \( \alpha \) and \( n \) values. Figure 3 gives plots of the percent differences in average prices over a range of \( \alpha \)'s, given a constant \( n \), for both cost functions considered. All other parameter values are the same as given above. The top two panels in Figure 3 compare the cooperative prices with the prices obtained under IFQ management with no cooperation. The bottom two panels of Figure 3 compare the IFQ and no-IFQ cases. As expected, the Table 1 and Figure 3 plots reveal that for a given \( \alpha \), \( n \) pair, the noncooperative IFQ price lies somewhere between the price without IFQs and the price obtained when the fishermen act cooperatively. With respect to a change from no IFQs to IFQs, it can be seen that the size of the ex-vessel price jump is dependent upon both the \( \alpha \) and \( n \) values. For instance, in the case of increasing time costs, the largest price differential between the IFQ case and the no-IFQ case happens when marginal time costs are low and the number of fishermen is large. This is primarily because when \( n \) is large and there are no IFQs the race for fish is more competitive, which drives ex-vessel prices down, regardless of the time costs (see Table 1). Conversely, when \( \alpha_1 \) is low and IFQs are implemented, competitive fishing behavior is diminished and ex-vessel prices move to or near the processor rent extraction price, \( p_{max} \). As \( \alpha_1 \) increases the competition among fishermen increases, even with IFQs implemented, and this competition is exacerbated by an increase in the number of participants. The end result of this competition is lower average ex-vessel prices.

A similar, but more pronounced, result occurs for the simulations with a stock effect cost. Here we see that when participation is low, the uncooperative IFQ holders reach a similar outcome to the cooperative case regardless of the level of the marginal stock effect. However, as \( n \) grows in the IFQ case, there is more competition on the stock effect cost margin, creating an equilibrium with more early season harvesting. In these cases, the simulations calculate that
imposing IFQs will result in only a small increase in average ex-vessel prices compared to the
fishery managed without IFQs.

4.3 Responses to Changes in $\alpha$’s and $s$

An often-discussed component in assessing ex-vessel pricing impacts associated with
imposing IFQs is the market structure of the corresponding processing sector (e.g., Matulich and
Sever 1999; Matulich, Sever and Inaba 2001; Halvorsen, Khalil and Lawarrée 2003). That is, an
important factor to consider in modeling fishermen’s harvesting strategies and the resulting ex-
vessel prices is their perceived notion of the competitive structure of the processing sector. As
stated above, that conjecture is modeled through the parameter $s$ in these simulations. We
therefore run simulations with various $\alpha$, $s$ pairs for both cost functions considered, with the
other parameter values as given above. Three different $s$ specifications are considered — $s_{low}$: $s_o = 0.20$, $s_u = 0.05$; $s_{med}$: $s_o = s_u = 0.10$; and $s_{hi}$: $s_o = 0.05$, $s_u = 0.2$. Given these specifications, $s_{low}$
corresponds to the case with low levels of competition in the processing sector (i.e., more
collusive), $s_{med}$ corresponds to medium processor competition, and $s_{hi}$ represents the high level of
processor competition scenario.

Table 2 provides the calculated $p_{avg}$’s for the simulations conducted with the varying $\alpha$, $s$
pairs. Figure 4 plots the percentage differences in the average ex-vessel price for the $\alpha$, $s$ pairs
used. Again, the top two panels of this figure give the percent difference in prices between the
case where the fishermen have IFQs but do not act cooperatively and the case where the
fishermen do act cooperatively to maximize the harvesting sector’s profit. The bottom two panels
compare the outcomes when the noncooperative fishermen do not have IFQs and when they are
granted IFQs.

For both cost functions considered, the difference between the average price with IFQs
and without IFQs increases as the level of processor competition increases. This highlights the
notion that as processors become more competitive, IFQs will increase the ability of fishermen to
extract rents from the processors. As marginal time costs/stock effect costs increase, the percent
difference in the pre- and post-IFQ ex-vessel prices declines. However, the rate of decline is a
function of the level of competition in the processing sector, with the ex-vessel price differential
falling more rapidly as processor competition decreases. This result follows the logic that as
processor competition declines there is a diminishing incentive to act cooperatively and harvest
slowly, even with IFQs implemented. When marginal time costs/stock effect costs are increasing
there is an incentive to increase harvest rates early in the season. With IFQs, when both
processor competition is decreasing and marginal time costs/stock effect costs are increasing, the
two features exacerbate the incentive to harvest early and consequently the ex-vessel price does not appreciate much compared to the no-IFQ case.

Comparing the cooperative case to the noncooperative IFQ case results also highlights the result of increased competition among fishermen with IFQs when processors are collusive and marginal time costs/stock effect costs are high. This is particularly the case when stock-effect costs are included in the cost function. With a stock effect cost there is a production externality in the cost function as well as in the marginal value function (i.e., price dynamics). This creates the potential for competition on two margins even with IFQs. If fishermen can act cooperatively they can alleviate competition and, as shown in the simulation, push the ex-vessel price to, or near, the maximum level even if the processing sector is highly collusive and marginal stock effect costs are high. With increasing time costs, even when acting cooperatively there is still an incentive to harvest early as marginal time costs increase. This diminishes the ability of a cooperative fleet to extract all rents from the processing sector.

4.4 Responses to Changes in $\alpha$'s and $Y$

Another concern for processors is that a race for fish under regulated open-access management creates a corresponding race to process. This leads to overcapitalization in the processing sector and a higher optimal throughput for processors. In turn, this could leave the processing sector vulnerable once IFQs are implemented since slower-paced fishing could induce higher ex-vessel prices as processors try to meet their excess demand. We explore this theme by running simulations with various $\alpha$, $Y$ pairs.

Table 3 provides the calculated $p_{avg}$’s for the $\alpha$, $Y$ pairs considered.

Figure 5 shows the percent differences in the average ex-vessel prices derived from simulations under various $\alpha$, $Y$ pairs for the different management systems and the cooperative fishing assumption. In comparing the no-IFQ prices to the IFQ prices, a similar result is obtained for both cost functions considered. Regardless of the level of capitalization, the percentage increase in ex-vessel prices post-IFQ implementation declines as marginal time costs or marginal stock effect costs increase. However, similar to the varying processor competition case, the rate at which the price differential between the no-IFQ– and IFQ-managed fisheries declines is a function of the level of optimal throughput. This result follows a simple logic. As stated above, fishermen without IFQs do not alter harvesting strategies much as marginal time costs or marginal stock effect costs increase since their strategies are dominated by the race for fish. For fishermen with IFQs, increasing the $\alpha$’s gives a greater incentive to harvest early, leading to an
equilibrium strategy with increased early season harvesting. The effect this harvesting strategy has on ex-vessel prices will be a function of the optimal throughput. When $Y$ is large, the change in harvesting strategies for the fishermen with IFQs brought about by increasing $\alpha$’s does not diminish the large price differential between the IFQ price and the no-IFQ price. This is because the demand for fish is so high. However, when $Y$ is relatively low, the IFQ fishermen who increase early harvesting due to increased marginal time costs or marginal stock effect costs will keep ex-vessel prices low, thereby diminishing the price increase compared to the no-IFQ case. Additionally, one can see that if the fishermen with IFQs act cooperatively they stand to benefit greatly as $\alpha$’s increase and $Y$ decreases.

5. Conclusion

In TAC-regulated fisheries without individually secured harvesting rights the common outcome is excessive competition for TAC among the fishermen, resulting in severe rent dissipation for the harvesting sector. IFQs have been promoted as a way to alleviate this harvesting competition. However, the value and cost of harvesting many fish species can be dependent on multiple attributes. IFQs provide a formal property system for one of these attributes, namely quantity of fish caught. Therefore, in the absence of a coordinated fishing effort, IFQs alone do not remove fishermen competition as these other attributes of fish harvesting are still in the public domain. This possibility for competition among fishermen with IFQs can inhibit the harvesting sector’s ability to extract rents from the corresponding processing sector.

In this paper, I demonstrate this situation in a game-theoretic context. We model within-season fishermen behavior in a TAC-regulated fishery, where the season length is a free time horizon, under the situations in which fishermen do not have IFQs, where IFQs are implemented, and when the fishermen act cooperatively to maximize the seasonal profit of the harvesting sector. To include multiple attribute effects, we analyze cost functions, which include increasing harvesting costs over time and a stock effect cost, respectively. Additionally, we model the ex-vessel price as a function of the total harvesting rate, the processing sector’s optimal throughput, and the perceived level of competition among processors. A GA approach is used to compute open-loop Nash equilibria for numerical simulations run under various assumed parameter values.

The results of these simulations show that the harvesting strategies and the resulting ex-vessel prices are highly dependent upon the parameter values used. In the case of the fishery managed without IFQs, ex-vessel prices are determined largely by the number of participants.
Conversely, for a given number of participants and particularly when the number of participants is large, we calculate relatively small fluctuations in average ex-vessel prices as other model parameters are varied. This is as expected since the harvesting strategy without secured harvest rights is dominated by the race for fish. This race for fish intensifies with additional participants and consequently creates downward pressure on ex-vessel prices.

In the simulations run with IFQs implemented, our results indicate that the equilibrium harvest strategies and resulting ex-vessel prices are highly dependent on the parameter values used on multiple margins. For instance, given the number of participants we calculated decreasing average ex-vessel prices as the marginal time costs or marginal stock effect costs are increased. However, the rate at which the ex-vessel prices decrease in response to increasing marginal time costs/stock effect costs is largely dependent upon the number of participants. This indicates that these margins can create significant competition among fishermen with IFQs and, like the race for fish, this competition is more intense with a greater number of participants. Similarly, we find that the interaction of price dynamics parameters, such as the perceived level of processor competition or the level of processor capitalization, and marginal time costs/stock effect costs, can greatly impact the harvest strategies of the fishermen with IFQs. More specifically, we see that when the ex-vessel price is less responsive to supply shortages, due to either low competition among processors or low levels of processor capitalization, fishermen move more of their harvest to the early part of the season as marginal time costs/stock effect costs increase, creating lower average ex-vessel prices. The instances of competition among fishermen with IFQs are further highlighted when the average ex-vessel prices computed under IFQ management are compared to those derived under the assumption of a cooperative harvesting sector. For the parameter values tested, we find that the profit-maximizing harvest profile is one that leads to an average ex-vessel price that is always equal to or greater than the noncooperative IFQ average ex-vessel price and is often at or near the maximum price processors will pay.

The major policy implication of this work is that these simulations show that the effect IFQs have on ex-vessel prices, and consequently on the processing sector, can be a multidimensional problem. Fishery managers should consider issues such as the impact of time-specific costs, stock effect costs, pre- and post-IFQ implementation participation, and processor competition and capitalization when assessing the impact IFQs will have on ex-vessel prices. Unfortunately, identifying the magnitude of these issues empirically may be quite difficult. For instance, if there are shifts in the spatial distribution of the fish population over time, it may be difficult to identify stock effect costs. Likewise, since these results show so much of the pre-IFQ
harvest strategy is dominated by a race for fish, it will be difficult to determine if ex-vessel prices are suppressed by low levels of processor competition or competitive fishing levels. Obviously, these challenges and others like it will have to be addressed on a fishery-to-fishery basis.

Several extensions and application of this work are also readily apparent. Within the framework provided here, multiple cost specifications could be applied to portray specific fisheries. One could also experiment with a time-dependent processor reservation price, as wholesale market prices may vary within seasons. Expanding upon this framework, a more realistic characterization of the fishery would include a heterogeneous fishing fleet. However, solving the problem as done in this paper with a heterogeneous fishing fleet would be considerably more computationally taxing. A next step to this line of research would also be to formally incorporate a processing sector into the model, in which the processors react not only to the fishermen’s behavior but also to each other’s actions. From an empirical standpoint, an application of this research would be to test if the derived comparative statics results hold for fisheries that have undergone IFQ implementation.
References


### Tables and Figures

#### Table 1. $p_{avg}$ for $\alpha$, $n$ Pairs

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### Table 3. $p_{avg}$ for $\alpha, Y$ Pairs

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Figure 1: Harvest Paths with Varying Time Costs
Figure 2: Harvest Paths with Varying Stock Effect Costs
Figure 3: Percentage Difference in $p_{\text{avg}}$ Varying $\alpha$'s and $n$

Figure 4: Percentage Difference in $p_{\text{avg}}$ Varying $\alpha$'s and $s$
Figure 5: Percentage Difference in \( p_{\text{avg}} \) Varying \( \alpha \)'s and \( Y \)