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Private Access Fees and Congestion

Is There a Role for the Government After All?

Stephen Salant and Nathan Seegert

1616 P St. NW
Washington, DC 20036
202-328-5000 www.rff.org

RESOURCES FOR THE FUTURE

Private Access Fees and Congestion: Is There a Role for the Government After All?

Stephen Salant* Nathan Seegert†‡

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Abstract

We reframe an important debate between Pigou and Knight about the need for government intervention in allocating congestible resources like roads. Knight showed that private toll-setting would achieve an efficient allocation if some motorists commuted to work on an alternative route that was uncongestible. Government intervention was unnecessary. Others have shown that Knight's *laissez-faire* solution fails if the other road was also congestible but marred their demonstration by excluding private toll-setting on that road as well. We consider a game with simultaneous toll-setting on *every* congestible road. When we discover that the *laissez-faire* allocation is inefficient, we consider how the government can improve the private allocation by providing motorists an actual or potential alternative to the privately-priced, congestible resources. Our results apply to a wide range of allocation problems involving congestion: simultaneous tuition setting in private (or charter) schools when students can instead attend a public school or simultaneous prize-setting in contests to cure diseases when researchers can instead work at NIH.

Keywords: Congestion externality, congestible roads, Knight-Pigou debate, potential competition, mixed economy

JEL Classification: H21, H23.

*Department of Economics, University of Michigan, 254 Lorch Hall, 611 Tappan Street, Ann Arbor, MI 48109-1220. Office phone: (734) 764-2370. ssalant@umich.edu

†Department of Finance, University of Utah

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1 Introduction

Consider a market where N identical buyers each purchase one unit of a good from one of $n(<< N)$ heterogeneous producers. Producers set prices and sell imperfect substitutes. A seller can steal a fraction of his rivals' customers by marginally reducing his price. Buyers impose externalities on each other in the sense that, at fixed prices, a decision by one buyer can harm other buyers. Given the externalities and the price-setting, the sufficient conditions of the First Welfare Theorem are clearly violated, and, from the perspective of the Arrow-Debreu model, efficiency of the market allocation seems unlikely.

Knight (1924), one of the intellectual founders of the Chicago School, provided an example of an economy with both price-setting and a particular class of externalities but where the decentralized equilibrium is nonetheless efficient. Knight considered the allocation of motorists (buyers) between a free, uncongestible road and a faster road subject to a congestion externality. Pigou (1920) had pointed out that if each motorist simultaneously chooses his own route to work, the resulting Nash equilibrium allocation of motorists fails to minimize the total wages lost commuting since a motorist choosing the congestible road disregards the impact of his choice on the commute time of fellow motorists. Pigou had suggested that efficiency could be restored through a government tax at the "Pigouvian level" on motorists using the congestible road. In response, Knight pointed out that government intervention is unnecessary: if ownership of the congestible road were assigned to a self-interested private agent, he would offer to sell to each motorist one-time access to his property and at the toll-revenue maximizing price an efficient allocation would result without any government intervention.

Knight appeared to win the debate. Pigou dropped the congestible roads example from subsequent editions of his textbook, *The Economics of Welfare*.[1] In our view, Pigou's apparent capitulation in this debate was far too hasty. The debate overstated the efficiency of the market solution and understated the value of government intervention.

Knight's idea has been applied in countless other contexts. If fishermen can choose whether

[1] Cheung (1973, footnote 2) interprets Pigou's withdrawal of the highway example from his opus as an attempt to avoid further criticism by Knight.

to fish on one of several congestible lakes or to take a job at a low fixed wage, for example, the fishermen will allocate themselves among the lakes and the low wage job in a way that equalizes earnings from each activity. But this allocation fails to maximize their aggregate earnings since no one takes account of the negative impact his own presence has on the productivity of the others fishing on the same lake. Instead of a lake-specific government tax to restore efficiency, however, disciples of Knight recommend empowering one individual per lake to set a fee that must be paid to gain access to the lake since the revenue-maximizing fee on each lake would supposedly be set equal to the lake-specific Pigouvian tax.

Or, to take a thoroughly modern example, suppose there are N biomedical researchers, each of whom can be deployed to discover the cure for one of $n(<< N)$ diseases. Discovering a cure for each disease has a specified social value, as does working for NIH. This is a congestion problem since when any researcher seeks the cure for a given disease he reduces the chance that another person will be the first to discover that cure, an effect that the planner would take into account but that a private agent would ignore when deciding which activity to pursue. A planner could work out how many of the researchers should be deployed in each activity to maximize expected social welfare and the government could set "Pigouvian prizes" to achieve this efficient allocation. However, disciples of Knight would argue that the government is not needed; a private agent would find it in his interest to offer a Pigouvian prizes to the first discoverer of a cure in exchange for the property right to produce the resulting drug. Will competing prize setters in theory achieve the efficient allocation?[2]

Following Pigou's argument for government taxes and Knight's rebuttal in favor of private toll setters, a large literature has developed on both sides (Lindsey 2006). Early work by Buchanan (1956) and Mills (1981) identified that Knight's argument depended upon competition because in its absence a private road supplier would have some degree of monopoly power (Edelson 1971). Seminal papers by Vickrey (1963) and Walters (1961) formalized Pigou's initial argument that roads are goods that are misallocated in the absence of a market. Buchanan (1965) reimagined this argument by framing roads as club goods, an intellectually attractive approach that many

[2]Prizes are becoming an important mechanism in many areas. See Brennan et al. (2011) for a partial list.

others followed (Ellickson, Grodal, Scotchmer and Zame, 1999).[3] Following Knight's critique, work by Rothbard (1978), Block (1979, 1980, 1996), and Cadin and Block (1997) continued to describe the benefits private roads would have on efficient capacity and operations. Fielding and Klein (1993) reimagined the private roads argument suggesting franchising as a practical mechanism. These arguments were continued by Roth (2006), Klein and Majewski (2006), Button (2004), and Foldvary (2006). The support of privatization of roads also stemmed from work following Coase (1960) (Winston and Yan 2011).[4]

Edelson (1971) finds that Knight's private solution is inefficient when it is extended to have two congestible roads but his demonstration is marred by his assumption that only one of the congestible roads has a toll setter. This formulation ensures that Knight's solution fails. Consider, for example, the very special case of two identical roads. Since they are congestible, a planner would put half the motorists on each of them. The *identical* allocation would arise if each road had its own toll setter since Bertrand competition would result in a unique and symmetric Nash equilibrium. Thus, in this case, Knight's idea continues to work. However, Edelson would conclude that Knight's proposal would fail because, with no toll setter allowed on the second road, the exercise of unchecked market power by the toll setter results in inefficiently low use of that road. To analyze Knight's private solution fairly, *every* congestible road should have its own independent toll setter.

In our formulation, we assume there are n congestible roads each with its own profit-maximizing toll setter. They set tolls simultaneously and engage in Bertrand competition to attract the N motorists. We assume these motorists also have access to an uncongestible alternative with exogenous cost c. By varying c we are able to investigate (1) the Knight-Pigou case (sufficiently low c), (2) the case where no motorist would ever use the uncongestible alternative (prohibitively high c) and (3) an intermediate case. This allows us to clarify whether competition between two, or more, toll

[3]The competitive equilibria of clubs are also discussed by Berglas (1976, 1981), Berglas and Pines (1981), Boadway (1980), Berglas, Helpman and Pines (1982), Hillman and Swan (1983), Sandler and Tschirhart (1980), Scotchmer (1994), and Scotchmer and Wooders (1987). Seegert (2014) discusses the equilibrium in clubs in the context of cities.

[4]There is a large literature following Coase (1960) that considers the problem of congestion as one of ill defined property rights. Solutions include transferable discharge permits (Crocker 1966, Dales 1968), accounting for concentration in the permit and product market (Malueg 1990), and transaction costs (Stavins 1995).

setters always restores efficiency.[5]

We conclude that Knight's private solution is inefficient even when there is a toll setter on every congestible road. Toll-setting *does* result in efficient allocation in the limit of a large economy, an interesting complement to Edgeworth's (1881) result about core convergence in replica economies. But in the more realistic circumstance of a finite economy, toll-setting will not in general achieve efficiency.

We then go on to show how this inefficient purely private solution can be improved if the government offers motorists an uncongestible alternative. Our results are important for the allocation of other congestible resources including the airline market discussed by Brueckner (2002) and Pels, Nijkamp and Rietveld (2000) and the electrical power industry discussed by Borenstein, Bushnell and Stoft (2000). The government can sometimes improve allocative efficiency by supplying consumers with an alternative to what the private sector provides. If the congestion in the busy New York–Washington corridor were resolved by allowing independent toll-setting on alternative congestible routes, allocative efficiency could be improved if the government provided a train running with sufficient frequency that motorists considered it a viable alternative to driving on congested toll roads and then parking in congested lots. The express package delivery services (FedEx, UPS, DHL, etc.) may function more efficiently if the US postal service provided consumers with an alternative, private and charter schools may function more efficiently if students have available the public school alternative, private contests to discover disease cures function more efficiently because contestants have the alternative of working for NIH instead, and so forth. In the body of the paper, we confine our attention to Kinght's original context—highway congestion. We will return to other applications in the concluding section and the appendix.

We proceed as follows. In section 2, we introduce notation, set up the model, and show that Knight's response to Pigou generalizes to the case with n congestible toll roads and an outside option inexpensive enough to be utilized. In section 3, we clarify that with no outside option (or, equivalently, with a sufficiently unattractive outside option), the allocation need not be efficient.

[5] Previous studies have considered duopoly cases with congestible goods, for example Braid (1986) and Van Dender (2005).

Section 4 contains a positive and normative analysis of duopoly toll-setting as the attractiveness of the outside option is varied exogenously; comparative statics for the generalization of this model to oligopoly toll-setting is relegated to the appendix. In section 5, we use our findings to suggest a new role the government can play in a mixed economy: supplying an uncongestible alternative sufficiently attractive to provide either actual or potential competition to alter the behavior of private toll setters. In our concluding section, we consider other applications of our results.

2 Model Setup

Consider drivers trying to get from the same starting point to the same destination using one of several roads. Assume that each motorist earns the same wage. Measure the time cost of delay in each driver's commute in dollars of forgone wages. Classify each road the motorists might take as either congestible or uncongestible. On a congestible road, the commute time depends on the number of drivers using the same road. Denote the cost per motorist to commute on road i (again denominated in dollars) as $A_i(x_i)$ where x_i is the number of drivers on road i.[6] We assume below that (1) $A_i(\cdot)$ is differentiable, (2) it is strictly increasing, and (3) $x_i A_i(x_i)$ is strictly convex for $i = 1, \ldots, n$. Thus as the number of drivers on road i increases so does the cost each driver pays in lost wages.[7] On an uncongestible road, the commute time is constant regardless of the number of drivers on the road. We denote the cost in wages forgone of using the fastest of the uncongestible roads as c; if there are slower uncongestible roads, we disregard them as motorists would never use them.

2.1 The Planning Problem

As a benchmark, consider how a planner would allocate N motorists to the $n << N$ congestible and one uncongestible roads to minimize the total cost of commuting in terms of forgone wages (or, equivalently, work hours lost commuting): $\sum_{i=1}^{n} x_i A_i(x_i) + (N - \sum_{j=i}^{n} x_i)c$. Since the minimand in the planning problem is strictly convex, the solution to the n first-order conditions is unique.

[6] Throughout we treat the number of drivers on road i (denoted x_i) as a real number rather than an integer.

[7] We discuss the role of the uncongestible road in section 4 by exogenously varying its commute time.

The planner can equivalently be regarded as maximizing the time saved over sending everyone on the uncongestible, slow road: $cN - \{\sum A_i(x_i)x_i + (N - \sum x_i)c\}$. To maximize the time saved, the planner would minimize the commute time (the expression in braces). Alternatively, by using road i instead of the uncongestible road, x_i motorists save $R_i(x_i) = x_i(c - A_i(x_i))$. The planner can be regarded as maximizing the aggregate time saved by using the n congestible roads: $\sum_i R_i(x_i)$.

If the planner finds it optimal to use the uncongestible road ($N - \sum_{i=1}^{n} x_i > 0$), then he will set the marginal cost equal across all roads:

$$\begin{aligned} A_i(x_i^s) + x_i^s A_i'(x_i^s) &= c \text{ where} \\ N - \sum_{i=1}^{n} x_i &> 0. \end{aligned} \tag{1}$$

2.2 The Knight-Pigou Controversy

Pigou and Knight focused on the case where there are two roads, one congestible and the other uncongestible.

Pigou pointed out that the free movement of drivers across the two roads would lead to the commute times being equal, $A_1(x_1) = c$. Since it violates equation (1), this allocation is inefficient. The inefficiency is due to an externality each driver on the congestible road imposes on the other drivers on the congestible road by slowing down traffic. To restore efficiency, Pigou proposed that the government set a toll per motorist of $\theta_1^P = x_1^s A_1'(x_1^s)$ for each motorist using the congestible road. Faced with this "Pigouvian" tax, drivers would allocate themselves across the two roads such that the full cost of using each road was the same: $A_1(x_1) + \theta_1^P = c$. This would ensure efficiency since, given that $\theta_1^P = x_1^s A_1'(x_1^s)$, the number of motorists on the two roads would satisfy equation (1). Clearly, if there are $n > 1$ congestible roads, Pigou's solution generalizes.

Knight replied that government intervention was unnecessary to achieve efficiency. If the congestible road were privately owned, its owner would charge an entry fee to maximize his toll revenue and would inadvertently induce the efficient allocation of motorists between the two roads. To

demonstrate Knight's claim, consider the toll setter's maximization problem:

$$\pi_1(\theta) = \theta_1 x_1(\theta_1) = x_1(\theta_1)(c - A_1(x_1(\theta_1)) \text{ with respect to } \theta_1. \tag{2}$$

Inadvertently the toll setter maximizes the savings in commute time from using the congestible road, $R(x_i)$, which, as shown previously, is the social planner's objective function. Hence, the toll setter's objective function is a strictly convex function that achieves its global maximum at x_1^s. One intuition for Knight's result is that for any θ_1 chosen by the toll setter, the payoff of the N motorists does not change: collectively, the motorists always lose \$ Nc (in wages and tolls) and the other toll setters receive the same payoffs. So when the toll setter varies his decision variable to maximize his own payoff, he inadvertently maximizes the sum of everyone's payoff.[8]

Knight's argument generalizes. Suppose instead there were n roads congestible to different degrees and one uncongestible road. Competitive toll setters will again inadvertently induce self-interested motorists to allocate themselves in a way that minimizes the total time lost commuting. Once again, private toll setters would set exactly the same tolls as the government tax authority.

To verify this claim, assume that every toll setter chooses the Pigouvian toll (denoted θ_j^P) where $\theta_j^P = x_j^s A_j'(x_j^s)$ for $j = 1, \ldots, n$. Suppose one of the n players (player i) considers a unilateral deviation. If he lowers his toll, motorists are attracted to his roadway until the increased congestion makes it no longer more attractive than the uncongestible, toll-free road; if he raises his toll, motorists leave his roadway until its reduced congestion makes it just as attractive as the uncongestible, toll-free road. When he varies his toll, motorists reallocate themselves between his road and the uncongestible road. But as long as any motorist remains on the uncongestible road, changes in his toll affect neither (1) the number of motorists using each of the other $n-1$ congestible roads nor (2) the revenue each of the $n-1$ other toll setters collects.

As long as any motorist remains on the uncongestible road, $A(x_i(\theta_i)) + \theta_i = c$ and the number of motorists on road i just depends on the toll θ_i. As in Knight's case ($n = 1$), $x_i(\theta_i)$ is a strictly decreasing function of the one variable, θ_i.

[8] The maximum point found in this section assuming a "sufficiently fast" uncongestible road is the global maximum, which we show in Appendix A.

The toll setter i ($i = 1, \ldots, n$) wants to maximize:

$$\pi_i(\theta_i) = \theta_i x_i(\theta_i) = x_i(\theta_i)(c - A_i(x_i(\theta_i)) \text{ with respect to } \theta_i. \tag{3}$$

Once again, the toll setter's maximand is $\pi_i(\theta_i) = R_i(x_i(\theta_i))$, which reaches a global maximum when the number of motorists on the congestible road is socially optimal because it is the planner's objective function.[9]

3 Assigning Property Rights Does Not Always Eliminate Inefficiency

The assignment of property rights cannot *always* solve this externality problem, however. Consider the case in which there are two roads, both congestible. To test Knight's proposal fairly, we depart from Edelson and assume that there is an independent toll setter on *each* road. Will competing, self-interested toll setters achieve an efficient allocation without government intervention?

The planner would divide the N motorists across the two roads so that the marginal cost of adding an additional motorist to either road was the same: $A_1(x_1^s) + x_1 A_1'(x_1^s) = A_2(x_2^s) + x_2^s A_2'(x_2^s)$ where $x_1^s + x_2^s = N$. In equilibrium motorists choose the cheaper road, causing the cost per motorist to equalize on the two roads, $A_1(x_1) + \theta_1 = A_2(x_2) + \theta_2$. The planner could, therefore, achieve the efficient solution by setting the Pigouvian tolls $\theta_1^P = x_1^s A_1'(x_1^s)$ and $\theta_2^P = x_2^s A_2'(x_2^s)$.

What allocation would occur in the Nash equilibrium of the toll-setting game? Toll setter 1 would entertain a conjecture ($\bar{\theta}_2$) about the toll on the other road and would maximize his revenue ($\theta_1 x_1$). At an optimum, $\theta_1 + x_1(\frac{d\theta_1}{dx_1}) = 0$. That is, if he lowered his toll enough to attract one more motorist, the gain in revenue from that one motorist (θ_1) must exactly balance the loss incurred on the x_1 inframarginal motorists. To obtain an explicit expression for $\frac{d\theta_1}{dx_1}$, we totally differentiate the equation indicating that the cost per customer is identical on the two roads and the equation indicating that all motorists use one of these two roads, concluding that $-\frac{d\theta_1}{dx_1} = A_1'(x_1) + A_2'(x_2)$.

[9] The details are the same as in the case of one congestible road and are explicitly given in Appendix A.

The change in the toll required to attract one more motorist differs from the earlier case with an uncongestible road ($-\frac{d\theta_1}{dx_1} = A_1'(x_1)$) because the motorists now must be attracted from the other congestible road. Further, as the congestion lessens on the other congestible road a deeper cut in the toll is required to attract the same number of motorists.

Since each player's conjecture about the toll of his rival is correct in a Nash equilibrium, the following 4 equations must hold.:

$$\begin{aligned} \theta_1^* &= x_1^*(A_1'(x_1^*) + A_2'(x_2^*)) \\ \theta_2^* &= x_2^*(A_1'(x_1^*) + A_2'(x_2^*)) \\ A_1(x_1^*) + \theta_1^* &= A_2^*(x_2^*) + \theta_2^* \\ x_1^* + x_2^* &= N. \end{aligned} \tag{4}$$

The pair of private tolls defined in the first two equations no longer leads to the efficient allocation of drivers (except in that artificial but instructive case where the two roads are identical).[10]

We have shown that private toll-setting is inefficient if there exists no uncongestible road. But the inefficiency remains if we added an uncongestible third road provided it was sufficiently slow. We make this statement precise in the next section.

3.1 The Uncongestible Road and "Competitive Conditions"

Therefore there is something important about having an uncongestible road fast enough to be part of the planner's solution. Its presence allows the assignment of property rights to solve the market inefficiency. We conclude the section by illuminating this issue.

Knight's elaboration of Pigou's example is frequently taken as evidence that the assignment of property rights ensures efficiency. But Knight (1924, p. 591) refers to "competitive conditions" that

[10] If the roads are identical, the planner would put an equal number of motorists on each of them. Since in that case, $x_1^* A_2'(x_2^*) = x_2^* A_1'(x_1^*)$, an equal division is what would occur in the unique Nash equilibrium as well. Note that even if the roads were identical, Edelson would have concluded that Knight's solution is inefficient since less than half of the motorists would choose the one road with the toll. A finding of inefficiency would occur in this case not because Knight's proposed solution fails but because Edelson has introduced an *artifact* by treating the identical roads asymmetrically. The role of heterogeneity is discussed further in Appendix E.1.

seem to acknowledge that it is not the mere assignment of property rights that ensures efficiency. Indeed, Knight and later followers implicitly recognize that efficiency is unlikely to result from use of the price system unless competitive conditions prevail. But it is unclear what would constitute competitive conditions when both roads are to some extent congestible.

Suppose there is no uncongestible road but instead there are M times as many motorists and M times as many congestible roads of each type. We know from the current section that if $M = 1$, toll setters competing for the business of the motorists will not in equilibrium achieve an efficient allocation of motorists. In Appendix B, we consider the case where M grows without bound. We show that in this case the incentive for toll setters to deviate unilaterally from an efficiency-inducing toll profile disappears. This provides one concrete definition of competitive conditions. Intuitively, for any M, the efficient number of motorists on any road of a given type does not change. Moreover, for any profile of tolls, the number of motorists choosing a roadway of a given type does not change. What changes is the rate at which a toll setter on a road of a given type anticipates that he will lose customers if he marginally increases his toll. For small M, this rate of loss is dampened since every competing road is rendered less attractive as the motorists fleeing the increased toll add to its congestion. But as M grows, the fleeing motorists locate on so many different roads that the additional congestion imposed on any one of them becomes negligible. In the limit, therefore, it is *as if* the alternative to any individual toll setter's road is a completely uncongestible alternative.

A thick market provides the necessary "competitive setting" (to use Knight's phrase) for property rights to eliminate the inefficiency because the effect of a change in toll in one road has increasingly less effect on the number of drivers on other roads until in the limit $dx_j/d\theta_i \to 0$. In this case the second term that produces the wedge in $-\frac{d\theta_1}{dx_1} = A_1'(x_1) + A_2'(x_2)$ goes to zero and efficiency is restored.

4 An Analysis of Duopoly Toll-Setting as the Cost of the Uncongestible Road Is Varied Exogenously

4.1 Overview

The analysis in section 2.2 implicitly assumed that the exogenous value of c was so low that some the motorists utilized the uncongestible road. It concluded that private toll-setting leads to an efficient allocation. Define $\underline{c}$ as the highest cost for which the uncongestible road would be utilized in equilibrium. The analysis in section 3 implicitly assumed an exogenous value of c so high that it is as if the uncongestible road did not exist: no motorist would use it and the common cost of commuting on the two toll roads, including tolls, was strictly cheaper than the cost of using the uncongestible road. It concluded that, contrary to Knight's conclusion, private toll-setting by duopolists is typically inefficient. Define $\bar{c}$ ($> \underline{c}$) as the lowest cost for which the equilibrium has these characteristics. It is natural to ask what happens in between ($c \in (\underline{c}, \bar{c})$.)

In this section, we analyze the subgame-perfect equilibrium of the duopoly toll-setting game systematically for every exogenous value of c. For c in the intermediate interval ($c \in (\underline{c}, \bar{c})$), potential competition from the uncongestible road leads to toll-setting equilibria where the common cost on the two roads equals the exogenous cost c even though no motorist uses the uncongestible road. As we show, potential competition from the uncongestible road is likely to improve the social welfare that would be generated by the private duopoly toll-setting in the absence of such competition.

We conclude the section by reinterpreting this result: although private toll-setting will lead to an inefficient allocation of motorists if the two roads are congestible, providing motorists with an uncongestible government alternative to the toll roads can improve on the free-market allocation even if the government alternative is not used.

We begin by considering the best-choice problem of a toll setter given his conjecture about the toll on the other road. This permits us to derive the shape of a toll setter's best-reply function. The best reply of each toll setter consists of three regions. For simplicity, we assume that each congestion function is linear when drawing our figures although our conclusions do not depend on linearity. Given this assumption, each best reply turns out to be piecewise linear, with a

strategic-complements segment and a strategic-independence segment, separated from each other by a strategic-substitutes segment. We then show how this best reply *shifts* if the exogenous cost c is larger. Finally, we show how the Nash equilibrium changes as the exogenous value of c changes.

To determine the best choice of toll setter 1 for any given c, we first determine, for every conjectured toll on road 2, how many motorists will use road 1 for each choice of θ_1, and hence the size of toll setter 1's revenue ($\theta_1 x_1$). Suppose, given the conjectured toll on road 2, that the toll on road 1 induced some of the motorists to use each of the three roads. If toll setter 1 marginally reduced his toll, motorists would switch to his road until the increased congestion they cause raises the cost of traveling on it back to c. None of these additional motorists would come from toll road 2 since the conjectured toll on that road is, by assumption, unchanged and hence congestion on that road must also be unchanged in order for its overall cost per motorist to remain the same as on the uncongestible road (c). Since $A_1(x_1) + \theta_1 = c$ throughout this range, $dx_1/d\theta_1 = -1/A_1'(x_1) < 0$.

Eventually, however, there will be no more motorists to attract from the uncongestible road. At that point, a further reduction in the toll on road 1 will begin to attract motorists from road 2. On the interior of this region the two congestible roads have a common cost that is strictly smaller than the exogenous cost of commuting on the uncongestible road. Since $A_1(x_1)+\theta_1 = A_2(N-x_1)+\bar{\theta}_2 \leq c$ throughout this range, $dx_1/d\theta_1 = -1/[A_1'(x_1)+A_2'(N-x_1)] < 0$. So the budget constraint is kinked at the boundary between these two regions and has a strictly smaller magnitude to the left of the kink than to its right. Intuitively, a given reduction in θ_1 will attract fewer motorists when they come from the other toll road rather than from the uncongestible road because the other toll road becomes more attractive as motorists leave it and consequently the commute time on it shortens.

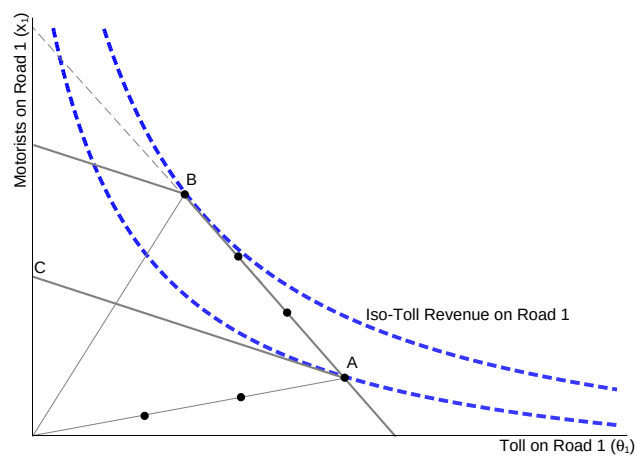
Assume $A_i(x_i) = a_i x_i + b_i$, where a_i and b_i are exogenous constants.[11] Then, with θ_1 on the horizontal axis and x_1 on the vertical axis, the budget constraint will be concave and will consist of two downward-sloping linear segments—a flatter segment when θ_1 is sufficiently small and a steeper segment when θ_1 is larger. If the toll conjectured for road 2 increases, the flatter portion of the budget constraint shifts out since the same toll on road 1 will attract more motorists from road 2

[11] The magnitude of the slope on the steeper segment is therefore $1/a_i$ for $i = 1, 2$ and on the flatter segment is $1/(a_1 + a_2)$.

because its conjectured toll is higher; the position of the steeper part of the budget constraint will not shift since, for any toll on road 1, the same level of congestion is required to raise the full cost on road 1 to c. Figure 4.1 displays in (θ_1, x_1) space the kinked budget constraint associated with a given toll on road 2 and shows how the flatter piece of it would shift up if the conjectured toll is higher.

We superimpose on these constraints toll setter 1's isorevenue curves: $\theta_1 x_1 =$ constant for different constants. They can be represented as a field of rectangular hyperbolas in the first quadrant. Since the slope of an indifference curve at a point (θ_1, x_1) is $\text{MRS}(\theta_1, x_1) = -x_1/\theta_1$, the preferences are homothetic and convex: any given ray from the origin cuts every indifference curve at the same slope and a flatter ray cuts the field of indifference curves where their common slope has a smaller magnitude. Two such rays are displayed in Figure 4.1. Ray OA cuts every indifference curve with the slope of the flatter part of the budget constraint $(-1/[A_1' + A_2'])$. Ray OB cuts every indifference curve with the slope of the steeper part of the budget constraint $(-1/A_1')$. Given the shape of the indifference curves and the budget constraint, the best choice for each conjectured toll on road 2 is unique.

Figure 4.1: Optimal Responses (θ_1, x_1) to Conjectured Toll on Road 2 (θ_2) for an Unchanged Cost (c) on Uncongested Road



Fix the cost (c) of commuting on the uncongestible road. We now examine the *locus* of best choices (θ_1, x_1) as the toll conjectured on road 2 increases. If θ_2 is very low, ray OA intersects the

budget constraint on its flatter portion. This intersection point must be optimal since the indifference curve through it will be tangent to the flatter portion of the constraint. As the conjectured toll 2 increases, the flatter portion of the budget constraint intersects ray OA farther from the origin. The reader can imagine lines with the same slope as AC passing through the three dots on ray OA. Each dot would denote the optimal point since the indifference curve would be tangent to the constraint there.

Eventually the flatter portion of the budget constraint shifts out so far that it intersects ray OA on its steeper portion. The intersection of ray OA and the constraint is then no longer optimal since a northwest movement along the steeper part of the constraint is strictly preferred to point A. The optimum in this case occurs at the kink point on the budget constraint. As the conjectured toll on road 2 increases in this region, the best choice of toll setter 1 moves up the line segment between point A and point B. The dots along that segment indicate the best choice as the conjectured toll on road 2 increases. Eventually, the kink point reaches B.

If the road 2 toll is conjectured to be even higher, ray OB continues to cut the budget constraint at point B since the position of the steeper portion of the constraint does not change. In that case, the indifference curve through point B remains tangent to the constraint at B and point B remains optimal.

4.2 A Toll Setter's Best Reply

As the conjectured toll on road 2 increases and toll setter 1 best responds, the number of motorists that patronize road 1 (x_1) monotonically increases. As for the toll he sets in response to the conjectured increase in θ_2, it first increases (until point A is reached), then decreases (until point B is reached), and finally remains constant. As a result, the best reply of toll setter 1 has a strategic-complements segment, a strategic-substitutes segment and finally a segment, reflecting strategic independence.

We depict this best reply in Figure 4.2. If $A_i(x_i)$ for $i = 1, 2$ is linear, the slope on this first segment is $d\theta_2/d\theta_1 = 2$. Hence, the slope on the strategic-complements segment is constant and

independent of c.[12] Points on the strategic-substitutes segment of toll setter 1's best reply must satisfy the following equations:

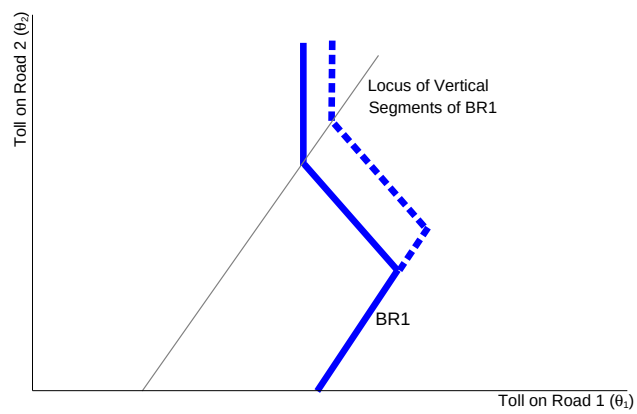
$$x_1 + x_2 = N \tag{5}$$

$$A_1(x_1) + \theta_1 = c \tag{6}$$

$$A_2(x_2) + \theta_2 = c. \tag{7}$$

If the congestion functions are linear, equations (6) and (7) will be linear and the system can be solved, for any given c, to determine the optimal θ_1 for any given θ_2. Differentiating, we conclude that on this segment $d\theta_2/d\theta_1 = -A_1'/A_2'$. Hence, the slope on this segment is also constant and is independent of c.[13] In the strategic-independence region, the best reply of toll setter 1 in $\theta_1 - \theta_2$ space is a vertical line.

Figure 4.2: How the Best Reply of Toll Setter 1 (BR1) Shifts as the Exogenous Cost (c) Increases



The best reply of toll setter 2 to any conjectured θ_1 can be deduced in the same manner. It will be piecewise linear, with the piece closest to the origin reflecting strategic complements, the

[12]The flatter piece of the budget constraint satisfies $A_1(x_1) + \theta_1 = A_2(N - x_1) + \theta_2$. The tangency condition is $1/[A_1'(x_1)+A_2'(x_2)] = x_1/\theta_1$. Assuming the two congestion functions are linear, we can eliminate x_1 using the tangency condition and can differentiate to obtain $d\theta_2/d\theta_1 = 2$; the slope of the other best reply in the strategic-complements region is $d\theta_2/d\theta_1 = 1/2$.

[13]The characterization of the equilibria in the linear example is done in Appendix C.

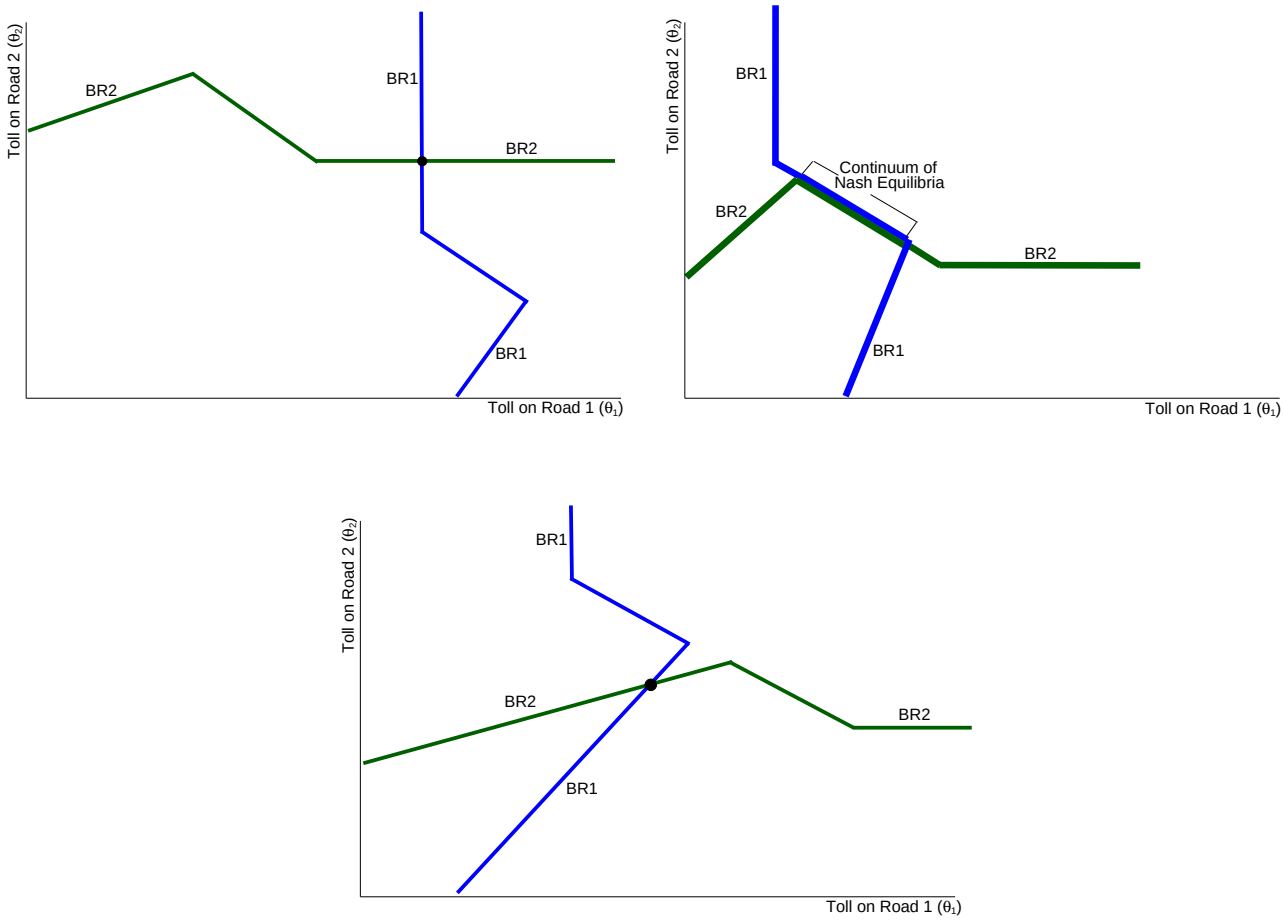
next piece reflecting strategic substitutes, and the piece farthest from the origin reflecting strategic independence. Every point on the strategic-substitutes segment of toll setter 2's best reply must satisfy exactly the *same* three equations (5)-(7). Hence, both the strategic-substitutes segment for toll setter 1 and the strategic-substitutes segment for toll setter 2 must lie on the same line in (θ_1, θ_2) space. But the two segments may be disjoint or may only partially overlap. If $A_i(x_i)$ for $i = 1, 2$ is linear, toll setter 2's best reply has a slope of 1/2 in its strategic-complements region, A_1'/A_2' in its strategic-substitutes region, and a slope of zero in its strategic-independence region.

4.3 Nash Equilibrium for a Given Exogenous c

The three panels of Figure 4.3 depict the two toll setters' best-reply functions and their intersections. The two functions are drawn for some $c < \underline{c}$ and therefore intersect where each function reflects strategic independence. This is the region discussed by Knight and Pigou and in section 2.2. At the opposite extreme ($c > \bar{c}$) each function would intersect in its upward-sloping region. In this region, private sector toll-setting is inefficient as shown in section 3.

In between ($c \in (\underline{c}, \bar{c})$), the two best replies overlap in their strategic-substitutes region. Consequently, there is a continuum of equilibria in this case. No motorist uses the uncongestible, slow road. But if either toll setter raised his toll unilaterally even slightly, some of his customers would switch to the uncongestible road.

Figure 4.3: Best Replies When c Is Low, Middle, and High



The Nash equilibrium is unique in two of the three regions. If $c < \underline{c}$, toll setter 1's best reply continues up vertically and toll setter 2's best reply continues to the right horizontally, so they cannot cross a second time with one or both tolls larger. Moreover, on the interior of this region the two strategic-substitutes segments must be disjoint. If $c > \bar{c}$, the best replies intersect in their respective strategic-complements segments. Their respective strategic-substitutes regions are again disjoint and so there can be no other Nash equilibrium.

If $c \in (\underline{c}, \bar{c})$, the two downward-sloping pieces *partially* overlap. Hence, in this range, a given c can generate a *continuum* of equilibria. Consider two equilibria associated with the same c. These can be visualized as lying on line segment AB in Figure 4.1. The equilibrium with the smaller

toll on road 1 will have more motorists on road 1, a larger toll on road 2, and fewer motorists on road 2. In each equilibrium, the total cost on each road (inclusive of the tolls) will be c. Since the planner's optimization problem results in a unique number of motorists on each road, all (or all but one) of the Nash equilibria in the strategic-substitutes region are inefficient.[14] Thus, there remains an efficiency rationale for government intervention.

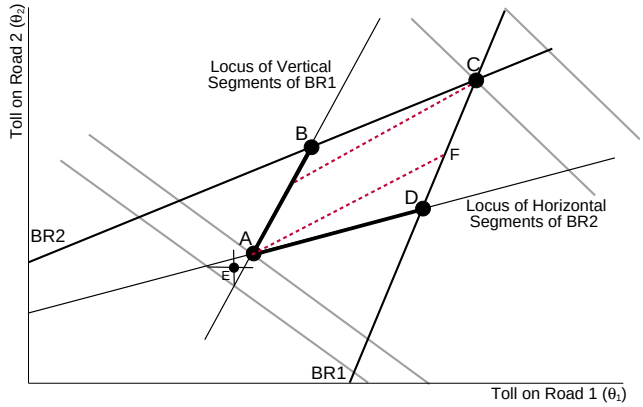
It is instructive to understand how the equilibrium changes as c varies. If c is higher, the steeper piece of the budget constraint in Figure 4.1 will be farther from the origin but will still have the same slope. Consequently, this shifted-out steeper portion of the constraint will intersect ray OB at a larger θ_1. Moreover, the largest θ_2 that generates an optimum where ray OB intersects the budget constraint is also larger. Finally, this shifted-out portion will intersect ray OA farther from the origin.

Hence, the new strategic-complements portion coincides with the previous strategic-complements portion but ends at a point farther from the origin. The strategic-substitutes segment is therefore farther from the origin and has the same slope as before; but the kink where the reaction function becomes vertical occurs at a larger θ_1 and a larger θ_2. We illustrate how the best reply of toll setter 1 shifts as c varies in Figure 4.2. For later use, we have indicated the locus of kinks (where the best reply of toll setter 1 turns vertical) that would be traced out as c varies.

As for the best reply of toll setter 2, as c is increased, the new strategic-complements portion also coincides with the previous strategic-complements portion but ends farther from the origin. The strategic-substitutes segment has the same slope as before and is again a segment of the same line from which toll setter 1's strategic-substitutes piece is extracted. The kink where the reaction function of toll setter 2 becomes horizontal occurs at a larger θ_2 and a larger θ_1. Figure 4.4 shows how the equilibrium tolls change as c varies. For later use, we have indicated the locus of kinks (where the best reply of toll setter 2 turns horizontal) that would be traced out as c varies.

[14] Hence, Knight's belief that self-interested toll-setting by the private market must ensure efficiency is mistaken.

Figure 4.4: Tolls Set by Duopolists in Response to Actual or Potential Government Competition



4.4 Nash Equilibria as c Varies

Figure 4.4 plots the Nash equilibria in (road 1 toll, road 2 toll) space for any c. For any c, the best reply of toll setter 1 can be constructed as follows: begin at BR1 on the diagram and follow the positively-sloped linear segment rising toward point D; when that segment intersects the negatively-sloped segment determined by the given c, follow that segment to the northwest until it intersects the "Locus of Vertical Segments of BR1"; the remaining segment of toll setter 1's best reply extends vertically upward from that intersection point. Similarly, for any c, the best reply of toll setter 2 can be constructed as follows: begin at BR2 on the diagram and follow the positively-sloped linear segment rising toward point B; when that segment intersects the negatively-sloped segment determined by the given c, follow that segment to the northwest until it intersects the "Locus of Horizontal Segments of BR2"; the remaining segment of toll setter 2's best reply extends horizontally to the right.

For any c, the pair of tolls that forms a Nash equilibrium is at the intersection point(s) of the two best replies. If the strategic-substitutes segment induced by c is on the origin side of point A, then the best replies intersect once in their strategic-independence region and nowhere else. If the strategic-substitutes segment induced by c is northeast of point C, then the best replies intersect

once in their strategic-complements region and nowhere else. Finally, if the strategic-substitutes segment induced by c lies in the intermediate range between points A and C, then there is a continuum of Nash equilibria associated with that exogenous c.

The continuum of equilibria is contained in the area ABCD, which is bounded on two sides by the strategic-complements segment of each best reply and on the other two sides by the locus of kinks between the strategic-substitutes and strategic-independence segment of each best reply. Within area ABCD, toll pairs lying on a line of slope 1 induce subgame-perfect equilibria with an unchanging number of motorists on each road. Two such lines are depicted in the diagram. As one moves from point A to point F, for example, each toll setter increases his toll by the same amount and therefore motorists have no incentive to switch to the other road. Hence, aggregate commute time remains minimized. However, toll pairs on this line closer to point F induce equlibria where the payoffs to motorists are reduced by exactly as much as payoffs to toll setters are increased. We have also drawn a line parallel to AF through point C, the situation with no uncongestible road. The allocation of motorists is unchanging in all the equilbria induced by toll pairs lying on this line since it slopes up at 45°. Hence they all have the same aggregate commute time as in the equilibrium where no uncongestible road is available. Now consider any point lying strictly north of a point on this line but still within area ABCD. Since toll 2 is higher and toll 1 is unchanged, this equilibrium will have more motorists on road 1 and fewer on road 2 than at point C. In any such equilibrium the aggregate commute time is even longer than if there were no uncongestible road. This explains why some of the shaded region in Figure 4.5 lies above the higher of the two horizontal lines.[15]

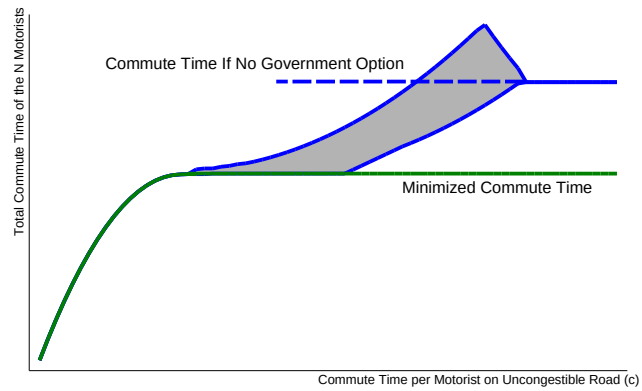
Figure 4.5 depicts the aggregate commute time (total man-hours lost commuting) in the efficient solution and the toll-setting equilibrium as the exogenous speed on the uncongestible road is changed. The aggregate commute time in the planning solution is strictly increasing in c until $c = \underline{c}$ and is constant thereafter since the planner finds the uncongestible road too slow to utilize. Aggregate commute time in the toll-setting equilibrium coincides with that in the planning solution

[15] In the case where both congestible roads are symmetric points A and C lie on the 45° line from the origin. In this case, the efficient allocation is feasible for any $c \in [\underline{c}, \bar{c}]$.

for $c \leq \underline{c}$. At the other extreme ($c > \bar{c}$), aggregate commute time is strictly higher than in the planning solution.

In the intermediate region, there is a continuum of equilibria. For some c, there exists a pair of tolls that forms a Nash equilibrium and that achieves the aggregate commute time of the planning solution. But there are also equilibria, for other values of c with aggregate commute times even longer than in the solution with no uncongestible road.

Figure 4.5: Commute Time as Speed on Uncongestible Road Varies



If c is in the intermediate third interval, some novel benefits of the uncongestible road appear. In this case, all of the motorists utilize the congestible roads, but nonetheless the total cost on the roads is equal to that on the uncongestible road. Therefore, there is a kink in the condition toll setters face. If they raise their toll, they push drivers onto the uncongestible road; in contrast, if they lower their toll, they pull drivers off of the other congestible roads.

Lower Toll:

$$\theta_i \frac{\partial x_i}{\partial \theta_i} + x_i > 0$$

$$A_i'(x_i)dx_i + d\theta_i = -dA_j'(x_j)dx_i \qquad \frac{\partial x_i}{\partial \theta_i} = \frac{-1}{(A_i'(x_i) + A_j'(x_j))} \tag{8}$$

Raise Toll:

$$\theta_i \frac{\partial x_i}{\partial \theta_i} + x_i < 0$$

$$A_i'(x_i)dx_i + d\theta_i = 0 \qquad \frac{\partial x_i}{\partial \theta_i} = \frac{-1}{A_i'(x_i)} \tag{9}$$

Nash Equilibrium Tolls:

$$\theta_i \in \left[A_i'(x_i)x_i, (A_i'(x_i) + A_j'(x_j))x_i\right] \tag{10}$$

These conditions produce a set of toll-setting equilibria that can be supported when $c \in (\underline{c}, \bar{c})$. It can easily be shown that (i) for any $c \in (\underline{c}, \bar{c})$ there always exists at least one equilibrium; (ii) in any such equilibrium, the uncongestible road is always empty; (iii) in any such equilibrium, the total cost to motorists equals the time cost on the uncongestible road; and (iv) for any given $c \in (\underline{c}, \bar{c})$ there is no equilibrium such that the uncongestible road is ignored by the toll setters. The total commute time in this scenario depends on the specific equilibrium.

To summarize, Knight's contention that government intervention is unnecessary in congestion problems because private toll setters can be relied upon to set access fees on the various congestible resources at the same levels as the government is mistaken. As Figure 4.5 makes clear, his novel contention is correct for $c \leq \underline{c}$ and is surprising given the first welfare theorem. But it is not generally true. In particular, it is false if $c > \underline{c}$.

When "competitive conditions" exist assigning property rights *does* restore efficiency. As we have seen, in this context "competitive conditions" mean either that there exists a sufficiently attractive uncongestible option or that there exist enough competing toll setters (formalized as a replica economy). However, there are many real-world examples of congestion where such "competitive conditions" do not exist.

In these cases, assigning property rights alone will, contrary to Knight's contention, fail to restore efficiency and there is a role for the government. As Pigou originally asserted, the government could restore efficiency by imposing road-specific taxes. Our analysis suggests, however, a *new* type

of government intervention: the government can provide an uncongestible option.

5 Lessons for Government Intervention

Suppose there were several congestible routes connecting Washington, DC, and New York. If competing private toll setters required payment for access to each route, competition among the toll setters would not minimize aggregate time wasted commuting. Suppose, however, that the government provided a slow but uncongestible alternative—a train. A train is uncongestible since it can be any length. If the time between one train and the next were sufficiently short, the average commute time on the uncongestible road would be fast enough that the toll setters would set their tolls at the Pigouvian levels. If trains were scheduled less frequently, however, fewer commuters would use them. Suppose the frequency were diminished just to the point where no commuter used the train: $c = \underline{c}$.

In this case, the toll-setting equilibrium is still efficient. What stops either toll setter from unilaterally raising his toll is his recognition that he would lose too many motorists to the uncongestible road for this to be profitable. Suppose now that the time between trains became longer so that on average motorists would lose Δ more in wages by taking a train. It is our hypothesis that each toll setter would raise his toll by Δ, reasoning that he could safely do so without losing any motorists to the uncongestible alternative provided the other toll setter reacted in the same way. That is, the cost of using the uncongestible road serves as a focal point for each toll setter. If this hypothesis is correct, then as we raise c from $\underline{c}$ toward $\bar{c}$, the toll setters will respond by raising their tolls in a way that maintains aggregate commute time at the efficient level. The government would incur fewer costs since it would be running fewer trains and the motorists would pay more in tolls to get to work at the same time as before.[16] Toll setters would enjoy higher revenues.

[16] If the two roads are equally congestible, even a cost increase of $\bar{c} - \underline{c}$ can induce an efficient subgame perfect equilibrium with each toll setter raising his toll by $\Delta = \bar{c} - \underline{c}$. If the roads are asymmetric, however, there will exist a maximum $c^* < \bar{c}$ beyond which the equilibrium is necessarily inefficient. If the two toll setters raised their tolls by $\bar{c} - c^*$, at least one of the toll setters would have an incentive to steal his rival's customers by cutting his toll. For example, point F on line AF in Figure 4.4 is not an equilibrium because toll setter 1 has an incentive to lower his toll since his best reply passes to the left of point E. By doing so, he expects that the number of motorists he can attract will rise by a larger percentage than the percentage reduction in his toll.

The potential competition from the government train would maintain efficiency. Despite this, no commuter would ride the train.

This analysis demonstrates an attractive alternative to Pigouvian taxes, in which the government provides an uncongestible outside option to discipline the market. With perfect information the Pigouvian tax always achieves efficiency. However, there is disagreement about the practicality of Pigouvian interventions: "Virtually every author points out that we do not know how to calculate the ideal Pigouvian tax or subsidy levels in practice, but because the point is rather obvious rarely is much made of it" (Baumol and Oates, 1971). Providing an uncongestible outside option requires far less information and can produce the efficient allocation even when the outside option is unused. Further, there are circumstances in which the government providing an outside option may be preferable, such as providing citizens with a low-cost health care alternative to the higher priced private system, engineers with a modest employment alternative at a national laboratory, students with a public school alternative to privatized schools, or fishermen with steady alternative employment.

6 Conclusion

Since congestion problems abound, our analysis also applies to other contexts and not merely to highway commuting. To remind readers of this, we close with an application that is seemingly very different.[17] Suppose N medical researchers must be assigned to one of $n < N$ independent research projects, each of which seeks a cure for an different disease. A planner would choose the labor allocation across these projects to maximize expected social surplus.

In reality, this labor allocation is performed by the private market. Private contests to stimulate solutions to such problems are becoming ubiquitous. Suppose each private prize setter offers a monetary prize to the first researcher to make a well-specified discovery in exchange for the property right to the discovery. Assume researchers can enter at most one such contest and each researcher is equally likely to find a cure for a particular disease (although some diseases are more intractable

[17] We elaborate on this application in Appendix D.

than others so simply assigning the same number (N/n) of researchers to every disease is not optimal). Since a researcher's chance of winning a prize in contest i is strictly decreasing in the number of *other* researchers with whom he is competing, this is a congestion problem.

There exists a set of prizes that would induce the planner's labor allocation. But private prize setters would typically not choose these "Pigouvian prizes" and the Nash equilibrium is inefficient. A unilateral increase in one prize would not affect that prize setter's own payoff but it would attract additional researchers and that would strictly benefit the N researchers and would strictly injure each of the $n-1$ other prize setters.[18] Just as toll setters would tend to set their tolls inefficiently high in the absence of government intervention, prize setters would tend to set their tolls inefficiently low. The toll setters would exercise their oligopoly power; the prize setters would exercise their oligopsony power. In such cases, government intervention might improve on the private market allocation.

The government can eliminate or at least reduce the exercise of market power by providing an alternative to the private market. If the government also has a task requiring scientists with similar training and it is of sufficient social value that a planner would also assign some of the N researchers to it, then (by an extension of Knight's finding about the effect of a sufficiently fast, uncongestible road) the Nash equilibrium of prize-setting would be efficient. But even if the social value of the task were so low that the planner would assign no labor to it, the government can still improve on the *laissez-faire* allocation. The potential competition created by the presence of the government alternative (even if the wage is so low that the vacancy is never filled) can raise expected social surplus compared to the *laissez-faire* solution and might even duplicate the planner's labor allocation.

University of Michigan

University of Utah.

[18]These effects would be offsetting only when the probability function for winning is the same for every contest and so it is optimal for every contest to have the identical number of researchers.

Appendix A Global Maximum with "Sufficiently Fast" Uncongestible Road

The assumption that it is never advantageous for the toll setter to drive *every* motorist away from his road or attract every motorist away from the uncongestible road holds for an uncongestible road that is "sufficiently fast." In section 4 the optimization is done in generality for all values of the uncongestible road. The maximum point found assuming a "sufficiently fast" uncongestible road is the global maximum. To show this consider the first order condition in the case where there are no motorists on the uncongestible road. In this case $\partial x_1/\partial\theta_1 = -1/(A_1'(x_1(\theta_1)) + A_2'(x_2(\theta_2))) > -1/A_1'(x_1(\theta_1))$. Therefore given the necessary condition for a maximum $x_1 + \theta_1\partial x_1/\partial\theta_1 = 0$ it is straightforward to show that a global deviation such that the uncongestible road is empty fails the necessary condition for a maximum, specifically $x_1^D + \theta_1^D\partial x_1/\partial\theta_1 > 0$.

$$
\begin{aligned}
x_1^D &> x_1^s \quad \text{Def. deviation} \\
x_1^s &= \theta_1^s/(A_1'(x_1(\theta_1^s)) \quad \text{FOC} \\
\theta_1^s/(A_1'(x_1(\theta_1^s)) &> \theta_1^D/A_1'(x_1(\theta_1)) \quad \text{know } \theta_1^D < \theta_1^s \\
\theta_1^D/A_1'(x_1^s(\theta_1)) &> \theta_1^D/(A_1'(x_1^D(\theta_1)) + A_2'(x_2^D(\theta_2))) \quad \text{know } A_1'(x_1(\theta_1)) + A_2'(x_2(\theta_2) > A_1'(x_1(\theta_1^s) \\
\Rightarrow \quad x_1^D &> \theta_1^D/(A_1'(x_1^D(\theta_1)) + A_2'(x_2^D(\theta_2))) \quad \Rightarrow x_1^D + \theta_1^D\partial x_1/\partial\theta_1 > 0
\end{aligned}
$$

Appendix B Efficiency Restored in the Toll-Setting Game under "Competitive Conditions"

Suppose instead of N motorists spread across n motorways congestible to different extents there are MN motorists spread across n types of congestible roads, with M identical roads of each type. We wish to show that as $M \to \infty$, the efficient allocation of motorists can be supported as a subgame-perfect equilibrium of the toll-setting game.[19]

The efficient solution in this case uniquely solves the following $n+1$ equations defining $\{x_i^s\}_{i=1}^n$ and $\lambda^s : A_i(x_i^s) + x_i^s A_i'(x_i^s) = \lambda^s,\ i = 1, \ldots, n$ and $\sum_{i=1}^n x_i^s = N$, where λ^s denotes the value of the Lagrangean multiplier at the social optimum.

Define the Pigouvian tolls as $\theta_i^P = x_i^s A_i'(x_i^s)$ for $i = 1, \ldots, n$. Suppose at the first stage of the two-stage game every player but one type i chooses the Pigouvian toll while this one remaining type i player chooses some $\theta_i \in [\theta_i^{\min}, \theta_i^{\max}]$. Define these minimum and maximum tolls follows: $\theta_1^{\max} = \mu - A_1(0)$ and $\theta_1^{\min} = \mu - A_1(MN)$ where μ in the first condition is equal to $min\{A_j(0) + \theta_j^P\}$ and in the second equal to the common cost given by $A_j(x_j) + \theta_j^P$. Then in the Nash equilibrium of the second stage, the motorists will allocate themselves so the full cost (the toll plus the lost wages) is the same on every motorway. Denote this common cost as μ. Then the following $n+2$

[19] Seegert (2014) demonstrates the efficiency of this system in the limiting case in the context of a system of cities.

equations define the $n+2$ variables $x_i^d, \{x_j\}_{j=1}^n,$ and μ

$$\begin{aligned} A_i(x_i^d) + \theta_i &= \mu && \text{(B.1)} \\ A_j(x_j) + x_j^s A_j'(x_j^s) &= \mu \text{ for } j = 1, \ldots n && \text{(B.2)} \\ x_i^d + (M-1)x_i + M\sum_{j \neq i} x_j &= N. && \text{(B.3)} \end{aligned}$$

Anticipating these second-stage responses and conjecturing that the other toll setters will maintain their tolls at the Pigouvian level, the deviating type i player can set any toll θ_i in the closed interval. A marginal increase in his toll would then cause his profits to increase at the following rate:

$$\frac{\partial \pi_i}{\partial \theta_i} = \frac{\partial\big(\theta_i x_i^d(\theta_i)\big)}{\partial \theta_i} = x_i^d + \theta_i \frac{\partial x_i}{\partial \theta_i}. \tag{B.4}$$

Let $\theta_i = \epsilon_i + \theta_i^P = \epsilon_i + x_i^s A_i'(x_i^s)$. Differentiating equation (B.1) we obtain:

$$\frac{\partial x_i}{\partial \theta_i} = \left(\frac{\partial \mu}{\partial \theta_i} - 1\right) \frac{1}{A_i'(x_i^d)}. \tag{B.5}$$

Substituting for θ_i and recognizing that as $M \to \infty$, $\frac{\partial \mu}{\partial \theta_i} \to 0$, we conclude that

$$\frac{\partial \pi_i}{\partial \theta_i} = \frac{x_i^d A_i'(x_i^d) - x_i^s A_i'(x_i^s) - \epsilon_i}{A_i'(x_i^d)} \tag{B.6}$$

If $\epsilon = 0$, then the toll under consideration is the Pigouvian toll. By definition, $x_i^d = x_i^s$. So the expression in (B.6) equals zero. If $\epsilon > 0$, then the toll exceeds the Pigouvian level ($\theta_i > \theta_i^P$), less than the efficient number of motorists take the route ($x_i^d < x_i^s$), and the expression in (B.6) is strictly negative. Finally, if $\epsilon < 0$, then the toll is smaller than the Pigouvian level ($\theta_i < \theta_i^P$), more than the efficient number of motorists take the route ($x_i^d > x_i^s$), and the expression in (B.6) is strictly positive.

It follows that for any toll θ_i in the closed interval, the payoff of player i is single-peaked at the Pigouvian toll. Moreover, any toll so high that no one uses the motorway or so low that raising it would not alter the number of motorists using his route is clearly suboptimal for toll setter i. But these arguments apply to *every* toll setter. Hence, in the limit no toll setter has a strict incentive to deviate unilaterally from the profile of Pigouvian tolls even when there is no uncongestible motorway.

Appendix C Linear Example

Suppose the congestion on the roads is given by $A_i(x_i) = a_i x_i + b_i$. The best response functions are piecewise linear consisting of three parts: strategic complements, strategic substitutes, and strategic independence. Figure 4.1 demonstrates the optimal choices for toll setter 1 as the toll on road 2 changes. When the toll on road 2 is low the optimal point is given by the tangency point between the iso-toll revenue curve and the constraint if all motorists prefer two toll roads. In this range the best reply function for toll 1 is a strategic complement to the toll on road 2. When the

toll on road 2 is an intermediate level, such that in Figure 4.1 the flatter constraint is between the two depicted, the optimal toll on road 1 is given by the kink point of the steeper and flatter constraints. In this range the best reply function for toll 1 is a strategic substitute with the toll on road 2. Finally, if the toll on road 2 is large, such that the flatter constraint is above the constraints drawn in Figure 4.1, then the optimal point is given by point B, the tangency between the iso-toll revenue on road 1 and the steeper constraint characterized by all motorists indifferent between toll road 1 and the uncongestible road. In this range the best reply function is strategically independent of the toll on road 2.

Deriving Strategic Complements: Low Values θ_2

$$\max \ \theta_1 x_1$$

$$\text{subject to } \ a_1 x_1 + b_1 + \theta_1 = a_2 x_2 + b_2 + \theta_2 \ \ N = x_1 + x_2$$

First-order condition

$$\frac{-2\theta_1}{a_1 + a_2} + \frac{a_2 N}{a_1 + a_2} + \frac{b_2\theta_2 - b_1}{a_1 + a_2} = 0$$

$$\theta_1^{BR} = \frac{a_2 N + b_2 + \theta_2 - b_1}{2}$$

$$\theta_2 = 2\theta_1^{BR} + b_1 - a_2 N - b_2$$

Deriving Strategic Substitutes: Intermediate Values θ_2

The best reply function for intermediate values of θ_2 is characterized by the point at which motorists are indifferent among being on toll road 1, toll road 2, and the uncongestible road but all motorists are on the two toll roads.

$$\begin{aligned} a_1 x_1 + b_1 + \theta_1 &= a_2 x_2 + b_2 + \theta_2 \\ a_1 x_1 + b_1 + \theta_1 &= c \\ N &= x_1 + x_2 \end{aligned}$$

Combining these three constraints produces:

$$\theta_2 c \frac{a_1 + a_2}{a_1} - b_1 \frac{a_2}{a_1} - \frac{a_2}{a_1}\theta_1 - b_2 - a_2 N.$$

In this range the best reply function for toll road 1 and toll road 2 are characterized by the same three conditions, and thus lie on the same line. The best reply functions of toll road 1 and toll road 2 can overlap or be disjoint in this region as depicted in Figure 4.3.

Deriving Strategic Independence: High Values θ_2

The best reply function in this region is characterized by point B in Figure 4.1. The best reply function can be derived from maximizing toll revenue given the constraint, $a_1 x_1 + b_1 + \theta_1 = c$. This leads to the best reply function,

$$\theta_1^{BR} = \frac{c - b_1}{2}.$$

<u>Reaction Function Road 1</u>
Strategic Substitutes: $\theta_2 = c\frac{a_1+a_2}{a_1} - b_1\frac{a_2}{a_1} - \frac{a_2}{a_1}\theta_1 - b_2 - a_2N$
Strategic Complements: $\theta_2 = 2\theta_1 + b_1 - a_2N - b_2$
Strategic Independence: $\theta_1 = \frac{c-b_1}{2}$

<u>Reaction Function Road 2</u>
Strategic Substitutes: $\theta_2 = c\frac{a_1+a_2}{a_1} - b_1\frac{a_2}{a_1} - \frac{a_2}{a_1}\theta_1 - b_2 - a_2N$
Strategic Complements: $\theta_2 = \frac{a_1N+b_1-b_2+\theta_1}{2}$
Strategic Independence: $\theta_2 = \frac{c-b_2}{2}$.

The strategic substitutes line is the same for both road 1 and road 2, but is sometimes disjoint and sometimes overlapping. The characterization of the best response functions is completed by finding the kink points among the three linear pieces. The kink point between the strategic substitutes and complement regions is found by their intersection. Similarly, the kink point between the strategic substitutes and independence regions is found by their intersection.

<u>Kink Point 1: Strategic Substitutes and Complements</u>
Road 1: $\theta_1 = (c - b_1)\frac{a_1+a_2}{2a_1+a_2}$, $\theta_2 = \frac{2a_1+2a_2}{2a_1+a_2}c - \frac{a_2}{2a_1+a_2}b_1 - a_2N - b_2$
Road 2: $\theta_1 = 2c\frac{a_1+a_2}{a_1+2a_2} - b_1 - b_2\frac{a_1}{a_1+2a_2} - a_1N$, $\theta_2 = (c - b_2)\frac{a_1+a_2}{a_1+2a_2}$

<u>Kink Point 2: Strategic Substitutes and Independence</u>
Road 1 Point: $\theta_1 = \frac{c-b_1}{2}$, $\theta_2 = \frac{2a_1+a_2}{2a_1}c - \frac{a_2}{2a_1}b_1 - b_2 - a_2N$
Road 1 Line: $\theta_2 = \frac{2a_1+a_2}{a_1}\theta_1 + b_1 - b_2 - a_2N$
Road 2 Point: $\theta_1 = \frac{a_1+2a_2}{2a_2}c - b_1 - a_1N - \frac{a_1}{2a_2}b_2$, $\theta_2 = \frac{c-b_2}{2}$
Road 2 Line: $\theta_2 = \frac{a_2}{a_1+2a_2}\theta_1 - \frac{a_2}{a_1+2a_2}b_2 + \frac{a_2}{a_1+2a_2}b_1 + \frac{a_1a_2}{a_1+2a_2}N$

Equilibria are characterized by one of three regimes. The first regime is duopoly, when the best response functions intersect in the region of strategic complements, when c is high. The second is the Knight-Pigou regime, when the best response functions intersect in the region of strategic independence, when c is low. Finally, the phantom road regime exists when the best response functions partially overlap in the strategic substitutes region, when c is an intermediate value. In the first two regimes the Nash equilibrium is unique and in the third there exists a continuum of equilibria.

The lower threshold value, $\underline{c}$, is found by intersecting the lines that characterize the kink point between the strategic substitutes and independence and the strategic independence value for road 1. The higher threshold value, $\bar{c}$, is found by intersecting the kink point 1 conditions. Together these two thresholds in combination with the value of c determine which of the three regimes the system is in.

<u>Intersection Lines Characterizing Kink Point 2</u>
$\theta_1 = (b_2 - b_1)\frac{a_1}{a_1+a_2} + \frac{2a_2^2a_1}{(a_1+a_2)^2}N$
$\theta_1 = \frac{c-b_1}{2}$
$\underline{c} = (b_2 - b_1)\frac{2a_1}{a_1+a_2} + \frac{4a_2^2a_1}{(a_1+a_2)^2}N + b_1$

Intersection Lines Characterizing Strategic Complements

$\theta_2 = 2\theta_1 + b_1 - a_2 N - b_2$

$\theta_2 = \frac{a_1 N + b_1 - b_2 + \theta_1}{2}$

$\bar{c} = \frac{b_1(a_1+2a_2)+b_2(2a_1+a_2)}{3(a_1+a_2)} + \frac{(2a_1+a_2)(a_1+2a_2)}{3(a_1+a_2)} N$

Appendix D The Case of Prize Setters Competing for N Biomedical Researchers

Appendix D.1 Overview

Suppose that there are n projects to develop vaccines. Any of $N > n$ biomedical researchers may make a breakthrough if he works on one of the vaccine projects. A breakthrough on vaccine project i has social value V_i. If x_i biomedical researchers work on problem i, there is probability $P_i(x_i)$ that the problem will be solved, where $P_i(\cdot)$ is twice differentiable, strictly increasing, strictly concave, exogenous function bounded above by 1. Assume a biomedical researcher works on at most one problem. If no biomedical researcher works on problem i, it will not be solved: $P_i(0) = 0$. Assume that a planner can assign biomedical researchers to any of these n projects or to the National Institutes of Health (NIH). At NIH, the social payoff per biomedical researcher is only c. For simplicity, assume that the Inada condition holds $P_i'(0) = \infty$. This is sufficient to ensure that having *some* biomedical researchers on every project i is socially optimal whether or not some researchers work at NIH. As more biomedical researchers are assigned to any given project, diminishing returns sets in. If the expected marginal social value of adding another researcher to any project is c and fewer than N researchers have been assigned, the remainder are assigned to NIH. If more than N researchers are required to drive the expected marginal social value down to c, then it is optimal to assign no one to NIH.[20]

We refer to the allocation of biomedical researchers to the n problems that *maximizes* the expected social payoff, $\sum_{i=1}^{n} V_i P_i(x_i) + c(N - \sum_{i=1}^{n} x_i)$ subject to $N - \sum_{i=1}^{n} x_i \geq 0$, as the socially optimal solution (denoted x_i^s). The first-order conditions for this Kuhn-Tucker problem are:

$$x_i \geq 0, V_i P_i(x_i) - (c + \lambda) \leq 0, \text{ with complementary slackness} \tag{D.1}$$

$$\lambda \geq 0, N - \sum_{i=1}^{n} x_i \geq 0, \text{ with complementary slackness.} \tag{D.2}$$

Given condition (D.1), the Inada assumption ensures that $x_i > 0$. Hence, there are two cases. In the first, some researchers work as NIH and since the constraint does not bind, condition (D.2) implies that $\lambda = 0$. Therefore, $V_i P_i(x_i) = c$. In the second, no researchers work at NIH and since the constraint binds, the multiplier is weakly positive. Therefore, $V_i P_i(x_i) = c + \lambda \geq c$ for $i = 1, \ldots n$. Note that in either case, it is socially optimal for the expected marginal social benefit to be equal across projects. Otherwise, a marginal expected social gain could be achieved by moving a researcher from a project with a strictly lower marginal expected social benefit to one with a strictly higher marginal expected social benefit.The maximand in the planning problem can also

[20]For additional background see Kremer and Glennerster (2004), Salant (2007), and Brennan, Macauley and Whitefoot (2011).

be written as maximizing the social surplus on each project,

$$\sum_{i=1}^{n} x_i \left(\frac{P_i(x_i)}{x_i} V_i - c \right), \tag{D.3}$$

relative to assigning the researcher to work at NIH.

Now suppose there are n independent *prize setters*, one for each technological problem, that sets a prize W_i to the first biomedical researcher to solve problem i. Each prize setter anticipates being able to extract the full social surplus from any discovery but must pay the researcher who made the breakthrough in project i, W_i. Hence, the expected payoff of a private prize setter is $P_i(x_i)(V_i - W_i)$.[21] In the first stage, prize setter i sets the reward W_i and in the second stage biomedical researchers decide which project to work on. Assume that each biomedical researcher working on problem i attempts to solve it by himself and is as likely to solve it as anyone else working on that problem. Hence, his probability of winning the prize is $P_i(x_i)/x_i$. We denote this function as $A_i(x_i)$. Given our assumptions about $P_i(\cdot)$, each $A_i(\cdot)$ is differentiable and strictly decreasing, with $x_i A_i(x_i)$ strictly concave for $i = 1, \ldots, n$.

In the second stage equilibrium, researchers allocate themselves so that the expected payoff is the same across all projects being worked on. There are two possible conditions, one in which some biomedical researchers work for NIH,

$$W_i A_i(x_i) = c, \tag{D.4}$$

and one in which no biomedical researchers work for NIH,

$$W_i A_i(x_i) = W_j A_j(x_j). \tag{D.5}$$

The key difference with these constraints is the responsiveness of biomedical researchers to changes in a given prize. For the first constraint, when some biomedical researchers work for NIH the responsiveness of biomedical researchers to an increase in W_i is increasing but depends only on the total number of researchers working on that one project:

$$\frac{dx_i}{dW_i} = \frac{-A_i(x_i)}{W_i A_i'(x_i)} > 0. \tag{D.6}$$

This follow because, as the prize setter increases his prize, it attracts biomedical researchers away from NIH until the additional congestion on that project offsets the increased prize level; the number of researchers at the other $n-1$ projects does not change.

In contrast, the responsiveness of biomedical researchers when no biomedical researchers work for NIH depends not only on the congestion in the prize setter's project but also on the congestion at the other projects,

$$\frac{dx_i}{dW_i} = \frac{-A_i(x_i)}{W_i A_i'(x_i) + W_j A_j'(x_j)} > 0. \tag{D.7}$$

As prize setter i increases his prize, he attracts biomedical researchers from the other projects,

[21] The assumption of full surplus extraction by the prize setter is not essential. Any increasing monotonic transformation of his objective function would leave his best choice unaffected. For example, instead of payoff $J_i(W_i)$ he could receive $(1-\tau_i)J_i(W_i) - k_i$ where $\tau_i \in (0,1)$ is a tax imposed on the net proceeds of the prize setter and k_i is a payment he must make even if his contest has no winner.

reducing their congestion and making them more attractive. Consequently, when no researchers work at NIH, a unit increase in the prize at project i attracts fewer researchers than when they are drawn from the NIH workforce.

Appendix D.2 Comparing the Efficient and Market Allocations of Researchers

Just as the government can set a highway-specific Pigouvian toll on each congestible road in such a way that motorists will allocate themselves to solve the planning problem, so too the government can set project-specific prizes so that researchers will allocate themselves efficiently. That is, the set of socially efficient prizes is defined as the set that in the second stage induces researchers to move across projects subject to either equation (D.4) or (D.5) such that the first-order conditions for the social optimum $V_i P_i'(x_i^s) = c$ or $V_i P_i'(x_i^s) = V_j P_j'(x_j^s)$ hold, respectively. The set of socially efficient prizes, in both cases, is then given by $W_i^s = P_i'(x_i^s)V_i/A_i(x_i^s)$.

We ask whether a self-interested prize setter would have any incentive to unilaterally deviate from the government's solution of offering the socially optimal prize W_i^s in contest i and attracting x_i^s researchers as a result. Is the government needed as Pigou initially contended in the case of roads or, as Knight contended, is the government unnecessary in a world where self-interested independent agents set the prizes?

Consider first the case where some biomedical researchers work for NIH. In this case, the second stage constraint is given by equation (D.4) and the responsiveness of researchers to changes in the prize is given by equation (D.6).

Prize setter i earns profit $\pi_i = (V_i - W_i)P_i(x_i(W_i))$ if he chooses W_i. Hence,

$$\begin{aligned}
\partial\pi_i/\partial W_i &= -P_i(x_i) + (V_i - W_i)P_i'(x_i)\partial x_i/\partial W_i && \text{(D.8)}\\
&= \frac{1}{W_i A_i'(x_i)}\{-P_i(x_i)W_i A_i'(x_i) - A_i(x_i)V_i P_i'(x_i) + W_i A_i(x_i)P_i'(x_i)\} && \text{(D.9)}\\
&= \frac{1}{W_i A_i'(x_i)}\{-x_i A_i(x_i)W_i A_i'(x_i) - A_i(x_i)V_i P_i'(x_i)\\
&\quad + W_i A_i(x_i)[A_i(x_i) + x_i A_i'(x_i)]\} && \text{(D.10)}\\
&= \frac{A_i(x_i)}{W_i A_i'(x_i)}\{-x_i W_i A_i'(x_i) - V_i P_i'(x_i) + W_i A_i(x_i) + W_i x_i A_i'(x_i)\}, && \text{(D.11)}
\end{aligned}$$

where (D.8) is obtained by differentiating to determine the expected private gain from marginally increasing the prize, (D.9) is obtained by substituting in the researcher response in equation (D.6), (D.10) is obtained by taking account of the fact that $P_i(x) = x_i A_i(x)$, and (D.11) is obtained by factoring out $A_i(x_i)$. At $x_i^s, W_i^s = \frac{P_i'(x_i^s)V_i}{A_i(x_i^s)}$, this marginal gain from deviating from the Pigouvian prize is:

$$\frac{A_i(x_i^s)}{A_i'(x_i^s)}\left\{-V_i P_i'(x_i^s) + A_i(x_i^s)\frac{P_i'(x_i^s)V_i}{A_i(x_i^s)}\right\} = 0. \qquad \text{(D.12)}$$

Hence, whenever it is socially optimal for some researchers to work at NIH, no prize setter would have any private incentive to deviate unilaterally from the socially optimal prize in his contest provided the other prize setters adopted the socially optimal prizes in their respective contests. Hence, the profile of socially efficient prizes forms a Nash equilibrium in the first stage of the two-stage game whenever it is socially efficient for some researchers to work at NIH. This is the counterpart in contests to Knight's conclusion about congestible roads.

To understand this result, start with the profit-maximizing prize setter's objective function $P_i(x_i)(V_i - W_i)$ and replace W_i with $c/A_i(x_i)$ from the second stage constraint, given by equation (D.4). With some rearranging the profit-maximizing prize setter's objective function can be written as $x_i(A_i(x_i)V_i - c) = V_iP_i(x_i) - cx_i$, which is the social planner's maximand in equation (D.3) and when maximized by each independent prize setter yields the socially efficient set of prizes.

Now consider the case where *no* biomedical researcher works for NIH. In this case, the second stage constraint is given by equation (D.5) and the responsiveness of researchers to changes in the prize is given by equation (D.7). Researchers are less responsive to an increase in the prize since as they are attracted away from the other projects, these contests become less congested. To shorten notation, denote by Z the fraction $\frac{1}{W_iA_i'(x_i)+W_jA_j'(x_j)}$. Then,

$$\begin{aligned}
\partial\pi_i/\partial W_i &= -P_i(x_i) + (V_i - W_i)P_i'(x_i)\partial x_i/\partial W_i && \text{(D.13)}\\
&= Z\{-P_i(x_i)(W_iA_i'(x_i) + W_jA_j'(x_j)) - A_i(x_i)P_i'(x_i)(V_i - W_i)\} && \text{(D.14)}\\
&= A_i(x_i)Z\{-x_iW_iA_i'(x_i) - x_iA_j'(x_j)W_j - V_iP_i'(x_i) + P_i'(x_i)W_i\} && \text{(D.15)}\\
&= A_i(x_i)Z\{-W_i[x_iA_i'(x_i) - x_iA_i'(x_i) - A_i(x_i)] - x_iA_j'(x_j)W_j - V_iP_i'(x_i)\} && \text{(D.16)}\\
&= A_i(x_i)Z\{W_iA_i(x_i) - x_iA_j'(x_j)W_j - V_iP_i'(x_i)\}, && \text{(D.17)}
\end{aligned}$$

where (D.13) is obtained by differentiating to determine the expected private gain from marginally increasing the prize, (D.14) is obtained by substituting in the researcher response in equation (D.7), and (D.15) and (D.16) are obtained by taking account of the fact that $P_i(x) = x_iA_i(x)$.

Evaluated at $x_i^s, W_i^s = \frac{P_i'(x^s)_i)V_i}{A_i(x_i^s)}$, this expected marginal gain from deviating from the Pigouvian prize is:

$$A_i(x_i^s)Z\left\{\frac{-x_i^sA_j'(x_j^s)P_j'(x_j^s)V_j}{A_j(x_j^s)} - V_iP_i'(x_i^s) + \frac{A_i(x_i^s)P_i'(x_i^s)V_i}{A_i(x_i^s)}\right\} < 0, \qquad \text{(D.18)}$$

since the first factor is negative and the second factor (in braces) is strictly positive.

Hence, along the constraint where no researcher would work at NIH in the socially efficient solution, every private prize setter would have an incentive to reduce unilaterally the socially optimal prize in his contest if he conjectured that the other prize setters had adopted the socially optimal prizes in their respective contests. Hence, the profile of socially efficient prizes cannot form a Nash equilibrium in the first stage of the two-stage game in this case. Private prize-setting will be inefficient.

We have found one case in which independent profit-maximizing prize setters achieve the socially efficient allocation of biomedical researchers and another case in which they do not, depending on whether or not biomedical researchers worked at NIH.

So far, this element has been taken as exogenous. In what follows, however, we derive the best response of each prize setter as a function of the exogenous variable c. As we will see, there are not only the two regions we have discussed but an intermediate region where efficient equilibria are possible even though no researchers work at NIH. For simplicity we limit ourselves to the case where biomedical researchers can work at one of two projects or at NIH.

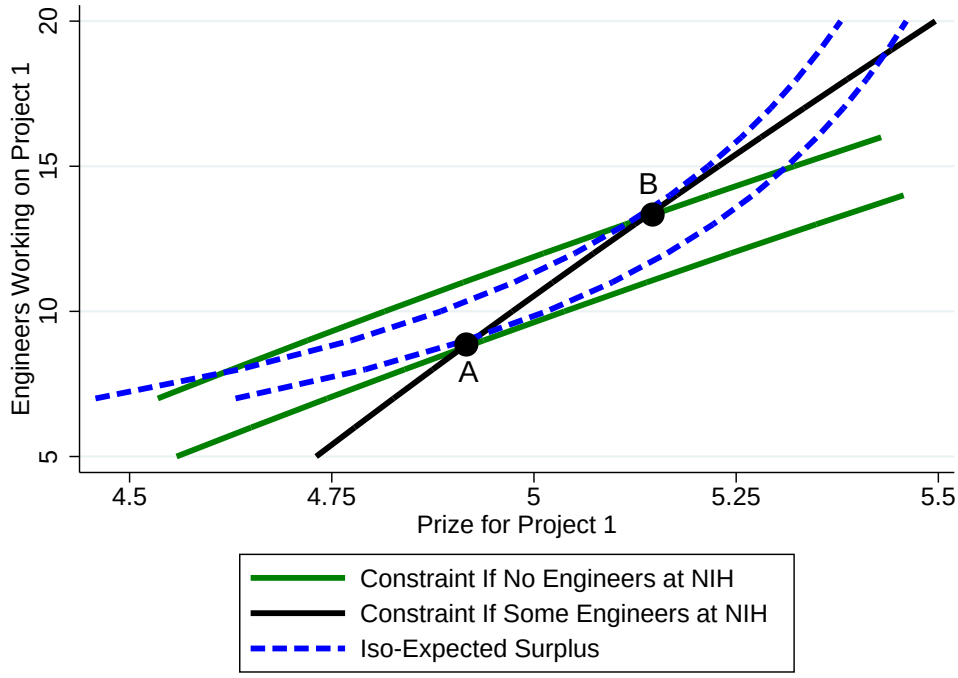
Appendix D.3 Endogenous Determination of Whether Researchers Work at NIH

First, consider that the prize for project 1, given a conjectured prize for project 2, induces biomedical researchers to work at both projects and NIH. If the prize setter for project 1 marginally increases the prize biomedical researchers will switch to working on project 1 until the expected benefit of working on project 1 decreases to equal the benefit of working at NIH. All of the additional biomedical researchers working on project 1 previously worked at NIH, since the conjectured prize for project 2 remained unchanged, and thus the probability the project is solved must also remain unchanged for the expected benefit to remain equal to the benefit of working at NIH. The case described here is exactly the case governed by equations (D.4) and (D.6) discussed above.

Eventually, however, as the prize for project 1 increases there will be no biomedical researchers left working at NIH. At that point, raising the prize on project 1 must cause researchers to switch from project 2. On the interior of this region the two projects have a common expected benefit for the biomedical researchers, which is strictly higher than the benefit of working at NIH. This region is governed by equations (D.5) and (D.7) where researchers are less responsive to changes in the prize on project 1 than they were in the region governed by equations (D.4) and (D.6). Therefore the constraint is kinked at the boundary between these two regions.

Figure D.1 graphs the two constraints from the second stage and the indifference curve from the first stage. The farther southeast of the two constraints, when no biomedical researchers work at NIH, corresponds to a larger prize for project 2. The constraint when some biomedical researchers work at NIH is steeper than the constraints if no biomedical researchers work at NIH because the biomedical researchers are more responsive if the change in researchers to project 1 is due to changes in researchers at NIH where the congestion on the other projects is unaffected, equations (D.6) and (D.7).

Figure D.1: Two Projects, NIH Constraints, and an Indifference Curve



In the first stage the prize setter maximizes his expected payoff, $P_i(x_i)(V_i - W_i)$. With some probability his project is solved, in which case he receives the social benefit V_i and must pay the biomedical researcher that solved the problem the prize W_i. Prize setters balance the benefits of increasing the probability the problem is solved with the cost of increasing the prize. The slope of the indifference curve for the prize setter is given by differentiating the prize setter's objective function $U = P_i(x_i)(V_i - W_i)$, which is

$$\frac{dx_i}{dW_i} = \frac{P_i(x_i)}{P_i'(x_i)(V_i - W_i)} > 0. \tag{D.19}$$

Since points to the northeast have both higher x_i and higher W_i, the numerator is larger, the denominator is smaller, and hence the quotient is larger. Each indifference curve is therefore positively sloped and strictly convex.

Point A in Figure D.1 represents the point at which the constraints intersect and the indifference curve is tangent to the constraint when all researchers work on congestible projects. Point B represents a different point where the constraints intersect (a lower prize for project 2) and the indifference curve is tangent to the constraint when some researchers work at NIH. Three cases exist: one with a low prize for project 2 where the constraints intersect northeast of point B, one with an intermediate prize for project 2 where the constraints intersect between points A and B, and one with a high prize for project 2 where the constraints intersect southwest of point A.

As the prize for project 2 decreases in the first case, where the prize for project 2 is low, the tangency of the indifference curve remains at point B. The best prize for the profit-maximizing

prize setter on project 1 to set remains constant for all prize levels for project 2 lower than the prize level with the constraint that intersects point B. In this region the best response function for the profit-maximizing prize setter on project 1 is strategically independent of prize 2.

In the second case, where the prize for project 2 is an intermediate level, the indifference curve intersects the constraint at the kink between the constraints. As the prize for project 2 decreases the kink point moves northeast implying the best reply for the profit-maximizing prize setter on project 1 is to increase his prize. In this region the profit-maximizing prize setter on project 1's prize is a strategic-substitute of the prize for project 2.

Finally, in the third case, where the prize for project 2 is high such that no researchers work for NIH, the indifference curve is tangent to the flatter constraint given by equation (D.5). In this region the prizes can be strategic substitutes or strategic complements.

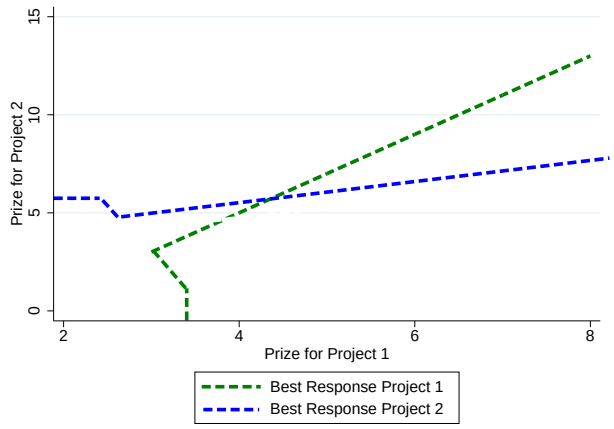
Appendix D.4 Best Response Functions

This section graphs the best response functions of the prize setters. The previous subsection demonstrated that the best response functions exhibit regions of strategic independence, strategic substitutes, and can exhibit regions of strategic complements. Figure D.2 graphs six cases for the best response functions. The panels on the left graph the best response functions when there is a region that exhibits strategic complements, while the panels on the right graph the best response functions when the best response functions exhibit only strategic-independent and strategic-substitutes regions. The panels on the top, middle, and bottom graph the best response functions when the wage at NIH is low, intermediate, and high, respectively.

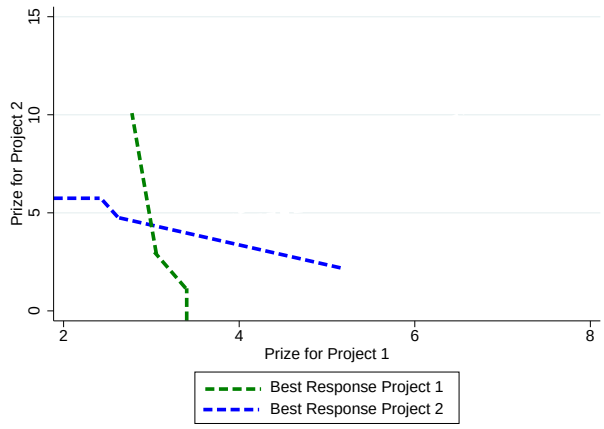
When the wage at NIH is low, depicted in the top two panels of Figure D.2, the best response functions intersect in the region of strategic complements (left graph) or strategic substitutes (right graph). In both cases all researchers work on projects 1 and 2 and no researchers work at NIH. When the wage at NIH is high, depicted in the bottom two panels of Figure D.2, the best response functions intersect in the region of strategic independence. In this case researchers work at both projects and at NIH.

When the wage at NIH is an intermediate level, depicted in the middle two panels of Figure D.2, the best response functions overlap in a region characterized by strategic substitutes. In this case all of the researchers work on projects 1 and 2 but the wage at NIH disciplines the market because if either prize setter unilaterally lowered his prize some researchers would switch and work for NIH. In this case there is a continuum of equilibria, which may or may not include prizes that induce the efficient allocation of researchers across projects.

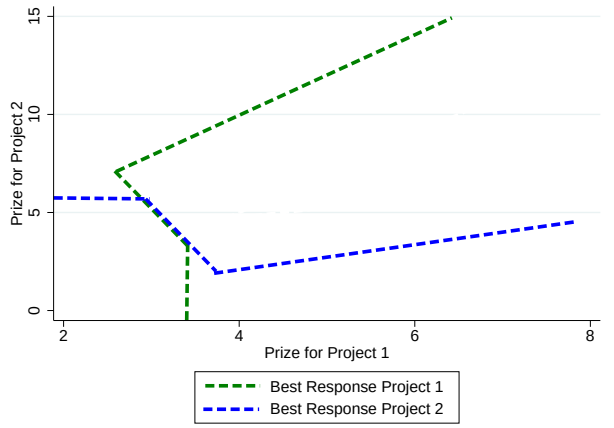
Figure D.2: Best Response Functions



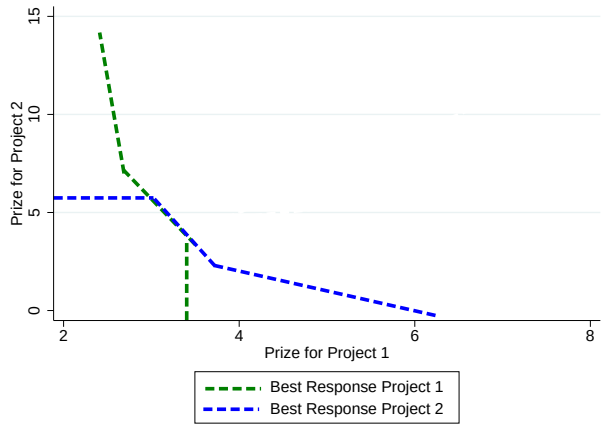
Low Wage NIH: Strategic Complements



Low Wage NIH: Strategic Substitutes



Intermediate Wage NIH: Strategic Complements



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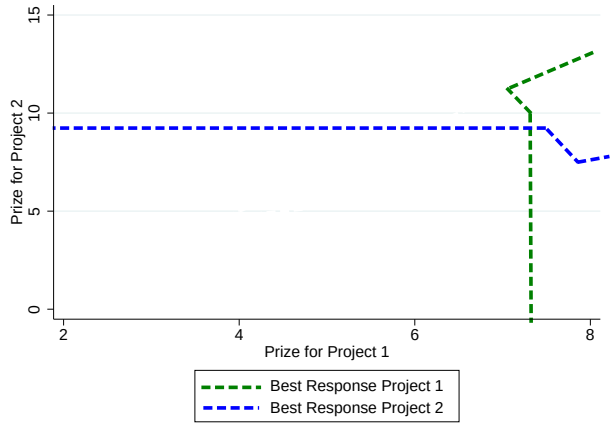
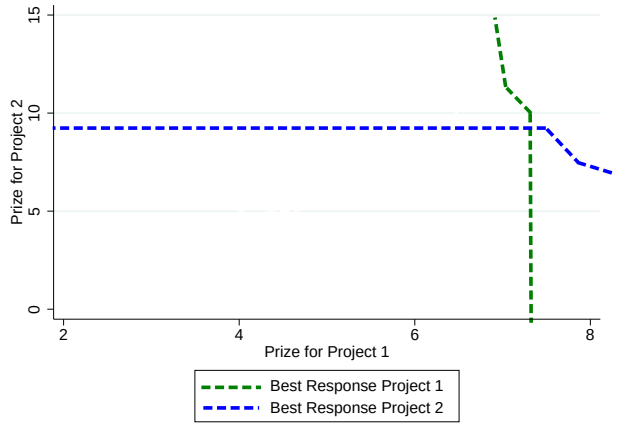


High Wage NIH: Strategic Complements



High Wage NIH: Strategic Substitutes

Appendix E Comparative Statics

This section discusses the impact of (1) heterogeneity in congestion across roads, (2) the number of motorists, and (3) the number of roads on the ability of the government to use the proposed intervention of providing an uncongestible alternative. The discussion on heterogeneity demonstrates under what circumstances government intervention is most useful—markets characterized by similar or dissimilar competitors. The analysis discussing the impact of changes in the number of motorists is important in both the short and long run as the number of motorists is likely to differ within a given day, during rush hour, and across years. The analysis discussing the number of roads is important in thinking about the effectiveness of the government's uncongestible option in different industries and dynamic industries where the number of firms is likely to change.

Appendix E.1 Impact of Heterogeneity across Roads

The efficient toll depends on the congestion on the road and therefore may differ as the congestion differs. As the congestion on a road increases, $x_i A_i'(x_i)$, the efficient toll increases causing the efficient allocation of motorists to have fewer motorists on road i and weakly more motorists on road j, because the allocation of motorists depends on the tolls. The allocation of motorists is not uniquely determined by a pair of tolls, in the strategic-complements case, but by the difference in tolls. Therefore, to consider the effect of heterogeneity on the allocation of motorists, we consider the difference in equilibrium tolls in the strategic-complements case without an uncongestible road.

$$\begin{aligned} \theta_1^* &= x_1 A_1'(x_1) + x_1 A_2'(x_2) \\ \theta_2^* &= x_2 A_2'(x_2) + x_2 A_1'(x_1) \\ \theta_1^* - \theta_2^* &= \theta_1^P - \theta_2^P + \underbrace{x_1 A_2'(x_2) - x_2 A_1'(x_1)}_{\text{Wedge}} \end{aligned} \tag{E.1}$$

When the difference in tolls is equal to the difference in "Pigouvian" tolls the motorist allocation is efficient, even if the tolls do not equal the "Pigouvian" tolls. Equation (E.1) demonstrates that the equilibrium difference in tolls, in the strategic-complements case without an uncongestible road, is equal to the difference in "Pigouvian" tolls plus a wedge term. Consider the case where both congestible roads are identical. In this case the wedge is zero and the allocation of motorists is efficient. This occurs because even though the equilibrium tolls are higher than the "Pigouvian" tolls the increase in toll is the same for both roads.

As the roads become more heterogeneous the wedge term increases in magnitude causing the allocation of motorists to become farther from the efficient allocation. This suggests the benefits from government intervention are highest in cases where congestion is heterogeneous. This analysis is consistent with the literature on dynamic city growth which demonstrates heterogeneity across cities has important implications for the effects of tax and zoning laws on the allocation of population across cities (Seegert (2014)).

Appendix E.2 Impact on Changing the Number of Motorists: Rush Hour

Consider the impact on the equilibria found in section 4 as the number of motorists increases but everything else is held constant—importantly the number of roads and the speed of the uncon-

gestible road c. Equilibria exist in three regions defined by the threshold levels $\underline{c}$ and $\bar{c}$, strategic independence ($c < \underline{c}$), substitutes ($c \in [\underline{c}, \bar{c}]$), and complements ($c > \bar{c}$). Increasing the number of motorists changes these threshold levels and therefore the set of equilibria. The threshold levels are defined as the level of c equal to the total cost to motorists on a congestible road where the equilibrium toll is set independently or strategically where tolls are strategic complements.

$$\begin{aligned} \underline{c} &= A_i(x_i) + \theta_i \quad \text{when } \theta_i = A_i'(x_i)x_i \\ \bar{c} &= A_i(x_i) + \theta_i \quad \text{when } \theta_i = (A_i'(x_i) + A_j'(x_j))x_i \end{aligned} \tag{E.2}$$

The effect of increasing the number of motorists on these threshold levels is determined by totally differentiating the conditions and rearranging.

$$\frac{d\underline{c}}{dN} = (2A_i'(x_i) + A_i''(x_i)x_i)\frac{\partial x_i}{\partial N} > 0$$

$$\frac{d\bar{c}}{dN} = (2A_i'(x_i) + A_j'(x_j) + A_i''(x_i)x_i)\frac{\partial x_i}{\partial N} + A_j''(x_j)x_i\frac{\partial x_j}{\partial N} > 0$$

The threshold levels both increase as the number of motorists increases. This implies that for a given speed of the uncongestible road, as the number of motorists increases the equilibria shift from strategic complements to strategic substitutes to strategic independence. If the increase in motorists is enough to shift the set of equilibria between these regions then it is possible for a government intervention that did not discipline the market at all with fewer motorists to discipline the market to the efficient allocation with more motorists. Even if the change is not large enough to discipline the market to the efficient allocation it may still increase efficiency.

If the change in motorists is not large enough to change the set of equilibria then efficiency may increase in the intermediate region if the uncongestible road disciplines the markets to a different set of equilibria that may be more efficient. The tolls will remain efficient in the strategic-independence region and remain inefficient in the strategic-complements regions. Within the context of rush hour, where there is a spike in the number of motorists on the roads, this analysis suggests the government intervention may become more effective, and will not become less effective, at disciplining the market during rush hour especially if the change is large.

Appendix E.3 Impact on Changing the Number of Roads

Now consider the impact of the equilibria when the number of roads increases by replicating the system of roads, but not the number of motorists, M times. The effect of replicating the number of roads on the threshold levels is given by totally differentiating the constraints in equation (E.2).

$$\frac{d\underline{c}}{dM} = (2A_i'(x_i) + A_i''(x_i)x_i)\frac{\partial x_i}{\partial M} < 0$$

$$\frac{d\bar{c}}{dM} = (2A_i'(x_i) + A_j'(x_j) + A_i''(x_i)x_i)\frac{\partial x_i}{\partial M} + A_j''(x_j)x_i\frac{\partial x_j}{\partial M} < 0$$

The threshold levels decrease as the number of roads increases. Intuitively, as the number of roads increases the time cost of commuting on any given road decreases as motorists are spread across more roads. Therefore, the threshold time cost for some motorists to be on the uncongestible road, $\underline{c}$, must be lower than before. As the number of roads increases, and the speed on the uncongestible road stays the same, eventually the uncongestible road that was being used, and disciplining the market, no longer is fast enough to be used in equilibrium. As the number of roads increases, the uncongestible road becomes less effective at disciplining the market.

As the number of roads increases, the government intervention becomes less effective but the benefit from disciplining the market also decreases. As demonstrated previously, as the number of replications increases to infinity the inefficiency caused by the market decreases to zero. However, if we consider the case in which the number of competitors decreases, for instance if one road is unused because of construction, then the government intervention becomes more effective. If market structure changes to consolidate the number of competitors in the market the government intervention automatically disciplines the market more, counteracting some of the possible negative impact of market power.

While these seem like two separate concerns it turns out that both can be summarized by the level of congestion in the market. Congestion in the market can be thought of as the level of negative externality motorists impose on other motorists, $x_iA_i'(x_i)$. As the number of motorists increases, either during rush hour or across many years, the congestion in the market increases. Conversely, as the number of competing roads increases, holding the number of motorists fixed, the congestion decreases.

These comparative statistics reinforce the usefulness of this new government intervention. First, demonstrated through the example of an increase in motorists, we see providing an uncongestible alternative disciplines the market more when the number of motorists increases and congestion is high. Second, as industries evolve, changing the number of competitors, roads in this example, we see the government intervention disciplines the market most when the level of competition is low, demonstrated through the example of an increase in roads. In both scenarios the government intervention's effectiveness at disciplining the market increases precisely when doing so is most beneficial—when congestion is high and competition is low. Third, the ability of this government intervention to adapt to these changing scenarios is automatic, requiring no changes in law or additional information on how congestion has changed.

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