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Makoto Hasegawa and Stephen Salant

1616 P St. NW
Washington, DC 20036
202-328-5000 www.rff.org

RESOURCES FOR THE FUTURE

Cap-and-Trade Programs under Continual Compliance*

Makoto Hasegawa† Stephen Salant‡

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Abstract

Price collars have frequently been advocated to restrict the price of emissions permits. Consequently, collars were incorporated in the three bills languishing in Congress as well as in California's AB-32; Europeans are now considering price collars for EU ETS. In advocating collars, most analysts have assumed (1) collars will be implemented by government purchases and sales from bufferstocks, just like bands on foreign exchange rates or commodity prices; and (2) firms must surrender permits whenever they pollute. In fact, however, no actual emissions trading scheme has conformed to these assumptions. In the current paper, we maintain the second assumption (continual compliance) and show that while a price collar supported by a sufficiently large bufferstock can restrict permit prices, a price collar supported instead by auctions with reserve prices cannot. In a companion paper (Hasegawa and Salant, 2012), we show that *neither* method works once account is taken of delayed compliance.

Keywords: emissions trading, marketable permits, price collar, safety valve, price ceiling, price floor

JEL Classification Numbers: Q54, Q58

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†Department of Economics, University of Michigan. E-mail: makotoh@umich.edu.

‡Department of Economics, University of Michigan, and Resources for the Future. E-mail: ssalant@umich.edu.

1 Introduction

Price ceilings, price floors, or "price collars" (a combination of the two) have frequently been advocated to restrict the price of emissions permits. Collars were incorporated in the three bills languishing in Congress as well as in California's AB-32; Europeans are now considering price collars for EU ETS.[1] In advocating collars, virtually all analysts have assumed (1) collars will be implemented by government purchases and sales from bufferstocks, just like bands on foreign exchange rates or commodity prices; and (2) firms must surrender permits continually as they pollute rather than after a delay.

There is an extensive literature that supports price collars or one-sided "safety valves" (either a price ceiling or a price floor) on efficiency grounds. Roberts and Spence (1976) demonstrated the expected welfare advantages of a price collar in a one-period model of competitive emissions trading when the regulator is uncertain about abatement costs. Since their model was static, the distinction between delayed and continual compliance did not arise; moreover, they proposed collars implemented by means of financial penalties and rewards rather than by bufferstock transactions or reserve price auctions. Jacoby and Ellerman (2004) summarize the origins of the safety-valve concept and suggest its possible application in future cap-and-trade programs to limit carbon emissions. Pizer (2002) finds that introducing a one-sided safety valve into a pure quantity control mechanism leads to significant welfare gains. More recently, Burtraw et al. (2010) and Fell and Morgenstern (2010) examine welfare consequences of introducing two-sided price collars. Burtraw et al. (2010) study a price collar for a cap-and-trade program by simulating a static model. They find that a price collar outperforms a one-sided "safety valve." Fell and Morgenstern (2010) simulate a stochastic dynamic model of a cap-and-trade program and compare several policies to reduce emissions: quantity policies with banking and borrowing, a price policy (an emissions tax), and hybrid policies (safety valve and price collar). However, both Burtraw et al. (2010) and Fell and Morgenstern (2010) assume that the price collar is implemented by a bufferstock policy and that this price collar can always be maintained. They also assume that

[1]For a recent discussion of this issue, see Robert Stavins's blogpost (www.robertstavinsblog.org) "Low Prices a Problem?" (April 25, 2012). Stavins regards the absence of a safety valve or price collar in the European system as a "design flaw." For a discussion of how a price floor could be introduced into the European system, see David Hone's blogpost (http://blogs.shell.com/climatechange/2012/06/auction/) "The Case for an Auction Reserve Price" (June 7, 2012) and the references therein. Hone notes that "it is too late for auctions to be held periodically throughout the commitment period for Phase III or the EU ETS (2013-2020) but it could be introduced as part of the expected legislative process to set the parameters for Phase IV (2021 and beyond, probably extending to 2030)." However, most of the literature on which Hones bases his advocacy (1) assumes price collars supported by bufferstock policies rather than by auctions with reserve prices and (2) assumes continual compliance rather than delayed compliance. One exception is Grubb. Grubb (2009, 2012) proposes setting a reserve price (floor price) on future EU ETS auctions to mitigate downside risks associated with low-carbon investment and to stabilize auction revenues.

firms surrender permits as soon as they emit ("continual compliance" instead of "delayed compliance").

In fact, however, the emissions trading schemes that emerged after much political debate and compromise do not conform to the assumptions of these theoretical analyses. While the three federal bills and California AB-32 propose a price collar, the Waxman-Markey[2] and Kerry-Boxer[3] bills do not propose that it be implemented by the government purchasing anything offered at the floor price and selling at the ceiling price anything demanded up to the limit of its bufferstock (Salant, 1983; Miranda and Helmberger, 1988). Instead, the two bills envision implementing the collar by means of reserve prices on bids at government auctions of emissions permits.[4]

For example, both the Waxman-Markey and Kerry-Boxer bills propose that the government hold quarterly auctions with minimum reserve prices and distinguish two types of auctions to implement the floor and ceiling, respectively. The minimum reserve price is expected to serve as a floor price (\$10 per ton of CO_2 equivalent in 2012).[5] In addition, they also propose another type of auction (a "strategic reserve auction" in the Waxman-Markey bill and a "market stability reserve auction" in the Kerry-Boxer bill) to implement a ceiling. A fraction (1-3%) of permits is to be placed each year in a government stockpile (the "strategic reserve account" in the Waxman-Markey bill and the "market stability reserve account" in the Kerry-Boxer bill) and auctioned quarterly with a minimum reserve price that is higher than the floor price (\$28 per ton of CO_2 equivalent in 2012).[6] The Kerry-Lieberman bill[7] proposes a price floor implemented by reserve auctions in the same manner as the Waxman-Markey and Kerry-Boxer bills.[8] California AB-32 also proposes quarterly reserve price auctions.

Sales from bufferstocks at fixed prices are contemplated in only two circumstances. California AB-32 envisions sales of specified numbers of permits at specified prices from an "Allowance Price Containment Reserve" shortly after each quarterly auction. In addition,

[2] The American Clean Energy and Security Act of 2009.

[3] The Clean Energy Jobs and American Power Act of 2009.

[4] Dallas Burtraw has pointed out that in principle there are really four possible policies rather than the two that we analyze, since a price ceiling supported in one way (either by a bufferstock transaction or by a reserve price auction) can be paired with a price floor supported in the other way.

[5] After 2012, the minimum reserve price increases by 5% plus the rate of inflation each year in both bills.

[6] The Waxman-Markey bill provides that the minimum reserve price for a ceiling increases by 5% plus the rate of inflation each year in 2013 and 2014, and that in 2015 and thereafter it be 60 percent above a rolling 36-month average of the daily closing price. On the other hand, the Kerry-Boxer bill provides that the minimum reserve price for a ceiling increases by 5% plus the rate of inflation each year in 2013 through 2017, and then increases by 7% plus the rate of inflation in 2018 and each year thereafter.

[7] The American Power Act of 2010.

[8] The reserve price starting from \$12 in 2013 increases by 3% plus the rate of inflation in 2014 and each year thereafter.

the Kerry-Lieberman bill proposes a price ceiling defended by direct sales of permits from a government reserve, the "cost containment reserve," at a ceiling price (\$25 per ton of CO_2 equivalent in 2013) for the 90-day period ending on the date on which covered entities are required to demonstrate compliance.[9]

We assume continual compliance throughout this paper and delayed compliance in a companion paper (Hasegawa and Salant, 2012). By comparing the results in the two papers, one sees that price collars have very different consequences under these two compliance regimes. By assuming continual compliance in the current paper, we facilitate comparison to the previous literature and in addition may provide guidance in the formulation of future price collar regulations.[10]

We show that under continual compliance, even if the auction succeeds in driving the permit price down to the reserve price, that price will subsequently pierce that ceiling. Thus, the analogy between a price band implemented by a bufferstock policy and a price collar implemented by auction reserve prices is false. The price paths induced by these two policies are dramatically different under continual compliance.[11] This difference is particularly striking in the limiting case where the ceiling and floor coincide (a price "peg"). Given a sufficiently large inventory, a bufferstock price policy could then stabilize the price and would therefore be equivalent to an emissions tax. In contrast, the proposed policy will almost never achieve this result.

The paper proceeds as follows. In Section 2, we analyze the optimal decisions of firms given an arbitrary path of prices and then characterize the competitive equilibrium. In Section 3, we examine the effect of a price collar implemented by a price band bufferstock policy. In Section 4, we consider the alternative policy of implementing the collar with reserve price auctions as specified in the legislation. Section 5 determines whether either way of implementing a price collar approximates an emissions tax as the collar tightens. Section 6 concludes the paper.

[9] The sales price at the ceiling starting from \$25 in 2013 increases by 5% plus the rate of inflation in 2014 and each year thereafter.

[10] In the future, continual compliance may replace delayed compliance. Alternatively, delayed compliance may be maintained, but the compliance date assigned to different regulated entities may differ so that on any given day some entity is just ending its period of delayed compliance. Dallas Burtraw has suggested that such staggering of compliance dates would make a delayed compliance regime resemble the regime of continual compliance we analyze in this paper.

[11] Murray et al. (2009) and Wood and Jotzo (2011) suggest that the price collar implemented by auctions do not guarantee the ceiling or floor price depending on the demand for permits at the time of the auctions and the number of initially grandfathered permits.

2 The Model

2.1 Demand for Permits to Be Used Contemporaneously

We consider a world with a competitive market for pollution permits in continuous-time $t \in [0, \infty)$. To facilitate comparison to previous analyses, we require firms to surrender continually permits to cover their contemporaneous emissions ("continual compliance").[12] That is, we require firm i ($i \in \{1, 2, \cdots, N\}$) to relinquish at time t one pollution permit for every unit of emissions at time t. The firms acquire the permits on the market at the going price $p(t)$ at time t. Each firm determines the quantity of emissions and the quantity of emissions permits that it sells or purchases at time t. We consider its decisions about emissions and the trade of permits separately.

Denote firm i's emissions at time t as $e_i(t)$ and the abatement cost for firm i by $C_i(e_i)$. We assume that $C_i(e_i)$ is a twice-continuously differentiable and strictly convex function with respect to e_i such that $C_i'(e_i) < 0$ and $C_i''(e_i) > 0$ and that $C_i'(0)$ is finite. Note that the abatement cost function is assumed to be stationary. We confine attention to the case of deterministic cost function since our aim is to make the consequences of the proposed collar transparent.

Because of the stationarity of the abatement cost function, firm i chooses its emissions at time t to minimize the sum of the abatement cost and the cost of purchasing the requisite permits at the price $p(t)$. Then this problem can be written as

$$\min_{e_i(t) \geq 0} p(t)e_i(t) + C_i(e_i(t)).$$

The first-order conditions for this problem are

$$\begin{aligned} p(t) + C_i'(e_i(t)) &= 0 \text{ if } e_i(t) > 0 &(1)\\ p(t) + C_i'(e_i(t)) &\geq 0 \text{ if } e_i(t) = 0. &(2)\end{aligned}$$

Condition (1) implies that, if firm i emits a positive level of emissions, we must have $p(t) = -C_i'(e_i(t))$. That is, firm i sets its emissions so that its marginal cost of pollution abatement equals the permit price. Condition (2) implies that, when it chooses to emit nothing, permits are so expensive that their price is at least as high as the marginal cost of even the first unit of emissions: $p(t) \geq -C_i'(0)$.

Because of the strict convexity of the objective function, these first-order conditions are

[12] The actual requirement of the three bills is that on a designated date each year, firms must surrender enough permits to cover their *cumulative* emissions over the previous year ("delayed compliance"). Hasegawa and Salant (2012) show that under delayed compliance not even a bufferstock policy can enforce the collar.

necessary and sufficient to identify $e_i(t)$ for $i = 1, 2, \cdots, N$. Conditions (1) and (2) uniquely identify the quantity of emissions of firm i as a function of the permit price $p(t)$ so that $e_i^*(t) = D_i(p(t))$, where $e_i^*(t)$ is the quantity of emissions of firm i at the price $p(t)$. We can interpret $e_i^*(t) = D_i(p(t))$ as the quantity of permits that firm i demands at the permit price $p(t)$ in order to emit the pollution level of $e_i^*(t)$ at time t, or the demand for permits to be used instantaneously by firm i at time t. From equation (1), we have (provided $e_i(t) > 0$)

$$\frac{de_i^*(t)}{dp(t)} = D_i'(p(t)) = -\frac{1}{C_i''(e_i^*(t))} < 0. \tag{3}$$

Thus, the demand function is strictly decreasing in the price of permits as long as firm i chooses positive emissions. From conditions (2) and (3), there is a "choke price" for firm i, $p_i^c = -C_i'(0)$, where $D_i(p(t)) = 0$ for $p(t) \geq p_i^c$. Then we have $D_i(p(t)) > 0$ for $p(t) < p_i^c, D_i(p(t)) = 0$ for $p(t) \geq p_i^c$, and $D'(p(t)) < 0$ for $p(t) < p_i^c$. The aggregate demand for permits to be surrendered contemporaneously at time t by all firms is defined by

$$D(p(t)) = \sum_{i=1}^{N} D_i(p(t)).$$

The aggregate demand function is also strictly decreasing in the permit price as long as at least one of N firms chooses a positive amount of emissions at the price. We can define the choke price for this aggregate demand function as $p^c = \max\{p_1^c, p_2^c, \cdots p_N^c\}$. The aggregate demand curve intersects the vertical axis at p^c. Then we have $D(p(t)) > 0$ for $p(t) < p^c$, $D(p(t)) = 0$ for $p(t) \geq p^c$, and $D(p) > D(p')$ for any $p < p' \leq p^c$.

2.2 Trade of Permits

We assume that firms can bank their permits without limitation and use stored permits any time they want.[13] Then they determine not only the quantity of permits used to emit $e_i^*(t)$ for $i = 1, 2, \cdots, N$ at time t but also the quantity of permits that they sell or buy so as to maximize their wealth. Let $x_i(t)$ denote firm i's net sales of permits at the permit price $p(t)$ at time t. $x_i(t) > 0$ implies that firm i sells $x_i(t)$ permits at time t, and $x_i(t) < 0$ implies that it purchases $|x_i(t)|$ permits at time t. As we will discuss in detail in the following sections, the government provides firms with permits by grandfathering and by direct selling at a fixed price or at auctions. We denote the set of instants when the government grandfathers or auctions permits for N firms as $\Omega = \{T_1, T_2, ..., T_{j-1}, T_j, T_{j+1}, ..., T_{J-1}, T_J\}$, where $0 \leq$

[13]We assume throughout this analysis that permits cannot be borrowed from the future. Allowing limited borrowing (as some programs do) eliminates or—if a borrowing constraint binds—reduces discontinuous drops of permit prices. The detail of the analysis of borrowing is available from the authors.

$T_j < T_{j+1}$ for $j = 1, 2, ..., J-1$, and $T_J < \infty$, which means that the government will provide permits J times in total by grandfathering or selling at auctions at times $T_1, ..., T_J$. We denote y_{it} as the quantity of permits provided for firm i by the government at time $t \in \Omega$. y_{it} can be negative when firm i sells $|y_{it}|$ permits at a floor price to the government. Then firm i will face the following problem

$$\max_{x_i(t)} \int_0^\infty p(t) x_i(t) e^{-rt} dt \tag{4}$$

subject to

$$s_i(0) = 0 \tag{5}$$

$$\dot{s}_i(t) = -x_i(t) \text{ for } t \in [0, \infty) \backslash \Omega \tag{6}$$

$$s_i(T_j) = \lim_{t \downarrow T_j} s_i(t) + y_{iT_j} \text{ for } j = 1, 2, ..., J \tag{7}$$

$$s_i(t) \geq 0, \tag{8}$$

where $s_i(t)$ is the stock of permits in the bank for firm i. The stock $s_i(t)$ discontinuously jumps up at the time when the government provides permits at time $t \in \Omega$. Equation (5) says that the stock of permits is initially zero. Equation (6) says that the rate of decrease in the stock is equal to the net sales of permits at time t except for the times when permits are provided by the government.[14] Equation (7) implies firm i's stock of permits at time T_j is the sum of the permits that firm i holds immediately before time T_j and y_{iT_j} permits provided by the government at time T_j. We denote the total net sales of permits by all firms at time t as $x(t)$: $x(t) = \sum_{i=1}^N x_i(t)$ and the total quantity of the permits provided by the government to all firms at time t as y_t: $y_t = \sum_{i=1}^N y_{it}$.

A competitive equilibrium in this economy consists of a permit price path $p(t)$ and net supply of permits by each firm $\{x_1(t), x_2(t), \cdots, x_N(t)\}$ such that [a] each firm solves its wealth maximization problem given the equilibrium price path as described by equations (4) to (8); [b] the demand for permits at every instant must be satisfied by the permits supplied by firms ($x(t) = D(p(t))$ for all $t \in [0, \infty)$); and [c] all of the permits provided to firms are eventually surrendered by firms: $\int_0^\infty x(t) dt = \sum_{t \in \Omega} y_t$.

The equilibrium conditions imply that the permit price increases at the rate of real interest (Hotelling's rule) when firms carry over ("bank") their permits to the future. Thus, firms will be indifferent between selling and buying any quantity of permits in the market. Given Hotelling's rule, the equilibrium properties [b] and [c] tell us that the equilibrium price path will be uniquely determined by the total quantity of permits provided by the

[14] At time $t \in \Omega$, $s(t)$ is not differentiable with respect to t.

government, regardless of the distribution to each firm, $\{y_{1t}, y_{2t}, \cdots, y_{Nt}\}_{t\in\Omega}$. Thus, as long as we focus on the equilibrium price path, we can ignore the distribution of the permits provided by the government among firms, $\{y_{1t}, y_{2t}, \cdots, y_{Nt}\}_{t\in\Omega}$, as well as the number of permits sold by each firm, $\{x_1(t), x_2(t), \cdots, x_N(t)\}_{t\in[0,\infty)}$.

3 Price Collar Implemented by Direct Purchases and Sales

We derive the equilibrium price path when the government imposes a price collar on the permit price. We examine two types of the government policies. First, we start by examining the case where the government is willing from the outset to purchase permits at a floor price and sell permits from the government stock at a ceiling price (a price band bufferstock policy). In Section 4, we will consider the case where the government defends floor and ceiling prices by selling permits at auctions with reserve prices. As we will show, the price paths induced by these two government policies are dramatically different.

In this section, we examine the price stabilization policy, in which the government offers to purchase permits at a floor price and offers to sell any quantity of permits in its possession at a ceiling price. That is, we assume the price collar is implemented like a price band used to stabilize commodity prices or exchange rates.[15]

Suppose that the government issues permits each year. We denote the total number of permits issued in year k as q_k, the beginning of year k as $t = t_k$, the end of the year (or the beginning of year $k+1$) as $t = t_{k+1}$. The government grandfathers g_k permits to firms at time t_k and deposits h_k permits in its reserve account such that $q_k = g_k + h_k$. Denote a floor price as p^f and a ceiling price as p^u where $p^f < p^u < p^c$. We assume that the ceiling price is lower than the choke price (otherwise the ceiling does not affect the permit price at all). Then the permit prices are never less than the floor (if the government has enough wealth to purchase all permits offered at p^f) and exceeds the ceiling only if the government exhausts its reserve.

Assuming that firms do not have any stock of permits at time t_k: $s_i(t_k) = 0$ for $i = 1, 2, \cdots, N$, we focus on the equilibrium price path between time t_k and t_{k+1}. Unless the total number of permits for year $k+1$ (q_{k+1}) is too small, all of the permits that are grandfathered and sold by the government in year k will be surrendered during that year. Then the permit price in year k can be determined independently of the stock of permits in

[15] Only the Kerry-Lieberman bill provided that a price ceiling be defended in this way and only for the 90-day period ending on the date of compliance (under delayed compliance).

previous and future years. Thus, we can consider each year's price path independently and uniquely depending on g_k, h_k, p^u, and p^f.

For the simplicity of our analysis, we assume provisionally $p^f < p^u e^{r(t_k - t_{k+1})}$, which means that the width of the price band $(p^u - p^f)$ is so large that the price does not hit the ceiling in the end of the year if it starts from the floor and increases at the rate of interest. This assumption will be relaxed in Section 5.

Case $D1$: If an intermediate number (g_k) of permits is grandfathered at time t_k satisfying the following inequality,

$$\int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{k+1})})dt < g_k < \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt, \tag{9}$$

then the price will start above the floor at time t_k, will rise at the rate of interest, and will be strictly below p^u at $t = t_{k+1}$. The initial price $p_{t_k} = p(t_k)$ is determined implicitly by

$$\int_{t_k}^{t_{k+1}} D(p_{t_k} e^{r(t-t_k)})dt = g_k. \tag{10}$$

Then by assumption (9), $p_{t_k} > p^f$ and $p(t_{k+1}) = p_{t_k} e^{r(t_{k+1}-t_k)} < p^u$. As illustrated in Figure I, neither the floor nor ceiling price is triggered.

Case $D2$: When the number of permits grandfathered is so small that the left inequality in (9) is violated,

$$g_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{k+1})})dt. \tag{11}$$

The price will start above the floor at time t_k and will grow at the rate of interest until the ceiling is hit at some time t' before time t_{k+1}. The two variables, p_{t_k} and t', are determined by the following two equations:

$$\begin{aligned} \int_{t_k}^{t'} D(p_{t_k} e^{r(t-t_k)})dt &= g_k & (12) \\ p_{t_k} e^{r(t'-t_k)} &= p^u. & (13) \end{aligned}$$

Equation (12) implies that firms surrender all of their grandfathered permits between t_k and t'. Carrying permits beyond t' is suboptimal since there is no capital gain to compensate for the loss of interest. After the price reaches the ceiling, the government will keep selling the permits in its reserve at the ceiling price p^u until either $t_{k+1}$is reached or the permits in its reserve are attacked at t''. In this case the price path depends on the amount of government reserve in year k.

Case $D2(a)$: When the government reserve is large enough that

$$h_k \geq (t_{k+1} - t')D(p^u) \tag{14}$$

holds, the government can defend the ceiling until the end of the year as shown in Figure II.

Case $D2(b)$: When $h_k < (t_{k+1} - t')D(p^u)$ holds, firms will buy up from the government all of the remaining reserve permits at some instant t'' in anticipation that the government's stock of permits would run out before time t_k. Between t'' and t_{k+1} firms will then sell the permits they acquired at prices growing up at the rate of interest starting at $p(t'') = p^u$. The time t'' is determined by the condition

$$\int_{t''}^{t_{k+1}} D(p^u e^{r(t-t'')})dt = h_k - (t'' - t')D(p^u). \tag{15}$$

The left-hand side of equation (15) is the total quantity of permits demanded between t'' and t_{k+1}. The right-hand side of equation (15) is the government stock of permits acquired at time t''. Then the ceiling price is broken due to the speculative attack by firms at time t'' as demonstrated by Salant and Henderson (1978) and Salant (1983). The price path is shown in Figure III.

Case $D3(a)$: When the quantity of grandfathered permits is large and the right inequality in (9) is violated:

$$g_k \geq \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt. \tag{16}$$

If the government did not intervene in the market, the price would start below the floor price. But since the government remains ready anytime to purchase any number of permits at the floor price, the price cannot fall below p^f. But the price cannot remain at the floor over an interval because firms would not have incentive to hold and sell their permits along the horizontal price path and then the market clearing condition [b] cannot be satisfied. Thus the market price has to start from p^f and increase at the rate of interest. Then the equilibrium conditions [a] - [c] require that in the equilibrium, firms sell at time t_k some stock of permits f_k and then the price starts to increase at the rate of interest. f_k is determined by

$$f_k = g_k - \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt, \tag{17}$$

the difference between the total quantity of permits grandfathered to firms and the number of permits demanded between t_k and t_{k+1} when the price starts to increase from $p(t_k) = p^f$. The price path is shown in Figure IV.

Note that the price path is independent of g_k since the total number of permits demanded in the year k does not exceed $\int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt$ and the associated price path $p(t) = p^f e^{(t-t_k)}$ is determined under condition (16) regardless of g_k as long as the government has enough wealth to buy f_k permits and defend the floor.

Case $D3(b)$: When the government's budget to support the floor is limited and it can purchase only f'_k permits at p^f where $f'_k < f_k$, even under condition (16), the initial price cannot start at p^f. Instead the firms sell f'_k permits at the floor price (p^f) to the government at time t_k but the market price drops below p^f to p_{t_k}, which is determined by the condition

$$\int_{t_k}^{t_{k+1}} D(p_{t_k} e^{r(t-t_k)})dt = g_k + (f_k - f'_k).$$

The price path is shown in Figure IV.

Under this policy, the government can defend both the price floor and ceiling as long as it has at all times enough wealth to buy any quantity of permits at p^f and has a sufficiently large number of permits in its stockpile to sell them whenever the price hits the ceiling. Hence the collar can confine the price. If more permits were exogenously grandfathered, the permit price path would fall uniformly. Once the initial price equals the floor (p^f), further increases in the number of permits grandfathered has no further effect on the price path.

4 Price Collar Implemented by Auctions with Minimum Reserve Prices

In this section, we consider the situation where the government defends a price collar by selling permits at auctions with minimum reserve prices. This is the policy to stabilize the permit prices that was actually specified in the two bills, Waxman-Markey and Kerry-Boxer. We assume that p^f and p^u are the same as in the previous section. We assume that the government grandfathers permits g_k to firms at the beginning of year k (at time t_k) and sells or makes available for sale the rest of permits in auctions.

The government holds two types of auctions. First there are "normal auctions" held to distribute up to a_k permits at the beginning of year k (at time t_k) and every year thereafter. This auction has a minimum reserve price at p^f, the floor of the price collar. We call the other type of auction the "market stability reserve auctions," following the terminology in the Kerry-Boxer bill. The market stability reserve auction is held to distribute up to h_k permits at some exogenous time $\widehat{t} \in (t_k, t_{k+1})$. It also has a minimum reserve price at p^u, the ceiling of the price collar, which we refer to as the "minimum market stability auction price," following the terminology in the Kerry-Boxer bill. Hence the government allocates up to q_k permits over the year such that $q_k = g_k + a_k + h_k$.

So far, we assume that the normal auction is held once per year (at time t_k) and the market stability reserve auction is held once per year (at time $\widehat{t}$). The essence of how these auctions work to defend the price collar can be demonstrated under this simplified setting, though we consider the case called for in the bills where the government holds both types of auctions quarterly in the appendix.

4.1 Price Floor Implemented by Auctions

First we look at the effect of the normal auction with the reserve price p^f. Under the assumption $p^f < p^u e^{r(t_k - t_{k+1})}$, we assume that any firm can join this auction to purchase the permits from the government with the minimum reserve price (p^f) at time t_k. Denote the price of the permits traded in the normal auction as p^a. Then we note that if the permit price in the market in time t_k is lower than p^f, none of the government's a_k permits will be sold in the normal auction because no one would bid as much as p^f to buy permits in the auction given $p(t_k) < p^f$. Thus, if some permits are sold at the auction, we must have $p(t_k) \geq p^f$.

In addition, if the market price were ever higher than the price in the auction ($p(t_k) > p^a$), firms would buy permits at the auction rather than in the market and they would bid up the auction price until the auction price equals the market price. On the other hand, if the market price were ever lower than the auction price ($p(t_k) < p^a$), no one would buy permits at the auction. Therefore, the permits prepared for the normal auction by the quantity of a_k must be traded at some price $\widetilde{p}_{t_k}$, which is the same price as that in the market: $\widetilde{p}_{t_k} = p^a$. Note that since under the assumption $p^f < p^u e^{r(t_k - t_{k+1})}$, the permit price will not be above p^u at time $\widehat{t}$ and no permits are sold at the market stability reserve auction. Thus we can ignore it when we examine the effect of the normal auction on the permit price path. Now we uniquely determine the equilibrium price path depending on g_k, a_k, and p^f.

Case $F1$: When a large number of permits is grandfathered so that

$$g_k > \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt \tag{18}$$

holds, the price starts below the reserve price p^f and no government permits are sold in the normal auction at t_k. Then the initial price p_{t_k} can be determined by the following condition:

$$\int_{t_k}^{t_{k+1}} D(p_{t_k} e^{r(t-t_k)})dt = g_k \tag{19}$$

where $p_{t_k} < p^f$ and the price simply increases at the rate of interest as shown in Figure V. The minimum reserve price in this auction cannot affect the price path at all regardless of the number of permits the government is prepared to auction (a_k).

Case $F2$: When a small number of permits is grandfathered so that

$$g_k \leq \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt \tag{20}$$

holds, in the absence of the normal auction at time t_k, the price would start above or at the same price equal to p^f, the initial price p_{t_k} would be determined by $\int_{t_k}^{t_{k+1}} D(p_{t_k} e^{r(t-t_k)})dt = g_k$ where $p_{t_k} \geq p^f$, and then some or all of a_k permits would be sold at the auction. The initial price (or the auctioned price) depends (as specified below) on the number of permits the government prepares for the auction (a_k).

Case $F2(a)$: When the government auctions a small number of permits such that

$$g_k + a_k \leq \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt$$

holds, then firms will buy all of the auctioned permits at time t_k at the price $\widetilde{p}_{t_k}$, which is determined by

$$\int_{t_k}^{t_{k+1}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt = g_k + a_k. \tag{21}$$

The initial price $\widetilde{p}_{t_k}$ is lower than that in absence of the auction (p_{t_k}), but it is still higher than p^f as shown in Figure VI.

Case $F2(b)$: When the government prepares for its normal auctions at time t_k so many permits that

$$g_k + a_k > \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt$$

holds, the government will sell at time t_k

$$\int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt - g_k$$

permits, which is strictly less than the total number of auctioned permits a_k. The market price is initially p^f and grows at the rate of interest as shown in Figure VI.

4.2 Price Ceiling Implemented by Auctions

Next, we look at the effect of the market stability reserve auction with the minimum reserve price p^u.This auction will be held at exogenous time $\widehat{t} \in (t_k, t_{k+1})$ to defend the ceiling price p^u. The government sells up to h_k permits from its reserve through the auction. We assume that any firm can join this auction to purchase the permits from the government at the minimum reserve price p^u at time $\widehat{t}$. Then, as in the case of the normal auction, if the market price at the time of this auction is lower than the minimum reserve price: $p(\widehat{t}) < p^u$, the government sells no permits in the auction. We again denote the initial price in the market without the auction by p_{t_k}, which is determined by condition (19). Note that as the quantity of the grandfathered permits (g_k) is larger, p_{t_k} becomes lower. To focus on the effect of the market stability reserve auction, we do not consider the normal auction here ($a_k = 0$).

Case $C1$: When g_k is sufficiently large so that

$$g_k > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{22}$$

holds, no bids are accepted by the government at time $\widehat{t}$ since $p(\widehat{t}) = p_{t_k} e^{r(\widehat{t}-t_k)} < p^u$. Thus the price just increases at the rate of interest starting from p_{t_k} as shown in Figure VII.

*Case C*2: When g_k is not large so that

$$g_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

holds, the permits are sold at the auction since the price at time $\widehat{t}$ would be above the minimum reserve price p^u in the absence of the market stability reserve auction. Then we consider the following cases (see below).

*Case C*2(*a*): When the government reserve h_k is so small that

$$g_k + h_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{23}$$

holds, all of h_k permits will be sold at time $\widehat{t}$. The initial price must be determined so that all of the permits $(g_k + h_k)$ are demanded and supplied between t_k and t_{k+1}. As in the previous section, we denote the initial price when the permits are sold at the market stability auction by $\widetilde{p}_{t_k}$. When g_k is not small enough so that

$$\int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt \leq g_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{24}$$

holds, the initial price $\widetilde{p}_{t_k}$ is determined by the following condition:

$$\int_{t_k}^{t_{k+1}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt = g_k + h_k. \tag{25}$$

Then the price at time $\widehat{t}$ is higher than p^u and the new price path goes down compared to that in the absence of the auction as shown in Figure VIII.

*Case C*2(*b*): Under condition (24), when the government reserve h_k is large enough so that

$$g_k + h_k > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{26}$$

holds, the initial price $\widetilde{p}_{t_k}$ is determined by the condition

$$\widetilde{p}_{t_k} e^{r(\widehat{t}-t_k)} = p^u. \tag{27}$$

Equation (27) implies that the initial price is determined so that the price path passes p^u exactly at time $\widehat{t}$ when it starts to increases from $\widetilde{p}_{t_k}$ at time t_k at the rate of interest. Under condition (26), the government will sell in the market reserve stability auction

$$\int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt - g_k \tag{28}$$

permits from the government reserve (h_k), and the price reaches the ceiling at p^u at time $\widehat{t}$ as shown in Figure VIII.

Case C3: When g_k is small enough so that

$$g_k < \int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt \tag{29}$$

holds, the initial price cannot start from $\widetilde{p}_{t_k}$ since, if so, g_k permits would be exhausted before time $\widehat{t}$. Then the initial price would need to be recalculated so that all of the initial stock of g_k permits is sold up in the market between t_k and $\widehat{t}$. We denote it as p'_{t_k}, which is determined as follows:

Case C3(a): When the government reserve is so small that

$$h_k \leq \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

holds, the initial price p'_{t_k} is determined by the following condition:

$$\int_{t_k}^{\widehat{t}} D(p'_{t_k} e^{r(t-t_k)})dt = g_k$$

so that all of the grandfathered permits are exhausted at time $\widehat{t}$. Then the price will drop to $p_{\widehat{t}}$, which is determined by

$$\int_{\widehat{t}}^{t_{k+1}} D(p_{\widehat{t}} e^{r(t-\widehat{t})})dt = h_k \tag{30}$$

as shown in Figure IX as long as $p_{\widehat{t}} < \widetilde{p}_{t_k} e^{r(\widehat{t}-t_k)}$ holds.[16]

[16] If it is violated because h_k is too small, the price will not drop to $p_{\widehat{t}}$ at time $\widehat{t}$. The initial price will

Case $C3(b)$: Given that condition (29) holds, when the government reserve is sufficiently large so that

$$h_k > \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{31}$$

holds, the initial price is the same as in *Case* $C3(a)$ but the government will sell in the market stability reserve auction

$$h_k - \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \tag{32}$$

permits from the government reserve (h_k) and the permit price will drop to p^u at time $\widehat{t}$ as shown in Figure IX.

4.3 The Combination of Two Types of Auctions

So far we have examined the normal auctions and the market stability reserve auctions separately. Now we can identify the price paths when the government holds the normal auction at time t_k and then the market stability reserve auction at time $\widehat{t}$ in year k. In Subsection 4.1, we demonstrated that the normal auction works when the government grandfathers a small number of permits at time t_k (condition (20)) and just lowers the permit prices uniformly as in *Case* $F2$. We can consider the price paths based on *Cases* $F1, F2$, and $C1 - C3$.

If condition (18) holds as in *Case* $F1$, the price will start below p^f and $p(\widehat{t}) < p^u$ since condition (22) in *Case* $C1$ is automatically satisfied under the assumption $p^f < p^u e^{r(t_k - t_{k+1})}$. Then no permits are sold at either the normal auction or the market stability reserve auction. Thus the price path is the same as that in *Case* $F1$.

If condition (22) holds but condition (18) is violated such that

$$\int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt < g_k \leq \int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt,$$

the permits are sold only at the normal auction. Thus the price path is the same as that in *Case* $F2$.

be determined by equation (24), and the price will increase at the rate of interest throughout the year as in *Case* $C2$.

If g_k is small and thus condition (22) fails to hold:

$$g_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt,$$

the permits are sold at the both auctions. The price path corresponds to one of *Cases* $C2(a), C2(b), C3(a)$, and $C3(b)$. We can determine the price path by checking which conditions among (23), (24), (26), (29), and (31) hold when we replace g_k in these conditions by $g_k + a_k$ as follows:

Case $C2(a')$: Replacing g_k by $g_k + a_k$ in conditions (23) and (24), when

$$g_k + a_k + h_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \quad \text{and} \quad \int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt \leq g_k + a_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

hold, the initial price $\widetilde{p}_{t_k}$ is determined by the condition

$$\int_{t_k}^{t_{k+1}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt = g_k + a_k + h_k \tag{33}$$

and the permit price will continue to increase at the rate of interest until t_{k+1} as in *Case* $C2(a)$.

Case $C2(b')$: Replacing g_k by $g_k + a_k$ in conditions (24) and (26), when

$$\int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt \leq g_k + a_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt \quad \text{and} \quad g_k + a_k + h_k > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

hold, the initial price is determined by the condition $\widetilde{p}_{t_k} e^{r(\widehat{t}-t_k)} = p^u$ so that the price path passes p^u exactly at time $\widehat{t}$ as in *Case* $C2(b)$, and the government will sell

$$\int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt - g_k - a_k$$

permits in the market stability reserve auction.

Case $C3(a')$: Replacing g_k by $g_k + a_k$ in condition (29), when

$$g_k + a_k < \int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-\widehat{t})})dt \quad \text{and} \quad h_k \leq \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

hold, where $\widetilde{p}_{t_k}$ is determined by condition (33) if $g_k + a_k + h_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$ holds or by the condition $\widetilde{p}_{t_k} e^{r(\widehat{t}-t_k)} = p^u$ if $g_k + a_k + h_k > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$ holds, a_k permits will be sold in the normal auction at time t_k. Then all of the initial stock of $(a_k + g_k)$ permits are exhausted between t_k and $\widehat{t}$. The initial price p'_{t_k} is determined by the condition

$$\int_{t_k}^{\widehat{t}} D(p'_{t_k} e^{r(t-t_k)})dt = g_k + a_k$$

and the permit price will drop at time $\widehat{t}$ to $p_{\widehat{t}}$, which is determined by condition (30) as in *Case* $C3(a)$.

Case $C3(b')$: Replacing g_k by $g_k + a_k$ in condition (29), when

$$g_k + a_k < \int_{t_k}^{\widehat{t}} D(\widetilde{p}_{t_k} e^{r(t-\widehat{t})})dt \quad \text{and} \quad h_k > \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

hold, the initial price is the same as in *Case* $C3(a')$, but the government will sell

$$h_k - \int_{\widehat{t}}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt$$

permits in the market stability reserve auction and the price will drop to p^u at time $\widehat{t}$ as in *Case* $C3(b)$.

In any of these cases above, the price path is the essentially the same as that which we derived in Subsection 4.2 except that the prices are uniformly lowered because a_k permits are additionally sold in the normal auction at time t_k.

In this auction setting, the auctioned permits a_k and h_k contribute to lowering the price path only when those permits are sold in these auctions. But unlike the price band policy implemented by the government direct purchases and sales of permits, if the market price for permits is below the minimum reserve prices, then nothing is sold at either auction regardless of the number of permits prepared for the auctions (a_k and h_k). This is illustrated in *Cases* $F1$ and $C1$. It can happen when the government grandfathers a large number of permits at time t_k.

In addition, even though a sufficiently large number of permits are prepared for these auctions as in *Cases* $F2(b)$, $C2(b)$, and $C3(b)$, after these auctions, the price continues to increase at the rate of interest. Especially, in *Cases* $C2(b)$ and $C3(b)$, after time $\widehat{t}$, the permit price breaks the ceiling and keeps growing by the rate of interest. The ceiling of the collar is not confining in these cases.

Although we assumed that the normal auction is held at time t_k and the market stability reserve auction at $\widehat{t}$ once in year k, these results are robust: they do not change even if the government holds normal and market stability reserve auctions more than once in each year. We consider the case where the market stability reserve auctions are held quarterly in each year in the appendix.

5 Price Collar versus Emissions Tax When the Price Band Is Narrow

It is widely believed that price collars implemented by auctions with reserve prices will successfully stabilize the permit price and keep it between the floor and ceiling prices. This belief presumably rests on the analogy between a price collar implemented by auctions and a price band implemented by purchases and sales from a bufferstock. However, as we demonstrated in Sections 3 and 4, this is a false analogy: the price paths induced by those two policies are dramatically different.

It also seems to be widely believed that, as the collar tightens, this policy will approach an emissions tax policy. But this contention is incorrect. Consider the situation where the bandwidth of the collar is very narrow so that $p^u = p^f + \varepsilon$, where ε is a positive but infinitesimal so that $p^u \approx p^f$.

Under the bufferstock policy that we studied in the previous section, as long as the government has sufficient reserve permits (h_k) to sell at the ceiling price (condition (14) holds) and has enough budget to purchase any permits at the floor price at time t_k, the price path will be similar to that under the emissions tax rate at $t = p^u(\approx p^f)$. Because the bandwidth is very narrow (ε), firms will sell almost all of their permits (f_k in (17)) at time t_k, the price will hit the ceiling immediately after t_k, and then the government will sell the reserve permits at p^u until time t_{k+1}. The price path is shown in Figure X.

Under the price collar implemented by auctions, however, the price path is very different. It may vary a lot depending on the number of grandfathered permits (g_k):

Case $T1$: If so many permits are grandfathered that

$$g_k \geq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt, \tag{34}$$

the initial price will start below p^f since the bandwidth is infinitely narrow ($p^u \approx p^f$) and the price will be below p^u at time $\widehat{t}$. Then no permits are sold at either the normal auction or the market stability reserve auction. Thus the initial price is determined by condition (19) and the price will simply increase from p_{t_k} at the rate of interest as shown in Figure XI.

Case $T2$: If the condition above is violated but g_k is not so small that

$$\int_{t_k}^{t_{k+1}} D(p^f e^{r(t-t_k)})dt \leq g_k < \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-\widehat{t})})dt, \tag{35}$$

the price would start below p^f at time t_k and would be above p^u at time $\widehat{t}$ in the absence of the auctions. Then only the market stability reserve auction affects the price path. Thus we consider the price path based on the cases in Subsection 4.2. Depending on which conditions among (23), (24), (26), (29), and (31) holds, the price path will change and the price will start below p^f and no permits are sold in the normal auction at time t_k. We draw the price path for *Case* $T2(a)$, where the conditions (24), (26), and (35) all hold, and *Case* $T2(b)$, where the conditions (29), (31), and (35) all hold in Figures XII and XIII, respectively.

When a small number of permits are grandfathered so that condition (34) is violated, the price will start above p^f and permits are sold in both auctions. Then, as we discussed in Subsection 4.3, we can identify the price path by checking which condition among (23), (24), (26), (29), and (31) hold when we replace g_k in these conditions by $g_k + a_k$. The price path will be similar to that in *Case* $T2$ except that the price starts above p^f.

It is clear that the price path will not be similar to that under the emissions tax policy with $t = p^u(\approx p^f)$ even if the government eliminates the width of the price collar. This result is robust: it does not change even if the government prepares more permits in the reserve or holds normal and market stability reserve auctions more than once in each year.

6 Conclusion

It is widely believed that price collars will successfully stabilize the price of emissions permits and will keep it between the floor and ceiling prices. This belief rests on the presumed analogy between price collars implemented by auctions and price bands implemented by bufferstocks. However, the analogy is false. As we showed in Sections 3 and 4, the price paths induced under these two policies are dramatically different. The price collars proposed in the federal bills and in programs implemented elsewhere utilize government auctions with minimum reserve prices.

Under such a policy, the normal auctions and market stability reserve auctions contribute to lowering the price path only when the auctioned permits are sold in these auctions. But unlike the price band policy implemented with government bufferstocks, if the permit price is below the minimum reserve prices at the time of an auction, no permits are sold. Then, even if the auction drives the permit price down to the reserve price, the permit price will subsequently pierce the ceiling.

These results indicate that the price collar policy does not necessarily approach an emissions tax policy as the collar tightens. We have shown that the price path will not be similar to that under the emissions tax policy even if the government narrows or eliminates the width of the price collar. This result is important because many believe that imposing a tight price collar on a cap-and-trade program will yield results virtually identical to an emissions tax policy.[17]

Most analyses of price collars conclude that they are more effective at restricting the price. They reach this conclusion because they assume that, contrary to the actual regulations, price collars will be implemented through government bufferstock policies rather than through reserve-price auctions. The government is assumed to purchase emissions permits at a floor price and to sell any quantity of permits in its possession at a ceiling price. As we have shown, the government can then defend both the price floor and ceiling as long as it has enough wealth to buy whatever permits are supplied at the floor price and enough permits to sell whatever is demanded at the ceiling price. Hence a collar implemented in this way would confine the price if firms were required to surrender enough permits to cover their contemporaneous emissions ("continual compliance"). However, every program of emissions trading, whether implemented or merely proposed, requires only delayed compliance: on a designated day firms must surrender enough permits to cover their *cumulative* emissions since the previous compliance date ("delayed compliance"). In our companion paper (Hasegawa

[17] The emissions tax policies are generally shown to be more efficient in limiting greenhouse gas emissions than quantity policies, though political economy issues force us to focus on quantity restrictions, a cap-and-trade system (Weitzman, 1974).

and Salant, 2012), we show that this relaxation of the compliance requirement renders even bufferstock implementation of the price collar ineffective.

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Appendix

Sequential Market Stability Reserve Auctions

In Section 4, we assumed that the government holds the market stability reserve auction at time $\widehat{t}$ once in year k and prepares h_k permits for only that auction. In this appendix, we examine whether and how the permit price path changes if the government holds the market stability reserve auctions quarterly.

Consider the situation where the government auctions h_k permits for the market stability reserve auctions in total and holds them at $t_k (= t_{Q1}), t_{Q2}, t_{Q3}$, and t_{Q4}. We assume that the government is willing to auction $(h_k/4)$ permits in each market stability auction. As in Subsection 4.2, we do not consider the normal auctions ($a_k = 0$) here. As the Kerry-Boxer bill specifies, we also assume that unsold permits at each market stability reserve auction are rolled over and added to the quantity available for sale in the following quarter. Unlike in Subsection 4.2, we do not list and examine every case. We focus on the case where the following conditions hold:

$$g_{t_k} > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_k)})dt \tag{36}$$

$$p(t_{Q2}) = p_{t_k} e^{r(t_{Q2}-t_k)} < p^u \tag{37}$$

$$p(t_{Q3}) = p_{t_k} e^{r(t_{Q3}-t_k)} > p^u, \tag{38}$$

where p_{t_k} is determined by $\int_{t_k}^{t_{k+1}} D(p_{t_k} e^{r(t-t_k)})dt = g_k$. Under these conditions, in the absence of the market stability reserve auctions, the permit price would start from $p_{t_k} < p^u$, increase at the rate of interest, be below p^u at t_k and t_{Q2}, and exceed p^u at t_{Q3} and t_{Q4} as illustrated in Figure XIV. In the presence of the market stability reserve auctions, though no permits will be sold at the auctions at t_{Q1} and t_{Q2} since the permit prices are below p^u at these time instants, some permits will be sold at both t_{Q3} and t_{Q4} or only at time t_{Q4} depending on h_k. Note that the government will auction $3h_k/4$ permits at t_{Q3} given that no permits are sold in the auctions at t_{Q1} and t_{Q2}. We can determine the price path depending on the following cases.

Case $S1$: Under conditions (36)-(38), if h_k is so small that

$$g_k + h_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{Q3})})dt \tag{39}$$

holds, all of permits (h_k in total) will be sold at the auctions at times t_{Q3} ($3h_k/4$ permits) and t_{Q4} ($h_k/4$ permits). The initial price $\widetilde{p}_{t_k}$ is determined by

$$\int_{t_k}^{t_{k+1}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt = g_k + h_k, \tag{40}$$

and the price will continue to increase at the rate of interest without any discontinuous drop at t_{Q3} or t_{Q4} as long as $g_k \geq \int_{t_k}^{t_{Q3}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ and $g_k + 3h_k/4 \geq \int_{t_k}^{t_{Q4}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ are satisfied.[18] Thus two market stability reserve auctions at t_{Q3} and t_{Q4} will lower the initial price to $\widetilde{p}_{t_k}$ compared to the initial price in the absence of these auctions (p_{t_k}), but the permit price at t_{Q3} is still above or equal to p^u ($\widetilde{p}_{t_k} e^{r(t_{Q3}-t_k)} \geq p^u$). The price path is shown in Figure XIV.

Case $S2$: Under conditions (36)-(38),

$$g_k + 3h_k/4 > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{Q3})})dt \ \text{ and } g_k + h_k \leq \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{Q4})})dt$$

hold. In this case, the initial price $\widetilde{p}_{t_k}$ is determined by condition (40) as long as $g_k \geq \int_{t_k}^{t_{Q4}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ holds.[19] Then all of h_k permits will be sold at the final market stability reserve auction at t_{Q4} though no permits are sold in the first three auctions at t_{Q1}, t_{Q2}, and t_{Q3}. Thus the last auction at t_{Q4} will lower the initial price to $\widetilde{p}_{t_k}$ compared to the initial price in the absence of these auctions (p_{t_k}), but the permit price at t_{Q4} is still above or equal to p^u ($\widetilde{p}_{t_k} e^{r(t_{Q4}-t_k)} \geq p^u$). The price path is shown in Figure XIV.

Case $S3$: Under conditions (36)-(38), if h_k is so large that

$$g_k + h_k > \int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{Q4})})dt \tag{41}$$

[18] If either $g_k \geq \int_{t_k}^{t_{Q3}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ or $g_k + 3h_k/4 \geq \int_{t_k}^{t_{Q4}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ is violated, the price can drop at time t_{Q3} or t_{Q4} respectively as in *Case* $C3$.

[19] Again, if the grandfathered permits are so small that $g_k \geq \int_{t_k}^{t_{Q4}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ is violated, the price will drop at time t_{Q3} or t_{Q4} as in *Case* $C3$.

holds, the initial price is determined so that

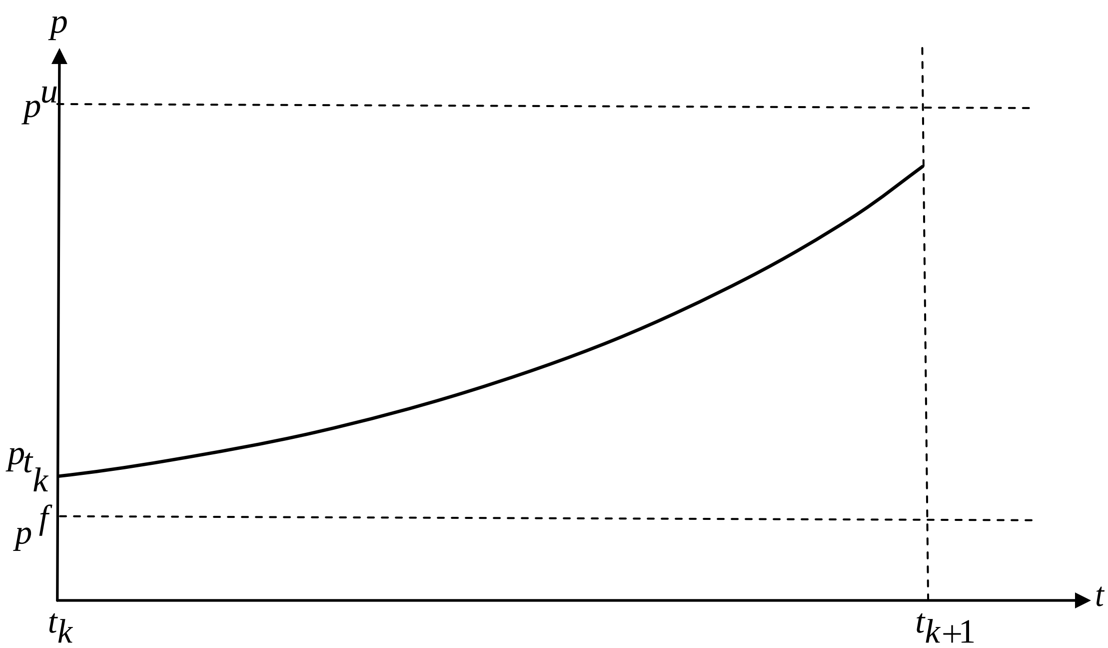
$$\widetilde{p}_{t_k} e^{r(t_{Q4}-t_k)} = p^u,$$

which means that the price path passes p^u exactly at time t_{Q4} when it starts to increase from $\widetilde{p}_{t_k}$ at time t_k at the rate of interest (as long as $g_k \geq \int_{t_k}^{t_{Q4}} D(\widetilde{p}_{t_k} e^{r(t-t_k)})dt$ holds). Under condition (41), the government will sell

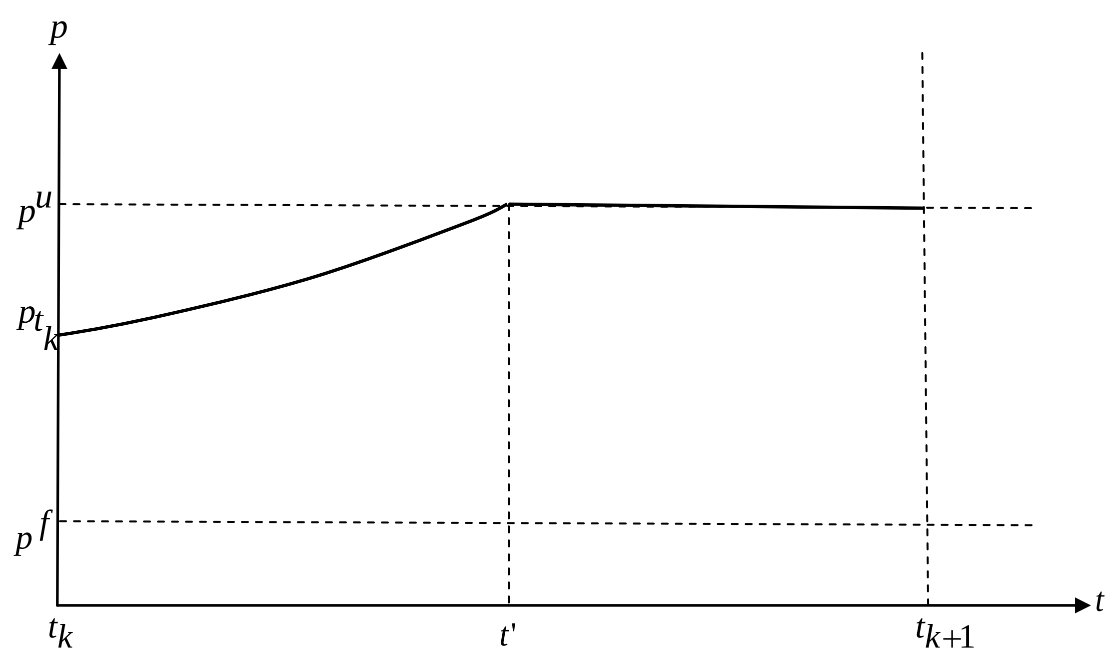
$$\int_{t_k}^{t_{k+1}} D(p^u e^{r(t-t_{Q4})})dt - g_k$$

permits in the last market reserve stability auction at time t_{Q4}. This is similar to *Case* $C2(b)$. The price path is shown in Figure XIV.

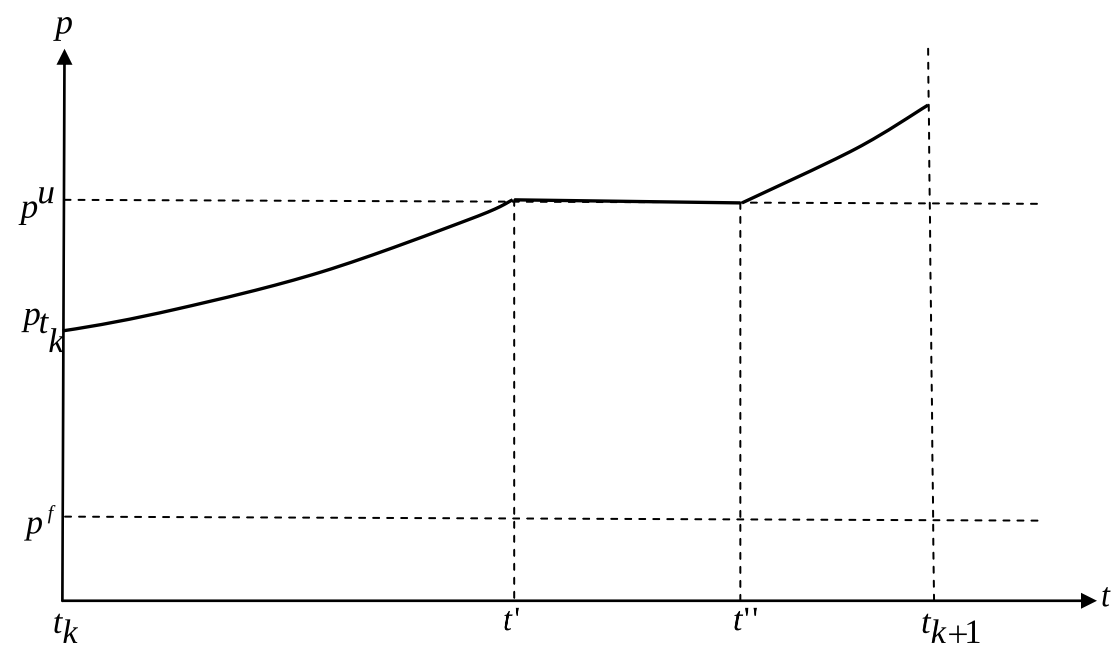
[Figure I] *Case D*1: Neither Floor nor Ceiling Price Is Triggered



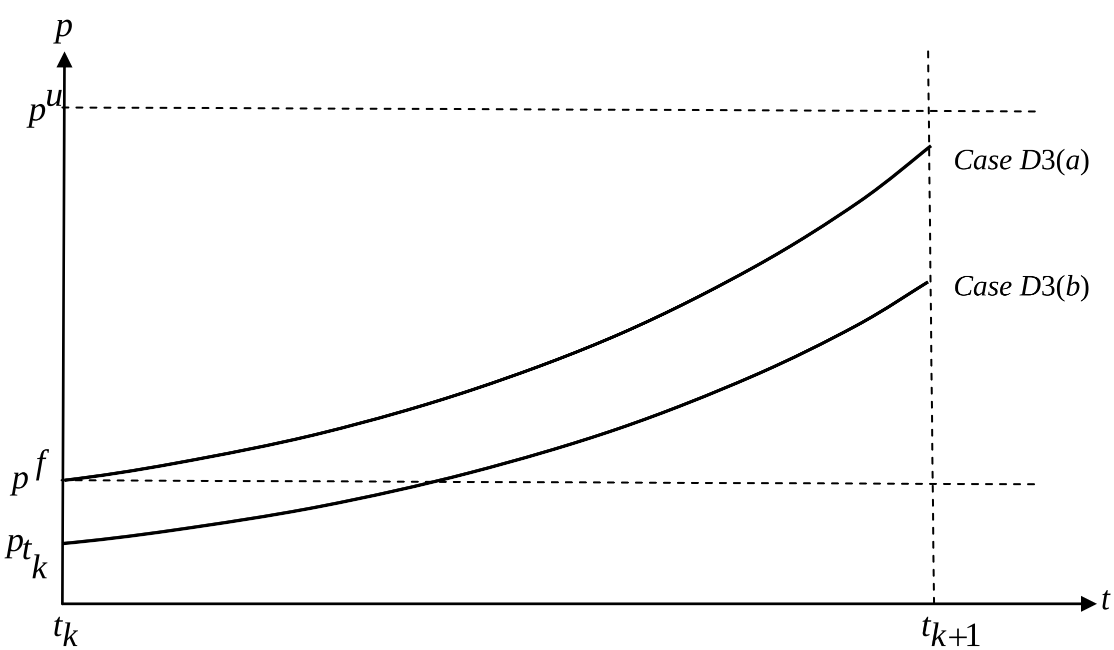
[Figure II] *Case D*2(*a*): The Government Defends the Ceiling by the Direct Sales of Permits



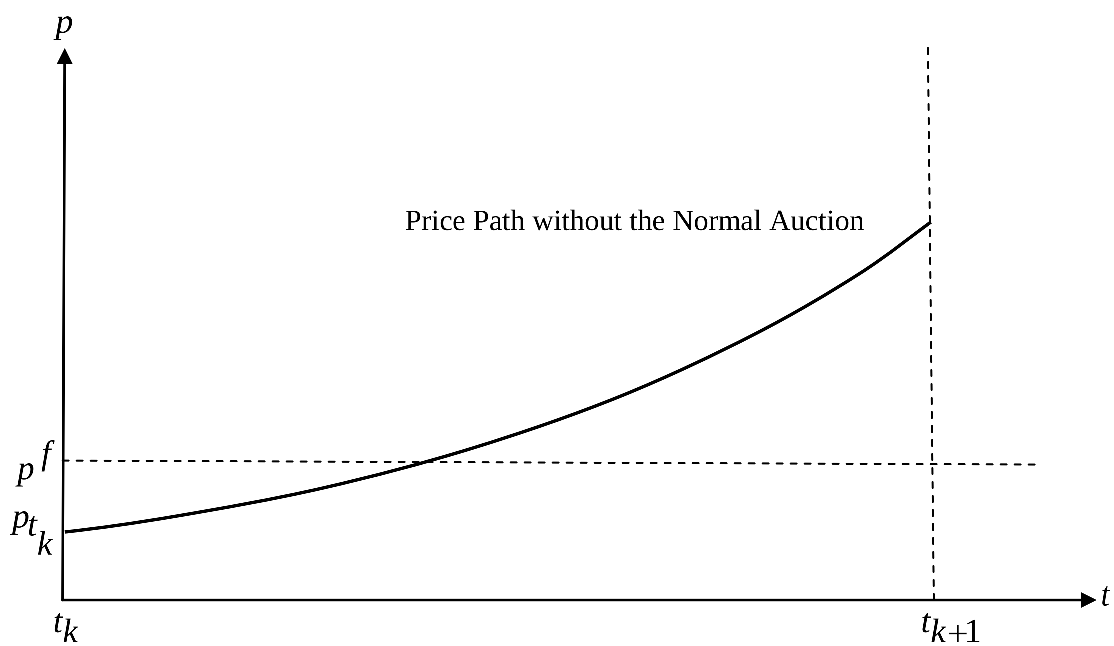
[Figure III] *Case D*2(*b*): Speculative Attack at t'' Due to Insufficient Government Stock of Permits



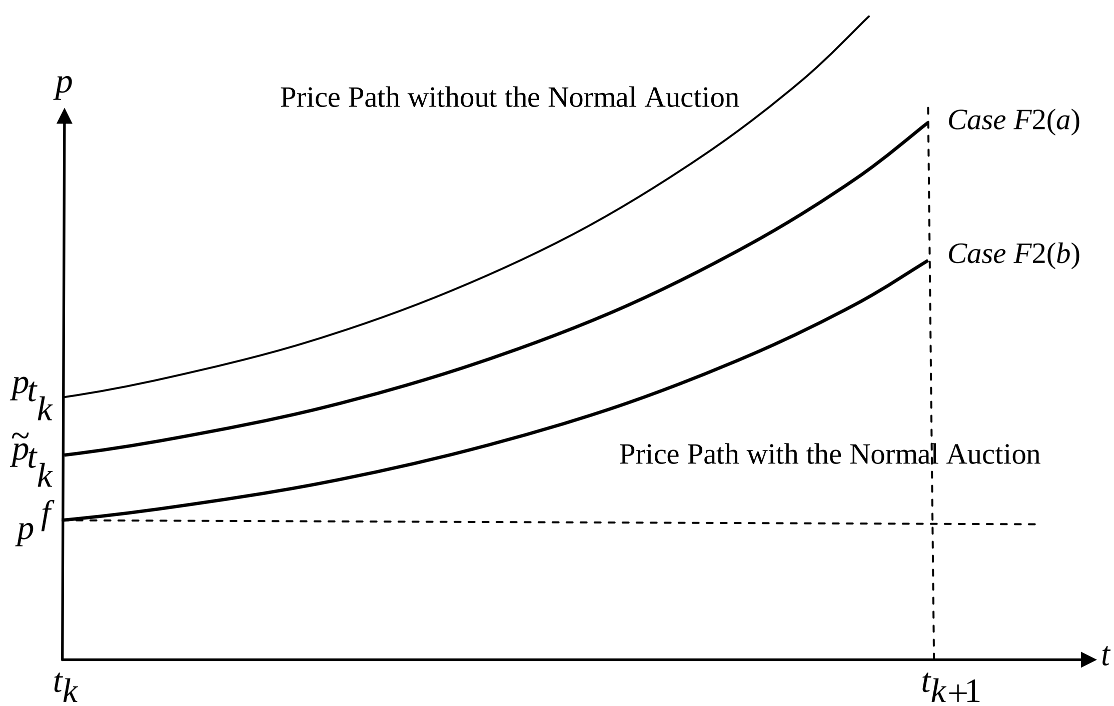
[Figure IV] *Case D*3: The Price Floor in the Cases of Unlimited and Limited Budget



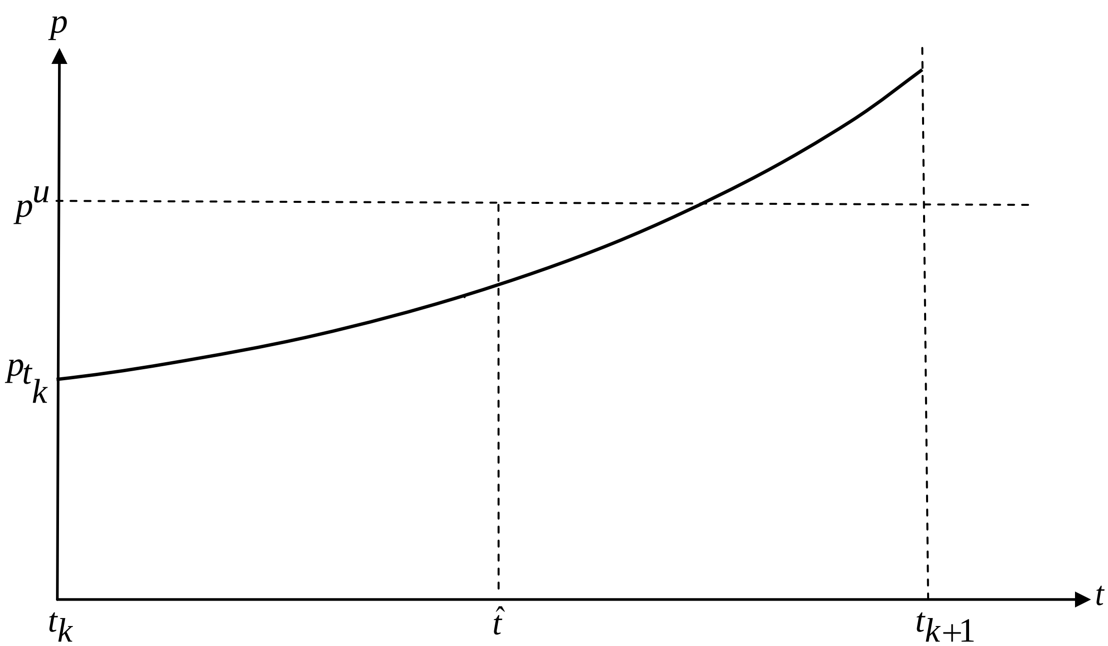
[Figure V] *Case F*1: Due to Low Market Price, No Bids Exceed the Reserve Price at the Normal Auction and Demand Is Satisfied from Grandfathered Permits



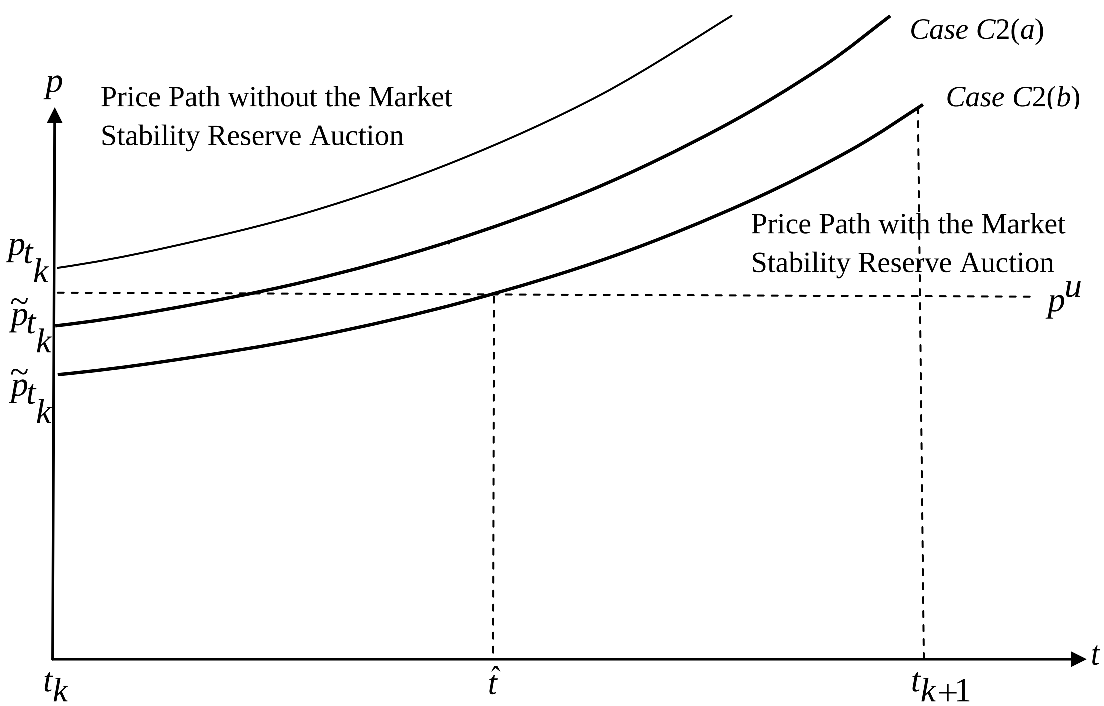
[Figure VI] *Case F*2: Price Floor Implemented by the Normal Auction



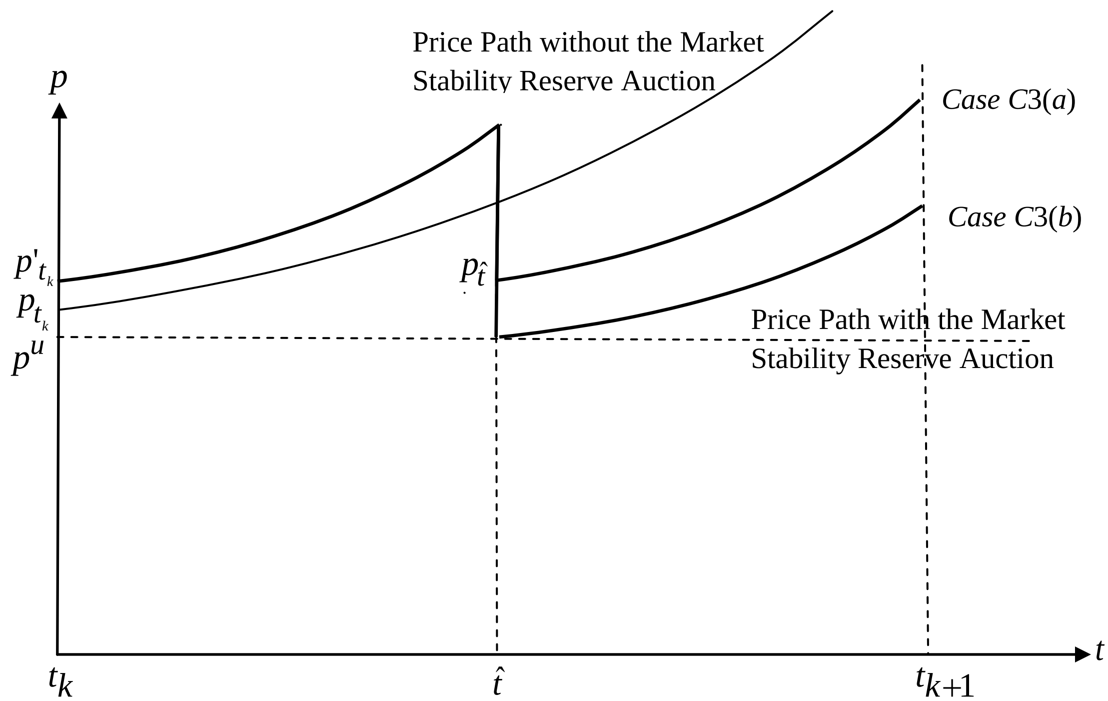
[Figure VII] *Case C*1: No Market Stability Reserve Auction



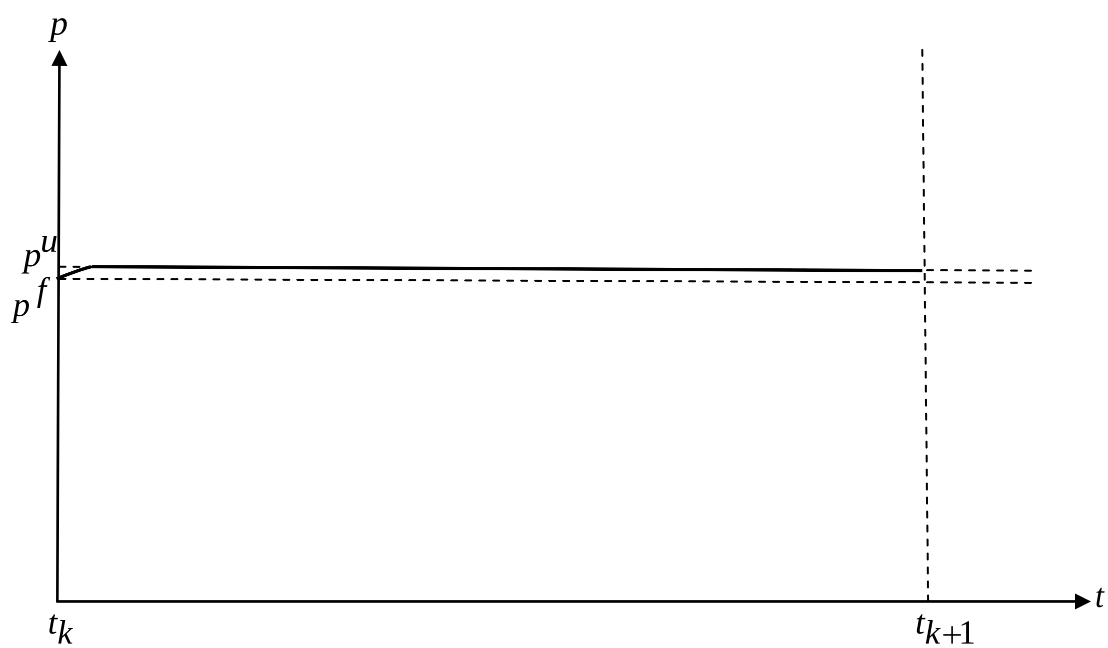
[Figure VIII] *Case C*2: Price Ceiling Implemented by the Market Stability Reserve Auction



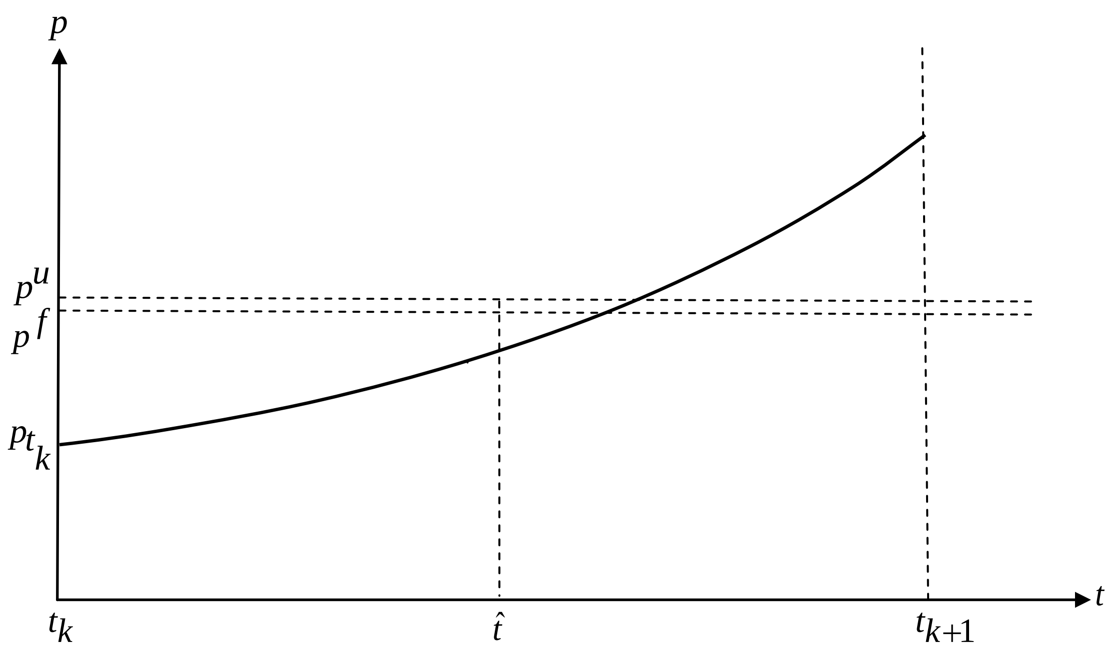
[Figure IX] *Case C*3: Price Ceiling with a Small Number of the Grandfathered Permits



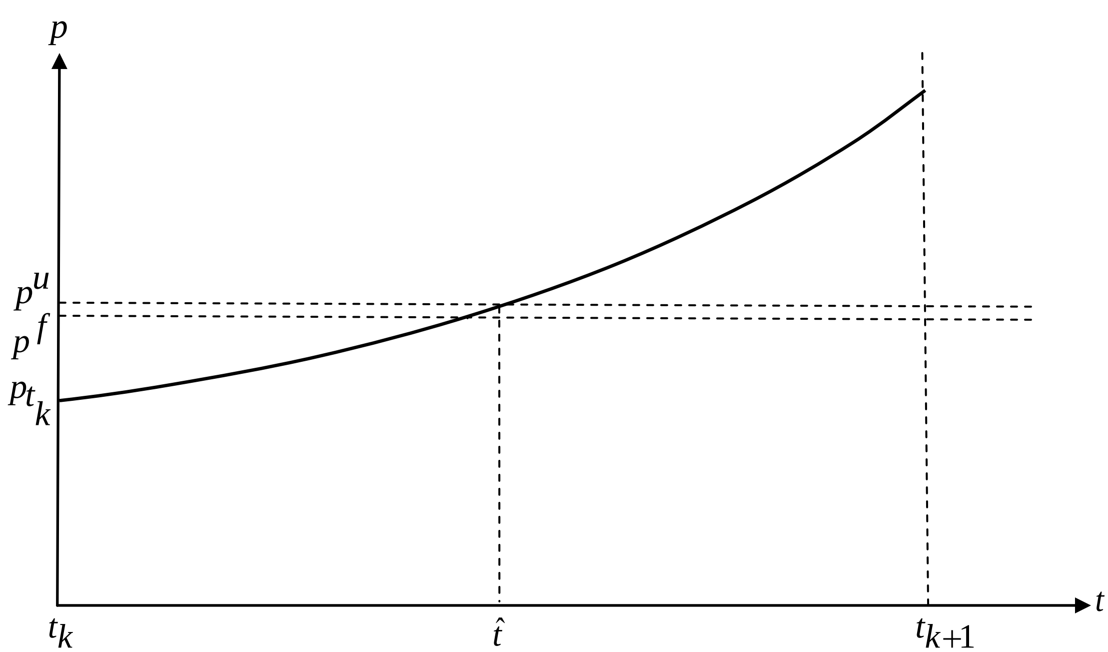
[Figure X] Emissions Tax and Buffer Stock Policy



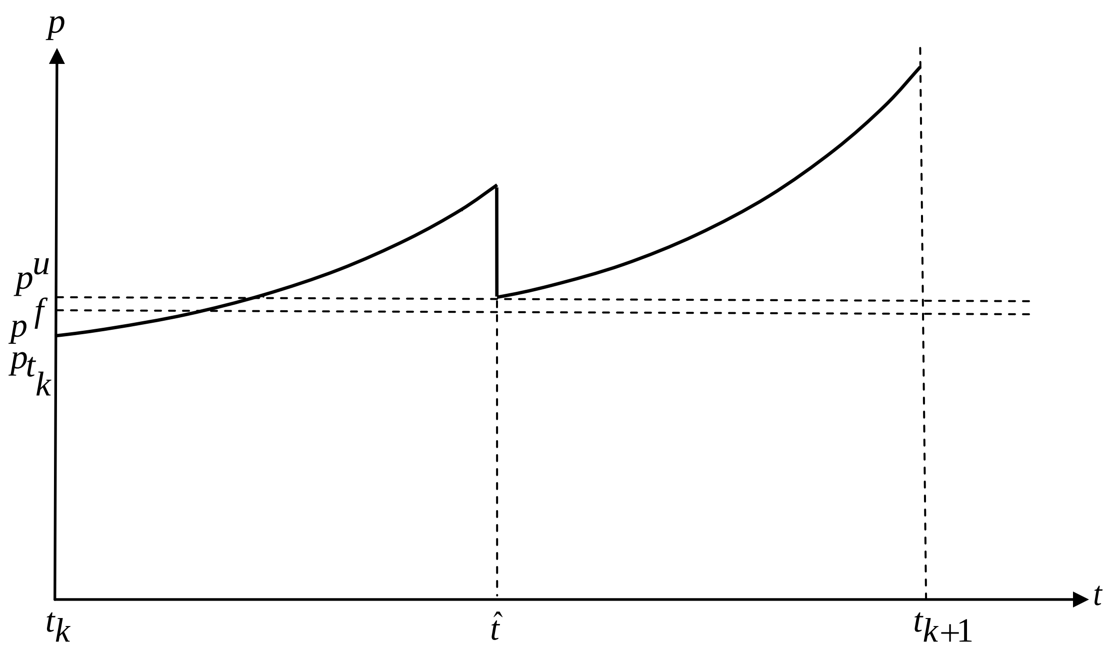
[Figure XI] *Case T*1: The Price Collar Is Completely Unbuttoned and Does Not Affect the Price Path



[Figure XII] *Case T*2 (*a*): The Price Path When Conditions (24), (26), and (35) Hold



[Figure XIII] *Case T*2(*b*): The Price Path When Conditions (29), (31), and (35) Hold



[Figure XIV] *Case S*: Sequential Market Stability Reserve Auctions