DISCUSSION PAPER

APRIL 2006 ■ RFF DP 06-18

Discounting and Relative Prices

Assessing Future Environmental Damages

Michael Hoel and Thomas Sterner

1616 P St. NW
Washington, DC 20036
202-328-5000 www.rff.org

RESOURCES
FOR THE FUTURE

Discounting and Relative Prices: Assessing Future Environmental Damages

Michael Hoel and Thomas Sterner

Abstract

Environmentalists are often upset at the effect of discounting costs of future environmental damage, e.g., due to climate change. An often-overlooked message is that we should discount costs but also take into account the increase in the relative price of the ecosystem service endangered. The effect of discounting would thus be counteracted, and if the rate of price rise of the item was fast enough, the effect might even be reversed. The scarcity that leads to rising relative prices for the environmental good will also have direct effects on the discount rate itself. The magnitude of these effects depends on properties of the economy's technology and on social preferences. We develop a simple model of the economy that illustrates how changes in crucial technology and preference parameters may affect both the discount rate and the rate of change of values of environmental goods. The combined effect of discounting and the change of values of environmental goods is more likely to be low—or even negative—the lower the growth rate of environmental quality (or the larger its decline rate), and the lower the elasticity of substitution between environmental quality and produced goods.

Key Words: discounting, future costs, scarcity, environment, climate change

JEL Classification Numbers: H43, Q32, Q54

Contents

1. Introduction **1**

2. Discounting and Relative Prices **3**

3. The Case of a Constant Elasticity of Substitution **8**

4. Development of the Discount Schedule over Time **12**

5. Conclusion **15**

Acknowledgments **17**

References **18**

Appendix A. Derivation of Equation (10) **20**

Appendix B. Derivation of Results in Table 2 ..... **21**

Derivatives of r 21

Derivatives of R 21

Derivatives of p 22

Appendix C. Proof that Long-run Declining U Implies $R-\rho<0$ in the Long Run **23**

$\sigma>1$ 23

$\sigma<1$ 23

$\sigma=1$ 23

Discounting and Relative Prices: Assessing Future Environmental Damages

Michael Hoel and Thomas Sterner*

1. Introduction

During the past 10 to 20 years, a vast economic literature has discussed various aspects of climate change and climate policies. Part of this literature gives cost-benefit analyses of various proposed measures to reduce greenhouse gases. In analyses of this type, economists add up all benefits and costs of any proposed policy. This summation of costs and benefits includes the current period and all future periods. In such cost-benefit analyses, future costs and benefits are discounted to present values through the use of a discount (or interest) rate. If some cost or benefit component at a future date t is of the magnitude Vt and the discount rate is r, the present value is $(1+r)^{-t}V_t$.

Environmentalists are often upset at the effect of discounting on such benefits and costs. There have been numerous debates—related to the climate problem as well as other environmental problems—in which environmentalists are dismayed to find the cost of some expected environmental damage is discounted to a present value that is basically insignificant.

The logic of discounting is in fact dramatic, but it is correct as long as the assumptions hold. With 5% discounting per year, a cost of $100 in 100 years is worth less than $1 today. There are various arguments in favor of low or nonlinear discount rates, and an extensive professional literature on the subject, but in this article we want to focus on another, separate, factor: that of changes in relative prices.[1] The cost Vt, for instance, of future environmental

* Hoel: Department of Economics, University of Oslo, P.O. Box 1095, Blindern, N-0317 Oslo, Norway. E-mail: m.o.hoel@econ.uio.no. Sterner: Department of Economics, University of Gøteborg, Sweden; and Resources for the Future. Tel: 46 31 7731377; Fax: 46 31 7731326; e-mail: Thomas.sterner@economics.gu.se

[1] Important contributions include Arrow et al. (1996), Cline (1992), Dasgupta (2001), Hasselmann (1999), Heal (1997), Horowitz (1996), Lind (1982), Nordhaus (1997), Portney and Weyant (1999), Schelling (1995), Shogren (2000), and Weitzman (1994, 1998).

damage, should be valued in real future prices. However we only know the prices today. Our best estimate of Vt is $V_t = V_0(1+p)^t$, where p is the expected rise per year in real price of the relevant good (i.e., the price relative to some general price level such as the consumer price index). For an environmental resource that will become scarce in the future, the relative price will typically rise—and this effect may again be exponential—and thus is potentially just as powerful arithmetically as is discounting. Indeed, combining the two formulae we see that the discounted, present, real cost of any given item of damage in the future will be $V_0(1+p)^t(1+r)^{-t}$. This general point is well known to economists, but is often overlooked.

The effect of future scarcity of environmental goods is difficult to study empirically for two reasons. First, it is difficult because we are speaking of a fairly recent phenomenon, and second, because many environmental goods are unpriced nonmarket goods and thus we are speaking more of a willingness to pay than of a regular price. However, there are market goods (partly related to environmental factors) that can illustrate the effect of scarcity on prices. One clear example is that of land and housing markets. We select this example because the stock of housing typically exhibits scarcity. Houses, of course, can be built and modernized, which no doubt explains a large part of the rise in price, but the underlying scarcity of space for housing lots is likely an important explanatory factor why housing should appreciate faster in densely populated areas. We choose to illustrate this with the housing market in the United Kingdom, where land scarcity is important. Table 1 shows that the Greater London area where land is scarcest has a rate of price escalation of 17.8%, compared with 11.5% in Scotland, and with the U.K. national average of 15.9%. The retail price index for the same period showed an inflation of 7.4%, far lower than even the lowest price increase in Table 1. The rate of interest was also much lower; the repo rate of the Bank of England for this period was between 3.5% and 7.5%.[2]

---

[2] Bank of England (2005).

Table 1. The Increase in the Price of Houses in the United Kingdom, by Region

Area	Annual % price increase 1993–2004	Area	Annual % price increase 1993–2004
Greater London	17.8	Wales	16.1
South West	16.7	Yorkshire/Humbershire	15.7
East Midlands	16.6	North	15.5
West Midlands	16.6	North West	15.2
South East	16.3	North Ireland	13.4
East Anglia	16.2	Scotland	11.5

Source: Halifax and Bank of Scotland (2005).

The simple, but often overlooked, message is that, in calculating the costs of some future damage (say, of excess carbon in the atmosphere), we should discount costs but also take into account the increase in the relative price of the ecosystem service endangered (biodiversity, clean air, water, arable land, coastal ecosystems, etc.). Thus the effect of discounting would be counteracted, and if the rate of price rise of the item were fast enough it might even be reversed. This point is in its simplest form well established in the economics literature, and has been explicitly or implicitly used in discussions of climate policy.[3] Matters are somewhat more complicated than they first appear, however. The scarcity that leads to rising relative prices for the environmental good will also have direct effects on the discount rate itself. The magnitude of these effects depend on the growth rate of the economy (or, more generally, on properties of the economy's technology) and on properties of the social preference function. In this paper we develop a simple model of the economy that illustrates how changes in crucial parameters may affect both the discount rate and the rate of change of values of environmental goods.

2. Discounting and Relative Prices

In economic models of dynamic optimization and cost-benefit analyses a frequently used objective function is of the type

(1) $$W = \int_0^T e^{-\rho t} U(C(t)) dt ,$$

[3] See, e.g., Hasselmann (1999), Nordhaus (1997), and Vennemo (1997).

where $C(t)$ stands for consumption at time t and U can be interpreted as a measure of well-being or utility. At the individual (person or household) level, the function given by (1) represents the person's preferences regarding alternative consumption profiles through life. In this case, it is natural to interpret the time horizon T as the (maximal) remaining lifetime of the person. The trade-offs between consumption at different points of time are given partly by the utility discount rate ρ, and partly by the utility function U. The larger is ρ, the more weight is given to the present relative to the future. Economists usually assume $\rho > 0$, implying that if one is faced initially with a situation where consumption is constant over time, and one is offered a particular increase in consumption in any year, one would prefer to have this consumption increase early rather than late. The function U is assumed to be strictly concave with $U(0)=0$, so that it increases less than proportionately with consumption. The more concave U is, the more weight is given to periods with low consumption relative to periods with high consumption. For the limiting case of $\rho = 0$, this means that if one is offered a particular increase in consumption in any year, one would prefer to have this consumption increase in a period when consumption is low rather than in a period when it is high.

In the context of the climate problem, the time perspective is longer than the lifetime for any particular generation. For such problems, the interpretation of Equation (1) is therefore slightly different from what we gave above. For issues with time perspective of a century or longer, it is natural to interpret the function (1) as a representation of society's preferences over distributions of consumption across generations. In this case, we interpret $C(t)$ as average per capita consumption at time t.[4] An assumption of $\rho > 0$ means that society gives the current generation more weight than it gives future generations, and that society gives future generations lower weight the more distant they are. In the economics literature, there has been an extensive discussion of whether a positive value of ρ can be given an ethical justification. For the points we make in the present article, it makes no difference whether $\rho > 0$ or $\rho = 0$.

The concavity of U measures inequality aversion: The more concave U is, the more weight is given to generations with low consumption relative to generations with high

[4] In this simple presentation we ignore all issues regarding distribution of consumption among different persons at any point of time.

consumption. In a situation with economic growth (rising $C(t)$), the future is thus given lower weight the more concave U is. In a situation with economic decline, however, the value of future consumption might even be given a higher weight than current consumption (we would thus have a negative discount rate if the rate of economic decline was sufficient to outweigh the effect of utility discounting ρ). Finally, because it is natural to let T be very large for analyses of climate issues and several other environmental problems, economists often choose an infinite time horizon.

The appropriate interest rate r for discounting consumption when preferences are given by (1) is

(2) $$r = \rho + \frac{-\frac{d}{dt}U'(C(t))}{U'(C(t))}.$$

The interpretation of this discount rate is, somewhat loosely, as follows: Imagine society makes some investment which causes present per capita consumption to go down by one unit (e.g., $1,000). What is the minimum increase in consumption one year ahead in order for W to not decline, provided there are no further changes in consumption after one year? The answer to this question is $1 + r$, where r is given by (2). This discount rate has two parts: one pure time preference or utility discounting ρ, and one related to the fact that additional money is less valuable to those (in the future) who will be richer than people are today.

With a concave utility function, U' is declining over time when consumption is growing, so that both terms in this expression are positive for this case. It is often assumed that the utility function has the simple form

(3) $$U(C) = \frac{1}{1-\alpha}C^{1-\alpha} \text{ for } \{\alpha > 0, \alpha \neq 1\} \text{ and } U(C) = LnC \text{ for } \alpha = 1.$$

This specification has the advantage that the elasticity of utility with respect to consumption is constant.[5] In this case, the appropriate discount rate *r* is (4), which is often called the Ramsey rate:

(4) $$r(t) = \rho + \alpha g_C(t),$$

where $g_C(t)$ is the relative growth rate of consumption. If, e.g., $\rho = 0.01$, $\alpha = 1.5$, and $g_C(t) = 0.025$, we find $r = 0.0475$, i.e., a discount rate of almost 5%. Notice that this discount rate will be constant over time only if the growth rate of consumption is constant over time. If we believe that consumption growth will be slower in the future than it is at present, future discount rates in our calculations should be set lower than present rates. For instance, Azar and Sterner (1996) show that limits to future economic growth imply lower discount rates and therefore higher values of damage per ton of carbon emitted.

In the debate on growth and sustainability, the so-called pessimists point to scarce resources as a reason for limits to growth, whereas the so-called optimists point to technology and new sectors as sources of growth. Clearly, communication and computing are two examples of phenomenal economic growth that use few scarce natural resources. But if future growth is concentrated in some sectors while other sectors do not grow, then this growth implies a changing output composition and presumably rising prices in the sectors that do not grow. These could include solitary access to unspoiled nature, but conceivably also include important goods such as clean water and other vital inputs. In studies of environmental issues it is therefore useful to explicitly distinguish between environmental goods and other consumption goods, which is not possible in the aggregate approach above. Let us use *E* to represent some aggregate measure of the environmental quality in society, while *C* is an aggregate measure of all other goods. Instead of the utility function given in (1), we assume $U=U(C,E)$, so that the objective function (ignoring time references to simplify notation) is changed to

(5) $$W = \int_0^\infty e^{-\rho t} U(C,E)\,dt \,.$$

---

[5] These utility functions are sometimes also referred to as constant relative risk aversion (CRRA) functions.

With this change, the appropriate discount rate r is changed from (2) to

$$r = \rho + \frac{-\frac{d}{dt}U_C(C,E)}{U_C(C,E)}, \tag{6}$$

where subscripts represent partial derivatives. As argued earlier, to calculate the future value of a change in environmental quality we must consider both discounting and the change in the relative price (or valuation) of the environmental quality. The valuation of the environmental good is given by U_E / U_C . This fraction tells us the amount that current consumption must increase to just offset a deterioration in current environmental quality of one unit (i.e., to make current utility or well-being the same before and after the change in environmental quality and consumption). The relative change in this price, previously denoted p, is thus

$$p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)}. \tag{7}$$

This price change will depend on the development over time of both consumption and the environmental quality. If C increases over time and E is constant or declines, p will be positive for most reasonable specifications of the function U. The combined effect of discounting and the relative price increase of environmental goods is given by $r - p$. If both r and p are positive, the sign of the combined effect is ambiguous without further specification of the utility function U.

Writing this paper in Oslo in January, an example that comes to mind as an effect of climate change is the loss in skiing areas. This would be sad enough but the potential consequences of climate change are, of course, much more serious. For the roughly half of humanity that lives in Asia, assessments speak of rising temperatures, which will cause decreasing water availability, drought, water and food shortages, spread of disease through several mechanisms (directly as a result of temperature, as well as through various disease vectors), coastal inundation, and population displacement.[6] In this context, we see that E is directly associated with major items such as water, shelter, land, homesteads, and food. With this definition, a fall in E or a rise in the marginal cost of obtaining these ecosystem resources obviously has major welfare consequences.

3. The Case of a Constant Elasticity of Substitution

The properties of the utility function $U(C,E)$ will, of course, depend on how we measure environmental quality. Nevertheless, it is useful to illustrate the points made above with a simple example. Consider the following constant elasticity of substitution utility function

$$U(C,E) = \frac{1}{1-\alpha}\left[(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}, \tag{8}$$

where σ is the elasticity of substitution, which is positive.[7] This elasticity is easiest to interpret if we consider the hypothetical case where environmental quality is a good that consumers can purchase in the market. If the price of this environmental good increases by 1% relative to the price of other consumer goods, the purchase of the environmental good will decline by σ% relative to the purchase of other consumption goods. The lower the elasticity of substitution, the less willing consumers are to substitute away from environmental quality as the price of environmental quality increases.

The parameter γ has no easy, direct interpretation. Consider, however, the following variable γ^* which will figure prominently in our derivation of the discount rate r based on (8):

---

[6] See, for instance, the impact assessments by the Intergovernmental Panel on Climate Change (IPCC 2001).

[7] If σ =1, we get a Cobb-Douglas function instead of (8): $U(C,E) = \frac{1}{1-\alpha}\left[C^{1-\gamma}E^{\gamma}\right]^{1-\alpha}$. If $\alpha = 1$, we get $U(C,E) = \frac{\sigma}{\sigma-1} Ln\left[(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}\right]$, and if $\alpha = \sigma = 1$, we get $U(C,E) = Ln\left[C^{1-\gamma}E^{\gamma}\right]$.

(9) $$\gamma^* = \frac{\gamma E^{1-\frac{1}{\sigma}}}{(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}}.$$

From (8) it follows that we could also write this as

(9) $$\gamma^* = \frac{U_E E}{U_E E + U_C C} = \frac{\frac{U_E}{U_C}E}{\left(\frac{U_E}{U_C}E\right) + C}.$$

Using the market analogy above, the variable γ^* may be interpreted as the value share of environmental quality. This tells us what share of their total consumption expenditures consumers would use on environmental quality if environmental quality was a good that could be purchased in the same manner as other consumption goods. An alternative interpretation of γ^* is that $\gamma^*/(1-\gamma^*)$, tells us by what percent the environmental quality must increase to offset a reduction in the consumption level by 1%.

By a suitable choice of units, we may set $\gamma^* = \gamma$ at our initial time (t = 0). From the interpretation above, this initial value of γ^* tells us somewhat loosely how large the value of the environmental quality is relative to the total consumption value (i.e., of environmental quality and other consumption goods). Over time, γ^* will change unless *C* and *E* change proportionately. Under the reasonable assumption that *C* is growing faster than *E*, γ^* will rise (decline) over time if $\sigma < 1$ ($\sigma > 1$). For the constant elasticity of substitution utility function with $\sigma < 1$, we have a case that Gerlagh and van der Zwaan (2002) call “poor substitutability” between environmental quality and other goods. If *C/E* grows without bounds in this case, the value share γ^* will approach 1 over time.[8]

[8] This could be illustrated if we consider for a moment *E* as a source of (or as equivalent to) food and water. In an economy where all sectors grew but food and water declined, it is clear that the relative price of food and water would increase so fast that the small amount of physical output in this sector would soon assume a share of close to 100% of total value in the economy.

Notice that the utility function (8) includes a parameter α that has an interpretation similar to that of α in (3). Moreover, (8) is identical to (3) for the special case of $\gamma = 0$. The same is true if *E* is proportional to *C*. For the general case of $0 < \gamma < 1$, with *E* and *C* growing at different rates, the utility function (8) generally gives a different interest rate than (4). From (6) and (8), tedious but straightforward derivations give[9]

$$(10) \qquad r = \rho + \left[(1-\gamma^*)\alpha + \gamma^* \frac{1}{\sigma}\right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma}\right)\right] g_E ,$$

where g_E is the relative growth rate of environmental quality. Notice that (10) is identical to (4) if $\gamma^* = 0$, $g_C = g_E$, or $\alpha\sigma = 1$. Generally, however, (10) may give a lower or higher discount rate than (4). For the reasonable case of $g_C > g_E$, (10) gives a lower or higher interest rate than (4) depending on whether $\alpha\sigma > 1$ or $\alpha\sigma < 1$. Notice also that if $\sigma \neq 1$ and $\alpha\sigma \neq 1$ the discount rate will not be constant over time, even if the growth rates for *C* and *E* are constant (because γ^* changes over time when $\sigma \neq 1$).

As for the relative price of environmental quality, it follows from (7) and (8) that

$$(11) \qquad p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)} = \frac{1}{\sigma}(g_C - g_E) .$$

The price change is thus positive, provided consumption increases relative to environmental quality over time, and that the price change is larger the smaller is the elasticity of substitution. If, e.g., the environmental quality is constant and consumption increases by 2.5% a year, and the elasticity of substitution is 0.5, this price will increase by 5% a year.

Using *R* to denote the combined effect of discounting and the relative price increase of environmental goods, i.e., $R = r - p$, it follows from (10) and (11) that

---

[9] See Appendix A for the derivation.

(12) $$R = \rho + \left[(1-\gamma^*)\left(\alpha - \frac{1}{\sigma}\right) \right] g_C + \left[\gamma^*\alpha + (1-\gamma^*)\frac{1}{\sigma} \right] g_E .$$

In Appendix B, we derive derivatives of *r, p*, and *R* with respect to the technology variables g_C and g_E, and the preference parameters α and σ. The signs are summarized in Table 2.

Table 2. Sign of Derivatives of *r*, *p*, and *R* with respect to g_C, g_E, α, and σ

	r	***p***	***R = r − p***
g_C	+	+	− if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$
g_E	− if $\alpha\sigma < 1$ + if $\alpha\sigma > 1$	−	+
α	Depends on γ^*, g_C and g_E (+ if $g_C > 0$ and $g_E \geq 0$)	0	Depends on γ^*, g_C and g_E (+ if $g_C > 0$ and $g_E \geq 0$)
σ	− (if $g_C > g_E$)	− (if $g_C > g_E$)	+ (if $g_C > g_E$)

The table reveals several interesting results. First, we see that if $g_C > 0$ and $g_E \geq 0$, the discount rate is higher the higher is the value of the parameter α (which measures the degree of inequality aversion). This is the same result we had for the simple case of only one good in the utility function. Moreover, the combined effect of the discount rate and the change in relative prices (i.e., *R*) is also higher the higher is α.

A second result is that for the reasonable case of $g_C > g_E$, an increase in the elasticity of substitution between environmental quality and other consumption will reduce the discount rate, but will increase the combined effect of the discount rate and the change in relative prices.

A third result concerns the effects of changes in growth rates. Changes in the growth rates g_C and g_E change the discount rate and the combined effect of the discount rate and the change in relative prices in the same direction if $\alpha\sigma > 1$. The case of $\alpha\sigma < 1$ (low elasticity of substitution and limited inequality aversion, which dose not seem unreasonable) is more interesting: In this case, changes in the growth rates g_C and g_E change the discount rate and the combined effect of the discount rate and the change in relative prices in the opposite direction. In particular, the higher is the consumption growth rate, the lower is the combined effect of discounting and price changes.

Notice that R is more likely to be low—or even negative—the lower the growth rate of environmental quality (or the larger its decline rate) and the lower the elasticity of substitution between environmental quality and produced goods. Notice also that if this elasticity is below 1 (and $g_C > g_E$), the variable γ^* defined by (9) will be rising over time. Over time, γ^* will approach 1, and it follows from (12) that R will approach $\rho + \alpha g_E$. In the numerical example above, we assumed $\rho = 0.01$ and $\alpha = 1.5$. If E declines by 0.67% per year in the long run, the value of R approaches zero in the long run, implying that the present value of specific future environmental damage should be approximately independent of how far into the future it occurs.[10]

It has been argued that conventional discounting may lead to too little mitigation today and therefore to unacceptably large climate changes in the future (see, e.g., Hasselmann et al., 1997). Although there may be some truth in this, it is important to remember that the discount rate should be an endogenous variable that is not independent of the evolution of the economy. This is particularly clear when we consider the simple aggregate economy with the discount rate given by (4). If the future is bad in the sense that aggregate consumption declines in the long run, the second term in the expression will be negative. In such a situation the long-run discount rate will therefore be negative, provided the utility discount rate ρ is sufficiently close to zero. In Appendix C we show that a similar result holds for the two-good economy we have considered in this paper. If the environment develops in such a bad way that utility or well-being is declining in the long run, then our combined discount and price change rate R will be negative, provided the utility discount rate ρ is sufficiently close to zero.

4. Development of the Discount Schedule over Time

In order to see the effects on discounting of changing shares in utility due to different growth rates, we carried out a simulation assuming that the consumption good–producing sector grows at 2.5%, but the supply of the environmental good is constant. Initial values of the parameters *E, C,* and γ are chosen so that γ^*, the value share of the environment, is initially = 0.1. (Had E been an ordinary good, then we would have spent 10% of our aggregate income on it.)

---

[10] Notice that utility is declining in the long run in this case. The reason why we in spite of this get a nonnegative value of R is that we have assumed that the utility rate of discount δ is positive.

Initial values for E and C are normalized to 1, but C grows at 2.5% while E is constant. Thus, in some sense C becomes dominant, but with a low elasticity of substitution, ($\sigma = 0.5$), E becomes increasingly important for our utility the more its relative scarcity increases. This is measured by γ^*, which, as shown above, increases as long as $\sigma < 1$. In our case, γ^* grows from 0.1 to 0.5 in 90 years and to 0.9 in 180 years.

Figure 1. Components of Discounting, ρ = 0.01, σ = 0.5, α = 1,5, γ^*_0 = 0,1, g_C = 0.025

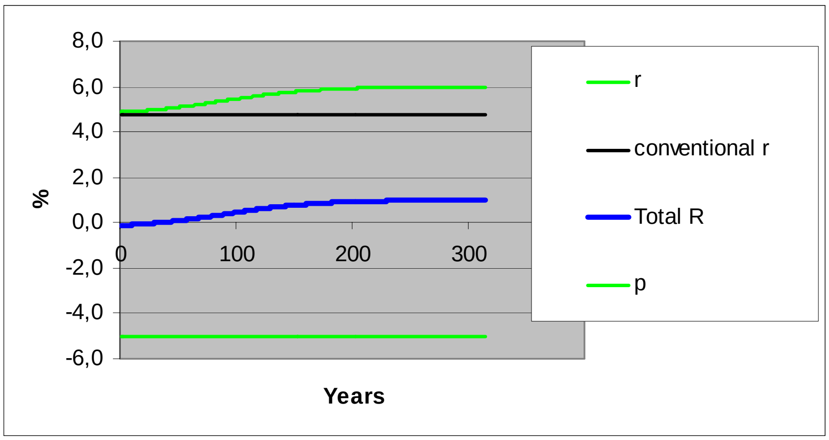

Figure 1 shows the components of discounting with a marginal elasticity of utility, α, of 1.5 and with an elasticity of substitution of 0.5. The conventional Ramsey rule discount rate would, as mentioned earlier, be 1.5*2.5 + 1 = 4.75%. Because $\alpha\sigma < 1$, our corrected discount rate is higher: It starts at 4.9% and rises slowly to 6%, but is counteracted by a relative price effect p of −5%, so that the effective actual discounting starts at −0.1% and rises gradually to + 1%. As mentioned above, the situation would be different if substitutability was easier. With an elasticity of substitution of 1 we have Cobb-Douglas, $\alpha\sigma > 1$, and a constant utility share γ^* of 0.1. Consequently, r is a constant 4.6%, lower than the conventional 4.75%. The price effect in this scenario exactly mirrors the growth rate of C (i.e., −2.5%) giving a total combined discount and price change rate of 2.1%.

The fact that the discount rates change over time due to the change in the composition of our utility basket (when C rises and E is constant) is an important result of our method, but it makes it hard to readily compare overall results of changing the parameters. We have therefore calculated the constant rates that are equivalent to our varying rates for 100 years.[11]

Table 3. A Comparison of Discount Rates

α	σ	**Convent *r***	r	p	**Total *R***
0.5	0.5	2.25	3.35	−5.00	−1.65
0.5	1	2.25	2.37	−2.50	−0.12
0.5	1.5	2.25	2.28	−1.67	0.61
1	0.5	3.5	4.24	−5.00	−0.76
1	1	3.5	3.50	−2.50	1.00
1	1.5	3.5	3.44	−1.67	1.77
1.5	0.5	4.75	5.12	−5.00	0.12
1.5	1	4.75	4.62	−2.50	2.13
1.5	1.5	4.75	4.60	−1.67	2.94

We see from Table 3 that, on the one hand, low values of σ give values of our discount rate r that are high compared with the conventional rates (which only depend on α). On the other hand, low values of σ also give high price effects. (The value of α does not affect p.) It is hard to say what reasonable values are. However, a value of σ of somewhere between 0.5 and 1 implies some reasonable degree of substitutability. Values above 1 would suggest that substitutability is so high we need hardly worry about the environment.

Our total discount factor including the price effect is, for the values chosen, always lower than the conventional rate. It is possible however to find combinations of values that give a higher discount rate than the conventional rate. For the range of values where both σ and α are between 0.5 and 1.5, our total combined discount and price change rate is in the interval − 1.6 to + 2.9%, all considerably below the conventional discount rates.

[11] The total result for our varying rates is compounded for 100 years and the corresponding fixed rate calculated. These calculations were in this case carried out for a growth rate of 2.5% in C with E constant. $\rho = \gamma^* = 0.1$.

5. Conclusion

We have in this paper taken as our starting point the concern felt by environmentalists that future environmental damage is given insufficient weight in economic calculations. We show that the broad intuition (that discounting should be complemented by a calculation of future prices that may well be rising if environmental goods are expected to become scarcer) is correct. By way of a simple example, we illustrate the importance of scarcity by looking at the market for real estate in the United Kingdom where house prices rise much faster in areas of land scarcity such as London than they rise in any other area. We built a model that is as simple as possible, but that still incorporates the following characteristics:

- Discounting is derived from a model of intertemporal utility maximization.
- Discounting rests on both pure time preference and the characteristics of the utility function.
- The utility function allows for different elasticities of utility.
- There is consumption substitution (to varying degrees) between a consumption good *C* and one environmental aggregate *E*.

With the help of this model we show that the formula for the rate of discount itself is different in a two-sector model with different growth rates than it is in an aggregate model of the economy. We can also directly relate price changes in the environmental good to scarcity and show that the same (substitution) parameter that is decisive for the rate of price appreciation also (together with other parameters) decides the discount rate. We show that future costs should be discounted at a rate that will not generally be constant—it will vary over time because the share of utility coming from consumption and environmental goods will vary over time. In order to compare our results with the traditional aggregate (Ramsey) formula, we calculate century–equivalent average rates and show that for some likely parameter values the total combined discount and price change rate should generally be considerably lower than the conventional discount rate—and in some cases even negative. Particularly in the case when climate change causes catastrophic change, the discount rate could be negative.

We believe that our results are sufficiently precise and detailed to be practicable at least at the level of numerical examples or sensitivity analyses conducted routinely as part of regular cost-benefit analyses or similar calculations, for instance in the framework of assessing investments and consequences in the area of climate economics.

Acknowledgments

Comments from Kjell-Arne Brekke, Reyer Gerlagh, Odd Godal, Richard Howarth, Åsa Löfgren, and Snorre Kverndokk are highly appreciated. This article was written during our participation in the project “Environmental economics: Policy instruments, technology development, and international cooperation,” which was conducted at the Centre for Advanced Study (CAS) at the Norwegian Academy of Science and Letters in Oslo, 2005 to 2006. The financial, administrative, and professional support of the Centre to this project is much appreciated. Thomas Sterner would also like to thank the Mistra CLIPORE program and Sida for support.

References

Arrow, K. J., W. R. Cline, K.-G. Mäler, M. Munasinghe, R. Squitieri, and J. E. Stiglitz. 1996. Intertemporal equity, discounting, and economic efficiency. In *Climate Change 1995*: *Economic and Social Dimensions—Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change*. J. P. Bruce, H. Lee, and E. F. Haites, eds., Cambridge, U.K.: Cambridge University Press, 125–144.

Azar, C., and T. Sterner. 1996. Discounting and distributional considerations in the context of global warming. *Ecological Economics* 19 (November): 169–184.

Bank of England. 2005. http://www.bankofengland.co.uk/statistics/index.htm. Accessed December 15, 2005.

Cline, W. 1992. *The Economics of Global Warming.* Institute for International Economics, ISBN 088132132X.

Dasgupta, P. 2001. *Human Well-being and the Natural Environment*, Part III. Oxford, UK: Oxford University Press.

Gerlagh, R., and B.C.C. van der Zwaan. 2002. Long-term substitutability and man-made goods. *Journal of Environmental Economics and Management* 44: 329–345.

Halifax and Bank of Scotland (HBOS). 2005. Halifax house price index, http://www.hbosplc.com/economy/includes/historic_data_01.11.2005.xls. Accessed November 2005.

Hasselmann, K. 1999. Intertemporal accounting of climate change—Harmonising economic efficiency and climate stewardship. *Climatic Change* 41: 333–350.

Hasselmann, K., S. Hasselmann, R. Giering, V. Ocana, and H. V. Storch. 1997. Sensitivity study of optimal CO_2 emission paths using a simplified structural integrated assessment model (SIAM). *Climatic Change* 37: 345–386.

Heal, G. M. 1997. Discounting and climate change—An editorial comment. *Climatic Change* 37: 335–343.

Horowitz, J. K. 1996. Environmental policy under a nonmarket discount rate. *Ecological Economics* 16: 73–78.

Intergovernmental Panel on Climate Change (IPCC). 2001. Working Group II Assessment. http://www.grida.no/climate/ipcc_tar/wg2/411.htm. Accessed December 15, 2005.

Lind, R., ed. 1982. *Discounting for Time and Risk in Energy Policy,* RFF Press, Resources for the Future, Washington, DC.

Nordhaus, W. D. 1997. Discounting in economics and climate change—An editorial comment. *Climatic Change* 37: 315–328.

Portney, P. R., and J. P. Weyant, eds. 1999. *Discounting and Intergenerational Equity*. Washington, DC: Resources for the Future.

Schelling, T.C. 1995. Intergenerational discounting. *Energy Policy* 23: 395–401.

Shogren, J. F. 2000. Speaking for citizens from the far distant future. *Climatic Change* 45: 489–491.

Vennemo, H. 1997. On the discounted price of future environmental services. Unpublished note, ECON, Oslo, Norway.

Weitzman, M. L. 1994. On the 'environmental' discount rate. *Journal of Environmental Economics and Management* 26: 200–209.

Weitzman, M. L. 1998. Why the far-distant future should be discounted at its lowest possible rate. *Journal of Environmental Economics and Management* 36: 201–208.

Appendix A. Derivation of Equation (10)

Using dots to denote derivatives with respect to time and El_x to denote an elasticity with respect to a variable *x,* differentiation of $U(C,E)$ gives

(A1) $$\frac{\dot{U}_C}{U_C}=\frac{U_{CC}\dot{C}}{U_C}+\frac{U_{CE}\dot{E}}{U_C}=\frac{U_{CC}C}{U_C}\frac{\dot{C}}{C}+\frac{U_{CE}E}{U_C}\frac{\dot{E}}{E}=\left(El_C U_C\right)g_C+\left(El_E U_C\right)g_E .$$

From (8), it follows that

(A2) $$U_C=\frac{\sigma}{\sigma-1}\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}-1}(1-\gamma)\left(1-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}},$$

so

(A3) $$El_C U_C=\left(\frac{(1-\alpha)\sigma}{\sigma-1}-1\right)(1-\gamma^*)\left(1-\frac{1}{\sigma}\right)-\frac{1}{\sigma}=\left(\frac{1}{\sigma}-\alpha\right)(1-\gamma^*)-\frac{1}{\sigma} .$$

Similarly, we find

(A4) $$El_E U_C=\left(\frac{1}{\sigma}-\alpha\right)\gamma^* .$$

Inserting (A3) and (A4) into (A1) gives

(A5) $$\frac{-\dot{U}_C}{U_C}=\left[\left(\alpha-\frac{1}{\sigma}\right)(1-\gamma^*)+\frac{1}{\sigma}\right]g_C+\left(\alpha-\frac{1}{\sigma}\right)\gamma^* g_E ,$$

which may be rewritten as (10).

Appendix B. Derivation of Results in Table 2

Derivatives of r

From (10) we find

$$\frac{\partial r}{\partial g_C} = (1-\gamma^*)\alpha + \gamma^* \frac{1}{\sigma} > 0$$

$$\frac{\partial r}{\partial g_E} = \gamma^* \left(\alpha - \frac{1}{\sigma} \right) > 0\,,$$

which has the same sign as $\alpha\sigma - 1$.

$$\frac{\partial r}{\partial \alpha} = (1-\gamma^*) g_C + \gamma^* g_E\,,$$

which is positive if both growth rates g_C and g_E are positive.

$$\frac{\partial r}{\partial \sigma} = -\frac{\gamma^*}{\sigma^2}(g_C - g_E)\,,$$

which has the opposite sign of $g_C - g_E$.

Derivatives of R

From (12), we find

$$\frac{\partial R}{\partial g_C} = (1-\gamma^*)\left(\alpha - \frac{1}{\sigma} \right),$$

which has the same sign as $\alpha\sigma - 1$.

$$\frac{\partial R}{\partial g_E} = \gamma^* \alpha + (1-\gamma^*)\frac{1}{\sigma} > 0\,,$$

$$\frac{\partial R}{\partial \alpha} = \frac{\partial r}{\partial \alpha}\,,$$

which is positive if both growth rates g_C and g_E are positive (see above).

$$\frac{\partial R}{\partial \sigma} = \frac{(1-\gamma^*)^2}{\sigma^2}(g_C - g_E) \, ,$$

which has the same sign as $g_C - g_E$.

Derivatives of p

The derivatives of p with respect to g_C, g_E, and α follow immediately from (11) and are not included in this appendix.

Appendix C. Proof that Long-run Declining *U* Implies $R-\rho<0$ in the Long Run

From (8) it follows that

(C1) $$\frac{\frac{d}{dt}U(C,E)}{U(C,E)}=(1-\gamma^*)g_C+\gamma^* g_E .$$

Clearly, if $g_C>0$ and $g_E\geq 0$, utility will be increasing over time. The interesting case is where $g_C>0$ and $g_E<0$, i.e., the environmental quality is declining at the same time that consumption is growing. We are particularly interested in the possibility of *U* declining, and will distinguish between the three cases of $\sigma>1$, $\sigma<1$, and $\sigma=1$.

$\sigma>1$

In this case, γ^* approaches zero in the long run, so that (C1) implies that the utility level must be increasing in the long run (although *U* might decline in the short run if *E* is declining at a sufficiently high rate).

$\sigma<1$

In this case, γ^* approaches one in the long run, so that (C1) and $g_E<0$ implies that the utility level must be declining in the long run. From (12) it follows that $R=\rho+\alpha g_E$ in the long run. For a sufficiently low utility discount rate ρ, the combined discount and price change rate must therefore be negative if the environmental quality is declining in the long run.

$\sigma=1$

In this case, γ^* is constant and equal to γ. For the utility level to be declining we see from (B1) that

(C2) $$g_E<\frac{1-\gamma}{\gamma}g_C,$$

so that (12) implies that

(C3) $$R<\rho+\left[(1-\gamma)(\alpha-1)\right]g_C+[\gamma\alpha+(1-\gamma)]\left(-\frac{1-\gamma}{\gamma}\right)g_C,$$

which may be rewritten as

(C4) $$R = \rho - \frac{1-\gamma}{\gamma} g_C < \rho .$$

We thus have a result similar to the case of $\sigma < 1$: For a sufficiently low utility discount rate ρ, the combined discount and price change rate must be negative if the long-run decline in the environmental quality is so strong that the utility level is declining.