

A Discounting Rule for the Social Cost of Carbon

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Abstract

We develop a discounting rule for estimating the social cost of carbon (SCC) given uncertain economic growth. Diminishing marginal utility of income implies a relationship between the discount rate term structure and economic growth uncertainty. In the classic Ramsey framework, this relationship is governed by parameters reflecting pure time preference and the elasticity of the marginal utility of consumption; yet disagreement remains about the values of these parameters. We calibrate these parameters to match empirical evidence on both the future interest rate term structure and economic growth uncertainty, while also maintaining consistency with discount rates used for shorter-term benefit-cost analysis. Such an integrated approach is crucial amidst growth uncertainty, where growth is also a key determinant of climate damages. This results in an empirically driven, stochastic discounting rule to be used in estimating the SCC that also accounts for the correlation between climate damage estimates and discount rates.

1 Introduction

Discounting theory and practice has long recognized the difference between positive, or descriptive, approaches based on observed market interest rates and normative, or prescriptive, approaches based on an appeal to broader social preferences. The debate over both the approach and the specific discount rate choice has been particularly pronounced in calculations of the social cost of carbon. The social cost of carbon equals the net present value of damages from an incremental ton of carbon dioxide emissions today. These damages, undiscounted, are a stream of annual monetized impacts that persist for at least several centuries due to the long residence time of carbon dioxide in the atmosphere. Thus, the SCC is highly sensitive to the chosen discount rate.

The United States and a number of other countries have largely embraced the descriptive approach, with government benefit-cost guidance using a small number of deterministic discount rates

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based on observed market rates. Yet, application of the descriptive approach using a deterministic rate runs into trouble when estimating the SCC with uncertainty about economic growth. If discount rates and uncertain economic growth rates are treated as uncorrelated, it implies that society would discount a given dollar of monetized impact in 2200 exactly the same when income per capita grows to \$150,000 versus \$5,000,000. This is the range of cases associated with long-term growth rates varying from 1% to 3%, a range consistent with recent studies of historic data. Such constant and uncorrelated discounting seems both instinctively incorrect and runs counter to decades of economic thought relating the discount rate in part to the lower incremental value of additional consumption to better-off individuals in the future.

With this in mind, a 2017 NAS committee report recommended a hybrid approach for discounting in the context of the social cost of greenhouse gases. In estimating the SCC, the NAS report recommended employing a Ramsey-like discount rate formula that is linear in the (uncertain) economic growth rate, implying higher discount rates under higher growth scenarios. That is, unlike the approach noted above, damage scenarios with different economic growth rates would be discounted differently, and expectations across these uncertain discounting and damage outcomes would be taken together. Further, the NAS report recommended the parameters of the Ramsey-like formula should be chosen to match observed near-term consumption rates of interest, to be consistent with other elements of benefit-cost analysis. However, the NAS recommendations did not provide much further guidance and, generally, there are a wide range of Ramsey parameters that match a given near-term rate for a given economic growth rate.

This paper implements the NAS discounting recommendations by proposing and demonstrating an approach that furthers the NAS hybrid descriptive/prescriptive recommendation with an additional descriptive component. In particular, we propose using empirical evidence on the shape of the long-run yield curve term structures derived from studies of long-run interest rate behavior—coupled with the NAS recommendation to match a near-term consumption interest rate—to identify a unique parameterization of the Ramsey-like discount rate formula. Thus, the descriptive element relates to both observed near-term interest rates and (modeled) long-term interest rate behavior. Moreover, the resulting discounting rule is calibrated using a distribution of future economic growth that itself is estimated in a manner consistent with a parallel NAS recommendation.

This approach also draws from, and provides a practical means to applying, a rich literature showing that introduction of uncertainty in the discount rate or its underlying drivers will lead to a certainty-equivalent discount rate that declines with the time horizon (e.g., Weitzman 1998, 2001; Newell and Pizer 2003; Groom et al. 2007; Gollier 2014; Freeman et al. 2015; Bauer and Rudebusch 2020*b*). The certainty-equivalent rate is the single rate delivering the same expected discount factor as the distribution of uncertain future discount rates. It also defines the term structure of discount rates, or the pattern of certainty-equivalent rates over different time horizons. This decline is a

natural result of Jensen’s inequality because of the convexity of discount factors, implying that the certainty-equivalent discount rate is lower than the expected discount rate. If uncertainty in the discount rate is persistent over time, the certainty-equivalent rate declines more over longer time horizons.

The empirical literature has focused on quantifying the declining term structures of long-term discount rates implied by the observed time-series behavior of market interest rates (Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015; Bauer and Rudebusch 2020b). These resulting term structures can be applied to a given stream of expected damages, but doing so implicitly assumes there is no correlation between uncertain discount rates and uncertain damages.

In a prescriptive, time-additive expected utility framework, the decline in the term structure of discount rates is governed by the degree of curvature in the utility function and the degree of persistent uncertainty in consumption growth. If these are non-zero, the term structure declines with the time horizon.

How? With isoelastic utility ($u(c) = \frac{c^{1-\eta}-1}{1-\eta}$), the discount rate in period τ for a given growth rate depends on two parameters: ρ , representing the rate of pure time preference (how much we discount utility) and η , representing the elasticity of the marginal utility of consumption (c) (capturing the elasticity of intertemporal substitution as well as risk/inequality aversion). The simple growth-conditional discounting rule is given by

$$r_\tau = \rho + \eta g_\tau. \tag{1}$$

This expression is referred to as the Ramsey equation based on the optimality condition for the Ramsey (1928) growth model, where the discount rate is also equal to the return on capital investment. Regardless of optimality, this rule can be used to determine the discount rate as a function of economic growth rates. It is important to emphasize that future economic growth is uncertain, meaning g_τ is a random variable. This in turns implies that the discount rate, r_τ , is also a random variable so long as $\eta \neq 0$. In particular, r_τ is the stochastic discount rate, leading to a stochastic discount factor, commonly used in the financial economics literature:

$$\begin{aligned} SDF_t &= e^{-\sum_{\tau=1}^t r_\tau} \\ &= e^{-\sum_{\tau=1}^t \rho + \eta g_\tau}. \end{aligned} \tag{2}$$

SDF_t is the stochastic discount factor that converts a dollar-valued cost or benefit occurring t periods in the future to a present value. The expected SCC,¹ which is the number typically used in regulatory analyses, is the expected sum of the stream of discounted climate impacts over the time

¹Throughout this paper, the expectation is taken with respect to the uncertain growth rate.

horizon. That is, $E[SCC] = E[\sum_{t=0}^{\infty} SDF_t \cdot MD_t]$ where MD_t represents the uncertain future marginal damages from an emissions pulse now, with both SDF_t and MD_t depending on stochastic growth rates g_t . In particular, with $\eta > 0$, high-growth states of the world will have higher discount rates and hence lower stochastic discount factors, and vice versa. Hence, the discount rate (and discount factor) will be uncertain whenever economic growth is uncertain.

Past quantitative modeling of the SCC using the prescriptive Ramsey approach has typically involved little to no uncertainty about economic growth. These efforts have often chosen the discounting parameters ρ and η to match a particular value for r_t in equation (1) for some near-term period t , perhaps based on recent market interest rates (e.g., Nordhaus 2014). For a given growth rate and discount rate, there will be many potential viable combinations, and the parameters are typically chosen to be roughly in line with priors on the values of those parameters. With deterministic and relatively constant growth rates, it matters little the degree to which the discount rate is governed by ρ or because of η , so long as the combination $\rho + \eta g$ remains fixed.

When growth is uncertain, however, the relative importance of η (versus ρ) has significant implications for the resulting discount rate and ultimately the SCC. As we noted in our motivation, it has significant implications for the shape of the term structure of discount rates. A relatively large (small) η leads to more (less) uncertainty in the discount rate. This, in turn, leads to more (less) decline in the term structure of long-term discount rates, a key point that motivates our approach.

Equally important, the magnitude of η allows for correlation of the discount rate with damages, which typically depend on the size of the economy and economic growth. The correlation will be very strong when η is large, and weak when it is small.² As we demonstrate in this paper, failure to account for this correlation would result in a significant overstatement of the level and variance of the SCC.

Taking advantage of recent efforts to quantify long-term economic growth uncertainty (Müller, Stock and Watson 2019), we propose a novel approach to choosing ρ and η while also matching a particular near-term rate. In particular, we propose to use evidence on the shape of the term structure of long-term interest rates from the descriptive literature to guide our parameter choice. This contrasts with past prescriptive approaches that have been guided by outside studies, surveys, and priors.

The fundamental contribution of this paper is a methodology to calibrate the parameters of this discounting rule (1)—that is, an approximation of the relationship between discount rates and

²A positive correlation between discount rates and damages works to offset the direct effect of implementing declining discount rates, which on its own would increase the SCC. For example, in the DICE model (Nordhaus and Sztorc 2013), climate damages are a percentage of economic activity, so cases of high economic growth feature both high damages and high discount rates, with offsetting impacts on the SCC. Ignoring this correlation by using a deterministic declining discount rate but stochastic growth would threaten to overstate the SCC—potentially by a large amount—by discounting high- and low-damage scenarios equally, despite the fact that high-growth/high-damage states should be discounted more.

economic growth rates. This calibrated relationship (in the form of two parameters, ρ and η) can then be used as an input into the computation of the SCC in an integrated assessment model (IAM) framework with uncertain economic growth, thereby allowing the discount rate to vary stochastically with growth. The advantage of our approach is that it can account for uncertainty in the discount rate (through uncertainty in the economic growth rate) in a way that is both internally consistent within an SCC IAM as recommended by the NAS, while also approximating additional empirical evidence on the term structure. By implementing the NAS recommendation for a Ramsey-like discounting approach that is fully parameterized by a descriptive approach, we satisfy a desire to relate different discount rates to scenarios with very different levels of economic growth, while hewing to the data-driven descriptive approach used in U.S. regulatory analysis and other benefit-cost analysis.

In the remainder of our paper we first present a detailed description of our approach, which involves matching the term-structures from descriptive and prescriptive models. The next section describes the input data we use. In particular, we need three key inputs: (1) evidence from the economic literature on the shape of the term structure of market interest rates; (2) near-term target discount rates that are consistent with regulatory guidance and observed market rates, to which we anchor the empirical term structures; and (3) projections of long-term uncertainty in economic growth. Finally, we present estimation results, as well as an illustrative example of the consequences for the SCC.

2 Methodology

This section lays out our approach to calibrate the parameters of the discounting equation (ρ and η in equation (1)). Broadly speaking, we calibrate ρ and η by finding the parameter values that, when applied to data on the distribution of economic growth via equation (1), produce a term structure that best matches the findings of the relevant empirical term structure literature.

Our methodology starts by noting that the values of ρ and η have implications for the stochastic nature of the discount factor and hence for the term structure of certainty equivalent discount rates. As is common in the term structure literature (Weitzman 1998, 2001; Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015), we focus on the *certainty-equivalent, risk-free discount rate* (denoted r_t^{ce})—the (time average) discount rate that is consistent with the expectation of the stochastic discount factor as presented in equation (2):

$$\begin{aligned} e^{-t \cdot r_t^{ce}} &= E[SDF_t] \\ &= E[e^{-\sum_{\tau=1}^t \rho + \eta g_\tau}]. \end{aligned} \tag{3}$$

Re-arranging this to solve for r_t^{ce} yields

$$\begin{aligned}
r_t^{ce} &= -\frac{1}{t} \ln \left(E[e^{-\sum_{\tau=1}^t \rho + \eta g_\tau}] \right) \\
&= -\frac{1}{t} \ln \left(e^{-\rho t} E[e^{-\sum_{\tau=1}^t \eta g_\tau}] \right) \\
&= \rho - \frac{1}{t} \ln \left(E[e^{-\sum_{\tau=1}^t \eta g_\tau}] \right). \tag{4}
\end{aligned}$$

For a given distribution of growth $\{g_\tau\}$ and set of parameters (ρ, η) , equation (4) defines a *term structure*: a set of rates $\{r_t^{ce}\}$ over time horizon t . Note that while the last term is subtracted, the natural logarithm of the expectation is typically a negative number, since it is the logarithm of something like a discount factor, which is less than one. Hence, the second term—accounting for the fact that it is subtracted—is typically positive. Indeed, in the case of no uncertainty, the equation reduces to the Ramsey equation. This positive second term tends to shrink as time progresses when there is uncertainty in g_τ due to Jensen’s inequality and as explained in Weitzman (1998).³ This leads to a declining discount rate.

The magnitude of this decline depends on two factors: the degree of uncertainty in economic growth (the distribution of $\{g_\tau\}$) and the magnitude of the η parameter, which multiplies the g_τ terms and hence scales that uncertainty up ($\eta > 1$) or down ($\eta < 1$). At one extreme if $\eta = 0$, the final term of equation (4) vanishes and the term structure of the certainty-equivalent rate would be perfectly flat: $r_t^{ce} = \rho$, which does not depend on t . As η increases, the term structure becomes more negatively sloped with time. In other words, for a given distribution of economic growth rates $\{g_\tau\}$, the slope of the term structure is governed by the value of η , and its level is governed by ρ . A large η corresponds to a steeply declining term structure, and a small η corresponds to a flatter one. This suggests a way to determine η , given (i) evidence on the shape of the term structure and (ii) evidence on the uncertain distribution of economic growth.

We use this relationship (i.e., equation (4)) to calibrate the values of ρ and η using evidence on the term structure ($\{r_t^{ce}\}$) from the relevant literature on long-term interest rate behavior (Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015; Bauer and Rudebusch 2020b) and new estimates of the distribution of economic growth $\{g_\tau\}$ (Müller, Stock and Watson 2019).⁴ Formally,

³For example, this can be seen clearly in equation (6) in Newell and Pizer (2003) and equation (4) in Gollier (2014), where in the special case of normally distributed growth rates, there is a negative term proportional to the average variance of the growth rate (Gollier’s equation (9)). As noted by Gollier (2014), if growth uncertainty is persistent, that average variance grows over time, leading to a declining discount rate.

⁴For the purposes of estimation, we use the projections of per-capita economic growth from MSW, as opposed to per-capita consumption growth. The distinction between economic growth and consumption growth owes primarily to climate damages creating a wedge between the two (e.g., see Kelleher and Wagner 2018). Because the appropriate “target” term structures were derived based on a lengthy time series of historical interest rates, they do not contain significant amounts of climate damages. Therefore, when calibrating the ρ and η parameters, the conceptually correct calculation should be based on growth projections that similarly do not contain damages—i.e., “business as usual”

we determine the values of ρ and η that are most consistent with equation (4) by non-linear least squares through the following estimating equation:

$$(\hat{\rho}, \hat{\eta}) = \arg \min_{\rho, \eta} \sum_{t=2021}^{2300} \left(r_t^{ce} - \left(\rho - \frac{1}{t} \ln \left(\hat{E} \left[e^{-\sum_{\tau=1}^t \eta g_{\tau}} \right] \right) \right) \right)^2 \quad (5)$$

subject to two constraints. The first constraint is that the fitted rates in the near-term match the target near-term rates on average.^{5,6} The second constraint is that ρ is non-negative.⁷ The expectation in equation (5) is calculated as the average of 2000 draws from the growth rate distribution provided by Müller, Stock and Watson (2019), henceforth referred to as MSW, that we describe in detail in section 3.3:

$$\hat{E} \left[e^{-\sum_{\tau=1}^t \eta g_{\tau}} \right] = \frac{1}{2000} \sum_{s=1}^{2000} e^{-\sum_{\tau=1}^t \eta g_{\tau, s}}.$$

The basic idea behind equation (5) is that any given choice of ρ and η parameters, when applied to the distribution of future economic growth, implies a certainty-equivalent term structure. This is the second component inside the square in (5). Not all pairs of ρ and η produce term structures that are consistent with the empirical evidence on from the term structure literature on r_t^{ce} . Hence, we

economic growth rates (corresponding to MSW’s projections), and not after-damage consumption growth rates. Further, the alternative approach of using after-damage consumption growth rates introduce an additional complicating factor, which is that the discounting parameters would be dependent on the IAM chosen to convert economic growth to consumption growth, including its damage function, climate model, emissions baseline, and so on.

As a robustness check, we re-ran the parameter estimation using after-damage consumption growth from the output of a DICE model run that used the MSW economic growth projections as an input. The calibrated parameters are very similar; for example, under a 3% near-term rate, the value of ρ is effectively unchanged and the value of η falls from 1.53 to 1.40.

⁵This constraint ensures that the fitted term structure reflects the near-term rate, as is logically necessary. We define “near-term” as the average of the first 10 years of the projection. This minimizes the importance of the choice of the initial year. The results are not very sensitive to this assumption of a ten-year versus shorter average.

⁶The equation is estimated via non-linear least squares using the `R.nlm` (non-linear minimization) function. For each guess at the value of η , we find the value of ρ that achieves the constraint that the fitted near-term rates match the target near-term rates on average. We then find the η value, and corresponding ρ value, that minimizes the loss function in equation (5).

⁷For very low near-term discount rates, it is possible that the (ρ, η) pair that minimizes equation (5) features $\rho < 0$. This raises a conceptual problem in the Ramsey framework, where a negative ρ implies social time preference of the “wrong sign”. While a negative ρ may be unreasonable, many have endorsed using a ρ of zero (e.g., see Drupp et al. 2018; Stern et al. 2006). Therefore we also impose a non-negativity constraint of $\rho \geq 0$. If this constraint binds, the calibrated η is simply the value that is consistent with the first constraint—that is, it returns the appropriate near-term rate in certainty equivalent terms. This is given approximately (this equation is approximate because the appropriate certainty-equivalent rate corresponds to the log of the expected discount factor, which not the same as the simple average rate—e.g., see equation (3)) by the simple formula

$$\eta \approx \frac{r_{\text{near-term}}^{ce}}{g_{\text{near-term}}}$$

which delivers the desired near-term discount rate of $r_{\text{near-term}}^{ce} = 0\% + \eta g_{\text{near-term}}$.

calibrate ρ and η by finding the parameter pair that reconciles the implied, “fitted” term structure with its econometric analogue.⁸

3 Data

The key data used in this estimation are (1) the term structure, r_t^{ce} , (2) near-term target discount rates, and (3) the growth rate distributions, $\{g_\tau\}$.

3.1 Term Structures

We use several sources for the term structure of interest rates based on estimates from the literature (Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015; Bauer and Rudebusch 2020b). These approaches involve estimating econometric models on historical market interest rate data, and then using simulations to project the term structures of the certainty-equivalent rate. The different methods in these papers vary primarily in the specification of the econometric model. We consider a total of six econometric models. First, we use the most up to date estimates of the term structure from Bauer and Rudebusch (2020b), which in turn is based on the model developed in Bauer and Rudebusch (2020a). Second, we use the three models from Newell and Pizer (2003). These model interest rates as (i) an uncertain rate, modeled in levels, with a permanent component and serially correlated shocks, (ii) a Random Walk (RW) model of the logarithm of the rate, and (iii) the Mean Reverting (MR) model of the log rate. We also use the State Space (SS) model of Groom et al. (2007) and the Augmented Autoregressive Distributed Lag (AADL) model of Freeman et al. (2015), each of which also model the logarithm of the rate.

While the empirical term structure literature (Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015; Bauer and Rudebusch 2020b) typically has focused on the *forward* or *instantaneous* discount rate (i.e., the instantaneous discount rate to discount from period t to period $t - 1$), we present results in terms of the *average* discount rate (i.e., the single, or constant, rate to discount from period t to period 0).⁹ This means the discount rates shown in our figures should not be

⁸A natural question is the structural compatibility of the Ramsey-like model in (1) and the model underlying r_t^{ce} in (5). We discuss our efforts to match the consistency of data and econometric models we use for calibration in the next section. In addition, one might ask why we favor an approach based on calibration to future interest rate term structures, rather than attempting to directly estimate a Ramsey-like model based on historical data on consumption growth and interest rates. That is, why not assume the market is behaving in accordance with (1) and try to recover the parameters underlying that data-generating process? The main reason is that there is little evidence (beginning with Hall 1988; see NAS 2017 page 173) that such a direct relationship can be accurately or precisely identified based on available market data. Moreover, our focus is long-term behavior and how the discount rate behaves over decades if not centuries. Estimation based on historical data would necessarily identify off relatively short-term co-variation of interest rates and consumption growth, but short-term and long-term relationships can be quite different.

⁹This average rate is the single (or constant) rate that one would take to the power of t to properly discount a future value occurring at time t . In the case of continuous time discounting where the instantaneous rate is the change in the

directly compared to those in the empirical term structure literature. A proper comparison would be to compare the term structures we present to the *average over time* (from period 0 to time t) of the term structures presented in that literature.

For each model, we obtain new simulated term structures centered at the set of near-term rates discussed in section 3.2: 1.5%, 2%, 3%, and 5%. For the Bauer-Rudebusch (BR) model, we used their equation (10) initiated at each starting near-term rate to simulate posterior predictive paths of interest rates, accounting for uncertainty in both the innovation itself and its variance (posterior data was generously provided by the authors). Because their model is a random walk in the level of the interest rate, some draws can become persistently and arbitrarily negative. As they discuss, nominal interest rates are typically constrained to be non-negative because of households' option to hold cash. Moreover, there is a general view that negative real rates are unlikely to persist for long periods. Therefore, we impose a constraint that the time-average interest rate (from time 0 to time t) cannot be negative, although year-on-year rates may be. In particular, we constrain all negative values of r_t^{ce} to zero in (5).¹⁰ This is less restrictive than the BR simulations, which bounds negative instantaneous interest rates to zero.

For the three NP models, we analogously re-used the code from Newell and Pizer (2003) to simulate interest rate paths, again recentering the interest rate at the beginning of the projection to each of the four near-term target rates. The term structure of forward (instantaneous) rates for the SS and AADL models were generously constructed and provided by Ben Groom, Katerina Panopoulou, and Theologos Pantelidis. In all cases, we then converted forward (instantaneous) rates to average rates (i.e., the rate to discount from period t to period 0) to coincide with equation (4), which represents the average rate. This procedure results in more than twenty different term structures. For each term structure, we calibrate the (ρ, η) parameters by minimizing equation (5) subject to the two constraints discussed above.

Our preferred term structure model is that of BR for three primary reasons. First, it reflects recent trends and state-of-the art modeling of real interest rates, distinguishing long-term and short-term behavior. Second, that model is specified in levels of the interest rate (as opposed to the logarithm of that rate), which is most conceptually consistent with the underlying behavior of the growth rate distribution from MSW discussed in section 3.3. The growth rate model underlying that distribution features additive shocks in the logarithm of GDP per capita (and hence the additive shocks translate

log of the discount factor, this average rate is the arithmetic average of the instantaneous rates. In the case of discrete time discounting where the instantaneous rate is the ratio of the current and previous values minus 1, this average rate is a geometric average. In this paper, we use the continuous time discounting approach. Regardless, under either approach the average rate is the single rate that delivers the same discount factor as applying the full path of instantaneous rates over time.

¹⁰Specifically, we implement this non-negativity constraint when re-simulating the BR term structure for each of the four starting rates by placing a ceiling on the discount factor of 1, since a discount factor of one corresponds to a (time average) discount rate of zero.

into percentage changes—i.e., the level of the growth rate rather than its logarithm). In other words, both MSW and BR effectively feature additive uncertainty in the *rate* (interest rate or growth rate), ensuring the two are conceptually consistent.

Third, the Bauer and Rudebusch (2020*b*) model was estimated on data from the same time period used by Müller, Stock and Watson (2019)—1900-2017—ensuring consistency between the two datasets underlying our calibration (interest rates and growth rates). While the Bauer and Rudebusch (2020*b*) paper focuses on the 1955-2017 sample period, the authors generously re-estimated their model over the longer 1900-2017 sample using an updated version of the Newell and Pizer (2003) dataset and provided us with the resulting posterior. While the Bauer and Rudebusch (2020*b*) term structure remains our preferred benchmark for the century-scale term structure, we present results for five other models from the literature to demonstrate robustness of the calibration process.

3.2 Near-term Discount Rates

For the purposes of this paper, we consider a range of “near-term” discount rates, in line with the approach to sensitivity analysis historically taken by the IWG (IWG 2016) and recommended by the NAS SCC study (NAS 2017). US federal rulemaking has historically employed two discount rates: 3% and 7%, as directed by the Office of Management and Budget (2003) in OMB Circular A-4. These rates were chosen to represent (1) the consumption rate of interest (as estimated as the real return on government debt) and (2) the opportunity cost of capital (estimated as the before-tax return to private capital). However, both the NAS and IWG emphasized that the before-tax, opportunity cost of capital was not an appropriate discount rate for climate change. They argued that the costs and benefits to be discounted are expressed in terms of consumption. For these and other reasons, the IWG (IWG 2010; IWG 2013; IWG 2016) used a set of rates centered on OMB’s 3% consumption rate: 2.5%, 3%, and 5%.¹¹ Moreover, consumption discounting is always appropriate over long horizons, regardless of whether costs and benefits fall on investment or consumption (Li and Pizer, 2019).

However, for the present purposes of calibrating a range of discount rates there are reasons not to focus on the same three rates used previously by the IWG. Since those specific values were last updated and the IWG approach was developed, the economy has changed (interest rates have fallen), and the theory behind the appropriate discount rate relevant for climate change has advanced. Reflecting this, the OMB is undertaking an effort to modernize the regulatory review

¹¹In addition, Circular A-4 itself explains that it is appropriate to use lower rates when discounting across generations and that “Estimates of the appropriate discount rate appropriate in this case, from the 1990s, ranged from 1 to 3 percent per annum”. Therefore, Circular A-4 explains, it is appropriate to consider rates below 3% as a further sensitivity analysis.

process, including revisions to Circular A-4.¹² The IWG’s past central rate of 3% was based on OMB guidance under Circular A-4, which we continue to consider in this paper. However, that 3% rate may not continue to be deemed a reasonable “central” value due to observed declines in market interest rates, among other reasons. Indeed, in early 2021 when the IWG adopted interim SCC values simply reflecting an inflation adjustment to the previous (IWG 2016) estimates at 2.5%, 3%, and 5% rates, they also notably no longer referred to the 3% value as “central.” Nonetheless, 3% remains current guidance under Circular A-4 and the rate used historically by the IWG, so we include a 3% rate among the near-term rates we consider in this paper.

The basis for the IWG’s “low” and “high” rates of 2.5% and 5% are no longer conceptually justified based on the NAS recommendations and the approach in this paper. Those rates were included to approximate the impact of uncertainty in the discount rate. In particular, the 2.5% rate represented the average certainty-equivalent rate from Newell and Pizer (2003) starting at a discount rate of 3%. In this paper, we account for discount rate uncertainty directly, so the past rationale for using 2.5% and 5% rates are not valid once the uncertainty is treated explicitly.

The recent literature has begun to converge towards considering a consumption discount rate of about 2% (rather than 3%), as has been adopted by New York State, if not lower. As explained in a Council of Economic Advisers (CEA) report (CEA 2017), risk-free interest rates have recently remained well below 2%, reflecting a lower rate at which households are willing to trade off consumption between the present and the future. In the very recent past, real 10-year Treasury yields during 2010-2020 were generally close to 0%, and even reaching negative territory for lengthy periods of time.¹³ While such near-zero rates are unlikely to continue in the long term, most macroeconomic forecasts project real yields below 2% for the foreseeable future (CEA 2017). For this reason, CEA (CEA 2017) suggests that 2% reasonably reflects the risk-free return to savings. As shown in Drupp et al. (2018), there is general convergence among experts that a rate of 2% is appropriate to consider. Indeed, New York State has formally adopted 2% as its central rate, resulting in a social cost of carbon of \$125 per ton (NYSERDA and RFF 2020), holding other aspects of the IWG (2016) estimation process constant.

Another recent study, Giglio, Maggiori and Stroebel (2015), also suggests using a rate below 3% for long-run impacts. That study estimates discount rates used by real estate investors in the very long run by comparing the prices of 100-year leases to the price of owning an equivalent property in perpetuity. The difference in those prices reflect the discounted value of flows beyond 100 years, from which they can derive the long-run discount rate used by investors. They find discount rates “below 2.6% for 100-year claims”. Because real estate assets are not risk free, this may overstate the

¹²<https://www.whitehouse.gov/briefing-room/presidential-actions/2021/01/20/modernizing-regulatory-review/>.

¹³See <https://fred.stlouisfed.org/series/DFII10>.

appropriate risk-free rate (see Giglio et al. 2015), providing additional support for the use of a rate lower than 2.6%, closer to the conclusion of 2% from CEA (CEA 2017) and Drupp et al. (2018). Based on these multiple lines of evidence, we also consider a 2% rate for this analysis, alongside Circular A-4’s current rate of 3%. We do not take a stand here on the correct central rate and instead consider both.

In addition, we consider “low” and “high” rates as sensitivity cases, analogous to past considerations by the IWG and the NAS. Historically, “low” and “high” rates are meant to bound reasonable ranges based on various relevant considerations such as observed market rates of different kinds of assets. For the purposes of this paper, we consider a “low” rate of 1.5% and a “high” rate of 5%, for the reasons described below. While our exploration of this range of rates should not be taken as an indication that we endorse that 1.5% and 5% should be the low and high discount rates sensitivity cases adopted by the IWG, a low and high range should be within these bounds. In addition, in the appendix, we also present results for a series of alternative rates ranging between 1.5% and 5% in increments of 0.5%.

First, because the rationale for the IWG’s previous “low” rate of 2.5% is no longer valid, and also because market interest rates have fallen since the relevant research and policy guidance was established, we look to the recent economics literature for evidence for a new “low” rate. As already discussed, the economics profession has exhibited gradually growing support for the use of a rate below 2%, as supported by the CEA and Drupp et al. (2018). In addition, the macro-finance literature has also found evidence for such a low rate. In particular, Bauer and Rudebusch (2020b) summarizes the evidence from six recent empirical models from the literature. For one-year bonds, all six models find that equilibrium interest rates have fallen well below 2% over the past two decades, and perhaps even below 1% in recent years. However, one-year bonds may not be the best data for estimating long-term rate behavior. When BR estimate their model using long-term rates, which arguably better reflect long-term expectations, they find evidence for equilibrium rates of around 1.5%. For these reasons, we therefore consider 1.5% as a “low” case. As the results will indicate, a near-term rate of 1.5% also happens to be the point at which the non-negativity constraint on ρ becomes binding, resulting in a corresponding ρ of zero.

Regarding the “high” rate, the IWG’s rationale for its high rate of 5% is similarly no longer valid, since that rate was also meant to approximate the uncertainty about future discount rates and their correlation with climate impacts, which we are modeling explicitly. An alternative motivation for a rate of about 5%, however, is that it corresponds approximately to the after-tax return to the stock-market and other equity investment. The adjustment for taxes is necessary to calculate the *consumption* rate of interest, since the after-tax rate is the rate actually received by consumers. While interest rates have fallen in recent years, there has not been a marked decline in the pre-tax return to capital, which remains about 7% (CEA 2017). Simply adjusting the historical pre-tax

return to a post-tax return implies that the 7% pre-tax return corresponds to approximately 5% after-tax.¹⁴ Based on this rationale, we consider 5% as a “high” case, while recognizing that this does not account for risk, which would entail a further downward adjustment.

3.3 Projected Distribution of Economic Growth

To calculate the expectation in equation (5) with respect to the uncertain growth rate, g_τ , we use the probabilistic projections of growth in global long-run GDP per capita estimated by Müller, Stock and Watson (2019). The MSW approach is a refinement of the low frequency forecasting framework developed by Müller and Watson (2016, 2017).

We censor some draws in the extreme tails of the raw MSW distribution that produce implausibly high or low (i.e., strongly negative) growth rates that are outside of the range of historical experience or, in the long run, expert opinion based on an expert elicitation of future economic growth. This affects less than one percent of country-year draws in either tail. Details about this censoring process and elicitation are in a forthcoming paper. The raw and censored data are both available upon request.

Figure 1 illustrates the resulting distributions, showing the average (over time) global economic growth rate from 2020 to each year through 2300.¹⁵ The figure shows the mean growth rate along with 50%, 80%, 90%, and 95% probability ranges.

¹⁴See IWG (2010), footnote 19.

¹⁵Note this should not be confused with the instantaneous growth rate in each year.

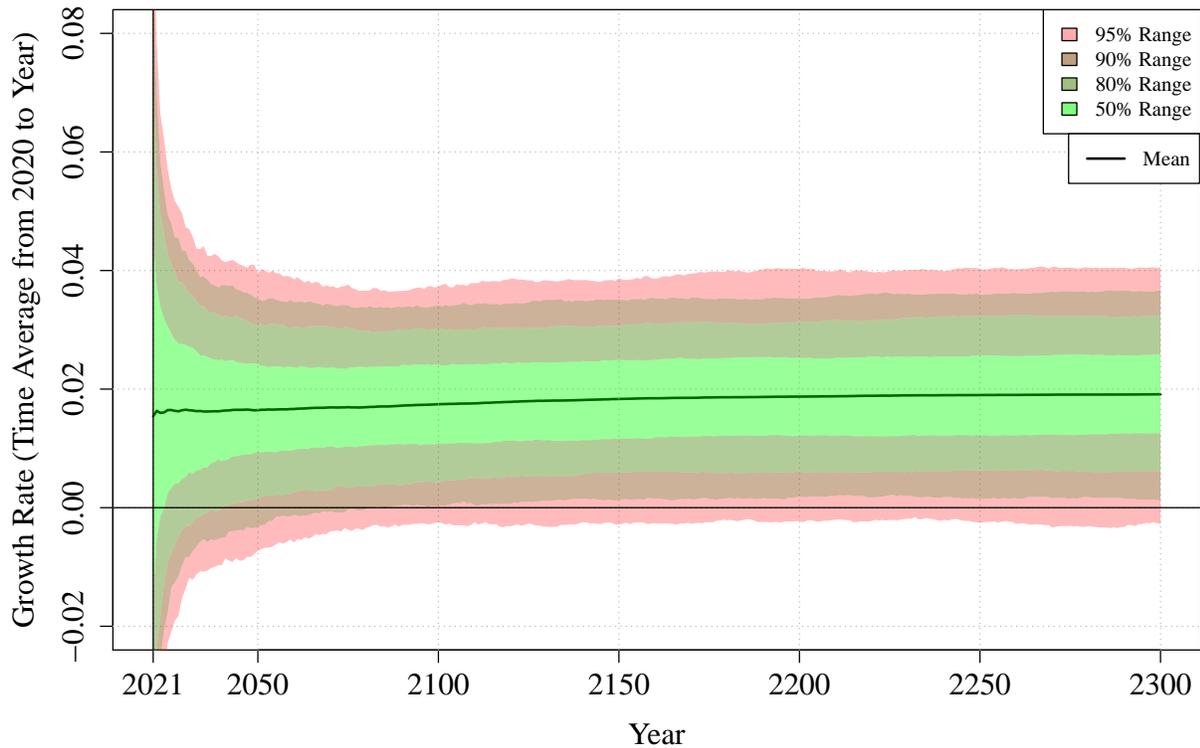


Figure 1: Distributions of MSW Global Per Capita Economic Growth Rates, based on Müller, Stock and Watson (2019)

3.4 Estimation Results for ρ and η

The calibrated parameter values are reported in Table 1. The calibration process generally yields values for ρ and η that scale up (down) with higher (lower) near-term target rates. The values are also comparable to the range of values found elsewhere in the literature (e.g., see (NAS 2017) Table 6-1 and Drupp et al. 2018). The top line shows our preferred values based on the BR model. As the near-term target rate rises from 1.5% to 5% , the preferred approach results in calibrated parameters that rise from $\rho = 0\%$ and $\eta = 0.99$ at a near-term rate of 1.5%, to $\rho = 2.4\%$ and $\eta = 1.86$ at a 5% rate, with a value of $\rho = 0.8\%$ and $\eta = 1.53$ for a 3% rate. Under this preferred model, the

Table 1: Discounting Parameters by Model and Near-Term Rate

	1.5% Rate		2% Rate		3% Rate		5% Rate	
	ρ	η	ρ	η	ρ	η	ρ	η
<i>Preferred Term Structure</i>								
Bauer & Rudebusch (2020)	0%*	0.99*	0.1%	1.25	0.8%	1.53	2.4%	1.86
<i>Other Term Structures</i>								
N&P (2003) Levels	0%*	1.00*	0.0%	1.34	1.0%	1.34	3.0%	1.34
N&P (2003) RW	0.2%	0.82	0.5%	0.99	1.1%	1.30	2.3%	1.94
N&P (2003) MR	0.8%	0.44	1.2%	0.51	2.0%	0.68	3.4%	1.09
Groom et al. (2007) SS	0.7%	0.50	0.9%	0.69	1.2%	1.23	2.1%	2.06
Freeman et al. (2015) AADL	0.7%	0.51	0.8%	0.81	1.1%	1.27	2.0%	2.19

Notes: *: These values are the results under the constraint that $\rho \geq 0$. This constraint is only ever binding under the 1.5% near-term rate. The unconstrained values are $\rho = -0.1\%$ and $\eta = 1.08$ for the BR term structure and $\rho = -0.5\%$ and $\eta = 1.34$ for the N&P Levels term structure.

discounting rule for use in the SCC under the four near-term rates would be:

$$r_t^{1.5\%} = 0\% + 0.99g_t, \quad (6a)$$

$$r_t^{2\%} = 0.1\% + 1.25g_t, \quad (6b)$$

$$r_t^{3\%} = 0.8\% + 1.53g_t, \quad (6c)$$

$$r_t^{5\%} = 2.4\% + 1.86g_t. \quad (6d)$$

The results are also depicted in Figure 2 (for the preferred BR term structure) and in Appendix Figure A.2 (for all term structures from the literature), which plot each fitted term structure versus its target, alongside the parameter estimates. For the 3% near-term rate, the BR term structure has the best quality of fit of all the term structure options from the literature, with an R^2 value of 0.995; versus 0.79-0.98 for the range of alternatives. Appendix Figure A.1 also shows results under the BR term structure for a series of alternative near-term rates ranging from 1.5% to 5% in increments of 0.5%.

The results reveal that using near-term rates lower than 2% can cause the non-negativity constraint on ρ to bind. For example, while the calibrated ρ value under a 2% near-term rate is 0.1% meaning the non-negativity constraint is non-binding, the constraint becomes binding at the slightly lower 1.5% rate. Without that constraint, the required ρ would be -0.1% (along with $\eta = 1.08$) under the BR 1.5% term structure. In the Ramsey framework, ρ represents the rate of pure time preference, reflecting time preferences over utility. With the non-negativity constraint, the η value that delivers the desired 1.5% near-term discount rate is close to one ($\eta = 0.99$) because the near-term growth rate in the MSW distribution is slightly above 1.5% on average.

Compared to the literature, our preferred values for η of 1.25 and 1.53 for near-term rates of 2% and 3% are within the range of commonly used values, such as Nordhaus's preferred value of

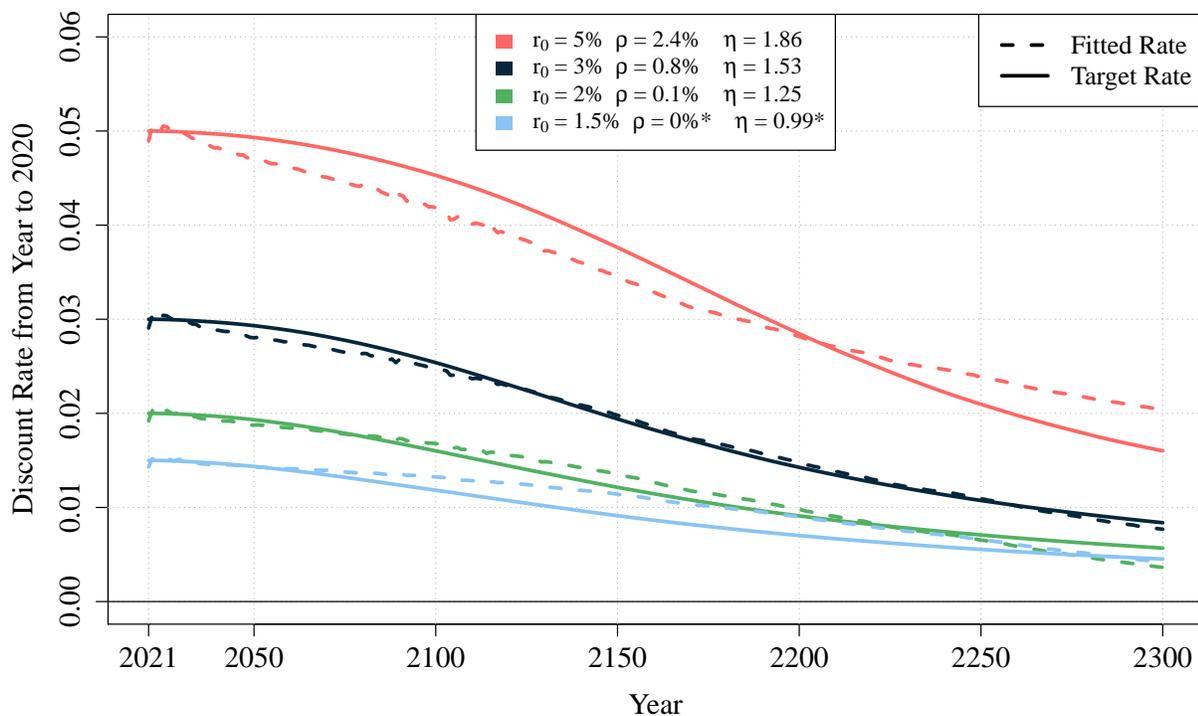


Figure 2: Estimated and Actual Certainty-Equivalent Paths, Bauer and Rudebusch (2020b) Model

Notes: Values indicated by an * are the results under the constraint that $\rho \geq 0$. This constraint is only ever binding under the 1.5% near-term rate. The unconstrained values are $\rho = -0.1\%$ and $\eta = 1.08$.

1.45 and the Stern review's value of 1, and the mean value of 1.35 found by Drupp et al. (2018). Hansen and Singleton (1983) suggest a range for η of 0 to 2, which straddles these values. Groom and Pr (2019); Groom and Maddison (2013); Evans (2005); Evans and Sezer (2005) tend to find η values between 1 and 2, derived using various approaches to infer revealed social preferences. If we were to apply an η value on the higher end of that range (e.g., 2) to the MSW data, it would imply term structures that decline much more steeply than the interest rate term structure literature suggests. Indeed, using an η value of 2 (while choosing ρ to achieve a 3% near-term rate) would result in negative certainty-equivalent discount rates for all years after 2260.

4 Impact on the Social Cost of Carbon (SCC)

The certainty-equivalent discount rate can eventually become quite low at long time horizons; it may even become negative if there is a non-zero probability of negative long-run economic growth. This is the inevitable result of (i) the Weitzman (1998) conclusion that the certainty-equivalent rate converges to the lowest possible rate, and (ii) the potential for the lowest possible discount rate to

be close to zero or even negative, which it will be when the lowest possible growth rate is negative. The reader may reasonably be concerned that such a low long-run certainty-equivalent discount rate would yield an unbounded social cost of carbon. For example, with the 3% near-term rate, the certainty-equivalent discount rate in 2300 is about 1 percent, which is below the central growth rate of about 2 percent, raising concerns that damages (if proportional to GDP as in DICE) may grow over time faster than they are discounted. In this section, we show that these low rates do not in fact lead to an unbounded SCC for our calibrated estimates (based on a near-term rate of 3%) due to the modeled correlation between damages and the discount rate.

In particular, the explanation for this result is that a separate discount rate is used in each state of the world with a different growth rate and damage estimate, whereas the certainty-equivalent rate is simply a summary of the distribution of these rates (and is actually not used directly). To illustrate this, suppose for the moment that undiscounted marginal damages, MD_t , are proportional to GDP at rate α , as in $MD_t = \alpha Y_t$. Denoting the annual growth rate g_τ , GDP at time t is given by $Y_t = Y_0 e^{\sum_{\tau=1}^t g_\tau}$. Then discounted period t damages are given by multiplying undiscounted marginal damages by the stochastic discount factor from (3):

$$SDF_t \times MD_t = e^{-\sum_{\tau=1}^t (\rho + \eta g_\tau)} \times \alpha Y_0 e^{\sum_{\tau=1}^t g_\tau}, \quad (7)$$

yielding state-contingent discounted damages.

Equation (7) shows that very low discount rates (the $(\rho + \eta g_t)$ term in the stochastic discount factor) only arise in states of the world where economic growth is very low or negative. In those low growth states, damages tend to be relatively small due to the common assumption that climate damages scale with the size of the economy. Hence, with our discounting rule, very low or negative discount rates are only applied when the damages to be discounted are relatively small. To build intuition for the mechanics of these effects, we pause to present a series of stylized examples of discounted damages under alternative discounting approaches. Then we numerically demonstrate these effects using the DICE model as an illustration.

4.1 Stylized Discounting Example

To build intuition for the effects of the discounting rule, consider the stylized discounting examples depicted in Figure 3. The top panel of that figure shows three stylized paths of undiscounted marginal damages over time. For simplicity, damages are assumed to scale proportionately with the size of the economy as they approximately do in many IAMs, corresponding to a climate beta of 1. Each line corresponds to a different stylized growth scenario, where the annual growth rate is fixed at either 2%, 0%, or 4%. Undiscounted marginal damages across paths diverge sharply given the different economic growth rates (top panel, noting the log scale). If one applies a constant discount

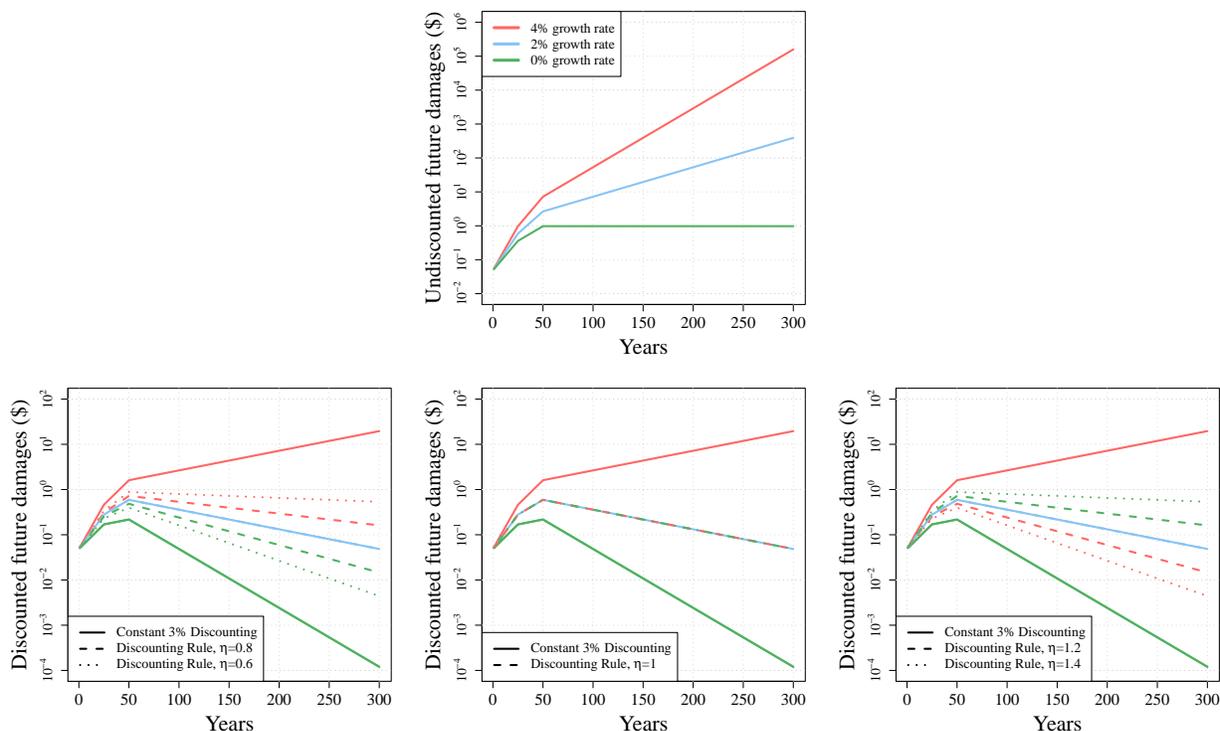


Figure 3: Stylized Paths of Marginal Damages, Undiscounted (top panel) and Discounted (bottom panels) under Alternative Approaches

rate of 3% across the scenarios, discounted marginal damages would also diverge strongly from each other (bottom panel, solid lines). Noting the log scale of the y -axis, the mean SCC in this case would be driven by the large damages in the high growth case (top line in red). This can make the mean SCC quite large.

If one instead discounts high growth scenarios more and low growth scenarios less using the Ramsey-like discounting rule, this dampens the effect of growth on discounted damages. The bottom three panels show discounted damages under alternative discounting rules. Each rule uses a different value of η and ρ , but nonetheless maintains a 3% near-term rate.¹⁶ In all cases, the discounting rule shrinks the variation in discounted damages across growth scenarios. This owes to the offsetting effects of growth on undiscounted damages and the discount rates. Focusing on the bottom left panel, where η is less than one, the discount rate scales up and down with the growth rate, but with $\eta < 1$ it does not completely offset the fact that damages are larger in high growth scenarios. As a result, high growth scenarios continue to have higher discounted damages than low growth scenarios (the red lines are above the green lines). However, the variation across scenarios is

¹⁶Specifically, as we vary η , we also vary ρ to achieve a 3% discount rate in the central 2% growth case. That is, we set $\rho = 3\% - \eta \cdot 2\%$, such that $r = \rho + \eta \cdot 2\% = 3\%$.

smaller than with constant discounting (solid lines). This narrowing of the variance also reduces the expected value of the SCC, as the SCC is a convex function of the growth rate.

Turning to the bottom right panel where η is larger than one, the variation across growth scenarios is once again smaller than with constant discounting. However, the pattern is reversed. With η larger than one, the higher discount rates in high growth scenarios more than offset the higher damages. As a result, high growth scenarios in fact feature smaller discounted damages (red dotted and dashed lines) than low growth scenarios (green dotted and dashed lines). But once again, the variation in discounted damages is smaller than with constant discounting.

The bottom middle panel shows discounted damages when η is exactly one. In this case, the higher damages in higher growth scenarios is exactly offset by a higher discount rate (due to the proportionality of damages to the size of the economy in this example). For example, the 4% scenario has undiscounted damages that grow 2% faster than the 2% scenario, but they are also discounted 2% more. The result is that, with $\eta = 1$, discounted damages are the same under each growth rate scenario. This corresponds with a complete collapse of discounted damages to the central case, in which they no longer vary with the growth rate.

Discounted damages are the same across growth rate states when $\eta = 1$ because of the assumption that damages are proportional to economic activity—that is, a climate beta of 1. More generally, this centering point is around $\eta \approx \beta$, as explained in more detail in Appendix B. Roughly speaking, if (stochastic) marginal damages scale up multiplicatively with the size of the economy at a rate β but the stochastic discount factor scales these damages down at a rate η , these two effects offset each other when η and β are similar in magnitude.

We do not take a stand on the “correct” climate beta in this paper because it does not affect the calibration of the (ρ, η) parameters, which depends on risk-free rates that are not driven by the climate beta. Our focus on this case in the preceding discussion, as well as the NAS recommendation for Ramsey-like discounting, stems at least in part from the fact that the climate beta is near 1 in the IAMs that have been used for SCC estimation. But our proposed approach would be equally appropriate for other models, including where the climate beta is negative. In that case, it would be important to compute growth inclusive of climate damages, as recommended by the NAS as a longer-term enhancement.

To summarize, the discounting rule reduces the variance of discounted damages across growth scenarios. Relative to constant discounting with uncertain growth, the discounting rule also reduces the expected SCC, by moderating the influence of extreme growth states on expected discounted damages. In the next section, we demonstrate these effects numerically using the DICE model.

4.2 Numerical SCC Impacts in DICE

While Figure 3 presents a simple stylized example, the general lesson holds true more generally in IAMs, which are considerably more complex. To show this, we numerically calculate the impact of using our discounting rule on the SCC in the widely used DICE model, which is also one of three models employed by the IWG in its 2010-2016 approach. We implement stochastic economic growth into DICE using Monte Carlo simulations in which we sample economic growth trajectories from the growth distribution, using 10,000 draws.¹⁷,¹⁸ In each draw, we discount marginal damages, which are also a function of the growth rate g_t (and hence written $MD_t(g_t)$), analogously to equation (7):

$$SCC = \sum_{t=0}^T [e^{-\sum_{\tau=1}^t (\rho + \eta g_{\tau})} \times MD_t(g_t)].$$

This state-contingent SCC is typically summarized by a summary statistic such as its mean or median, which we focus on here. For purposes of comparison, we show results both using a constant 3% percent discount rate (i.e., $\rho = 3\%$ and $\eta = 0$) and also using the parameters corresponding to a 3% near-term rate shown in equation (6c) ($\rho = 0.8\%$ and $\eta = 1.53$). The results show that, relative to a constant discount rate when economic growth is uncertain, implementing our discounting rule using the IWG’s 3% historically used central rate (i.e., setting $\rho = 0.8\%$ and $\eta = 1.53$) reduces both the mean SCC and its variance. This replicates the stylized result. At the same time, jointly implementing our discounting rule and uncertain economic growth *increases* the SCC relative to the case with constant discounting and *deterministic* growth.

We can see these different effects more clearly by calculating the SCC using the DICE model under a step-wise series of scenarios. We start with the actual SCC calculations performed by the IWG (2016), adjusted for inflation to 2020\$, then incrementally add a number of changes to incorporate the stochastic MSW economic growth projections (in a Monte Carlo analysis) and our discounting rule. We incrementally add each change one-by-one to decompose the effects of each individual change on the SCC. The results are shown in Table 2. All values correspond to a pulse of CO₂ in 2015, presented in 2020\$ per ton of CO₂, using a near-term discount rate of 3% (either constant or stochastic according to the ρ and η parameters in Figure 2).

¹⁷In this calculation, we re-weight the MSW distributions based on a structured expert judgement elicitation described in more detail in Rennert et al. (2021). The re-weighting approach serves to combine the econometrically based growth projections from MSW with expert judgment about very long-run economic growth (i.e., through 2300), as recommended by the NAS (NAS 2017). While we apply these weights to the economic growth distribution in the DICE simulation, we have used the raw (unadjusted) MSW distributions to calibrate the ρ and η parameters. We leave the development of discounting parameters calibrated to re-weighted growth projections to future work.

¹⁸In each draw, we also sample the climate sensitivity from the Roe-Baker distribution (Roe and Baker 2007), in line with the IWG’s 2010-2016 approach. Note, however, that this example does not include the full set of NAS-recommended updates to the IWG SCC estimation process, so it should not be taken as an indication of what those resulting estimates would be.

Table 2: Impacts of Modeling Changes on the DICE Social Cost of Carbon (2020\$)

Scenario	SCC (\$/ton CO ₂)	
	Mean	Median
1) IWG (DICE only, 2015 pulse, inflation adjusted)	\$42	\$37
2) 1 + Deterministic Emissions	\$43	\$38
3) 2 + Adjust Mean Population Growth to UW	\$45	\$40
4) 3 + Adjust Mean Economic Growth to MSW	\$56	\$51
5) 4 + Stochastic MSW Economic Growth	\$136	\$50
6) 5 + Stochastic Discount Rate ($\rho = 0.8\%$, $\eta = 1.53$)	\$78	\$61

Note: All rows except the final one use a constant 3% discount rate ($\rho = 3\%$, $\eta = 0$).

We present both mean and median SCCs. The SCC values incorporating all our changes (most importantly, the MSW economic growth projections and our discounting rule) appear in the final row of that table: a mean SCC of \$78/ton in 2020\$, higher than the IWG value of \$42/ton (DICE-only mean for a 2015 pulse, in 2020\$). It is important to note, however, that while the final mean (i.e., expected) SCC is higher than the relevant IWG estimate, the decomposition reveals that implementing the discounting rule itself (the last step) actually reduces the mean SCC given the damage function of the DICE model. The primary reason the mean SCC is higher than the IWG value is that the MSW projections feature somewhat higher expected economic growth than the IWG assumed.

What are the intermediate steps? Row (1) simply reflects the IWG approach (DICE-only results for a 2015 pulse), converted to 2020\$ using the GDP Implicit Price Deflator.¹⁹ For simplicity and consistency across scenarios, in row (2) we fix the trajectory of baseline emissions at the average of the five scenarios used in the IWG.²⁰ The impact is minimal: the mean SCC changes slightly to \$43.

Next, row (3) shifts the global population projections from those in the IWG approach to the average population projections from Raftery and Sevcikova (forthcoming). This step is required to incorporate the MSW projections, which are for country-level GDP *per capita*. Specifically, we require country-level population projections, such as those calculated in Raftery and Sevcikova (forthcoming), to aggregate MSW’s country-level per capita values to global GDP. The effect of this

¹⁹<https://fred.stlouisfed.org/series/GDPDEF>.

²⁰This is for simplicity, but it also corresponds to the common approach in the literature of treating the stochastic growth rate as representing “emissions-neutral technological progress” (e.g., see Dietz, Gollier and Kessler 2018, which make explicit this often implicit assumption). We emphasize that this is for simplicity of illustrating the impacts of our discounting rule, and a full estimation of the SCC should account for a GDP-emissions relationship. A number of issues arise when implementing endogenous emissions in high growth scenarios that put this exercise beyond the scope of this paper. Allowing emissions to be uncertain through growth uncertainty would likely increase the mean SCC even more than we find here, due to the convexity of the damage function.

is again minor in this example using DICE, with the mean SCC rising from \$43 to \$45. The rise is attributable to a somewhat larger projected population, relative to IWG's approach. Whereas IWG generally assumed the population size would peak before 2100 and then decline thereafter, Raftery and Sevcikova (forthcoming) projects continued population growth for the first few decades of the 2100s.

Row (4) then shifts the average GDP per capita growth rates from IWG's assumptions to the average growth in the MSW distribution (while not actually implementing the full MSW uncertainty distribution—only its mean growth rate for each year). By shifting the mean growth rate, we isolate the impacts of MSW's higher average economic growth projections (relative to IWG) from the impact of implementing a fully stochastic growth rate (which we implement in the next step). This shift in the average growth rate increases the mean SCC due to higher expected economic growth rates: the mean rises from \$45 to \$56, and the median rises from \$40 to \$51. If we use our discounting parameters ($\rho = 0.8\%$, $\eta = 1.53$) before adding stochastic economic growth from MSW—the mean and median SCC values are \$68 and \$59 respectively. This increase reflects a lower effective long-run discount rate than the constant 3% value, due to a deterministic deceleration in growth over time. This is not any uncertainty-driven declining discount rate, which we add in the final step in row (6)

Row (5) then fully implements the MSW uncertainty distributions summarized in Figure 1, but retains a constant 3% discount rate (i.e., without implementing our discounting rule). The *mean* SCC increases substantially, from \$56 to \$136 per ton of CO₂, when we introduce stochastic economic growth but non-stochastic discounting. As explained in Appendix B, this large increase is to be expected with constant discounting (i.e., $\eta = 0$). This is because undiscounted damages are a strongly convex function of the uncertain growth rate. Specifically, undiscounted damages in DICE are proportional to GDP, implying an approximately exponential relationship between damages and economic growth rates that becomes increasingly convex at long time horizons. Other IAMs (e.g., FUND, PAGE) also typically feature convex damage functions, suggesting the result would likely be similar. Jensen's inequality implies that the expectation (i.e., mean) of a strongly convex function of g (as damages are) will strongly exceed the value (damages) computed at the expected g , as the mean is increasingly driven by outcomes by the extremes of the distribution. Unless large growth rate states are also discounted at high discount rates, the expectation can be quite large. As a result, this produces a mean SCC of \$136 per ton. While Jensen's inequality implies that the mean will increase, the median can remain stable. To illustrate this point, Table 2 also shows the median SCC, which shows a negligible change at \$50.

Finally, row (6) implements our discounting rule, allowing the discount rate to be stochastic through the stochastic economic growth rate. This accounts for both uncertainty in the discount rate and its correlation with damages, discounting high-growth states at higher rates. This results in the

more stable mean shown in row (6). Implementing our discounting rule causes the mean SCC to decline to from \$136 to \$78 per ton, where the median rises to \$61.²¹

This final mean SCC value of \$78 in row 6 is larger than the “deterministic” value of \$56 in row 4, indicating that the joint effect of uncertain economic growth and the discounting rule (i.e., using $\rho = 0.8\%$ and $\eta = 1.53$ instead of a constant 3% discount rate) is to increase the SCC values substantially. The comparison is similar if we apply the discounting rule without uncertain economic growth (i.e., row 4 but with $\rho = 0.8\%$ and $\eta = 1.53$), which leads to a mean SCC of \$68, still lower than \$78 in row 6. This illustrates the importance of the interaction of the discounting rule with the uncertain economic growth rate.

More generally, the absence of a particularly large effect accounting for growth in the discount rate (and with our calibrated η being similar to the DICE climate beta) stems from the absence of large, non-linear uncertainties in the climate change and damage models in DICE. For example, we are presently only considering uncertainty in growth (along with the climate sensitivity) and not other factors such as tipping points (e.g., Cai and Lontzek 2019; Lontzek et al. 2015). In general, the net impact of the discounting rule and uncertain economic growth will depend on the model structure (here, the structure of the DICE model, including the climate model, the damage function, and the climate beta), but typically the effect should be positive for the reasons discussed in Appendix B.

²¹The mean’s decline is tightly linked to the strong correlation between damages and the discount rate. If we “break” this correlation by discounting expected damages by the expected discount factor ($E[e^{-\sum_{\tau=0}^t \rho + \eta g_{\tau}}]E[MD_t]$), the mean SCC would be thousands of dollars per ton, while the median would remain around \$66/ton. This approach of “breaking” the correlation would be inappropriate because it ignores the risk characteristics of the payoffs to climate mitigation. Put differently, employing stochastic economic growth while ignoring the damage/growth correlation effectively discounts a risky potential payoff at the declining certainty-equivalent rate (which does not account for this risk), leading to a major overvaluation.

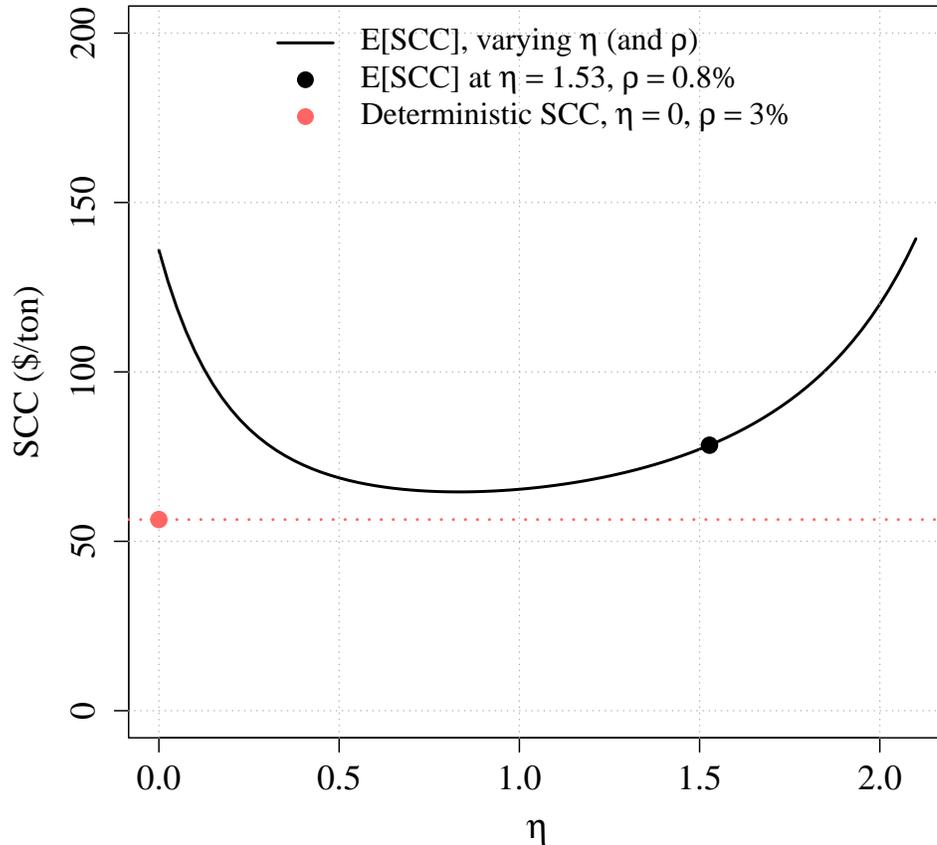


Figure 4: E[SCC] (2020\$) by Value of η , Varying ρ to Match 3% Near-term Rate

A sensitivity analysis shown in Figure 4 using DICE reveals that the mean SCC is a U-shaped function of η (where as we vary η we also vary ρ to hold the near-term rate fixed at 3%), where very small or very large η values can produce large mean SCC values, but the mean SCC is relatively “flat” along standard η values. For example, for all η values between 0.35 and 1.53, the discounting rule produces a mean SCC that is between the deterministic value of \$56 (as in row 4) and the stochastic value of \$78 (in row 6). The η values used in the literature typically fall within this range, as do all calibrated η values for the 3% rate shown in Table 1. Incidentally, under all those η values, the mean SCC estimates are all between \$65 and \$78, highlighting the stability of the mean SCC in this range. However, the mean SCC remains sensitive to more extreme η values. For example, the mean SCC exceeds \$100 for all η values smaller than 0.12 or larger than 1.85 (again, varying ρ to maintain a near-term 3% discount rate).

The range of η for which the discounting rule has a roughly constant effect on the mean SCC may, however, be different for different IAMs or damage functions, and in particular is driven by the assumed climate “beta”. Specifically, the climate beta is a key determinant of the location of the bottom of the “U” shape seen in Figure 4. For example, an alternative damage function featuring

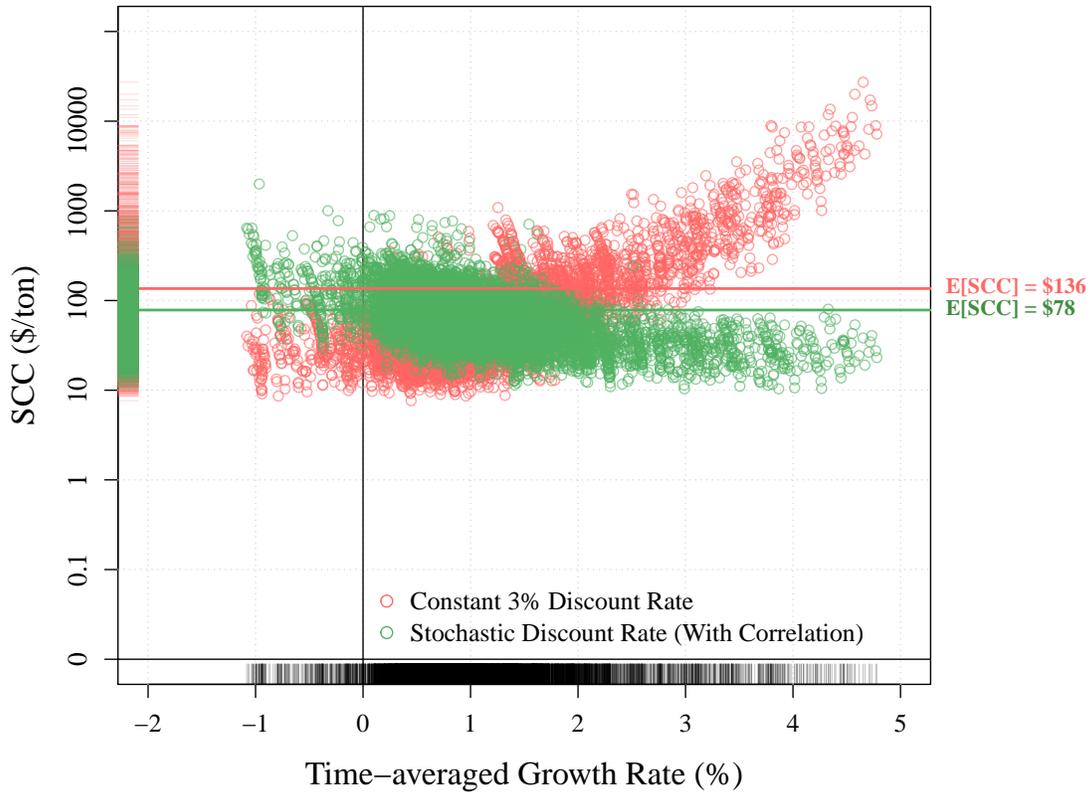


Figure 5: SCC (2020\$) and Long-run Average Consumption Growth across Monte Carlo Draws, by Discounting Method

a lower beta would likely shift the base of the curve left, although the effects on the shape of the function are otherwise unclear because they would depend on the other details of the alternative damage function.

The stabilizing effect of the stochastic discounting rule is further illustrated in Figure 5, which shows the individual SCC values for each of the 10,000 Monte Carlo draws, graphed against long-run average consumption growth rate in each draw. The SCC values are plotted separately in red for SCCs calculated using constant discounting (row 5 of Table 2) and in green using stochastic discounting with the full correlation structure (row 6). The nominal stream of damages is exactly the same between these two sets of points; the only difference is how those damages are discounted. Rug plots are shown along the axes to illustrate the distributions of the SCC draws and growth rates.

The figure illustrates how the discounting rule stabilizes the mean SCC. With constant discounting (in red), growth rate uncertainty creates substantial uncertainty in the SCC due to the scaling of damages with economic growth. Under constant discounting, higher growth draws result in large values of the SCC, with some SCC draws exceeding \$10,000 per ton. The individual SCC values exhibit an approximately exponential relationship with the growth rate—which is to be expected

because marginal damages are proportional to GDP, which is exponential in economic growth. As a result, the large SCC draws dominate the mean (though not the median).

The stochastic discounting rule (in green) dampens this effect, as higher nominal damages scenarios are discounted more, and low damage scenarios are discounted less. With the discounting rule, these large growth (and hence large damages) draws are discounted more, resulting in a more stable, less uncertain distribution of the SCC. Analogously, low and negative growth draws are discounted less than under constant discounting, increasing the discounted value of damages in those draws. Because $\eta = 1.53$ is larger than 1, and because DICE features damages proportional to GDP, there is a modest negative relationship between the SCC and the growth rate, and the SCC tends to be larger at lower or negative growth rates.

The implementation of our discounting rule decreases the overall variance around the SCC, as high damage draws are discounted more and low damage draws are discounted less. As a result, 95% of all SCC draws fall between \$20 and \$220 per ton when using our discounting rule, compared to the much wider range of \$15 to \$645 when using constant discounting.

5 Extensions and future work

Equation (1) can be interpreted either as a first order approximation to the relationship between the discount rate and economic growth or as directly representing the traditional Ramsey equation, where the η parameter corresponds to a representative agent's iso-elastic utility function. As is well known, a drawback of the standard Ramsey equation is that it uses a single parameter to represent both risk aversion and aversion to intertemporal inequality. The recursive preference framework of Epstein-Zin-Weil (EZW) is more flexible, allowing these parameters to differ (see Backus, Routledge and Zin 2004 for discussion). An interesting question is how the social cost of carbon might be further affected by such flexibility.

Despite such interest, there are several reasons why we do not pursue this question here but consider it a useful area for further work. Our objective here has been to develop an empirical, evidenced-based approach to discounting stochastic simulations of climate damages as recommended by the NAS. In particular, over periods of hundreds of years we know that persistent uncertainty about discount rates lowers the certainty-equivalent rate (Weitzman 1998; Newell and Pizer 2003). Defining the discount rate as a linear function of economic growth, as in the Ramsey relationship, we can choose parameters to approximate the term structure of long-term interest rates (Newell and Pizer 2003; Groom et al. 2007; Freeman et al. 2015; Bauer and Rudebusch 2020*b*) based on probabilistic empirical evidence regarding long-term economic growth (e.g., Müller, Stock and Watson 2019).

Meanwhile, EZW preferences were developed in large part to help explore certain paradoxes that exist in a traditional Ramsey framework, such as the equity premium. That is, the goal was to explain broad features of the economy, not to provide parameter estimates specifically relevant to long-term discounting. Quantitative analysis using EZW preferences has largely focused on simulating calibrated model behavior using assumed parameter values. For example, Gollier and Mahul (2017) use data on aggregate consumption and while assume values for EZW preferences to show yield curves (to 20 years) for 248 countries. Caldara et al. (2012) review techniques for simulating macroeconomic behavior and asset prices, again assuming parameter values. Empirical estimation, where risk aversion and aversion to intertemporal inequality are chosen by the data, remains a more limited area of work (e.g., Van Binsbergen et al. 2008).

There are further challenges to implementing an EZW in our context. Van Binsbergen et al. (2008) use macroeconomic data on consumption, investment, and hours worked as well as bond yields with up to 30-year terms. It is unclear how to modify the data requirements associated with such approaches to focus on the much longer time horizon relevant for the SCC. Finally, we are not aware of an existing framework for using EZW preferences to provide state-contingent discount rates that could be applied to stochastic simulations in the same way that equation (1) provides a simple rule for each random pathway and period. For example, Daniel, Litterman and Wagner (2019) reduce uncertainty to six bifurcating periods. They then solve recursively from the last (final) period rather than making use of a rule.

Specifically, it has been a significant advancement in SCC estimation to development empirically-driven, stochastic projections of aggregate outcome over several centuries (Müller, Stock and Watson 2019). We believe a useful further line of research could explore (a) how EZW preferences could be approximated as a discounting rule and applied to simulated data of the sort in Müller, Stock and Watson (2019); and (b) how EZW preferences, particularly risk aversion and aversion to intertemporal inequality, could be estimated in a relevant way to this particular long-term problem.

6 Conclusion

Ongoing work to improve the methodology for estimating the social cost of carbon requires a discounting rule to relate the discount rate to economic growth rates in a manner that consistently represents their joint uncertainty. While much of the effort behind parameterizing a Ramsey-like discounting rule has been derived using normative approaches, we propose an empirically derived approach based on evidence on the term structure of certainty-equivalent discount rates and econometrically estimated distributions of economic growth rates.

This approach leverages a conceptual relationship between the term structure of interest rates and the distribution of economic growth rates. For a given economic growth rate distribution, the

parameters of this discount rate/growth rate relationship determine the level and slope of the term structure. We find the parameter values that best match empirical evidence on the term structure and the growth rate distribution. We calibrate these parameters for each of a set of near-term target discount rates, resulting in an empirically driven set of discounting parameters. These parameters can then be used in an integrated assessment framework for the Social Cost of Carbon where uncertain economic growth drives both damage estimates and discount rates. The advantage of this approach is that it allows modelers to account for uncertainty in the discount rate (through uncertainty in the economic growth rate) in a way that is both internally consistent within an IAM while also approximating the empirical literature of the term structure of discount rates.

We further implemented this stochastic discounting rule in the DICE model to correctly account for the linkage between discount rates and growth rates, and hence the correlation between the discount rate and climate damages. The results demonstrate that accounting for this relationship is not only important for determining the expected value of the SCC, but it also reduces the overall uncertainty around it. Employing such a rule linking the discount rate to growth rates is hence crucial for the stability of the SCC when incorporating stochastic economic growth into an IAM framework.

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A Alternative Term Structures

A.1 Fitted Term Structures under Alternative Near-term Rates

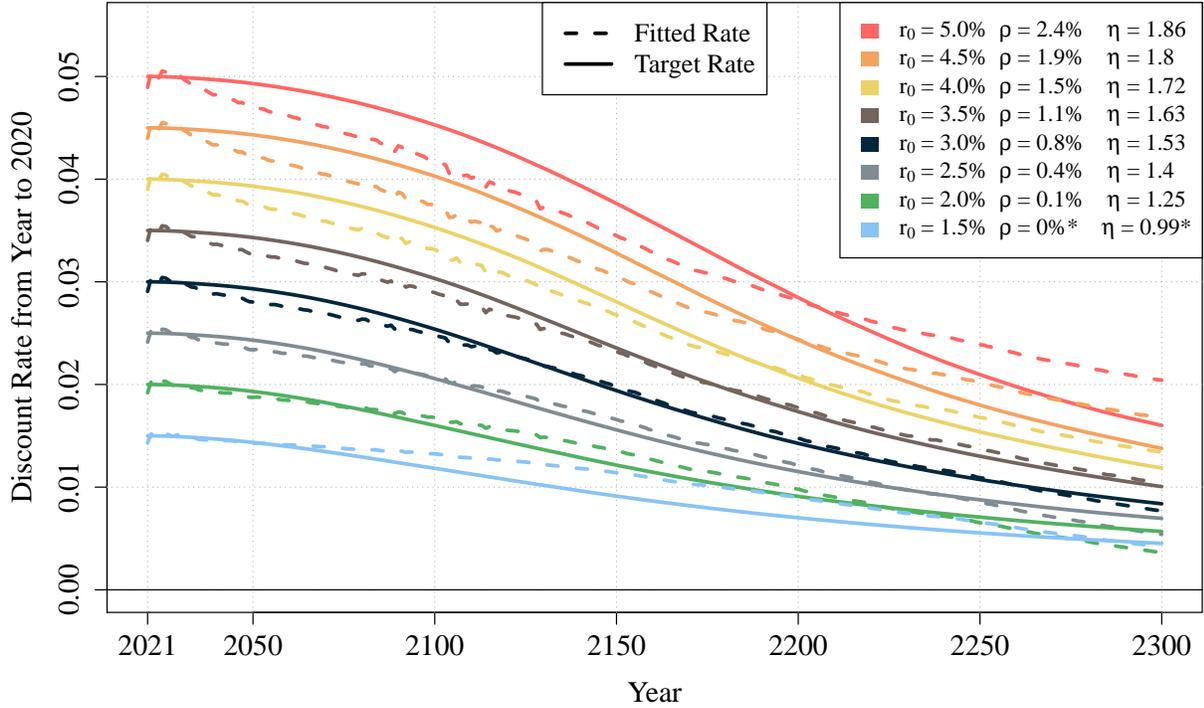


Figure A.1: Estimated and Actual Certainty-Equivalent Paths, Bauer and Rudebusch (2020b) Model, Alternative Near-term Rates

Notes: Values indicated by an * are the results under the constraint that $\rho \geq 0$. This constraint is only ever binding under the 1.5% near-term rate. The unconstrained values are $\rho = -0.1\%$ and $\eta = 1.08$.

A.2 Fitted Term Structures under Alternative Target Models

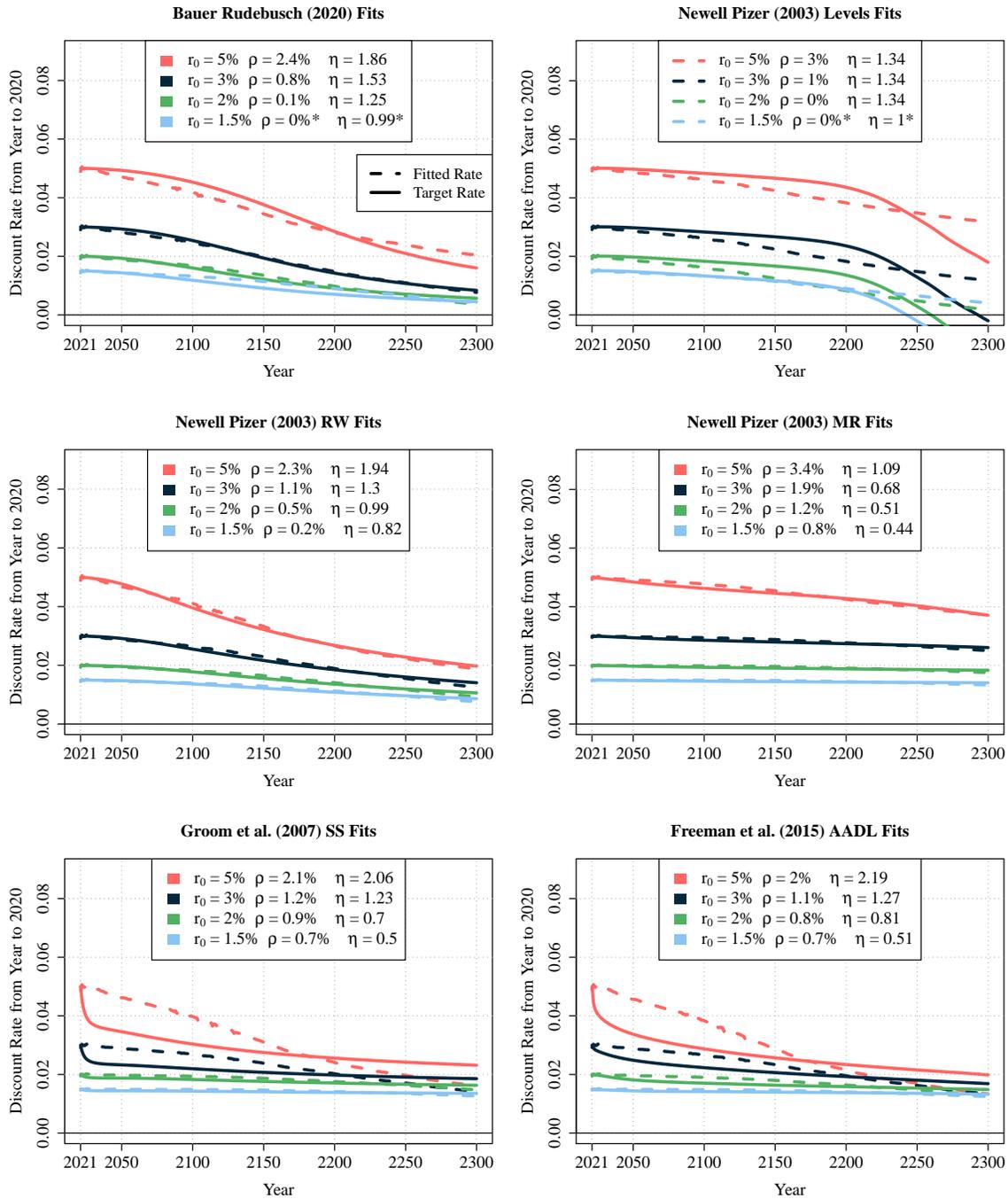


Figure A.2: Estimated and Actual Certainty-Equivalent Paths, All Models from Literature

Notes: *: These values are under the $\rho \geq 0$ constraint, which only binds under the 1.5% rate. The unconstrained (ρ, η) values are $(-0.1\%, 1.08)$ and $(-0.5\%, 1.34)$ for the BR and N&P Levels term structures respectively.

B Explanation for the Growth Uncertainty Premium

The SCC results illustrate that uncertainty in economic growth can lead to large increases in the expected value of the SCC (an uncertainty premium), but that the discounting parameter η mediates that premium. This appendix explores and explains that effect mathematically. Specifically, denote damages as a percentage of GDP t years into the future as Ω_t , GDP that year as Y_t , and time-average economic growth as $\tilde{g}_t = \frac{1}{t} \sum_{\tau=1}^t g_\tau$. We assume that $E[\tilde{g}_t]$ exists but otherwise make no specific distributional assumptions. For simplicity, assume that Ω_t is deterministic (or alternatively, not correlated with economic growth). Therefore, damages in period t are equal to $\Omega_t Y_t = \Omega_t Y_0 e^{\tilde{g}_t}$. When using a constant discount rate denoted r , expected discounted damages are $E[\Omega_t Y_t e^{-rt}]$, which are discounted according to the deterministic discount factor e^{-rt} . Therefore, by Jensen's inequality, expected discounted damages t years ahead using a constant discount rate are given by

$$\begin{aligned} E[\Omega_t Y_t e^{-rt}] &= E[\Omega_t Y_0 e^{\tilde{g}_t} e^{-rt}] \\ &> \Omega_t Y_0 e^{E[\tilde{g}_t]t} e^{-rt}, \end{aligned} \quad (8)$$

where expectation is taken over economic growth rates. The last expression in (8) is the discounted damages that would result from ignoring the stochastic nature of economic growth. The use of the expected growth rate corresponds to calculating damages with a deterministic growth path that is nonetheless centered around the same rate, such as using a fixed (say) 2% growth rate based on historical averages.²² We refer to this as the “deterministic growth” case and it is qualitatively similar to past iterations of the IWG, which featured very little uncertainty in economic growth but nonetheless has a mean rate around 2% for most of the time horizon, similar to the mean rate in MSW.

Equation (8) shows that economic growth uncertainty results in larger discounted damages than would be calculated if one assumes deterministic growth. This can be seen numerically in Table 2, where row (4) shows the SCC at the mean growth rate, and row (5) shows the expected SCC with stochastic growth, both using a constant discount rate of $r = 3\%$. The inequality follows from the convexity of the $e^{\tilde{g}_t}$ function in growth rates. The more convex this function, the larger is the inequality.

Extending this Jensen's inequality result reveals how η in a Ramsey-like discounting framework mediates this effect. With the Ramsey-like discounting rule ($r_\tau = \rho + \eta g_\tau$ for one-period growth rate g_τ), the discount factor now takes the stochastic form $SDF_t = e^{-\sum_{\tau=1}^t r_\tau} = e^{-\sum_{\tau=1}^t \rho + \eta g_\tau} =$

²²For example, if the growth rate were distributed around 2% with some variance $\sigma^2 > 0$, the left side of the inequality would represent damages taking account this uncertainty, whereas the right side would represent damages using the same 2% mean but collapsing σ^2 to zero.

$e^{-(\rho+\eta\tilde{g}_t)t}$. Discounted damages are now:

$$\begin{aligned}
E[\Omega_t Y_t e^{-(\rho+\eta\tilde{g}_t)t}] &= E[\Omega_t Y_0 e^{\tilde{g}_t t} e^{-(\rho+\eta\tilde{g}_t)t}] \\
&= E[\Omega_t Y_0 e^{(1-\eta)\tilde{g}_t t} e^{-\rho t}] \\
&\geq \Omega_t Y_0 e^{(1-\eta)E[\tilde{g}_t]t} e^{-\rho t}.
\end{aligned} \tag{9}$$

It is straightforward to see this equation is a more general case of equation (8). Setting $\eta = 0$ and $\rho = r$ in equation (9) yields (8). Further, for $\eta = 1$, the expression no longer depends on the growth rate. We thank a referee for noting an intuitive interpretation of this result: when $\eta = 1$, the influence on discounted damages of the intertemporal substitution effect (η) and the income effect (the climate β) are equal and opposite to each other.

Analogous to equation (8), the inequality now follows from the convexity of this $e^{(1-\eta)\tilde{g}_t t}$ function. The degree of convexity is now given by $\frac{d^2}{d\tilde{g}_t^2} e^{(1-\eta)\tilde{g}_t t} = (1-\eta)^2 t^2 e^{(1-\eta)\tilde{g}_t t} \geq 0$. While this still convex, it is typically less convex than in the constant discounting case in equation (8). Comparing the two expressions reveals convexity is now smaller by a factor of $(1-\eta)^2 e^{-\eta\tilde{g}_t t} \leq 1$. With η between 0 and 2 (a normal range) and $\eta \neq 1$ this factor is strictly less than 1 at all time horizons ($t > 0$). (At sufficiently long time horizons, this shrinking effect arises for any $\eta \neq 1$, since the exponential term $e^{-\eta\tilde{g}_t t}$ ultimately dominates the expression.) In other words, the stochastic discounting rule “shrinks” the convexity of discounted damages, thereby reducing the “uncertainty premium” on the SCC.

Interestingly, the uncertainty premium is proportional to $(1-\eta)^2$, which is weakly positive for any value of η . I.e., its sign is positive and does not depend on whether η is larger or smaller than one. Specifically, the uncertainty premium is U-shaped as a function of η , explaining the simulation results seen in Figure 4. This is because discounted damages are proportional to $e^{(1-\eta)\tilde{g}_t t}$, which is a convex function of growth regardless of whether η is larger or smaller than 1.

When η is small, it is an increasing convex function of growth, and discounted damages are highest when growth is large (see Figure 3, bottom left panel). The intuition in this case is straightforward—high growth states feature large GDP, leading to the traditional damage scaling effect. With small η , this effect is only partially offset by a higher discount rate. High growth scenarios thus loom large in the expectation, and low growth scenarios that feature very small damages contribute little to the expectation.

What is less intuitive is that when η is large, damages are a *decreasing* convex function, i.e., discounted damages are higher when growth is low (see Figure 3, bottom right panel). To see why, consider a 1 percentage point increase in the growth rate. Damage scaling increases undiscounted damages by 1 percent each year, but they are discounted by $\eta > 1$ percent more each year. On net, discounted damages fall. Now low growth scenarios loom large, and the heavily discounted high

growth scenarios contribute little to the expectation. Indeed, Figure 5 illustrates that with $\eta = 1.53$, the SCC is typically higher when growth is lower.