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The Shadow Price of Capital: Accounting for Capital Displacement in Benefit–Cost Analysis

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About the Project

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Abstract

Government analysts have long used discount rates based on investment rates of return to approximate the effect of capital displacement. However, we show how this approach is not well grounded in economic theory and produces highly biased results, particularly in the context of decisions involving long-lived impacts such as climate change. We demonstrate how analysts can use the conceptually correct shadow price of capital (SPC) approach in a straightforward manner to account for concerns about capital displacement in federal regulatory analysis. We derive a formula for the SPC as a function of four key parameters and propose a central SPC value of 1.1, with a reasonable range of 1.1 to 1.2. We then illustrate how the SPC approach could be easily implemented in practice using the example of the 2015 Clean Power Plan Regulatory Impact Analysis, showing that estimated net benefits are far less sensitive to capital displacement concerns under the analytically correct SPC approach as compared to the inappropriate approach of using a 7 percent investment rate or return. Our work is particularly important given the ongoing efforts to revise federal guidance for benefit-cost analysis and discounting.

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1. Introduction

A longstanding 1993 executive order, EO 12866, requires any economically significant federal regulation to be accompanied by a benefit–cost analysis (BCA). BCAs require a choice of discount rates to compare costs and benefits that accrue at different periods of time, and for impacts with long-lived consequences, the result of a BCA can be highly sensitive to the choice of the discount rate. For 20 years, federal guidelines for BCAs, including appropriate approaches to discounting, have been set by a guidance document known as Circular A-4 (OMB 2003), which the federal government is now proposing to update.[1] In 2003, Circular A-4 recommended two discount rates to adjust estimated future costs and benefits to present-day equivalents: 3 percent and 7 percent. The 3 percent consumption discount rate is meant to reflect the discount rate applicable to impacts on individual households (as measured by their consumption), with individuals being the ultimate concern of economic welfare analysis. In contrast, the 7 percent investment rate of return—which is sometimes called the opportunity cost of capital—is meant to reflect the possibility that costs may displace capital investment, which has a higher rate of return than the consumption rate due to economic distortions such as taxes and transaction costs (Boardman et al. 2017), as well as a risk premium. Importantly, both the consumption and investment rates for the purposes of this discussion should reflect risk-free returns. If not, additional work must be done to adjust for the potential difference in risk between the source of discount rate data and the proposed policy benefits. As we discuss later, there is reason to believe that the estimated 7 percent investment rate of return is not truly risk free, but we set that point aside for the moment for the purposes of discussion.

Circular A-4 notes, however, that the economic literature has shown that the "analytically preferred" method to account for the higher investment rate of return is instead to use the "shadow price of capital" (SPC) approach, in which impacts affecting capital investments are converted to consumption-equivalent values. Once all impacts are measured consistently in terms of consumption, all costs and benefits are appropriately discounted at the consumption discount rate. "Analytically preferred" is another way of saying "welfare-grounded," meaning based in the concepts of welfare economics which takes as its starting point the well-being of households as measured by their consumption over time.

Nonetheless, the analytically preferred SPC approach is rarely used in practice. Instead, federal BCAs typically include a sensitivity case that discounts all costs and benefits at the investment rate of return, set at 7 percent in 2003 by Circular A-4. The use of the investment rate was intended to serve as a simplified way to account for capital displacement, perhaps because the shadow price of capital rate was seen as too complex for use by agency analysts. However, in this paper we show that the SPC approach is actually both simpler and much better grounded in welfare economics than the current 7 percent approach and can be implemented with currently available information.

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[1] https://www.federalregister.gov/documents/2023/04/07/2023-07364/request-for-comments-on-proposed-omb-circular-no-a-4-regulatory-analysis.

Using an investment rate of return as one of the discount rate sensitivity cases is common practice in federal regulatory impact analyses (RIAs), but economists have demonstrated that it is only conceptually consistent with the analytically preferred SPC method under very restrictive and unrealistic conditions that are almost never satisfied (Li and Pizer 2021). Simply discounting benefits at an investment rate of return ignores the differences between the time pattern of the benefits and that of the capital returns being displaced. That is, discounting future benefits at the investment rate of return to account for immediate cost impacts on capital investment mismeasures the value to households if those time patterns differ. This reflects well-known problems with using investment rates of return to compare policy options, rather than an appropriate consumption discount rate.

To see why the use of an investment rate yields incorrect conclusions, consider a simple example of a policy that costs $1 in capital today and creates a benefit of $100 delivered 100 years into the future. Households would value that benefit at $5.20 in present value based on a 3% discount rate ($=\$100/1.03^{100}$). The cost of the policy as felt by consumers is not the $1 in capital itself, but rather the lost stream of returns that capital would have earned, which for the moment we will assume is 7% annually in perpetuity. The costs to consumers therefore take the form of a perpetuity of $0.07 annually, the present value of which, discounted at the consumer's discount rate of 3%, is $0.07/3% = $2.33. Hence, the net benefits of this policy are positive: $5.20 - $2.33 = $2.87. By contrast, if one were to instead try to account for capital displacement by discounting benefits at a 7% rate as Circular A-4 suggests, one would instead mistakenly calculate *negative* net benefits because the present value of benefits would be merely $0.12 ($=\$100/1.07^{100}$). This is just one example of how using an investment discount rate in a benefit-cost analysis can yield wrong conclusions when costs and benefits have different time patterns.

Recent work not only confirms that the longstanding approach of using a 7 percent discount rate is inconsistent with the welfare-grounded SPC approach, but also shows that the degree of embedded inaccuracy tends to compound the longer the time frame of the policy being evaluated (Li and Pizer 2021). It is therefore particularly inaccurate for actions with long-term consequences, such as actions that reduce greenhouse gas emissions. This paper explains why the discounting sensitivity case using a 7 percent investment rate of return is not theoretically grounded and can yield extremely misleading estimates of the costs and benefits of policies with long-lived impacts, such as climate change.

Moreover, this paper explains how to move beyond the inconsistent but common practice of applying a 7 percent discount rate to address concerns about capital displacement. Instead, we show how the welfare-grounded SPC approach is simple to implement in practice, would not involve major changes in analytical procedures, and would simplify federal BCAs by dispensing with multiple internally inconsistent discount rates within a given BCA. We demonstrate how one would implement this by re-evaluating the final RIA for the 2015 Clean Power Plan using the SPC framework and Circular A-4's 2003 recommended 3 percent consumption discount rate. This demonstrates that the SPC approach would be simple to implement, account for concerns about capital displacement without resorting to the inconsistent 7 percent discount rate approach, and yield results similar to those reached by recently conducted RIAs in cases where the time horizon is not particularly long. We also explore scenarios that use a 2 percent consumption discount rate, which evidence suggests is now a more appropriate estimate than 3 percent going forward. For example, the federal government's recent revisions to Circular A-4 proposed a value of 1.7%, as discussed in more detail below.

These results have clear policy implications given the Biden administration's ongoing efforts to update Circular A-4 to "reflect new developments in scientific and economic understanding"[2] and OMB's recently proposed revisions. While the concept of the SPC is not a new idea (e.g., Bradford 1975; Lind 1990; Lyon 1990; Moore et al. 2004), it was not widely understood in 2003, when Circular A-4 was drafted, just how large the bias would be from the use of an investment rate of return for discounting in the case of policies with long-lived impacts, such as regulations addressing greenhouse gas emissions.

In April 2023, OMB proposed a comprehensive revision of Circular A-4, which includes changes to discounting as well as other issues such as distributional effects. OMB has proposed two specific changes relating to discounting. First, OMB has proposed updating the estimate of the consumption rate of interest to 1.7%, which reflects average real returns to 10-year US Treasury notes from 1993 to 2022.[3] This 30-year average approach is conceptually analogous to the calculation underlying the 3% estimate developed in the 2003 version of Circular A-4.

Second, OMB has eliminated the recommendation to discount using a 7 percent discount rate to account for capital displacement and instead suggests using the shadow price of capital. In particular, OMB suggests using two values of the shadow price—1.0 (reflecting an open economy with perfect capital mobility) and 1.2 (reflecting a closed economy with limited international capital flows). These SPC values would then be applied to the estimated share of benefits and costs accruing to capital if the analyst has such an estimate; if no estimate is available, OMB has recommended two "outer-bound" cases: one assuming all benefits and no costs accrue to capital, and another assuming the reverse. These proposed revisions are broadly consistent with our recommendations in this paper, which we would note was drafted *before* OMB's proposed revisions were published. The main difference is that our central SPC value is 1.1, as compared to OMB's proposed value of 1.2, which we discuss below.

The paper proceeds as follows: First, we explain the conceptual basis that motivated the use of an investment rate for discounting recommended in Circular A-4, when it is and is not equivalent to the welfare-grounded SPC approach, and how much the two approaches can differ. Second, we derive a formula for the shadow value of capital as a function of four parameters: the consumption discount rate, the investment rate of return, the depreciation rate of capital, and the savings rate. Third, we use reasonable estimates of those parameters to propose SPC values for use in regulatory analysis and other decision contexts. Finally, we demonstrate how the SPC could be implemented in practice, using the Clean Power Plan RIA as an example. In this example, we are balanced in applying the SPC to both costs and benefits—that is, including the possibility that benefits may augment capital, alongside the traditional concern that costs may displace capital investment.

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[2] **https://www.federalregister.gov/documents/2021/01/26/2021-01866/modernizing-regulatory-review**.

[3] The 30-year average spanning the years 1991 to 2020 yields a value very close to 2 percent.

2. Rationale for Discounting at the Investment Rate to Account for Capital Displacement Is Simplistic and Theoretically Unsupported

This section describes the mathematical logic underlying Circular A-4's approach of incorporating capital displacement concerns by simply discounting benefits at a rate of 7 percent. This section also illustrates why it does not accurately reflect the fundamental concern except in very limited situations. Circular A-4 notes that the SPC approach is analytically preferred, but for simplicity, A-4 nonetheless recommends two rates for sensitivity analysis—a consumption discount rate (estimated historically at 3 percent) and an investment rate of return (estimated historically at 7 percent), notionally as a bound on how capital displacement might affect BCAs. The latter rate is sometimes referred to as the "opportunity cost of capital." The use of the consumption rate is appropriate when all costs and benefits fall on consumption and no costs or benefits displace capital.

However, the 7 percent investment rate of return is only an appropriate opposing bound under very specific assumptions. In particular, discounting benefits at the investment rate is only appropriate if (1) the capital is displaced immediately and permanently and both the displaced returns and benefits to be discounted are paid out as perpetuities, or (2) more generally, if the pattern of benefits and investment returns are the same, and (3) only costs (and not benefits) impact investment.[4] A matching time pattern is a necessary condition to the validity of using the investment rate, implying that the use of the investment rate to reflect capital displacement is only appropriate in very limited circumstances, such as those summarized above. By contrast, the costs of regulations or other policies tend to play out over time (say due to changes in regulated entities' variable costs), but these changes do not typically precisely parallel the time profile of benefits (say due to reduced long-term damages from climate change).

To illustrate when the use of an investment rate is and is not appropriate, we walk through the special case in (1) above and compare using the investment rate of return as a substitute for the welfare-grounded SPC approach. For permanent capital displacement, each dollar of immediate costs leads to a permanent $1 loss in capital. This has no immediate impact on consumers, but suppose that dollar of capital would have returned the investment rate of return, denoted r_i, to consumers every year in perpetuity. That is, the return is assumed to go entirely to consumers and does not

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[4] We later show that in addition to accounting for capital displacement by compliance costs, a more balanced approach to applying the SPC would allow for the possibility that benefits may also augment capital.

further affect capital through savings. Therefore, a dollar of permanently displaced capital implies consumers face a cost equal to r_i every year into the future. The present value of this stream of consumer costs equals

$$PV(Consumer\ Costs) = \frac{r_i}{1+r_c} + \frac{r_i}{(1+r_c)^2} + \frac{r_i}{(1+r_c)^3} + \cdots = \frac{r_i}{r_c},$$

where r_c is the consumption discount rate and the final equality, r_i/r_c, derives from the equation for the present value of a perpetuity. Thus, a dollar of permanently displaced capital is equivalent to r_i/r_c dollars of lost consumption today and the hypothetical shadow price is r_i/r_c. For $r_i = 7$ percent and $r_c = 3$ percent, this ratio would be $0.07/0.03 = 2.33$. (Note, however, that we will show later in the paper that an $\text{SPC} = 1.1$ is a more generally appropriate value once one properly accounts for savings and depreciation, which we assume away for the purposes of this thought experiment.)

This shadow price can easily be used to calculate net benefits within this extreme scenario. Assume a regulation that generates $\$B$ of consumer benefits annually for each $1 of permanently displaced capital costs. Then, under the SPC approach in this stylized example, the net present value is given by

$$PV(Net\ Benefits_{SPC}) = PV(Consumer\ Benefits) - PV(Consumer\ Costs) = \frac{\$B}{r_c} - \$1\frac{r_i}{r_c}. \quad (1)$$

Using the shadow price equal to r_i/r_c, this project passes a benefit-cost test if the right-hand side of equation (1) is greater than zero, or $\$B > r_i$.

However, government analyses do not currently account for capital displacement using the SPC approach. The alternative approach to account for capital displacement commonly used by government analysts, per Circular A-4, is to discount the flow of benefits ($\$B$) at the investment rate of return, r_i. Despite there being little conceptual reason to change the benefit calculation to reflect the nature of costs, the intuition given is that the benefits from a project or policy should deliver the same or greater return than if the costs were invested, essentially comparing the internal rate of return. In this simple extreme case of permanent capital displacement and a perpetuity benefit, using the investment rate of return yields the same result as the SPC approach regarding whether a project passes the benefit-cost test. To see why, note that the net benefits would be equal to the perpetuity value of the benefit flows, $\$B$, discounted to the present at the investment rate of return r_i, minus the immediate $1 of costs:[5]

$$PV\left(Net\ Benefits_{r_i}\right) = \frac{\$B}{r_i} - \$1 \quad (2)$$

Comparing the expressions for net benefits under the two approaches—equation (1) for the SPC approach and (2) for the investment rate of return—they both reach the same directional conclusion that the project passes a benefit-cost test if, and only if, $\$B > r_i$. This is the implicit logic underlying Circular A-4's recommendation to

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[5] In this special case, the $1 of costs is assumed to accrue immediately, so it need not be discounted.

discount all costs and benefits at the 7 percent rate: that is, it can mimic the result of the SPC approach under certain conditions, such as when costs permanently displace capital and benefits are paid out as a perpetuity.

Discounting both costs and benefits at the investment rate yields an equivalent result as the SPC approach in this specific case—but not more generally. The equivalency breaks down as soon as the time pattern of regulatory benefits and ordinary capital investment returns differ. It is particularly problematic when regulatory benefits are much longer lived than ordinary capital investment returns. In the above special case, they are both assumed to be perpetuities.

To illustrate how the investment rate of return approach goes wrong as a proxy for the correct SPC approach, suppose the benefits of this \$1 in regulatory cost do not pay out as a perpetuity with $\$B$ paid every year, but rather are a fixed payment of $\$B = \$10$ received at $T = 40$ years. For this example, we will again use $r_c = 3\text{ percent}$ and $r_i = 7\text{ percent}$ as our consumption and investment discount rates, respectively. Therefore, the present value of those benefits would be $\$10/1.03^{40} = \$3.07$. As previously shown, using the SPC approach in this example, the \$1 in immediate investment-displacing cost is valued at $\$1 \cdot r_i/r_c = \$1 \cdot 7\text{ percent}/3\text{ percent} = \$2.33$. Hence, this investment yields positive net benefits of \$0.73 (equaling \$3.07 in benefits minus \$2.33 in costs), thereby passing the benefit-cost criteria. Had we instead discounted benefits at the investment rate of return of 7 percent as in the OMB guidance (in lieu of the SPC approach), we would have computed a present value of benefits of only $\$10/1.07^{40} = \$0.67$, yielding the opposite conclusion of negative net benefits of -\$0.33 (equal to \$0.67 in benefits minus \$1 in costs), erroneously concluding that the policy failed on benefit-cost grounds. This illustrates how discounting at the investment rate goes particularly wrong analytically for situations with near-term costs and long-term benefits.

A subtle point is that in addition to the difference in sign, the 7 percent investment discounting approach is implicitly calculating net benefits in terms of "capital equivalents." In terms of consumption equivalents, the result of using the 7 percent approach to which one should compare the \$0.73 in net benefits from the SPC would be -$\$0.78\ (\text{i.e.}, -\$0.33 \times 2.33)$—even further from the correct answer than initially apparent.

Another way to illustrate how using 7 percent goes awry is to convert the net benefit result from the SPC approach to capital equivalents, which would be \$0.31 (i.e., $\$0.73/2.33$). If one were to solve for the single discount rate that yields the analytically correct conclusion in terms of capital—positive net benefits of \$0.31—the resulting rate would be about 5.2 percent,[6] which has moved toward the 3 percent consumption rate. For longer time horizons, such as those relevant to climate change, the rate will move even closer to the consumption discount rate (3 percent in this example).

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[6] Specifically, $\$10/1.052^{40} - \$1 = \$0.31$.

In general, the discount rate ρ^* that should be used to discount benefits in year T to correctly replicate the results of the SPC approach is the solution to the following equation:

$$\frac{\$B}{(1+r_c)^T} - SPC = \left(\frac{\$B}{(1+\rho^*)^T} - \$1\right)SPC.$$

The left side of this equation is net benefits in consumption terms under the SPC approach. The term in parentheses on the right-hand side is net benefits in capital terms, which is then converted to consumption equivalents by multiplying by the SPC. Simple algebraic manipulation yields a value for ρ^* given by

$$\rho^* = (1+r_c)(SPC)^{\frac{1}{T}} - 1. \tag{3}$$

This replicates equation (16) in Li and Pizer (2021) and demonstrates that ρ^* converges to r_c as the time horizon grows larger. For the case of $T = 100$, SPC=7/3, we find $\rho^* = 3.9 \text{ percent}$.

This exercise demonstrates another important point: the SPC is a simpler way to correctly account for capital displacement than discounting at a higher investment rate of return. Namely, if one wishes to account for investment displacement in the initial period by applying a higher discount rate (that is, above the consumption rate) to future costs and benefits, then one must first determine the appropriate rate ρ^*. As we have shown, the appropriate rate in this circumstance is generally lower than the investment rate of return for sufficiently long time horizons, and both depend on the time pattern of both benefits and costs and on the SPC—even in this very simple example. Furthermore, determining the rate ρ^* requires conducting a full SPC analysis in any event.

One might imagine that a shortcut would be to use a bounding value for ρ^*—but this would be incorrect. For example, over a single year, $\rho^* = SPC - 1 + r_c(SPC)$, which will be considerably larger than r_i for any typical $SPC \geq 1.1$. Hence, the only straightforward and correct approach is to simply conduct the SPC analysis from the start. We turn next to showing how the SPC approach can be used straightforwardly in practice.

3. Derivation of the Shadow Price of Capital Formula

The welfare-grounded approach is to employ the SPC and use a consumption discount rate, so the natural question is what numeric value should be used for the SPC in BCAs. The SPC, which reflects the welfare value lost from displaced capital investment, depends on how long it remains displaced in the economy. The degree of the displacement's persistence is determined by broad economic equilibrium dynamics, including depreciation and savings, suggesting that the SPC should be guided by macro-derived models of savings and investment. Li and Pizer (2021) present such a model reflecting the degree of permanence of capital displacement, demonstrating that the SPC depends on four parameters:

1. μ, the depreciation rate of capital, which determines how quickly capital would have decayed over time had it not been displaced;
2. s, the savings rate (gross of depreciation), which replenishes capital over time;
3. r_i, the investment rate of return (net of depreciation), which determines the annual income (savings and consumption) lost per dollar of displaced capital; and
4. r_c, the consumption discount rate, which converts future consumption into equivalent present values.

Li and Pizer (2021) and Pizer (2021) derive an analytical expression for the SPC as a function of these parameters, generalizing methods developed by Marglin, Bradford, and others in the 1960s and 1970s (e.g., Marglin 1963ab and Bradford 1975). We repeat a version of that derivation for reference.

Here, we pause to highlight an important conceptual linkage between the consumption and investment rates as we have described them. As previously noted, for the purposes of calculating the shadow price of capital, both the consumption and investment rates must represent risk-free rates. Working with r_c and r_i as risk-free rates, in the absence of economic distortions such as taxes, arbitrage would ensure that r_i and r_c are equal (Boardman et al. 2017).[7] Taxes introduce a wedge between the pre-tax capital return r_i and the rate of return received by consumers. That is, given a tax rate τ, there is a direct relationship between the two rates: $r_i = r_c/(1-\tau)$. While in principle the following derivation could replace all values of r_i with $r_c/(1-\tau)$, we nonetheless retain the r_i notation in the following derivation for simplicity and intuition.

Moving on to the derivation, we start by noting that the SPC is defined as the change in immediate consumption equivalent to the present value of the stream of consumption losses associated with the immediate displacement of $1 of capital, discounted at the consumption discount rate. Computing this requires considering the effect that such an immediate displacement of capital would have on consumption over time, given savings rates, investment returns, and depreciation.

We denote an immediate exogenous change in capital in period t as ΔK_t. Given a net (after depreciation) investment return r_i and a depreciation rate μ, K_t produces a change in gross returns (before depreciation) in the next period of $(r_i + \mu)\Delta K_t$. Of that amount, a fraction s is saved, leading to an augmentation of the change in capital formation of $s(r_i + \mu)$. The resulting change in the capital stock in the next period, denoted ΔK_{t+1}, will thus be the direct change in capital stock less depreciation, plus the indirect change in savings induced by the change in capital $s(r_i + \mu)\Delta K_t$:

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[7] Boardman et al. (2017) notes that transaction costs could also play a role, but in the modern liquid financial system it is difficult to believe these would be large. As in the literature, we focus on taxes as the main driver.

$$\Delta K_{t+1} = (1-\mu)\Delta K_t + s(r_i+\mu)\Delta K_t = [s(r_i+\mu)+(1-\mu)]\Delta K_t.$$

Similarly, in period $t+2$, ΔK_{t+1} will in turn produce gross returns of

$$\Delta K_{t+2} = [s(r_i+\mu)+(1-\mu)]\Delta K_{t+1} = [s(r_i+\mu)+(1-\mu)]^2\Delta K_t.$$

And more generally,

$$\Delta K_{t+h} = [s(r_i+\mu)+(1-\mu)]^h\Delta K_t.$$

The portion of the increased returns that are not saved yields consumption benefits of $(1-s)(r_i+\mu)\Delta K_t$. This change in consumption in period $t+1$ is given by

$$\Delta C_{t+1} = (1-s)(r_i+\mu)\Delta K_t.$$

An analogous equation holds for period $t+2$, into which we substitute the above expression for ΔK_{t+1}, yielding

$$\Delta C_{t+2} = (1-s)(r_i+\mu)\Delta K_{t+1} = (1-s)(r_i+\mu)[s(r_i+\mu)+(1-\mu)]\Delta K_t,$$

and more generally, by recursive substitution we find an expression for the change in consumption for each time period into the future:

$$\Delta C_{t+h} = (1-s)(r_i+\mu)[s(r_i+\mu)+(1-\mu)]^{h-1}\Delta K_t.$$

The SPC is the present value of these consumption losses, discounted at the consumption discount rate, per unit of displaced capital, ΔK_t:

$$SPC = \sum_{h=1}^{\infty}\frac{\Delta C_{t+h}}{\Delta K_t} = \sum_{h=1}^{\infty}\frac{(1-s)(r_i+\mu)[s(r_i+\mu)+(1-\mu)]^{h-1}}{(1+r_c)^h}$$

$$SPC = \sum_{h=1}^{\infty}\frac{(1-s)(r_i+\mu)}{1+r_c}\left(\frac{s(r_i+\mu)+1-\mu}{1+r_c}\right)^{h-1}$$

$$SPC = \frac{(1-s)(r_i+\mu)}{r_c+\mu-s(r_i+\mu)}. \tag{4}$$

4. Estimation of a Numerical Value for the Shadow Price of Capital

Equation (4) demonstrates that the SPC is a function of four parameters: the consumption discount rate r_c, the investment rate of return r_i, the savings rate s, and the depreciation rate μ. We draw on recent work to obtain estimates of these parameters. As in Li and Pizer (2021), Pizer (2021), and Moore et al. (2004), we use a depreciation rate of 10 percent. We use the 50-year average US savings rate of about 22 percent as measured by the gross fixed capital formation as a percentage of GDP from the World Bank.[8] We vary estimates of the consumption discount rate and the investment rate of return, as estimates of these have varied in recent years. We focus on a value of 2 percent for the consumption discount rate, which reflects a growing consensus that such a value is a more appropriate estimate of the consumption discount rate than Circular A-4's 3 percent value (CEA 2017; Rennert et al. 2021b, 2022; Newell, Pizer, and Prest 2022; EPA 2022; Carleton and Greenstone 2022). This is also roughly in line with OMB's consumption rate estimate of 1.7% in its recently proposed update to Circular A-4. We also include Circular A-4's existing benchmark of 3 percent for the consumption discount rate for comparison. There is less evidence that the investment rate of return has changed materially from the 2003 Circular A-4's 7 percent estimate (CEA 2017). Updated data based on the approach of Gomme, Ravikumar, and Rupert (2011) also suggests the pre-tax return on capital remains about 7 percent without any adjustment for risk.[9]

As previously noted, the assumed wedge between the risk-free consumption and investment rates of return owes to the economic distortion introduced by taxes. However, the implied tax wedges that would rationalize a 7 percent investment rate with a 2 percent or 3 percent consumption rate are 71 percent and 57 percent respectively, which is implausibly high compared to the average historical capital tax rates of around 35 percent in the Gomme data (i.e., [7 percent – 2 percent]/7 percent=71 percent and [7 percent – 3 percent]/7 percent=57 percent).[10] In any numerical estimation of the SPC, one should therefore assess whether the tax rate implied by the spread between r_c and r_i is reasonable, and these implied rates are clearly not. This suggests an inconsistency in the triplet values of $r_c = 2$ percemt, $r_i = 7$ percent, and $\tau \approx 35$ percent.

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[8] Data from **https://data.worldbank.org/indicator/NE.GDI.FTOT.ZS?locations=US** (accessed March 15, 2023). 22 percent (more specifically, 21.6 percent) represents the average from 1972–2021, which is the longest period available. The 30-year average (1992–2021) is 21.0 percent.

[9] Data available at **https://paulgomme.github.io/#data** (accessed March 17, 2023). Over the longest time period available (Q2 1947–Q4 2021), the pre-tax return to capital averaged 7.4 percent. The 30-year average (Q1 1992–Q4 2021) was 6.7 percent.

[10] According to the Gomme data, the tax rate on capital has averaged 35 percent over the longest available time span (Q1 1947–Q1 2020). The 30-year average (Q2 1990–Q1 2020) is 30 percent.

The somewhat obvious source of this inconsistency is that the estimated historical capital return of 7 percent is not risk free, and because of this embedded risk premium, it would not be appropriate to use 7 percent as the relevant investment return for the purposes of estimating the SPC. Best practice in BCAs warrants separating discounting from risk and discounting using relatively risk-free rates (Lind et al. 2011). Indeed, estimates of the equity risk premium are substantial; the dataset maintained by Damodaran (2022) suggests that the risk premium has averaged about 4 percent historically.[11] Subtracting this 4 percent risk premium from the unadjusted 7 percent investment rate of return yields a risk-adjusted pre-tax investment return of about 3 percent (not to be confused with Circular A4's current 3 percent consumption discount rate). Subtracting taxes from this risk-adjusted 3 percent investment rate using the above 35 percent average capital tax rate yields 1.95 percent, which closely aligns with the 2 percent consumption discount rate favored above and validates the internal consistency of these values. For this reason, we focus on a central triplet of $r_c = 2$ percent, $r_i = 3.1$ percent, and $\tau = 35$ percent in calculating the SPC (where all discount rates are in real terms). We also show alternative sets of SPC calculations using variations in the key parameters, where we vary the tax rate and infer the investment rate accordingly as $r_i = r_c/(1 - \tau)$.

Table 1 shows the results. Panel A shows the results under Circular A-4's 3 percent consumption rate. We start in row (1) using both of Circular A-4's recommended rates, 3 percent and 7 percent, which yields an SPC value of 1.43. As previously noted however, the implied tax wedge between the two rates is implausibly high at 57 percent. Rows (2)–(4) retain the 3 percent consumption rate but instead calculate the investment return consistent with reasonable estimates of the capital tax rate (30 percent, 35 percent, and 40 percent), yielding SPC values of 1.13, 1.16, and 1.20, respectively.

Panel B updates the consumption rate to 2 percent to correspond with the aforementioned consensus that risk-free interest rates have trended downward since Circular A-4 was written in 2003. If we use this lower consumption rate but retain Circular A-4's 7 percent investment rate, we would find an SPC of 1.6, but again with an implausibly high implied tax wedge of 71 percent. Calculating the investment rate by grossing up the 2 percent rate according to our three capital tax rates yields SPC values of 1.09, 1.12, and 1.15. We take as our central estimate the SPC value of 1.1, corresponding to a 2 percent consumption discount rate, a 35 percent tax rate, and a 3.1 percent pre-tax risk-adjusted investment rate of return (all in real terms). Using OMB's proposed 1.7 percent discount rate and our three tax rates would yield very similar SPC values of 1.08, 1.10, and 1.13 (not shown in Table 1), all of which similarly round to our central value of 1.1.

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[11] The average over the longest available period (1960–2022) of Damodaran's central risk premium estimate is 4.2 percent (using the "Implied ERP [FCFE]" column of his "histimpl.xls" dataset, available at https://pages.stern.nyu.edu/~adamodar/ [accessed March 17, 2023]). The 30-year average (1993–2022) is 4.4 percent.

In Panel C, we compare these values to the SPC estimates from Moore et al. (2004), which presents estimates under two approaches to calibrating the parameter that corresponds to consumption rate of interest in equation (4).[12] Moore et al. use slightly different rates of return from our central case, and they use a lower savings rate of 17 percent, although the SPC estimates are not very sensitive to the savings rate. They find two SPC values: 1.33 and 1.09, with 1.09 representing their preferred estimate. The parameters underlying their 1.33 estimate once again imply an implausibly high implied tax wedge of 67 percent.

Overall, Table 1 suggests a range of SPC estimates tied to the chosen consumption and investment rates with SPC values of between 1.09 and 1.60, with a central value of about 1.1. If we limit ourselves to scenarios with plausible tax wedges of 30–40 percent, the range of reasonable SPC values narrows to between 1.1 and 1.2. In summary, we propose an SPC value of 1.1 based on the parameter values in row (7) of Table 1. This value is the same as the recommendation in Moore et al. (2004) and is very close to the value proposed in Pizer (2021).[13]

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[12] These two cases both use the SPC but vary in the parameter used in place of the consumption rate of interest. The first approach is the approach described in this paper: the consumption rate of interest combined with the SPC, or CRI-SPC. The second replaces the consumption rate of with the equilibrium condition of a Ramsey-style optimal growth rate model, or OGR-SPC.

[13] Pizer (2021) recommend an SPC value of 1.2, which is based on a 40 percent tax rate, as in rows (4) and (8), and a slightly different savings rate, which together yielded an estimate of 1.2.

Table 1. Estimates of the SPC Under Different Parameters

	Description (percent)	Consumption Rate of Interest (r_c) (percent)	Investment Rate of Return (r_i) (percent)	Savings Rate (s) (percent)	SPC	Tax Wedge (percent)	Is the Tax Wedge Plausible?
Panel A: Using a 3 Percent Consumption Rate							
(1)	$r_c = 3$ and $r_i = 7$	3	7	22	1.43	57	X
(2)	$\tau = 30$	3	4.3	22	1.13	30	✓
(3)	$\tau = 35$	3	4.6	22	1.16	35	✓
(4)	$\tau = 40$	3	5.0	22	1.20	40	✓
Panel B: Using a 2 Percent Consumption Rate							
(5)	$r_c = 2$ and $r_i = 7$	2	7	22	1.60	71	X
(6)	$\tau = 30$	2	2.9	22	1.09	30	✓
(7)	$\tau = 35$	2	3.1	22	1.12	35	✓
(8)	$\tau = 40$	2	3.3	22	1.15	40	✓
Panel C: Comparison to Moore et al. (2004)							
(9)	Moore et al. (2004) CRI-SPC	1.5	4.5	17	1.33	67	X
(10)	Moore et al. (2004) OGR-SPC	3.5	4.5	17	1.09	22	✓

Notes: All calculations use a depreciation rate of 10 percent, as in Li and Pizer (2021), Pizer (2021), and Moore et al. (2004). Rows (1), (5), (9), and (10) assume the investment return and compute the implied tax wedge, whereas rows (2)–(4) and (6)–(8) assume the tax wedge and compute the implied investment rate of return.

5. A Simple Approach to the Shadow Price of Capital, with an Illustrative Example

With a value for the SPC in hand, its application is straightforward: simply adjust any capital-displacing costs (or benefits) in any year upward by multiplying by the SPC. Formally, assuming that a share α of costs displaces capital, and share $(1-\alpha)$ of costs displaces consumption, then adjusted costs would be calculated as

$$Adjusted\ Costs = \alpha SPC(Unadjusted\ Costs) + \ (1-\alpha)(Unadjusted\ costs).$$

While historically the focus has been on adjusting costs for capital displacement, we take a more balanced approach in what follows, also accounting for the possibility that benefits may augment investment, rather than consumption. Given this, the approach for adjusting benefits is the same as adjusting costs. These calculations convert costs and benefits into consumption equivalents, so all such adjusted costs and benefits can then be directly compared and discounted at the consumption discount rate. This allows analysts to dispense with the common but inappropriate approach of using the investment rate of return and avoid the inconsistency of different discount rates in a single analysis.

The use of an investment rate further assumes all costs displace capital, and no benefits accrue to capital, which may or may not be a reasonable assumption. By contrast, the SPC approach allows for a more nuanced assessment of capital impacts by applying the SPC only to the estimated share of costs and benefits that fall on capital. In some applications, the appropriate shares may be clear. As an alternative, we propose three general cases. The first is a default, central case that assumes that the share of all costs and benefits impacting investment is equal to the savings rate, which we have set to 22 percent in the preceding analysis based on historical averages. Savings augment investment, and therefore assuming as a central benchmark that 22 percent of costs and benefits impact investment (rather than consumption directly) is consistent with overall economic conditions.

For the other two cases, we propose the inclusion of two extreme cases that assume either that all costs displace capital or that all benefits augment capital. These two extremes would bound the central case that assumes that 22 percent of costs and benefits fall on investment. All cases use the consumption discount rate, bringing clarity and consistency to the appropriate discount rate.

This SPC sensitivity approach is both simpler and much better grounded in welfare economics than the current 7 percent approach and can be implemented with currently available information. Moreover, it is much less biased in its assumptions with regard to whether costs displace investment or benefits augment investment—the latter possibility being underappreciated (Li and Pizer 2021). For example, reduced damages from climate change may prevent destruction of long-lived assets, such as coastal infrastructure, and reduced mortality avoids the destruction of human capital as well as leads to more savings and investment.

This approach also addresses an important set of recommendations of a formative report by the National Academies on the social cost of greenhouse gas (NASEM 2017) that discounting approaches in BCAs having climate impacts should apply a consumption discount rate to consumption-equivalent impacts and be internally consistent across various categories of benefits and costs (Rennert et al. 2021b; Prest et al. 2021).

To demonstrate the simplicity of the SPC sensitivity approach, we show how analysts could have implemented the SPC in the RIA for the 2015 Clean Power Plan. That RIA compared costs and benefits calculated using different discount rates, raising concerns about analytical consistency. The SPC approach would avoid such concerns but still account for the important issue of capital displacement.

Figure 1 shows the main panel of the original table from the Clean Power Plan RIA for reference, illustrating the problematic mixing of different discount rates. Table 2 shows a recreation of the 2015 Clean Power Plan BCA using the SPC approach. We show the net benefits calculation using the “snapshot” approach for three specific years—2020, 2025, and 2030—as was done in the original RIA. The original RIA mixed 3 percent and 7 percent discount rates, but we dispense with the need for the 7 percent discount rate by accounting for the potential for capital displacement using the SPC. This greatly simplifies the net benefits table.

If the 10 percent premium is only applied to 22 percent of the costs or benefits (as per the above central recommendation), the adjustment to costs is smaller. For costs, this yields $2.55, $1.02, and $8.58 billion (row 4a). The adjustment to benefits is similarly modest, changing benefits in 2030 from $34–$54 billion to $34.7–$55.2 billion (row 3a).

Putting these together, the estimated net benefits are similarly little changed when we apply the SPC on both sides of the ledger, increasing net benefits in 2030 from $25.6–$45.6 billion to $26.2–$46.6 billion (row 6). That is, net benefits in 2030 change by 2.2 percent, as the 10 percent premium is applied to the 22 percent of both costs and benefits affecting capital.

Our bounding cases are shown in rows (7) and (8). In the first extreme case of applying the SPC to 100 percent of costs (but not to benefits), net benefits in 2030 change from $25.6–$45.6 billion (row 5) to $24.8–$44.8 billion (row 7). In the other extreme sensitivity case, we apply the SPC to 100 percent of benefits to account for the potential capital impacts of environmental improvements, such as reduced mortality bolstering the value of human capital, or individual savings. This increases estimated net benefits in 2030 from $25.6–$45.6 billion (row 5) to $29–$51 billion (row 8).

Stepping back, it is notable that even with a wide range of assumptions about capital displacement, the estimated annual benefits across rows (5) through (8) are generally within relatively narrow ranges of each other. Focusing on 2030, across all scenarios the maximum difference between estimates is 15 percent—the two extreme estimates of $24.8 billion in row (7), assuming all costs displace capital, versus $29.0 billion in net benefits in row (8), assuming all benefits augment capital. This contrasts with the major sensitivity exhibited when using a 7 percent discount rate for climate benefits instead of 3 percent, which can change gross benefits by about a factor of 9 (see Li and Pizer 2021, Appendix Figure A-1).

Note that even if we were to use an SPC on the upper end of the plausible range—1.4 in row 1 of Table 1, consistent with a 3 percent consumption rate and a 7 percent investment rate—the net benefits would be similarly much less sensitive under the SPC approach than under the approach of simply discounting at those two rates. For example, using a 7 percent discount rate for benefits would reduce net benefits in 2030 by approximately $19–$21 billion: $20 billion versus approximately $\$20/9 = \$2\ \text{billion}$ in climate benefits, plus $1–3 billion in reduced health benefits (see Figure 1). By contrast, applying an SPC of 1.4 to 100 percent of the $8.4 billion in costs in 2030 would reduce net benefits by a much smaller $3.4 billion. This again demonstrates the inappropriate nature of relying on a 7 percent investment discount rate instead of using the welfare-grounded SPC approach, particularly when a BCA includes long-lived impacts like climate benefits that are sensitive to the discount rate.

Indeed, most of that disparity between the two approaches (changing net benefits by $18–$20 billion when using a 7 percent discount rate versus $3.4 billion under the SPC approach) is attributable to changes in long-lived climate benefits as measured by the social cost of carbon. For shorter-lived, non-climate benefits, the effects of the two approaches are much closer to each other. As previously noted, the SPC approach applied to all costs using an SPC of 1.4 reduces net benefits by $3.4 billion in 2030, whereas the use of a 7 percent discount rate reduces health benefits, which are near-term impacts, by the similar amount of $1–$3 billion (see Figure 1). This reinforces the earlier analytical result that the use of the SPC is especially important when impacts to be discounted are long-lived.

Table 2. Monetized Benefits, Compliance Costs, and Net Benefits of the 2015 Clean Power Plan, under 2015 RIA and Updated SPC Approach, 3 Percent Consumption Discount Rate

		Value (billions of 2011$)			Source
		2020	2025	2030	
	Benefits				
(1)	Climate benefits (3 percent rate)	2.8	10	20	Table ES-9
(2)	Air quality health co-benefits	0.7–1.8	7.4–18	14–34	Table ES-9
(3)	Total benefits (no SPC adjustment)	3.5–4.6	17.4–28	34–54	(1) + (2)
(3a)	Adjusted benefits, assuming 22 percent capital augmentation	3.6–4.7	17.8–28.6	34.7–55.2	(3)*22 percent*SPC +(3)*(1–22 percent)
(3b)	Adjusted benefits, assuming 100 percent capital augmentation	3.9–5.1	19.19–30.8	37.4–59.4	(3)*SPC
	Compliance costs				
(4)	Costs (no SPC adjustment)	2.5	1.0	8.4	Table ES-9
(4a)	Adjusted costs, assuming 22 percent capital displacement	2.6	1.0	8.6	(4)*22 percent*SPC +(4)*(1–22 percent)
(4b)	Adjusted costs, assuming 100 percent capital displacement	2.8	1.1	9.2	(4)*SPC
	Net Benefits				
(5)	No SPC adjustment	1.0–2.1	16.4–27.0	25.6–45.6	(3)–(4) and Table ES-9
(6)	22 percent of costs and benefits impact capital	1.0–2.1	16.8–27.6	26.2–46.6	(3a)–(4a)
(7)	100 percent of costs displace capital	0.8–1.9	16.3–26.9	24.8–44.8	(3)–(4b)
(8)	100 percent of benefits augment capital	1.4–2.6	18.1–29.8	29.0–51.0	(3b)–(4)

Notes: We use SPC = 1.1 in all cases. Some values differ slightly from those in Figure 1 due to rounding. EPA Clean Power Plan RIA available at **https://www3.epa.gov/ttnecas1/docs/ria/utilities_ria_final-clean-power-plan-existing-units_2015-08.pdf.**

Figure 1. Original Clean Power Plan RIA Table

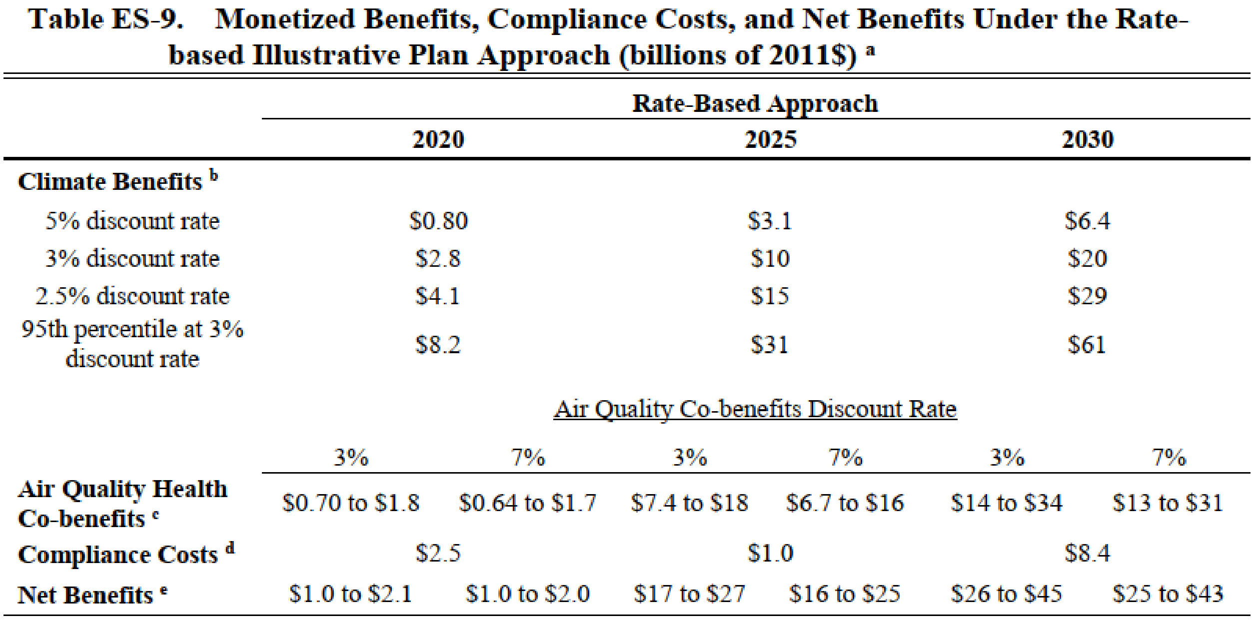

Table ES-9. Monetized Benefits, Compliance Costs, and Net Benefits Under the Rate-based Illustrative Plan Approach (billions of 2011$) [a]

	Rate-Based Approach					
	2020		**2025**		**2030**	
Climate Benefits [b]						
5% discount rate	$0.80		$3.1		$6.4	
3% discount rate	$2.8		$10		$20	
2.5% discount rate	$4.1		$15		$29	
95th percentile at 3% discount rate	$8.2		$31		$61	
	Air Quality Co-benefits Discount Rate					
	3%	7%	3%	7%	3%	7%
Air Quality Health Co-benefits [c]	$0.70 to $1.8	$0.64 to $1.7	$7.4 to $18	$6.7 to $16	$14 to $34	$13 to $31
Compliance Costs [d]	$2.5		$1.0		$8.4	
Net Benefits [e]	$1.0 to $2.1	$1.0 to $2.0	$17 to $27	$16 to $25	$26 to $45	$25 to $43

6. Conclusion

Government analysts have long used discount rates based on investment rates of return to approximate the effect of capital displacement. However, as we discuss, this approach is very inappropriate and produces highly biased results, in particular in the context of decisions involving long-lived impacts like climate change. We demonstrate how analysts can use the welfare-grounded shadow price of capital (SPC) approach in a straightforward manner to account for concerns about capital displacement in federal regulatory analysis. We propose a central SPC value of 1.1, with a reasonable range of 1.1 to 1.2. This estimate could easily be implemented in regulatory analysis by multiplying any costs or benefits that fall on capital by the SPC. If the share of costs or benefits is not known, then analysts can conduct sensitivity analyses to bound the effect of capital by alternatively assuming all costs and no benefits fall on capital, and vice versa. We illustrate how it could be easily implemented in practice using the example of the 2015 Clean Power Plan RIA. This shows that estimated net benefits results are far less sensitive to capital displacement concerns under the welfare-grounded SPC approach as compared to the inappropriate approach of using an unadjusted 7 percent investment rate of return. Our work is particularly important given the ongoing efforts to revise the federal guidance document for best practices in benefit-cost analysis and discounting, Circular A-4, to “reflect new developments in scientific and economic understanding.”

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