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# **Disentangling the Roles of Growth Uncertainty, Discounting, and the Climate Beta on the Social Cost of Carbon**

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# 1. Introduction

Getting the discount rate right is essential for estimating the social cost of carbon (SCC). Changing the discount rate from 3 to 2 percent—a change approximately consistent with recently proposed updates to federal guidance (OMB 2023a,b)—can more than double the SCC (see, e.g., Rennert et al. 2022; Barrage and Nordhaus 2023). Further, when estimating the SCC, it is common to adjust discount rates to account for uncertainty in future consumption growth and its covariance with uncertain climate impacts (or, alternatively, climate impacts’ covariance with market returns), often called the “climate beta” (Gollier 2014; Dietz et al. 2018). Yet disagreement remains as to whether this adjustment should result in a higher or lower discount rate, largely due to disagreement about the magnitude and sign of the climate beta (see, e.g., Groom et al. 2022; Drupp et al. 2023; Lemoine 2021; Dietz et al. 2018). While major integrated assessment models (IAMs) like William Nordhaus’s DICE model feature a positive climate beta and therefore employ a higher discount rate (Barrage and Nordhaus 2023), others have argued for a negative beta, implying lower discount rates (e.g., Howard and Schwartz 2022; Lemoine 2021). This debate has major consequences for estimates of the SCC, wherein a positive risk adjustment to the discount rate (positive beta) is commonly presumed to correspond to a lower SCC (e.g., Barrage and Nordhaus 2023), whereas a negative risk adjustment to the discount rate (negative beta) is presumed to correspond to a higher one (e.g., Howard 2023).

This paper demonstrates that those presumptions are generally incorrect because they consider only one side of the ledger—how uncertainty affects discount rates—while ignoring the offsetting effect of how the same uncertainty affects the value of the object being discounted: expected marginal damages from an incremental ton of carbon dioxide (CO<sub>2</sub>) emissions. In short, this paper shows that uncertainty in future consumption growth generally increases the SCC, except in one edge case where the effect is zero. This result arises because with a nonzero climate beta, uncertainty in economic growth affects not only the variance but also the expected value of climate impacts, and amid persistent growth uncertainty, this effect is particularly large for impacts occurring far in the future. As I show in this paper, this effect on expected values easily dominates the effect on the discount rate. This result implies that using risk-adjusted discount rates to discount expected climate impacts without accounting for growth uncertainty’s effect on those same expected impacts will yield highly biased estimates of the SCC. In models with a positive beta, this bias leads to substantial underestimates of the SCC, whereas in models with a negative beta, the bias leads to substantial overestimates.

Despite this result, the economic literature and applied analysis both give disproportionate and often exclusive attention to risk adjustments to the discount rate, with little or no attention to corresponding adjustments to the expected values being discounted. Indeed, it is common in cost-benefit analysis to calculate costs and benefits in a deterministic model, but then apply risk-adjusted discount rates to those deterministic values based on the idea that those costs and benefits are, in reality,

uncertain with some risk profile. This approach is correct only if the deterministically modeled costs and benefits are representative of expected values embodying the same uncertainties that motivate risk adjustments in discount rates, but analysts typically do not seem to consider this in practice.

For example, recently proposed revisions to Circulars A-4 and A-94 (OMB 2023a,b) dedicate entire sections to accounting for effects of uncertainty and risk aversion, but those discussions focus principally on risk adjustments to discount rates. There is no mention of how those same uncertainties may similarly affect expected values. This is also true of Nordhaus's (2023) critique of the US Environmental Protection Agency's treatment of risk and uncertainty in the discount rate (EPA 2022). The only study I am aware of that acknowledges the effect of growth uncertainty on expected values is that of Ni and Maurice (2021), who note that uncertainty affects the growth rate of expected impacts in a manner governed by the beta; nonetheless, they focus on the discount rate. In general, the common inattention in the literature to growth uncertainty's effect on expected values has likely contributed to its widespread omission in applied analysis.

To derive these results, this paper begins by defining risk-adjusted and certainty-equivalent (also sometimes referred to risk-free) discount rates in the consumption capital asset pricing model, illustrates various conceptual features of those rates, derives analytical expressions for them under certain structural assumptions, and shows how key parameters affect the levels and trajectories of each discount rate. While many of the expressions derived herein are not completely new to the literature (e.g., related expressions are derived in Weitzman 1998; Gollier 2014; and Dietz et al. 2018), this paper synthesizes key insights from across the literature to illuminate the drivers of the term structures of risk-free and risk-adjusted discount rates and their implications for the SCC. Further, it shows the certainty-equivalent and risk-adjusted rates implied by the Greenhouse Gas Impact Valuation Estimator (GIVE; Rennert et al. 2022), demonstrating that GIVE's relatively high central estimate of the SCC at \$185 per ton of carbon dioxide is nonetheless consistent with a risk-adjusted discount rate that rises over time, with a risk premium reaching 2.7 percent by the end of its time horizon (2300).

The key takeaways of this paper are that, under an assumption about the persistence of uncertainty in economic growth consistent with recent literature (Müller, Stock, and Watson 2022; Rennert et al. 2021),

1. The certainty-equivalent discount rate has a term structure that declines approximately linearly with the time horizon at a rate determined by the elasticity of marginal utility,  $\eta$ .
2. Assuming a constant climate  $\beta$  (meaning a 1 percent increase in economic growth corresponds to a  $\beta$  percent increase in marginal damages) and abstracting away from trends in the mean and variance of growth rates, the risk-adjusted term structure rises with the time horizon if and only if  $\beta > \eta/2$ . This condition is satisfied in most standard integrated assessment models.
3. Notwithstanding the slope of the term structure, growth uncertainty increases the SCC except in the edge case of  $\beta = \eta$ , where it has no effect.
4. The same logic that suggests adding a risk premium to the discount rate also implies that expected marginal damages should similarly be affected strongly by uncertainty in an offsetting manner. This latter effect dominates the former, yet it is commonly ignored.
5. While reducing  $\beta$  reduces the risk-adjusted discount rate, it nonetheless generally reduces the SCC for reasonable parameter values, through its offsetting effect on marginal damages.

The paper proceeds as follows. Section 2 derives general expressions for risk-adjusted and certainty-equivalent discount rates and shows the term structures of those rates implied by the GIVE model. Section 3 derives analytical expressions for those discount rates and expected marginal damages under specific assumptions of the consumption growth process (normally distributed) and the functional form of marginal damages (constant beta). Section 4 decomposes and signs the effect of consumption growth uncertainty on the time path of expected discounted marginal damages. Section 5 further demonstrates that the SCC is typically an increasing function of the climate beta, even though a higher beta implies a higher risk-adjusted discount rate. Section 6 explains the role of persistence in growth uncertainty for the above results. Section 7 provides a brief discussion, and Section 8 concludes with a summary of the key takeaways.

## 2. Expressions for Risk-Adjusted and Certainty-Equivalent Discount Rates

The  $SCC$  represents the discounted present value of a stream of climate damages caused by a marginal ton of carbon dioxide ( $CO_2$ ) emissions released today. Mathematically, the  $SCC$  is the certain consumption loss that a representative agent would be willing to bear today, valued at today's marginal utility  $u'(c_0)$ , that is equivalent to the expected value of the discounted stream of marginal damages over time  $MD_t$ , valued at future levels of marginal utility  $u'(c_t)$ . That is,

$$SCC \cdot u'(c_0) = E \left[ \sum_{t=1}^T e^{-\rho t} u'(c_t) MD_t \right]$$

where  $\rho$  is the utility discount rate, and both  $MD_t$  and  $c_t$  are random variables that are potentially correlated. With the commonly used isoelastic utility function  $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$ , and hence  $u'(c) = c^{-\eta}$ , the  $SCC$  can be written equivalently as

$$\begin{aligned} SCC &= E \left[ \sum_{t=1}^T e^{-\rho t} \frac{u'(c_t)}{u'(c_0)} MD_t \right] = E \left[ \sum_{t=1}^T e^{-\rho t} \left( \frac{c_t}{c_0} \right)^{-\eta} MD_t \right] \\ &= E \left[ \sum_{t=1}^T e^{-(\rho+\eta g_t)t} MD_t \right] \end{aligned}$$

where  $g_t$  is the cumulative (continuous time) average growth rate of consumption per capita,  $g_t = \ln(c_t/c_0)/t$ . This is also sometimes referred to as the compound annual growth rate. One could in principle stop here and develop a model that produces estimates of  $g_t$  and  $MD_t$ , such as the GIVE model developed in Rennert et al. (2022) and estimate the  $SCC$  via a Monte Carlo analysis. Such an approach would inherently account for the uncertain distribution of growth rates and how they jointly affect both discount rates and marginal damages being discounted. However, this paper unpacks alternative perspectives by deriving analytical expressions for key objects of interest, such as risk-free and risk-adjusted discount rates.

For the purposes of this paper, I distinguish the  $e^{-(\rho+\eta g_t)t}$  term, which is the stochastic discount factor, and the  $MD_t$  term. For simplicity, the remainder of the paper focuses on the properties of year  $t$ 's expected discounted marginal damages, which are defined as

$$E[DMD_t] = E[e^{-(\rho+\eta g_t)t} MD_t].$$

The risk-adjusted discount rate is defined as the rate  $r_t^{ra}$  that, when used to discount expected marginal damages  $E[MD_t]$ , yields the same  $E[DMD_t]$  value:

$$e^{-r_t^{ra}t} E[MD_t] = E[e^{-(\rho+\eta g_t)t} MD_t].$$



Solving for this rate yields

$$r_t^{ra} = \frac{1}{-t} \ln \left( \frac{E[e^{-(\rho+\eta g_t)t} MD_t]}{E[MD_t]} \right).$$

With a sufficiently positive climate beta ( $MD_t$  and  $c_t$  positively correlated), this risk-adjusted rate is expected to be higher than the risk-free rate and may potentially even rise with the time horizon  $t$ .

This contrasts with the *certainty-equivalent* (or *risk-free*) discount rate, defined as the rate coinciding with the expected discount factor. This rate does not reflect the correlation between marginal damages and the discount rate (and hence discount factor), which is the source of the risk adjustment above. The certainty-equivalent rate  $r_t^{ce}$  is the rate that satisfies the following equation:

$$e^{-r_t^{ce}t} = E[e^{-r_t t}] = E[e^{-(\rho+\eta g_t)t}].$$

Solving for this rate yields

$$r_t^{ce} = \frac{1}{-t} \ln(E[e^{-(\rho+\eta g_t)t}]).$$

This rate is lower than the expected discount rate and may even *decline* with the time horizon, so long as there is persistent uncertainty in  $g_t$  (Weitzman 1998).

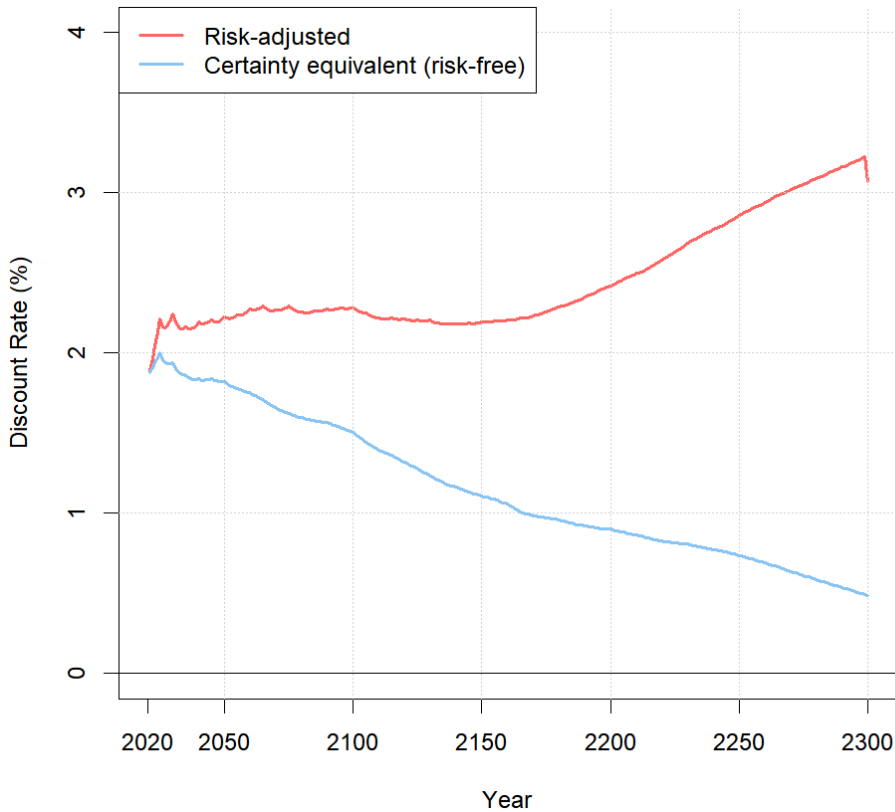
Note that if  $MD_t$  and consumption growth are independent, then  $r_t^{ra}$  collapses to  $r_t^{ce}$  because

$$\frac{E[e^{-(\rho+\eta g_t)t} MD_t]}{E[MD_t]} = \frac{E[e^{-(\rho+\eta g_t)t}]E[MD_t]}{E[MD_t]} = E[e^{-(\rho+\eta g_t)t}].$$

This is also true if  $\eta = 0$  (risk neutral preferences), in which case both rates are simply equal to  $\rho$ .

For illustrative purposes, I calculate  $r_t^{ra}$  and  $r_t^{ce}$  using the above formulas and the output of the GIVE model. I focus on GIVE because it was designed to comprehensively represent uncertainty across all components—including consumption growth, marginal damages, and their relationship—allowing for an internally consistent, bottom-up representation of the covariance between consumption growth and marginal damages. This calculation yields the results shown in Figure 1.

**Figure 1. Certainty-Equivalent and Risk-Adjusted Time-Average Discount Rates in GIVE**



The certainty-equivalent rate declines with the time horizon (à la Weitzman 1998), whereas the risk-adjusted rate rises, reflecting persistent growth uncertainty and a positive covariance between consumption growth and marginal damages (à la Gollier 2014). The climate risk premium ( $r_t^{ra} - r_t^{ce}$ ) grows over time and reaches 2.7 percent by the end of the time horizon. A simple average of this risk premium over the 280-year time horizon is roughly 1.3 percent.<sup>1</sup> By contrast, traditional applications of the consumption capital asset pricing model (CCAPM) approach commonly yield a risk premium that is instead constant over time (see, e.g., Ni and Maurice 2021), based on less presumed persistence in growth uncertainty (discussed in more detail in Section 6). Similarly, Barrage and Nordhaus (2023) propose a flat climate risk premium of 3.6 percent, corresponding to an assumed climate beta of 0.6 and a market risk premium of 6 percent ( $3.6\% = 0.6 \times 6\%$ ). The difference between risk adjustments in GIVE and those in DICE-2023 is due in part to the equity premium puzzle (discussed in Section 7), which reflects the divergence between risk premiums produced by the CCAPM and those based on market returns (Mehra 2008; Mehra and Prescott 1985), and in part to different values of the climate beta.

<sup>1</sup> Notably, this is a substantially larger risk premium than Nordhaus (2023) suggested is possible. Nordhaus claimed that “the maximum risk premium for climate investments” possible in GIVE “would be 0.2% per year” and that “the discount rate will be virtually identical to the risk-free discount rate for any climate beta in the [0,1] range.” The contrasting conclusion reached here owes to substantially less persistence in uncertainty than assumed in Nordhaus (2023).

### 3. Analytical Results under Normally Distributed Growth

This section further derives analytical expressions for certainty-equivalent and risk-adjusted rates under certain simple assumptions about the growth process and its relationship with marginal damages. This reveals the cases in which the risk-adjusted term structure rises or falls with the time horizon. However, deriving such analytical expressions requires imposing some structure on the distribution of consumption growth rates and the relationship of marginal damages to consumption growth.

#### 3.1. Certainty-Equivalent Rate

Starting with the equation for the certainty-equivalent rate from above,

$$r_t^{ce} = \frac{1}{-t} \ln(E[e^{-(\rho+\eta g_t)t}]),$$

suppose the cumulative average (continuous time) growth rate—sometimes referred to as the compound annual growth rate—is normally distributed as  $g_t \sim N(\mu_t, \sigma_t^2)$ .<sup>2</sup> As discussed in more detail in Section 6, this assumption is roughly consistent with recent literature on long-term probabilistic growth forecasts underlying the GIVE model (Müller et al. 2022; Rennert et al. 2021), through at least 2100. A well-known feature of the normal distribution and the exponential function  $e^x$  is that if  $x \sim N(m, s^2)$ , then for any constant  $a$ , the following equation holds:

$$E[e^{ax}] = e^{am + \frac{1}{2}a^2s^2}.$$

Applying this result to the equation for  $r_t^{ce}$  with  $a = -\eta t$ ,  $x = g_t$ ,  $m = \mu_t$ , and  $s^2 = \sigma_t^2$  yields

$$\begin{aligned} r_t^{ce} &= \frac{1}{-t} \ln(E[e^{-(\rho+\eta g_t)t}]) \\ &= \frac{\ln(e^{-\rho t} E[e^{-\eta g_t t}])}{-t} \\ &= \rho + \frac{1}{-t} \ln\left(e^{-\eta \mu_t t + \frac{1}{2} \eta^2 t^2 \sigma_t^2}\right) \\ r_t^{ce} &= \rho + \eta \mu_t - \frac{1}{2} \eta^2 \sigma_t^2 t. \end{aligned} \tag{1}$$

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<sup>2</sup> The cumulative average continuous growth rate is defined as  $g_t = \ln(c_t/c_0)/t$ , implying  $c_t = c_0 e^{g_t t}$ .

The last term in equation (1) reflects the well-known decline in the certainty-equivalent rate familiar from the extended Ramsey rule under uncertainty. Further, abstracting away from trends in the mean and variance of growth rates (i.e., assuming  $\mu_t = \mu$  and  $\sigma_t^2 = \sigma^2$  for all  $t$ ), the certainty-equivalent discount rate is *exactly linear* with the time horizon at a slope of  $-\frac{1}{2}\eta^2\sigma^2$ . This linearity is closely consistent with Figure 1, where the certainty-equivalent rate is approximately a straight line. The magnitude of this effect is smaller when risk aversion is smaller (as  $\eta$  gets smaller) and vanishes under risk neutrality ( $\eta = 0$ ). This is analogous to equation (4) of Gollier (2014), who uses the notation  $\gamma$  in place of  $\eta$  and  $Var(\ln c_t/c_0)/t$  in place of  $\sigma_t^2 t$ .<sup>3</sup>

### 3.2. Risk-Adjusted Rate

I now derive the analogous expression for the risk-adjusted rate. To do so analytically, it is necessary to also put some structure on  $MD_t$ . Similarly to Gollier (2014) and Dietz et al. (2018), I assume that marginal damages feature a constant  $\beta$ , defined as the income elasticity of marginal damages. A constant  $\beta$  assumption means that a 1 percent increase in economic growth corresponds to a  $\beta$  percent increase in marginal damages.<sup>4</sup> This holds when marginal damages are proportional to consumption to the power of  $\beta$ , as in  $MD_t = \alpha(c_t)^\beta$  where  $\alpha$  is some constant.<sup>5</sup> As noted in footnote 2,  $c_t = c_0 e^{g_t t}$ , which can be substituted into the marginal damages expression to yield marginal damages as a function of the growth rate:

$$MD_t = \alpha(c_0 e^{g_t t})^\beta.$$

<sup>3</sup> These are equivalent under the above assumptions about growth rates because  $Var(\ln c_t/c_0)/t = Var(g_t t)/t = \sigma_t^2 t$ .

<sup>4</sup> In principle,  $\beta$  could also be time varying, in which case its time-indexed analogue,  $\beta_t$ , would be substituted for  $\beta$  in all expressions and results in this paper. However, for simplicity in this paper we will assume it is not-time indexed because in most IAMs (including DICE and GIVE)  $\beta$  is typically fairly constant. See Appendix A for  $\beta_t$  values in GIVE, as well as the 2-degree scenario in Dietz et al. (2018).

<sup>5</sup> In principle,  $\alpha$  can also be time varying. The key relationship for the results in this paper is that marginal damages depend on period- $t$  consumption only through the  $(c_t)^\beta$  term and not through the  $\alpha$  term. The  $\alpha$  term can otherwise vary with other factors such as time to the extent that these factors are deterministic or at least uncorrelated with consumption growth. While this is a multiplicative form of damages—wherein damages scale multiplicatively with consumption at a rate governed by  $\beta$ —it is not the only possible functional form (Weitzman 2010, 2011). Nonetheless, the multiplicative form is commonly used and closely resembles the forms used in the DICE and GIVE models, on which this paper aims to shed light.

Returning to the definition of the risk-adjusted rate, and substituting this expression for  $MD_t$ , one can solve for an analytical expression for the risk-adjusted rate:

$$\begin{aligned} r_t^{ra} &= \frac{1}{-t} \ln \left( \frac{E[e^{-(\rho+\eta g_t)t} \cdot MD_t]}{E[MD_t]} \right) \\ &= \frac{1}{-t} \ln \left( \frac{E[e^{-(\rho+\eta g_t)t} \cdot \alpha(c_0 e^{g_t t})^\beta]}{E[\alpha(c_0 e^{g_t t})^\beta]} \right). \end{aligned}$$

The  $\alpha$  and  $c_0^\beta$  terms are deterministic, so they can come out of the expectations and thereby cancel, leaving:

$$\begin{aligned} r_t^{ra} &= \frac{1}{-t} \ln \left( \frac{E[e^{-(\rho+\eta g_t)t} \cdot e^{\beta g_t t}]}{E[e^{\beta g_t t}]} \right) \\ &= \frac{1}{-t} \ln \left( \frac{E[e^{-(\rho+(\eta-\beta)g_t)t}]}{E[e^{\beta g_t t}]} \right) \\ &= \rho - \frac{1}{t} \ln \left( \frac{E[e^{(\beta-\eta)g_t t}]}{E[e^{\beta g_t t}]} \right). \end{aligned}$$

The rule that  $E[e^{ax}] = e^{am + \frac{1}{2}a^2s^2}$  is applied twice, once for the numerator (with  $a = (\beta - \eta)t$ ) and again for the denominator (with  $a = \beta t$ ), yielding

$$\begin{aligned} r_t^{ra} &= \rho - \frac{1}{t} \ln \left( \frac{e^{(\beta-\eta)\mu_t t + \frac{1}{2}(\beta-\eta)^2 t^2 \sigma_t^2}}{e^{\beta\mu_t t + \frac{1}{2}\beta^2 \sigma_t^2 t^2}} \right) \\ &= \rho - \frac{1}{t} \ln \left( \frac{e^{-\eta\mu_t t + \frac{1}{2}(\beta^2 - 2\eta\beta + \eta^2)t^2 \sigma_t^2}}{e^{\frac{1}{2}\beta^2 \sigma_t^2 t^2}} \right) \\ &= \rho + \eta\mu_t - \frac{1}{2}(\eta^2 - 2\eta\beta)t\sigma_t^2 \\ &= \rho + \eta\mu_t - \frac{1}{2}\eta^2\sigma_t^2 t + \beta\eta\sigma_t^2 t \end{aligned}$$

$$r_t^{ra} = r_t^{ce} + \beta\eta\sigma_t^2 t, \quad (2)$$

where the last term in equation (2),  $\beta\eta\sigma_t^2 t$ , is the risk premium over and above the certainty-equivalent rate. Following are two comments about these results:

- Given that  $\eta$  is positive, the sign of the risk premium is the same as the sign of  $\beta$ .
- Abstracting away from trends in the mean and variance of growth rates (i.e.,  $\mu_t = \mu$  and  $\sigma_t^2 = \sigma^2$  for all  $t$ ), the risk-adjusted term structure is again a linear function of the time horizon with slope  $\left(-\frac{1}{2}\eta^2 + \beta\eta\right)\sigma^2$ . It is therefore increasing in the time horizon if and only if  $\beta > \frac{1}{2}\eta$ . In GIVE, under the default 2 percent Ramsey discounting parameters,  $\beta \approx 0.9$  (see Appendix A) and  $\frac{1}{2}\eta = \frac{1}{2}1.24 = 0.62$ , so this condition is satisfied. More generally, given the  $\beta$  value of  $\approx 0.9$ , it will be satisfied for any  $\eta$  below about 1.8.<sup>6</sup>

Equations (1) and (2) are accurate expressions for certainty-equivalent and risk-adjusted rates under the assumptions noted above regarding the normality of consumption growth and the constant-beta functional form. One may wonder, therefore, to what extent the approximations in equations (1) and (2) reflect the true values from GIVE shown in Figure 1. Appendix Figure B.1 shows this comparison, demonstrating that the expressions in equations (1) and (2) produce very accurate representations of the true certainty-equivalent and risk-adjusted rates consistent with GIVE through 2100, with some differences thereafter. In particular, beyond 2100, the true discount rates are modestly higher than suggested by the above approximations; this difference owes primarily to the distribution of consumption growth in GIVE gradually departing from a normal distribution and becoming increasingly right-skewed over time.

### 3.3. Expected Marginal Damages

With these functional forms, one can also derive a closed-form expression for  $E[MD_t]$ , which is the object to which the risk-adjusted discount rate is properly applied:

$$MD_t = \alpha(c_0 e^{gt})^\beta$$

$$E[MD_t] = \alpha c_0^\beta E[e^{\beta g t}].$$

Applying the rule  $E[e^{ax}] = e^{am + \frac{1}{2}a^2 s^2}$  with  $a = \beta t$ ,

$$E[MD_t] = \alpha c_0^\beta e^{\beta\mu_t t + \frac{1}{2}\beta^2\sigma_t^2 t^2}. \quad (3)$$

<sup>6</sup> The risk-adjusted rate in Figure 1 is not linear for the first half of the time horizon. This appears to owe to two factors: (i) the RFF Socioeconomic Scenarios used in GIVE feature declining economic growth rates in the first 150-200 years, meaning  $\mu_t$  is also declining, which reduces both  $r_t^{ce}$  and  $r_t^{ra}$ ; and (ii) the  $\beta$  in GIVE is time-varying. The cited value of  $\beta \approx 0.9$  is an average over the full time horizon.

In other words, if growth is normally distributed, marginal damages are log-normally distributed with a mean that rises not only exponentially but at an accelerating rate due to uncertainty in damage growth. If there were no uncertainty in the growth rate,  $\sigma_t^2 = 0$ , then expected marginal damages would grow at a constant exponential rate tied to the beta and the mean growth rate,  $\beta\mu_t$ . However, in the presence of uncertainty, damages are larger, and their growth rate accelerates over time because of the contribution of the  $\frac{1}{2}\beta^2\sigma_t^2t^2$  term. In other words, uncertainty introduces a right skew in marginal damages that would be ignored if expected damages were computed without accounting for the joint effect of  $\beta$  and growth uncertainty, and further, the effect of that uncertainty on expected marginal damages grows quickly and accelerates at the rate of  $\frac{1}{2}\beta\sigma_t^2t$  each year.

The implication of this result is striking. Recall that the risk-adjusted rate is the rate that is appropriate to use to discount *expected* marginal damages, and using it to discount anything else would be inappropriate. However, this result demonstrates that the logic implying a positive risk adjustment to the discount rate to address a nonzero correlation between consumption and marginal damages (as in equation (2), with  $\beta \neq 0$ ) also implies that the same uncertainty must also be reflected in the object being discounted,  $E[MD_t]$ . Including the  $\beta\sigma_t^2$  term in equation (2) but not (3) would be internally inconsistent, yielding highly misleading underestimates of discounted marginal damages. Given the exponential nature of this effect, it has the potential to be very large, particularly for impacts occurring far in the future, yet this effect is commonly ignored in applied analysis, as illustrated by the lack of any consideration in Circulars A-4 and A-94 (OMB 2023a,b).

## 4. Discounted Marginal Damages

The paper has thus far considered analytical expressions for expected undiscounted marginal damages separately from risk-adjusted discount rates. This revealed offsetting factors: a positive  $\beta$  suggests higher discount rates but also higher expected marginal damages. Considering those two factors in isolation does not reveal explicitly which of these two offsetting effects dominates. It is valuable, therefore, to consider their effects in tandem on the key object of interest: expected *discounted* marginal damages in each year, which when summed over time equals the SCC. Returning to the definition of this term, and using the constant- $\beta$  structure imposed on the form of marginal damages, yields the following expression:

$$\begin{aligned} E[DMD_t] &= E[e^{-(\rho+\eta g_t)t} \cdot \alpha(c_0 e^{g_t})^\beta] \\ &= \alpha c_0^\beta e^{-\rho t} E[e^{(\beta-\eta)g_t t}]. \end{aligned}$$

Once again using the rule for  $E[e^{ax}]$  with  $a = (\beta - \eta)t$ , this yields

$$E[DMD_t] = \underbrace{\alpha c_0^\beta e^{-(\rho+(\eta-\beta)\mu_t)t}}_{\text{Deterministic DMD}_t} \underbrace{e^{\frac{1}{2}(\beta-\eta)^2\sigma_t^2 t^2}}_{\text{Effect of Uncertainty}}. \quad (4)$$

The first bracketed term in equation (4) reflects discounted marginal damages absent uncertainty (bringing  $\sigma_t^2$  to 0). This deterministic contribution grows with consumption at rate  $\beta$  but is discounted at rate  $\eta$ , for a damage-growth-adjusted discount rate of  $\rho + (\eta - \beta)\mu_t$ . The overall effect of uncertainty in multiplicative terms is given by the second bracketed term, which is an exponential function that starts at 1 at time zero ( $t = 0$ ) and then grows an increasing rate given by  $\frac{1}{2}(\beta - \eta)^2\sigma_t^2 t > 0$ . This result means that while uncertainty does indeed imply a higher risk-adjusted discount rate when  $\beta > 0$ , as shown in equation (2), it also results in a strictly higher SCC so long as  $\beta \neq \eta$ . In the edge case when  $\beta = \eta$ , the impact of uncertainty on the discount rate and expected marginal damages are exactly symmetric, and uncertainty has no net effect on the SCC. Otherwise, growth uncertainty increases discounted marginal damages, and more so at long time horizons.

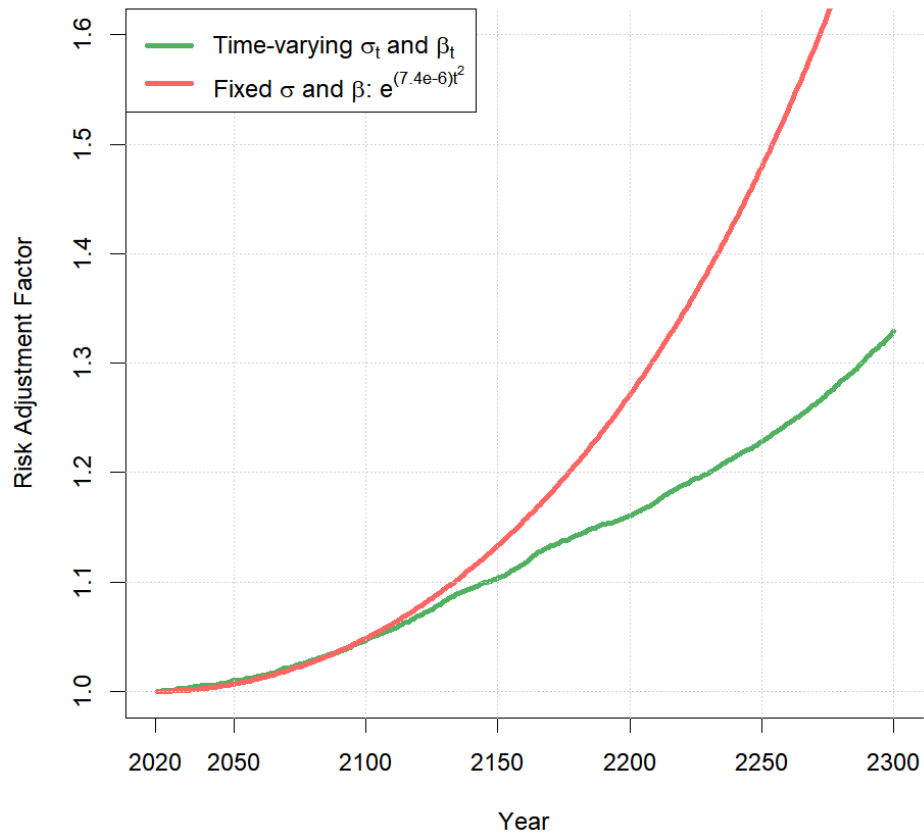
As an example, consider the time horizon of  $t = 80$  in GIVE corresponding to 2100. For that time horizon,  $\beta \approx 0.86$ ,  $\eta \approx 1.245$ , and  $\sigma_t^2 \approx 1\%$ . With these approximations, the uncertainty adjustment factor in equation (4) can be approximated as

$$e^{\frac{1}{2}(\beta-\eta)^2 t^2 \sigma_t^2} = e^{\frac{1}{2}(0.86-1.245)^2 t^2 (1\%)^2} \approx e^{(7.4e-6)t^2}.$$

This approximation is shown in red in Figure 2. In actuality,  $\beta$  and  $\sigma_t$  are time varying in GIVE, so this approximation will not hold precisely. In particular,  $\sigma_t$  declines modestly over time, whereas  $\beta$  rises slightly toward  $\eta$  from below. Both effects slow the growth rate of the uncertainty adjustment, particularly after 2150, as shown in Figure 2 in green, which uses the same expression from equation (4) but with time-varying analogues of  $\beta$  and  $\sigma_t$  from GIVE. That uncertainty adjustment term nonetheless rises to about 1.33× by the end of the time horizon. Over the first half of the time horizon, the curve largely resembles the simplified version, both of which increase the deterministic discounted marginal damages by about 10 percent by 2150.



**Figure 2. Uncertainty Adjustment Factor Using Formula in Equation (4), under Time-Varying and Simplified Parameters**



The key takeaway from this comparison is not the uncertainty adjustment's magnitude, but rather its sign: when considering uncertainty's impacts not only on the discount rate but also on the object being discounted, uncertainty increases the SCC regardless of the magnitude of  $\beta$ , so long as it is not exactly equal to  $\eta$ . By contrast, in DICE, for example,  $\beta$  interacts only with uncertainty in the discount rate and not with damages, resulting in uncertainty in consumption growth and reducing rather than increasing the SCC.

In addition to considering the uncertainty adjustment factor in equation (4), one can decompose the contributions of the three effects thus far considered: (i) the effect on the expected stochastic discount factor  $E[SDF_t] = E[e^{-(\rho+\eta g_t)t}]$  (or equivalently, on the certainty-equivalent discount rate); (ii) the contribution of the covariance of marginal damages and consumption; and (iii) the effect on expected marginal damages.<sup>7</sup> Specifically, the uncertainty adjustment can be factored as follows:

$$\begin{aligned}
 \underbrace{e^{\frac{1}{2}(\beta-\eta)^2\sigma_t^2 t^2}}_{\text{Effect of Uncertainty}} &= e^{\frac{1}{2}(\beta^2-2\beta\eta+\eta^2)\sigma_t^2 t^2} = \\
 \underbrace{e^{\frac{1}{2}\eta^2\sigma_t^2 t^2}}_{\text{Effect on } E[SDF_t]} &\quad \underbrace{e^{-\beta\eta\sigma_t^2 t^2}}_{\text{Risk Adjustment}} \quad \underbrace{e^{\frac{1}{2}\beta^2\sigma_t^2 t^2}}_{\text{Effect on } E[MD_t]} \quad (5)
 \end{aligned}$$

The three terms in equation (5) illustrate how the effect of uncertainty in equation (4) explicitly embodies the effects identified in equation (1) on the risk-free discount rate, equation (2) on the risk adjustment to the discount rate, and equation (3) on expected marginal damages. The first two bracketed terms on the right-hand side of equation (5) correspond to the effect of uncertainty on the discount rate, which, as previously demonstrated, serves to increase the risk-adjusted discount rate and therefore decrease the SCC if  $\beta > \frac{1}{2}\eta$ , a condition that typically holds in standard IAMs. This is the adjustment made by Barrage and Nordhaus (2023), for example. Overall, however, as the growth uncertainty increases the SCC on net, the increase in expected marginal damages represented in the third term of the right-hand side of equation (5) must dominate the first two terms. Hence including the effect of such uncertainty on expected marginal damages is essential for accurately representing the *sign* of growth uncertainty's effect on the SCC, let alone its magnitude.

## 5. Implications of Small or Negative $\beta$

Equation (4) is also informative about the effect of the climate beta on the SCC. Some have called for lower discount rates on the grounds of a potentially lower or even negative  $\beta$ . While it is true that a negative  $\beta$  would call for a negative risk premium on the discount rate, as is the case with a positive risk premium, it is not obvious that this lower discount rate would yield a higher SCC when the  $\beta$ 's offsetting effect on damages is taken into account. In equation (4), this offsetting effect manifests as a smaller  $E[DMD_t]$ , since when  $\beta$  is smaller, both time-zero damages ( $\alpha c_0^\beta$ ) and rate of growth ( $\beta\mu_t$ ) are smaller. On the other hand, the uncertainty adjustment factor is generally larger when  $\beta$  is smaller, since it is typically the case that  $\beta < \eta$ .

<sup>7</sup> I thank David Smith for this suggestion.

When does a larger  $\beta$  increase expected discounted marginal damages, and hence the SCC, on net? Taking the logarithm of equation (4) and differentiating with respect to  $\beta$  shows its marginal effect on (log) expected discounted damages, which is given by

$$\frac{d\ln(E[DMD_t])}{d\beta} = \ln(c_0) + \mu_t t + (\beta - \eta)t^2 \sigma_t^2.$$

The first two terms of this derivative are positive, whereas the third is typically negative because in most IAMs  $\beta < \eta$ . The interpretation is that a larger  $\beta$  increases damages through increasing period-zero deterministic damages ( $\alpha c_0^\beta$ ) and their growth rate ( $\beta \mu_t$ ), but it also shrinks the (positive) effect of uncertainty. Thus, on net, a higher  $\beta$  increases expected discounted damages, and hence the SCC, whenever the first two terms dominate the third, which occurs when

$$\beta > \eta - \frac{\ln(c_0) + \mu_t t}{t^2 \sigma_t^2}.$$

A related expression is derived by Dietz et al. (2018), who conclude that their version of this relation typically holds, implying that the SCC is increasing in  $\beta$  (see their Proposition 1).<sup>8</sup> The same is true with this version. To see why, note that it is *least* likely to hold for long time horizons, when  $t^2$  is large and the subtracted term, which is positive, is smallest in magnitude. Considering GIVE's longest time horizon of  $t = 280$  years (corresponding to 2300) and using parameter values from GIVE consistent with that horizon ( $c_0 \approx \$18,000$ ,  $\mu_{280} \approx 1\%$ ,  $\sigma_{280} \approx 0.8\%$ , and  $\eta = 1.24$ ) yields

$$\beta > 1.24 - \frac{9.8 + 0.01(280)}{280^2(0.008^2)} = -1.27.$$

This value is far below reasonable estimates for  $\beta$ , which typically range between 0.5 and 1, with a value of  $-1$  being on the extreme end. This implies that the inequality generally holds, and therefore higher  $\beta$  increases expected discounted damages in all periods and hence also increases the SCC. This is contrary to an apparent common belief that a lower or perhaps even negative  $\beta$  will result in a higher SCC. This can be seen in equation (4), where the  $\beta$  affects deterministic discounted damages more than it affects the uncertainty adjustment factor. Equations (2) and (3) also show this dynamic: while a higher  $\beta$  increases the risk-adjusted discount rate in equation (2), it increases expected damages to a larger degree through the multiple channels seen in equation (3), as period-zero expected damages ( $\alpha c_0^\beta$ ) and their growth rate ( $\beta \mu_t + \frac{1}{2} \beta \sigma_t^2 t$ ) both grow.

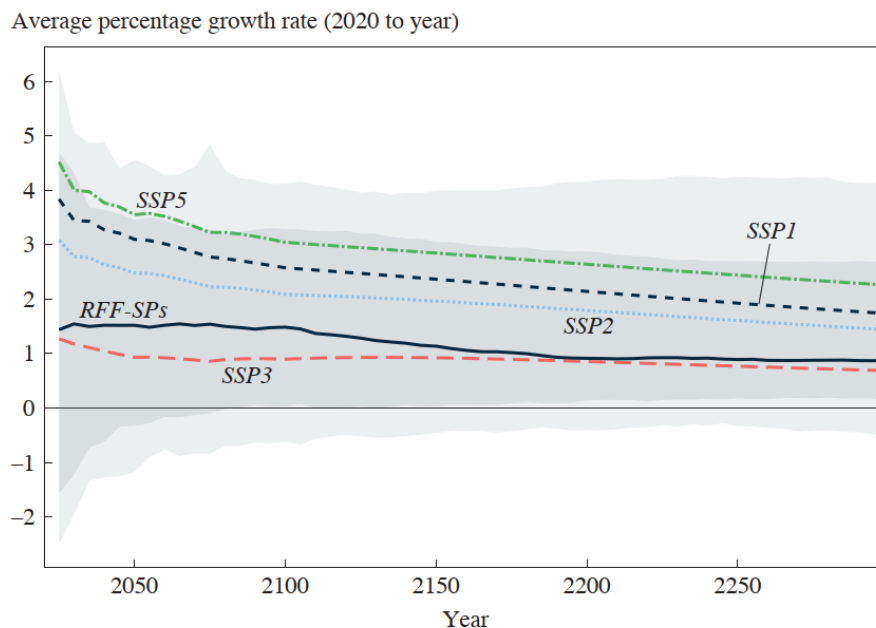
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<sup>8</sup> After converting to this paper's notation, the expression of Dietz et al. (2018) is that expected discounted marginal damages are increasing in  $\beta$  if  $\beta > \eta - (\mu_t / \sigma_t^2)$ . There are two differences between their expression and the one in this paper. First, the  $\ln c_0$  term is excluded in their expression because theirs is a sufficient condition for the result but not a necessary one. Second, the two papers assume different degrees of persistence in growth uncertainty, yielding a  $t^2$  term in the denominator in this paper's expression versus a  $t$  term in their expression that cancels out the  $t$  term in the numerator. The role of the persistence of growth uncertainty is discussed in Section 6.

## 6. The Importance of Distributional Assumptions about the Growth Rate Process

The risk premium in equation (2) increases over the time horizon at a rate governed in part by the variance of long-run consumption growth rates. Analogously, expected marginal damages in equation (3) have a growth rate based on that same variance. Importantly, these results derive from the assumption that *cumulative average growth rates*, defined as  $g_t = \ln(c_t/c_0)/t$ , are normally distributed with a time-varying mean ( $\mu_t$ ) and variance ( $\sigma_t^2$ ). This implies persistent uncertainty in economic growth that is roughly consistent with the results of Müller et al. (2022) and the RFF Socioeconomic Projections (RFF-SPs, documented in Rennert et al. 2021) used in GIVE (with the distinction that after 2100, the RFF-SPs feature a rightward skew in growth rates and therefore depart somewhat for a normal distribution). Figure 3 shows median and uncertainty ranges of global GDP per capita from the RFF-SPs. The figure indicates a somewhat declining central tendency and a modestly shrinking long-run variance (indicated by the gradual narrowing of the dark-shaded range), but most important, the uncertainty in long-run cumulative average growth rates remains persistent.

**Figure 3. Median and Percentile Ranges of Growth Rates of Global GDP per Capita**



Source: Authors' calculations.

Note: The solid line represents the median value, and dark and light shading represent the 5th to 95th (darker) and 1st to 99th (lighter) percentile ranges of the RFF-SPs.

Source: Rennert et al. (2021)

The assumption about the *cumulative average* growth rate being normally distributed as  $g_t \sim N(\mu_t, \sigma_t^2)$  is very different from an assumption that year-on-year growth rates, denoted as  $\tilde{g}_t = \ln\left(\frac{c_t}{c_{t-1}}\right)$ , are normal and independent, as in geometric Brownian motion. That alternative assumption that  $\tilde{g}_t \sim N(\tilde{\mu}_t, \tilde{\sigma}_t^2)$  and these year-on-year growth rates are independent across years is essentially what Gollier (2014) assumes to yield his equations (6)–(8) for the term structures of certainty-equivalent and risk-adjusted discount rates. In this case, the variance of the cumulative average growth rate is

$$\begin{aligned}\sigma_t^2 &= \text{Var}[g_t] = \text{Var}\left[\frac{\ln\left(\frac{c_t}{c_0}\right)}{t}\right] = \text{Var}\left[\frac{\ln\left(e^{\sum_{\tau=1}^t \tilde{g}_\tau}\right)}{t}\right] = \frac{1}{t^2} \text{Var}[\sum_{\tau=1}^t \tilde{g}_\tau] \\ &= \frac{1}{t^2} t \tilde{\sigma}_t^2 = \frac{\tilde{\sigma}_t^2}{t}.\end{aligned}$$

In other words, this alternative assumption implies that cumulative average growth rates are increasingly certain at longer time horizons, with a variance that converges to zero at rate  $t$ . What is driving this result is that high-growth and low-growth years cancel out in the long run, leading to little long-run uncertainty. For example, if the standard deviation of year-on-year growth rates were  $\tilde{\sigma}_t = 1\%$ , then the standard deviation of *cumulative average growth rates* would be  $\sigma_1 = \frac{1\%}{\sqrt{1}} = 1\%$  in year 1 but  $\sigma_{100} = \frac{1\%}{\sqrt{100}} = 0.1\%$  in year 100.

The notion of increasing certainty over the distant future does not seem particularly intuitive, and it is also inconsistent with both the econometric results from Müller et al. (2022) and the expert growth survey of Rennert et al. (2021), both of which exhibit persistent uncertainty. Visually, this increasing certainty would make the uncertainty ranges in Figure 3 look like a funnel, rapidly narrowing toward the central case, with a standard deviation of the cumulative average growth rate shrinking annually at rate  $1/\sqrt{t}$  before reaching a very narrow range in 2300. More concretely, the standard deviation and hence the confidence interval for a 300-year cumulative average growth rate would be  $\frac{1}{\sqrt{300}} \approx 1/17$  as wide as that of the 1-year growth rate.

## 6.1. Implications for Term Structures, Expected Marginal Damages, and Discounted Marginal Damages

While this alternative assumption about long-run growth uncertainty seems unintuitive, it nonetheless would have major implications for the term structure of discount rates. With all cumulative growth rate variance terms  $\sigma_t^2$  replaced with  $\tilde{\sigma}_t^2/t$ , equations (1) and (2) for  $r_t^{ra}$  and  $r_t^{ce}$  would conceptually align with equations (7) and (8) of Gollier (2014), albeit with some notational differences:

$$r_t^{ce} = \rho + \eta\mu_t - \frac{1}{2}\eta^2\tilde{\sigma}_t^2 \quad (1')$$

$$r_t^{ra} = r_t^{ce} + \beta\eta\tilde{\sigma}_t^2. \quad (2')$$

Abstracting away from the time-varying mean and variance (i.e., setting  $\mu_t = \mu$  and  $\tilde{\sigma}_t^2 = \tilde{\sigma}^2$  for all  $t$ ), these discount rates do not consistently slope up or down over the time horizon. They would be higher or lower than their deterministic analogues, but by a constant shift, not a trend. Hence this alternative assumption about the process underlying the growth rates is a key determinant of whether  $\eta$  and  $\beta$  lead to trends in the term structure or simply shifts.

This alternative assumption also alters equation (3) for expected marginal damages, but the exponential-growth form nonetheless remains

$$E[MD_t] = \alpha c_0^\beta e^{\beta\mu_t t + \frac{1}{2}\beta^2\tilde{\sigma}_t^2 t}. \quad (3')$$

The new expression changes the final term in the exponent from  $\frac{1}{2}\beta^2\sigma_t^2 t^2$  to  $\frac{1}{2}\beta^2\tilde{\sigma}_t^2 t$ , most importantly featuring a difference of a factor of  $t$ . In this case, expected marginal damages still grow exponentially but at a constant rate instead of an accelerating one (again setting aside potential trends in  $\mu_t$  and  $\tilde{\sigma}_t^2$ ).<sup>9</sup>

Similarly, expected discounted marginal damages would now take the form

$$E[DMD_t] = \underbrace{\alpha c_0^\beta e^{-(\rho+(\eta-\beta)\mu_t)t}}_{\text{Deterministic DMD}_t} \underbrace{e^{\frac{1}{2}(\beta-\eta)^2\tilde{\sigma}_t^2 t}}_{\text{Effect of Uncertainty}}. \quad (4')$$

Again, growth uncertainty continues to have a positive effect on the SCC, but now its effect grows at a constant exponential rate rather than accelerating over time, presuming that the values of  $\sigma_t^2$  and  $\tilde{\sigma}_t^2$  under the alternative assumed growth processes are of the same order of magnitude.

<sup>9</sup> That is, an annualized growth rate of  $\frac{\log(E[MD_t]/E[MD_0])}{t} = \beta\mu_t + \frac{1}{2}\beta^2\tilde{\sigma}_t^2$ . This also aligns with the structure of the growth rate of Ni and Maurice (2021).

## 7. Discussion

This paper has demonstrated that uncertainty in consumption growth, when considering its offsetting effects on the risk-adjusted discount rate and on marginal damages, should be expected to increase the social cost of carbon on net. These offsetting effects arise regardless of the climate beta—a larger beta leads to higher risk-adjusted discount rates but also higher damages, whereas a smaller or negative beta implies the opposite.

However, the model considered in this paper is based on the consumption capital asset pricing model (CCAPM). As noted by Nordhaus (2023) and Drupp et al. (2023), the CCAPM is known to produce risk premiums that are commonly smaller than those observed in financial markets, a feature called the equity premium puzzle (Mehra 2008; Mehra and Prescott 1985).<sup>10</sup> This puzzle has remained frustratingly unresolved for many decades, and there is no solution in sight.

Given the unresolved tension between the CCAPM and observed market returns, what is an analyst to do? Newell et al. (2022) propose calibrating the discounting parameters  $\rho$  and  $\eta$  to match evidence on risk-free rates, but this yields risk-adjusted rates that are arguably too low (see Figure 1). Alternatively, Barrage and Nordhaus (2023) suggest calibrating those parameters to match a risky rate, but this yields risk-free rates that are arguably too high. It appears that no clear way exists to be consistent with both risk-free and risk-adjusted rates without abandoning the classic CCAPM, and this will continue to be a concern unless a resolution to the equity premium puzzle emerges.

However, a key lesson from this paper is that if an analyst decides to use risk-adjusted discount rates, it is essential that they also make a conceptually similar adjustment to the expected values being discounted. Unfortunately, that adjustment has received little attention in the literature and regulatory guidance (e.g., Circulars A-4 and A-94), and as a result, it is often ignored or forgotten. But this omission threatens to lead to substantial underestimates of the SCC. In other words, an appropriate treatment of growth uncertainty would tend to increase the SCC, not decrease it.

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<sup>10</sup> As noted in Section 2, the risk premium implied by GIVE is generally in the 1–2 percent range, compared with the 3.6 percent premium proposed by Barrage and Nordhaus (2023).

## 8. Conclusions

This paper has explored the drivers of certainty-equivalent and risk-adjusted discount rates. Under an assumption about the persistence of uncertainty in economic growth consistent with recent literature (Müller et al. 2022; Rennert et al. 2021), the main takeaways are as follows:

1. The certainty-equivalent discount rate has a term structure that declines approximately linearly with the time horizon at a rate determined by the elasticity of marginal utility,  $\eta$ .
2. Assuming a constant climate  $\beta$  (meaning a 1 percent increase in economic growth corresponds to a  $\beta$  percent increase in marginal damages) and abstracting away from trends in the mean and variance of growth rates, the risk-adjusted term structure rises with the time horizon if and only if  $\beta > \eta/2$ . This condition is satisfied in most standard integrated assessment models.
3. Notwithstanding the slope of the term structure, growth uncertainty increases the SCC except in the edge case of  $\beta = \eta$ , where it has no effect.
4. The same logic that suggests adding a risk premium to the discount rate also implies that expected marginal damages should similarly be affected strongly by uncertainty in an offsetting manner. This latter effect dominates the former, yet it is commonly ignored.
5. While reducing  $\beta$  reduces the risk-adjusted discount rate, it nonetheless generally reduces the SCC for reasonable values of  $\eta$ ,  $c_0$ ,  $\mu_t$ , and  $\sigma_t^2$ , through its offsetting effect on marginal damages.

A central message of the landmark National Academies of Sciences report on the social cost of carbon (NASEM 2017) was the importance of a comprehensive assessment of uncertainty. Treating potentially correlated variables as certain, omitting important relationships, or ignoring the effects of uncertainty on expected values can yield highly misleading results. An example of this can be seen in Table 1 of Rennert et al. (2021), which shows that failing to account for uncertainty in discount rates via their relationship with growth rates would create inconsistencies that introduce enormous bias—as much as a factor of 9—into estimates of the SCC. Ignoring the effects of uncertainty on expected values similarly threatens to introduce enormous bias into estimates of the SCC.



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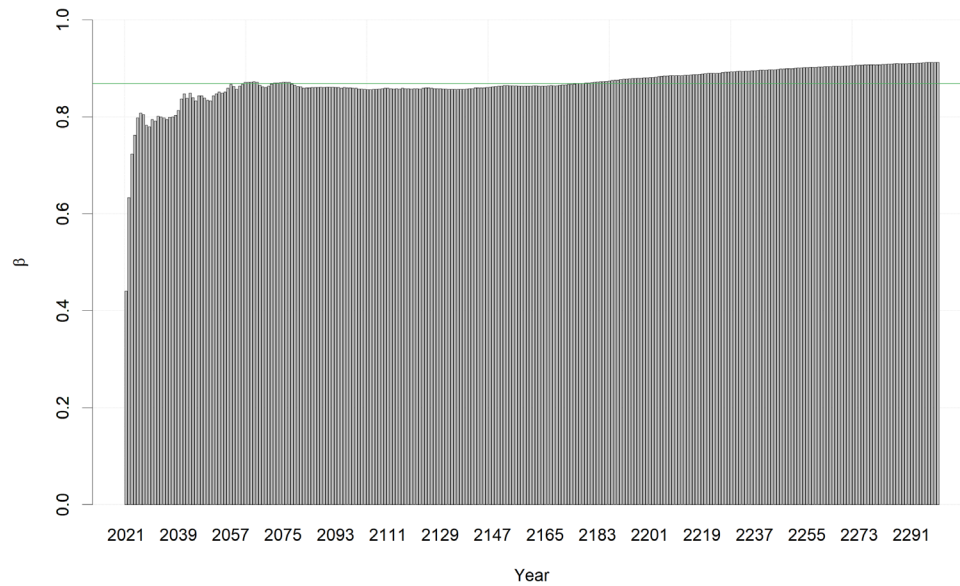
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# Appendixes

## Appendix A: Beta Values in GIVE

Figure A.1 shows the climate beta values in GIVE over time, derived by estimating an OLS regression of the log of the absolute value of marginal damages on log per capita consumption for each year. The horizontal green line is a simple average across years, yielding a value of 0.87, or about 0.9.

**Figure A.1. Climate Beta Values in GIVE, by Year**



## Appendix B: Validity of Normal Approximations

Figure B.1 compares the true values of the certainty-equivalent and risk-adjusted discount rates from GIVE with those implied by equations (1) and (2), which assume normally distributed growth and the simplified constant-beta functional form for marginal damages. The application of equations (1) and (2) use the true values of  $\rho$ ,  $\eta$ ,  $\beta$ ,  $\mu_t$ , and  $\sigma_t^2$  from GIVE, so the only distinction between the curves is due to those two assumptions. Because a constant-beta assumption is approximately accurate in GIVE, most of the difference between the solid and dashed lines is because of the departure from normality of GIVE's growth rate distributions at long time horizons. The relative flatness of the normal approximation to the risk-adjusted rate owes to mean growth rates ( $\mu_t$ ) that decline over time, offset by the positive effect of uncertainty  $\left(-\frac{1}{2}\eta^2 + \beta\eta\right)\sigma_t^2 t$ , which grows approximately linearly over time. This effect is augmented by the increasing right skew of growth rates in the true GIVE distribution. For the same reason, GIVE's certainty-equivalent rate is also somewhat higher than the normal approximation suggests.

**Figure B.1. Comparing the True CE and Risk-Adjusted Discount Rates from GIVE to Approximations Using Equations (1) and (2)**

